A Kalman Filter for Active Feedback on Rotating External Kink Instabilities in a Tokamak Plasma

Jeremy M. Hanson

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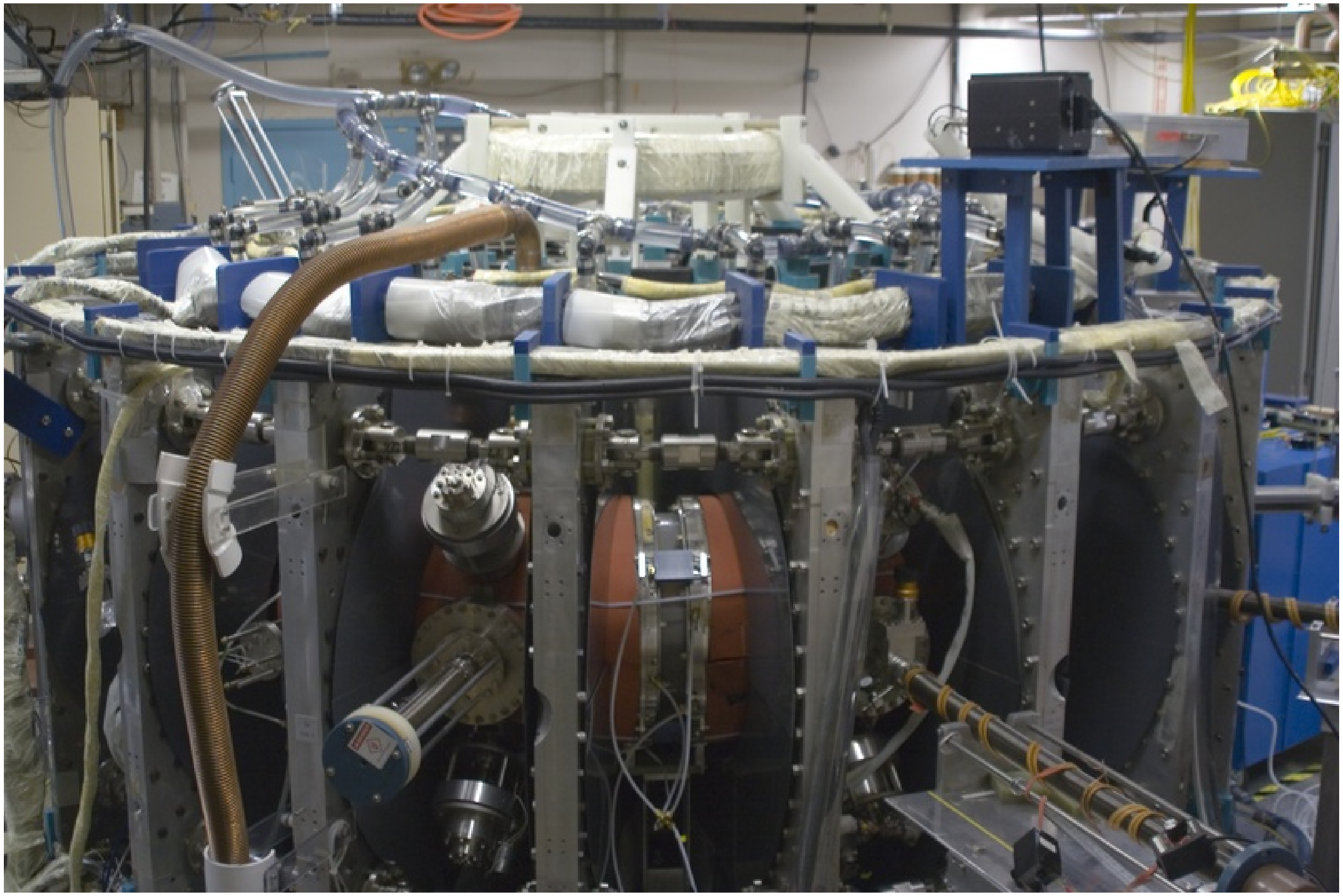
Workshop on Active Control of MHD Stability

Columbia University
in the City of New York

HBT-EP

HBT-EP – Collaborators

James Bialek, Bryan DeBono, Royce W. James (*Stevens Inst. Tech./USCG*), Jeff Levesque, Michael E. Mauel, David A. Maurer, Gerald A. Navratil, Steve Paul (*Princeton University*), Thomas Sunn Pedersen, Daisuke Shiraki



Overview

- Introduction
- MHD Control Hardware on HBT-EP
- Modeling Kink Mode Feedback
- Kalman Filtering
- External Kink Observation
- Experimental Results
- Conclusions and Future Work

How can the RWM be suppressed?

- The ideal external kink mode is the most stringent β-limit for tokamaks.
- In the presence of a close-fitting conducting wall, the kink becomes a resistive wall mode (RWM) that has a growth rate proportional to the inverse wall time $\gamma_w = R_w/L_w$.
- It's not practical to have $\gamma_w = 0$, but the RWM can be stabilized by plasma rotation or active magnetic feedback.
- It is uncertain whether the required amount of plasma rotation will be present in ITER and future tokamak reactors.
- Feedback with active coils works. Further improvements are possible if we can minimize sensor noise and pick-up from edge localized modes (ELMs).

Can Kalman filtering improve RWM feedback?

- The Kalman filter actively compares measurements of a dynamical system with the results of an internal model, producing an estimate that is optimal if the measurements are polluted with white, Gaussian noise.
- Several Kalman filters for RWM feedback have been proposed and modeled.
- However, no Kalman filter has yet been designed to make an estimate the RWM's phase.
- It is important to measure *both* the amplitude and phase of the RWM for feedback – falling out of phase with a rotating mode can result in positive rather than negative feedback.
- I present a Kalman filter that uses the simplest possible model for a growing, rotating RWM. There are only two parameters: the growth rate and rotation rate.

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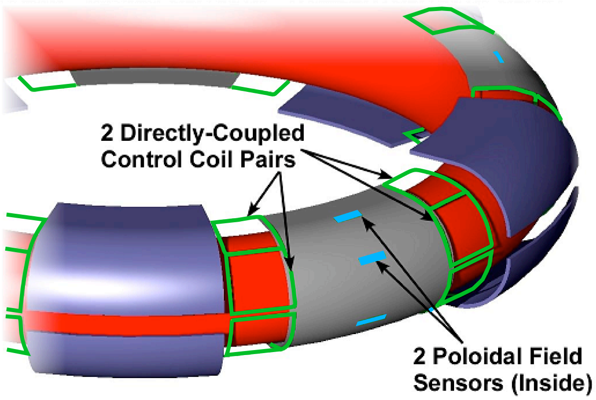
HBT-EP has passive and active stabilization hardware

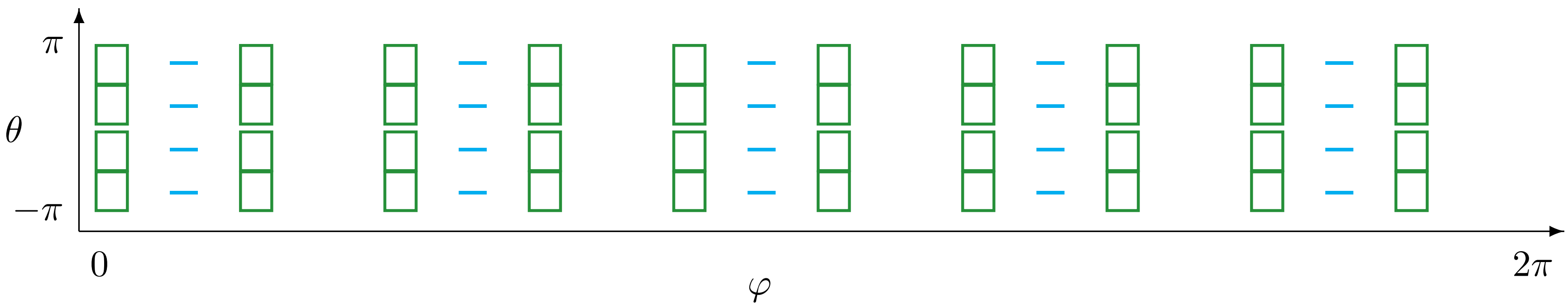
- There are 10 thick, aluminum shells with a long wall time (60 msec).
- There are 10 thin, stainless steel shells with a short wall time (300 μsec).
- For feedback experiments the Al shells are pulled back about 4 cm from the plasma surface, $\gamma_w \approx$ 3–5 msec^{-1}.

- Additionally, there are 20 poloidal sensor coils and 20 pairs of radial control coils.
- Four very low latency Field-Programmable Gate Arrays (FPGAs) are used as feedback controllers.

Small, localized control coils are used for feedback studies

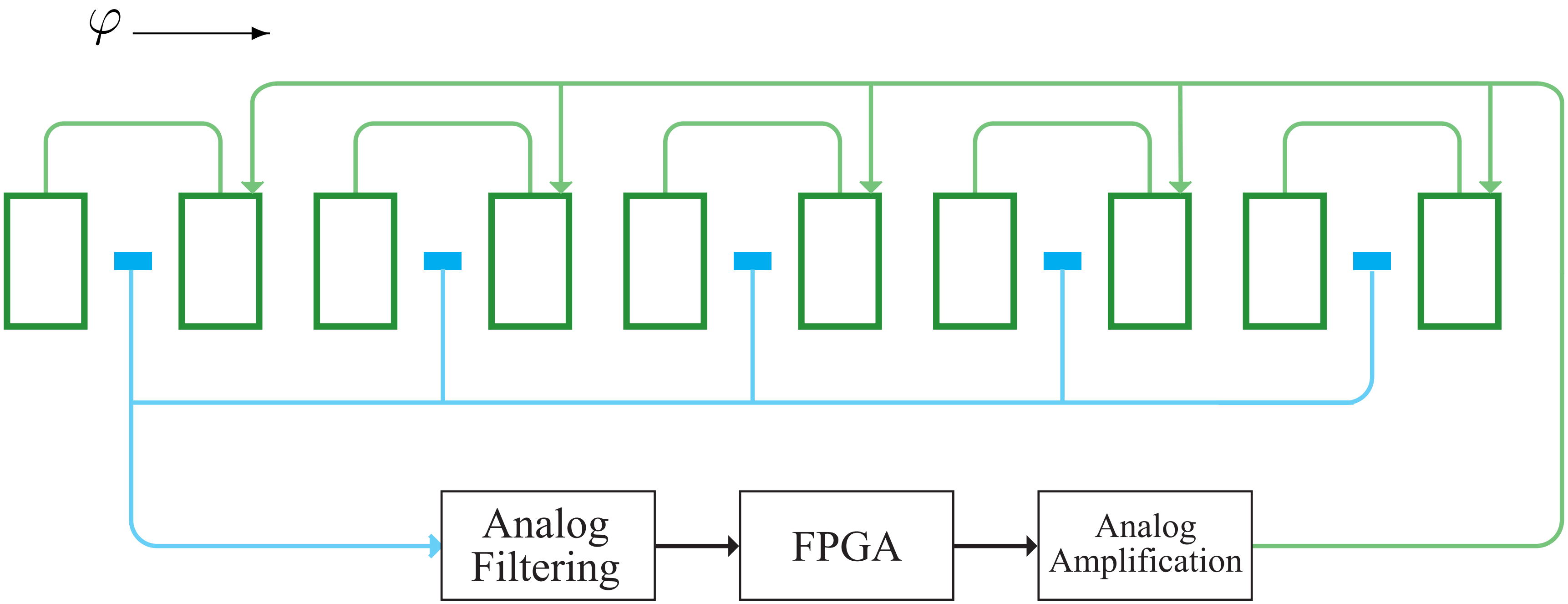


- There are five toroidal locations and four poloidal groups of coils.

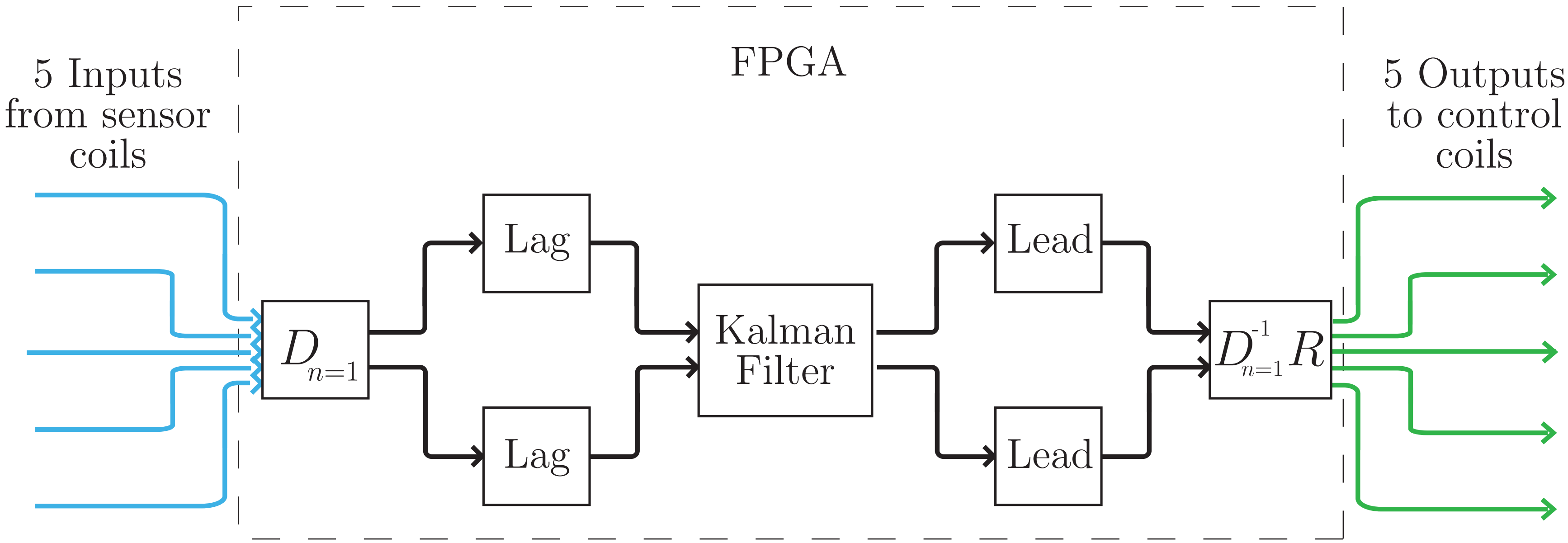


- The control coils are small and localized: they only cover 15% of the plasma surface.
- Each poloidal group is driven by a separate FPGA controller.

A feedback loop consists of 5 sensor coils and control coil pairs



The FPGA algorithm contains spatial and temporal filtering, plus the Kalman filter



- A spatial DFT is used to select the $n = 1$ mode.
- Lag and lead filters correct for the transfer functions of the control and sensor coils.
- The toroidal phasing of the output is adjustable.
- Total latency is $\sim$10 μsec.

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The Fitzpatrick-Aydemir model is used to test Feedback Controllers

- The reduced Fitzpatrick-Aydemir equations are used to simulate the $(m, n) = (3, 1)$ external kink mode.
- The Fitzpatrick-Aydemir model has been shown to accurately characterize experimental observations of the RWM on HBT-EP.[1]
- Growing, rotating, $n = 1$ plasma and wall modes are produced.
- Different feedback controllers can be tested in a loop similar to what we have on HBT-EP.

[1] M. Shilov, *et. al.*, Phys. Plasmas **11**, 2573 (2004).

The Reduced Fitzpatrick-Aydemir Equations describe a Growing, Rotating RWM

The fluxes at the plasma and the wall are given as a function of plasma parameters, coupling parameters and a control flux.[2]

$$\frac{\mathrm{d}\vec{y}}{\mathrm{d}t} = A\vec{y} + \vec{R}\psi_c,$$

where

$$\vec{y} = \begin{pmatrix} \psi_a \\ \psi_w \end{pmatrix}, \quad A = \begin{pmatrix} (1 - \bar{s} - i\bar{\alpha})\frac{\gamma_{mhd}^2}{\nu_d} & -\frac{\gamma_{mhd}^2}{\nu_d\sqrt{c}} \\ \frac{\gamma_w\sqrt{c}}{1-c} & -\frac{\gamma_w}{1-c} \end{pmatrix}, \quad \text{and} \quad \vec{R} = \begin{pmatrix} \frac{-c_f\gamma_{mhd}^2}{\nu_d} \\ \frac{\gamma_w(1-cc_f)}{1-c} \end{pmatrix}$$

With the normalized stability parameter set $\bar{s} = 1.0$ and the rotation-dissapation parameter $\bar{\alpha} = -\nu_d\Omega/\gamma_{mhd}^2 = -1.41$ (corresponding to a rotation rate of 5 kHz).

[2] M. E. Mauel, *et. al.*, Nucl. Fusion **45**, 285 (2005).

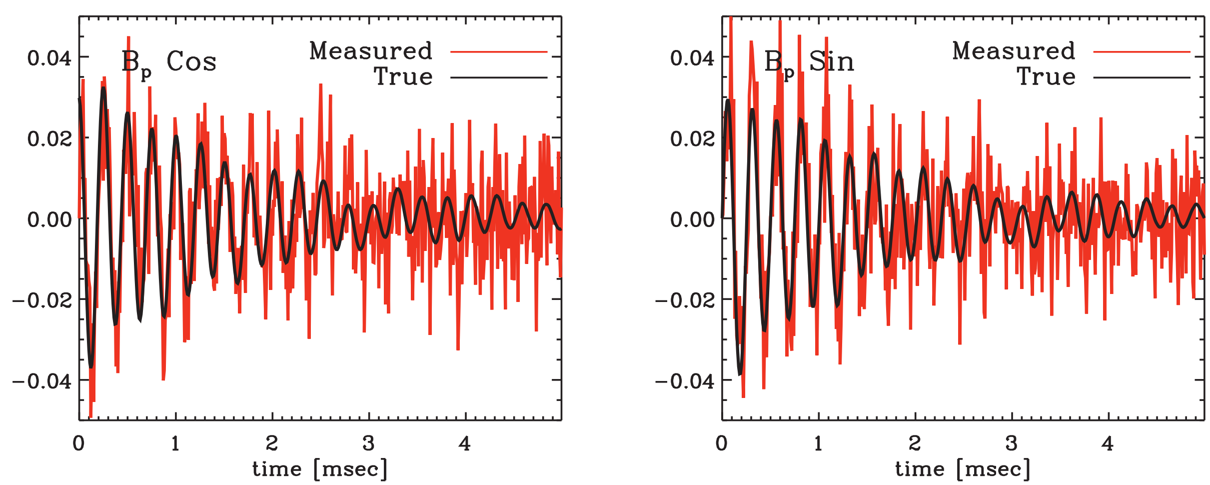
The poloidal field depends on the mode fluxes, wall radius, and a coupling coefficient

The poloidal magnetic field is calculated from a linear combination of the plasma and wall fluxes,

$$B_p = \frac{3}{r_w(1-c)}(2\sqrt{c}, -(c+1)) \cdot \begin{pmatrix} \psi_a \\ \psi_w \end{pmatrix},$$

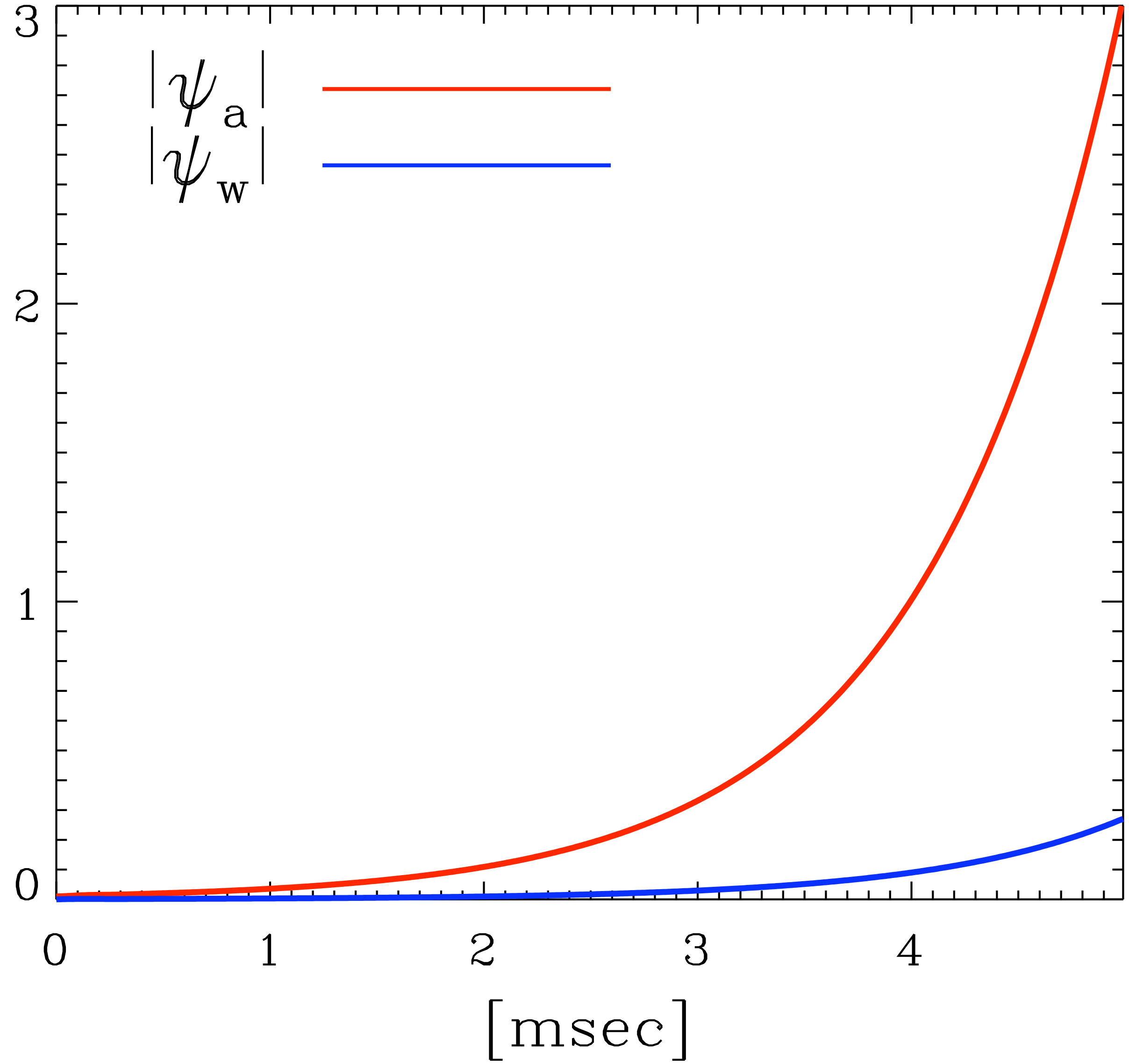
and decomposed into $\sin\varphi$ and $\cos\varphi$ modes with added Gaussian noise ν.

$$B_p^{\cos} = \mathrm{Re}[B_p + \nu]$$
$$B_p^{\sin} = \mathrm{Re}[\mathrm{e}^{-i\pi/2}(B_p + \nu)]$$

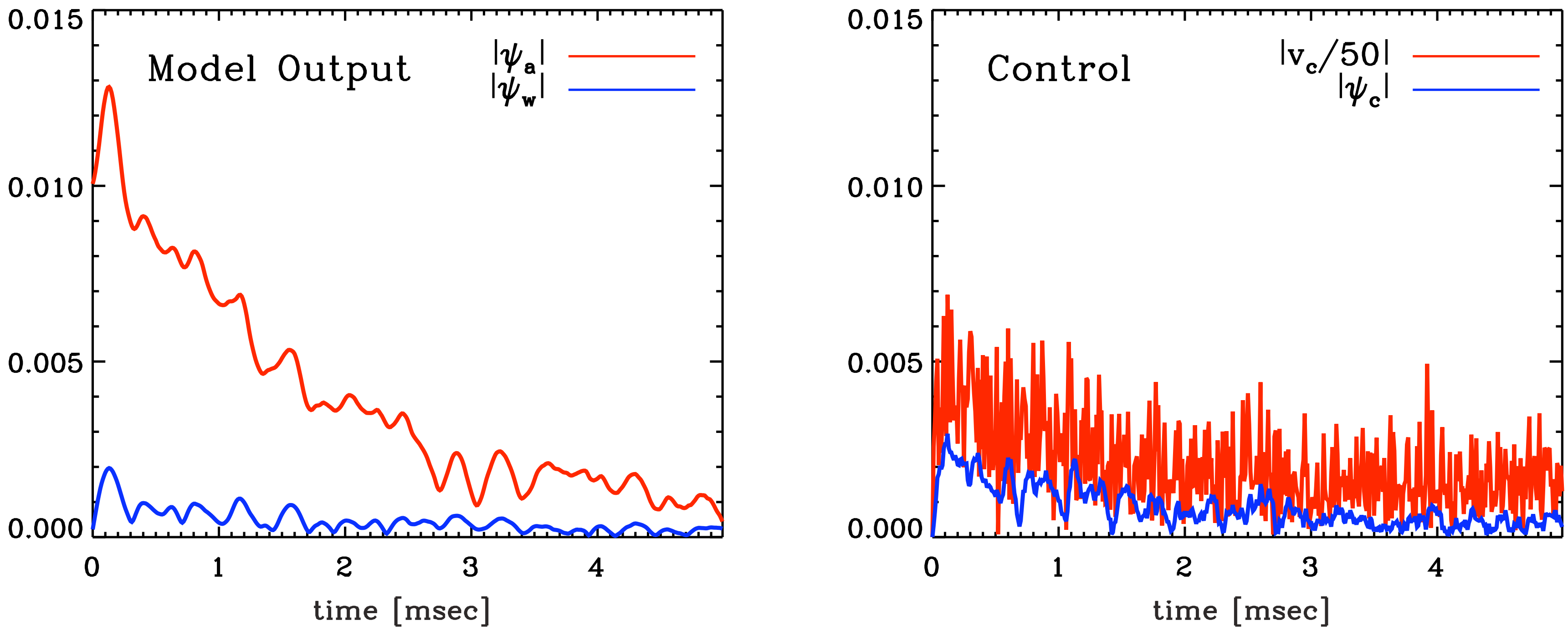


The control voltage V_c is calculated by applying proportional gain to these fields.

Without feedback, the reduced Fitzpatrick-Aydemir equations produce unstable plasma and wall modes



The modes can be stabilized using proportional gain feedback, but at a cost



- Quite a bit of noise makes it into the controller's output.
- A lot of control power is used, even after the mode is stabilized.

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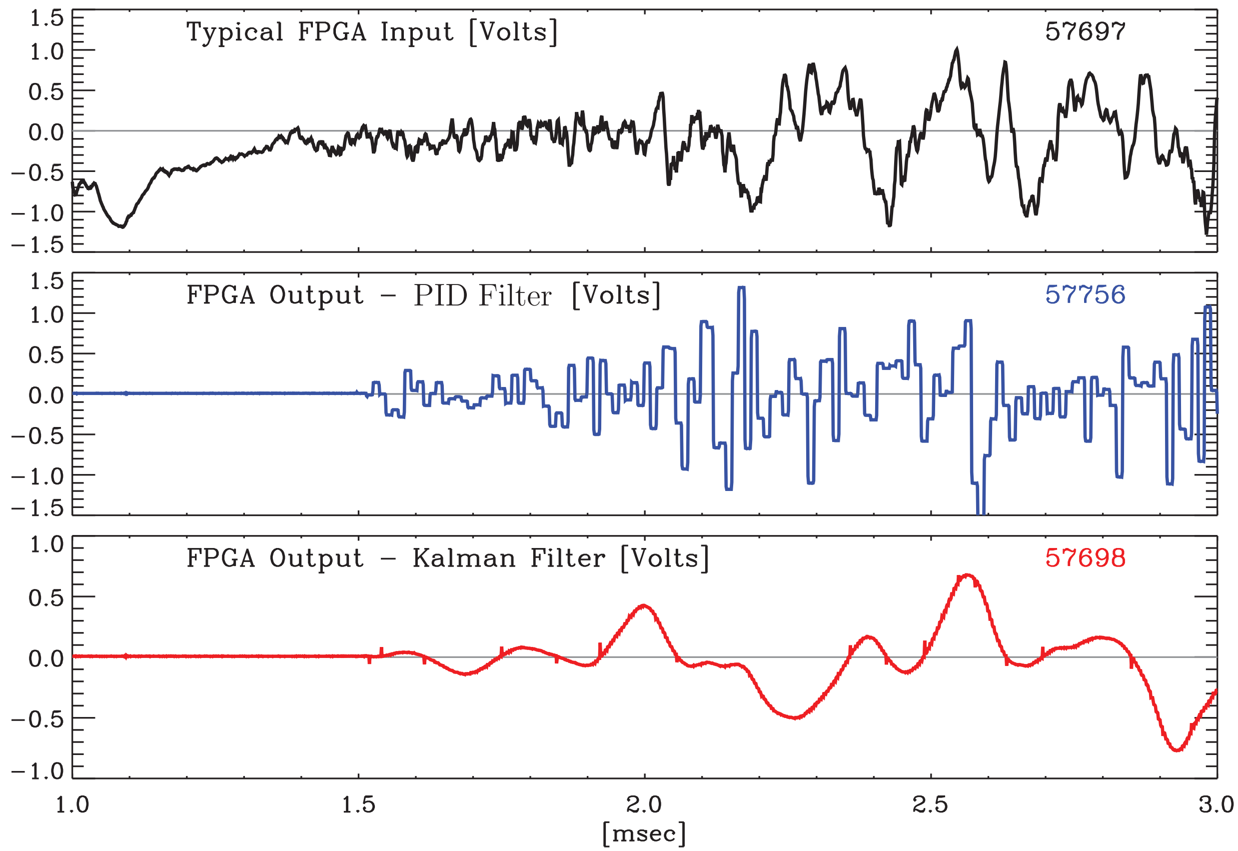
What is a Kalman filter?

- A Kalman filter produces an estimate for the state of a noisy system by comparing an internal model with measurements.
- The Kalman filter does an excellent job of removing noise from input signals. (Note: the data shown below is from three different shots.)



The Kalman filter is a simple matrix equation

In steady state,

$$\vec{x}_i = \Phi \vec{x}_{i-1} + K \vec{z}_i,$$

where $\vec{x}$ is the optimal estimate of the system state and $\vec{z}$ is a vector of measurements.

Here,

$$\begin{aligned} \Phi &= (I - KH)(A + BGH), \quad \text{and} \\ K &= (APA' + Q)H'(H(APA' + Q)H' + R)^{-1}. \end{aligned}$$

A	A model for the system dynamics
B	System response to a control input
G	Gain
H	Model for measurement dynamics
P	Error covariance of estimate
Q, R	System and measurement noise covariances

The system model depends only on the mode's growth and rotation rate

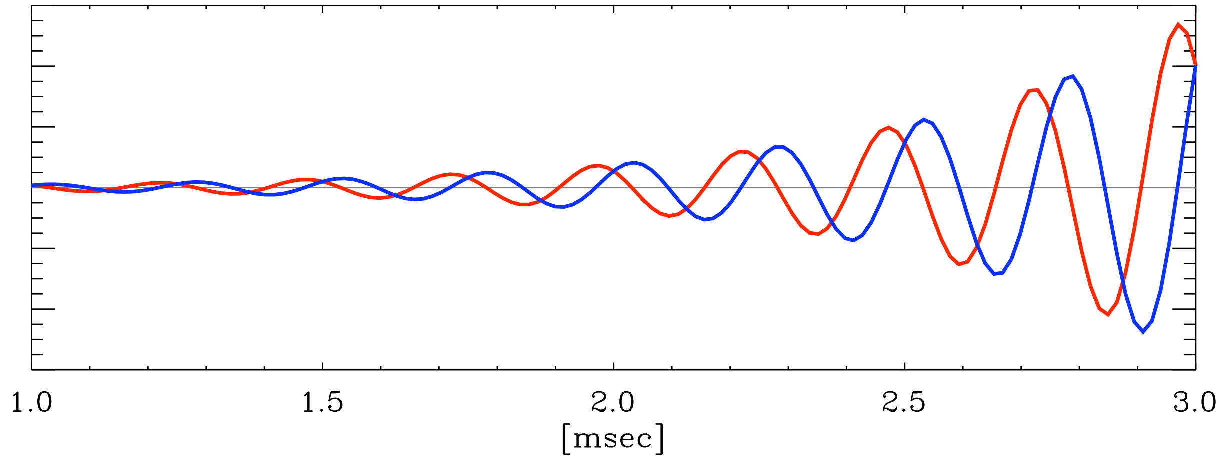
The idea is to model the $\cos\varphi$ and $\sin\varphi$ Fourier components of a rotating, growing mode. (These map to an amplitude and a phase.)

The time-domain equation is

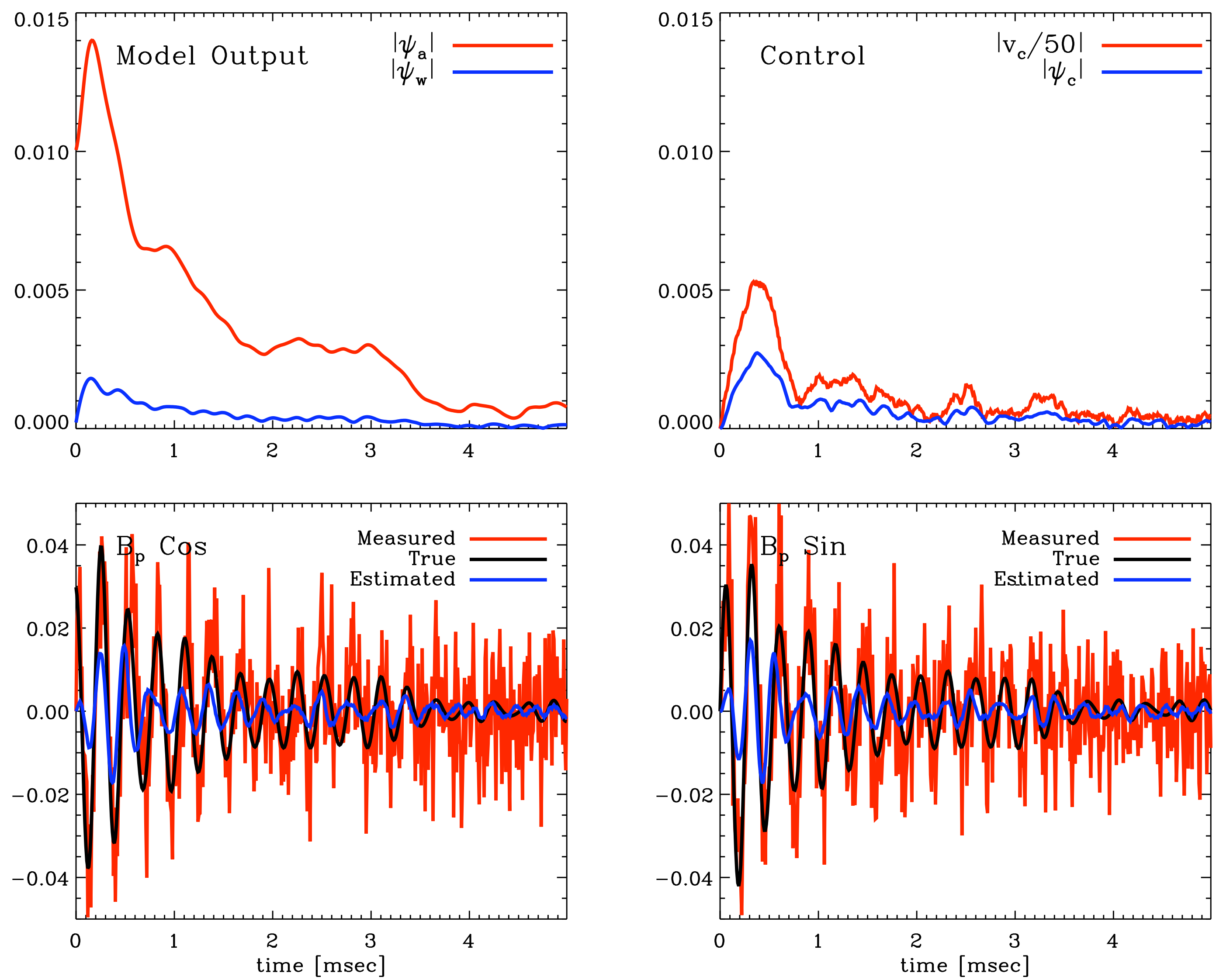
$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} B_p^{\cos} \\ B_p^{\sin} \end{pmatrix} = \begin{pmatrix} \mathrm{Re}\gamma_k & -\mathrm{Im}\gamma_k \\ \mathrm{Im}\gamma_k & \mathrm{Re}\gamma_k \end{pmatrix} \begin{pmatrix} B_p^{\cos} \\ B_p^{\sin} \end{pmatrix}.$$

The solution is

$$\vec{B}_p(t) = \exp(\mathrm{Re}\gamma_k\, t)\mathfrak{R}(\mathrm{Im}\gamma_k\, t)\vec{B}_p(0).$$



The Steady-State Kalman Filter stablizes the mode quickly and efficiently



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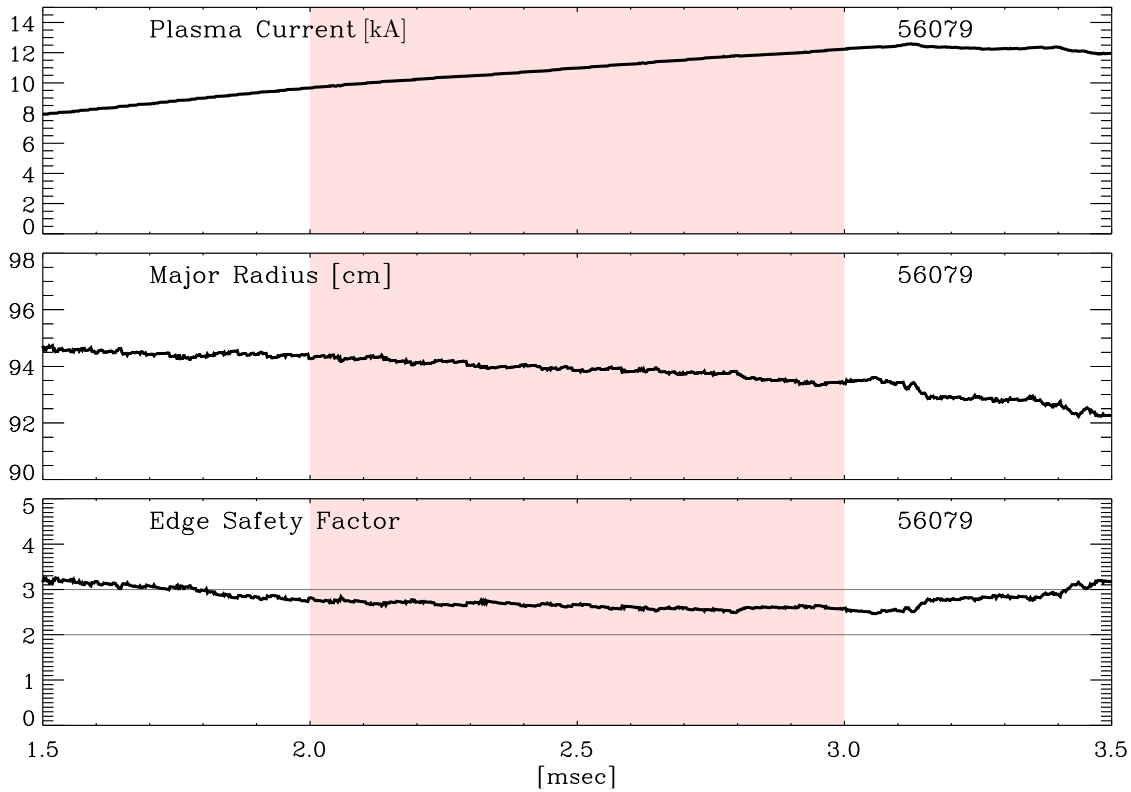
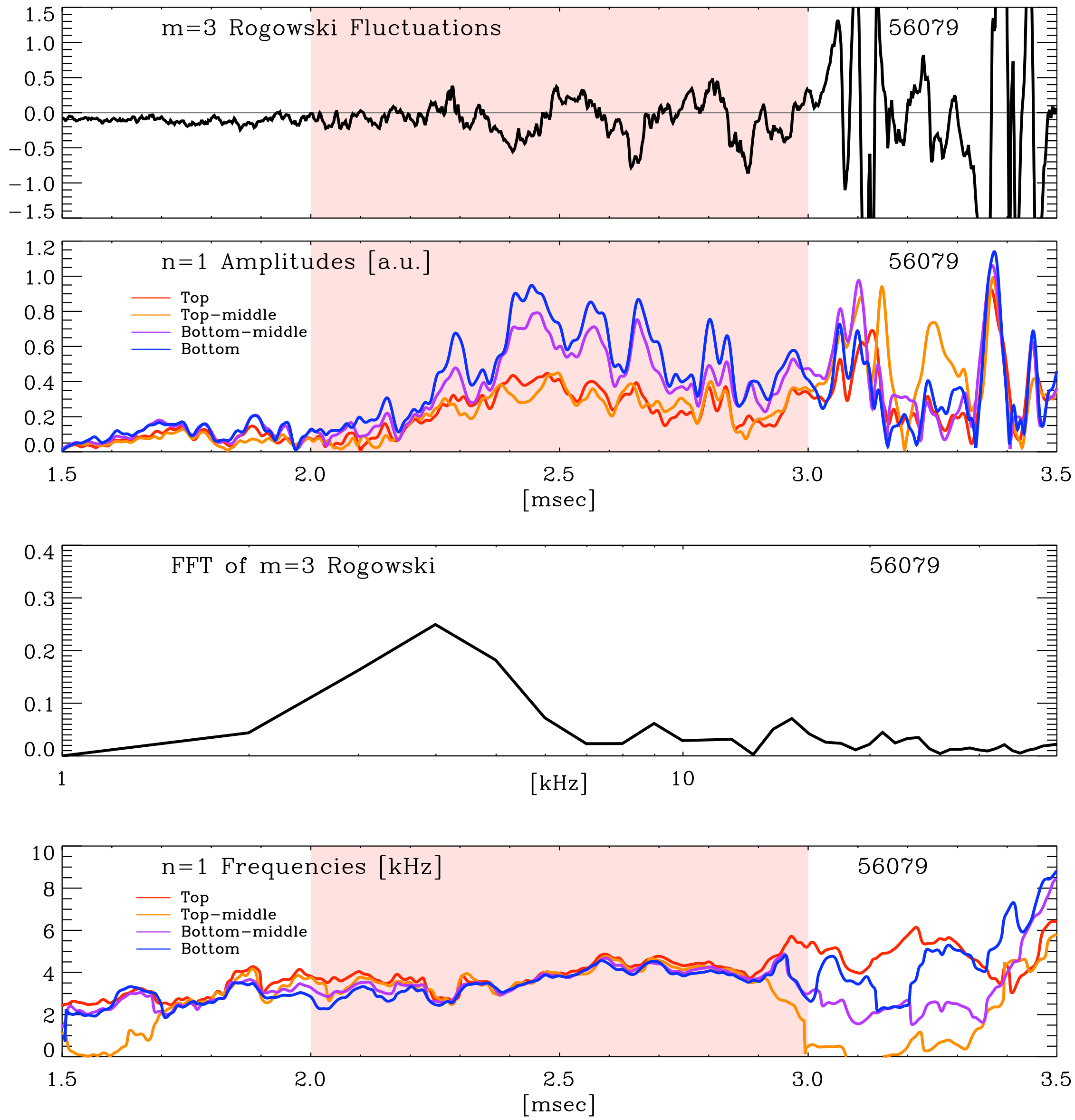
Ohmic heating is used to create a current driven mode



I_p is ramped at $\sim$3 MA/sec to create a current density gradient at the edge.

The $q = 3$ surface is pushed just outside the plasma edge.

The resulting external kink mode is robust and reproducible



We see a growing, rotating mode in the magnetic diagnostics.

The rotation frequency is 4–5 kHz.

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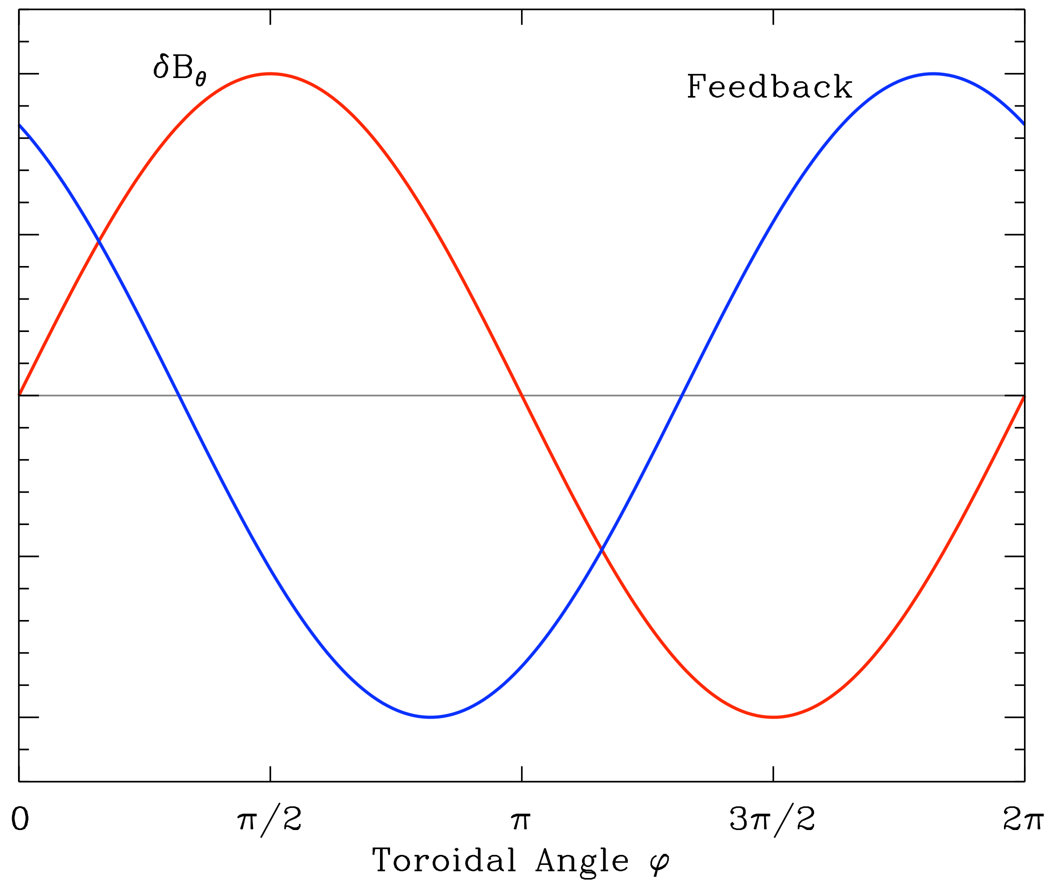
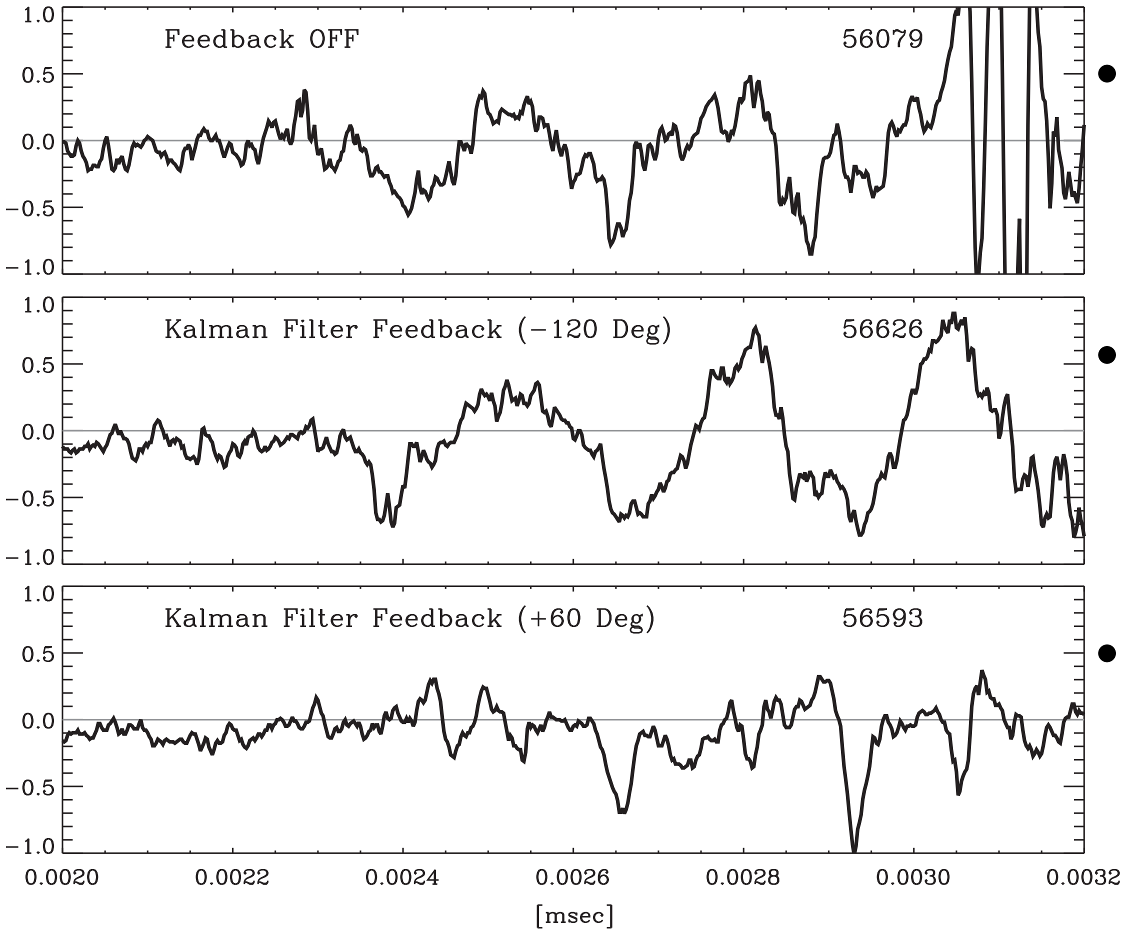
The toroidal phase of feedback is an adjustable control parameter

- Sensors measure the *poloidal* field, but feedback coils are *radial*. So a phase-shift is needed for negative feedback.
- Phase shifts also appear due to controller latency and imperfect optimization of the system transfer function.

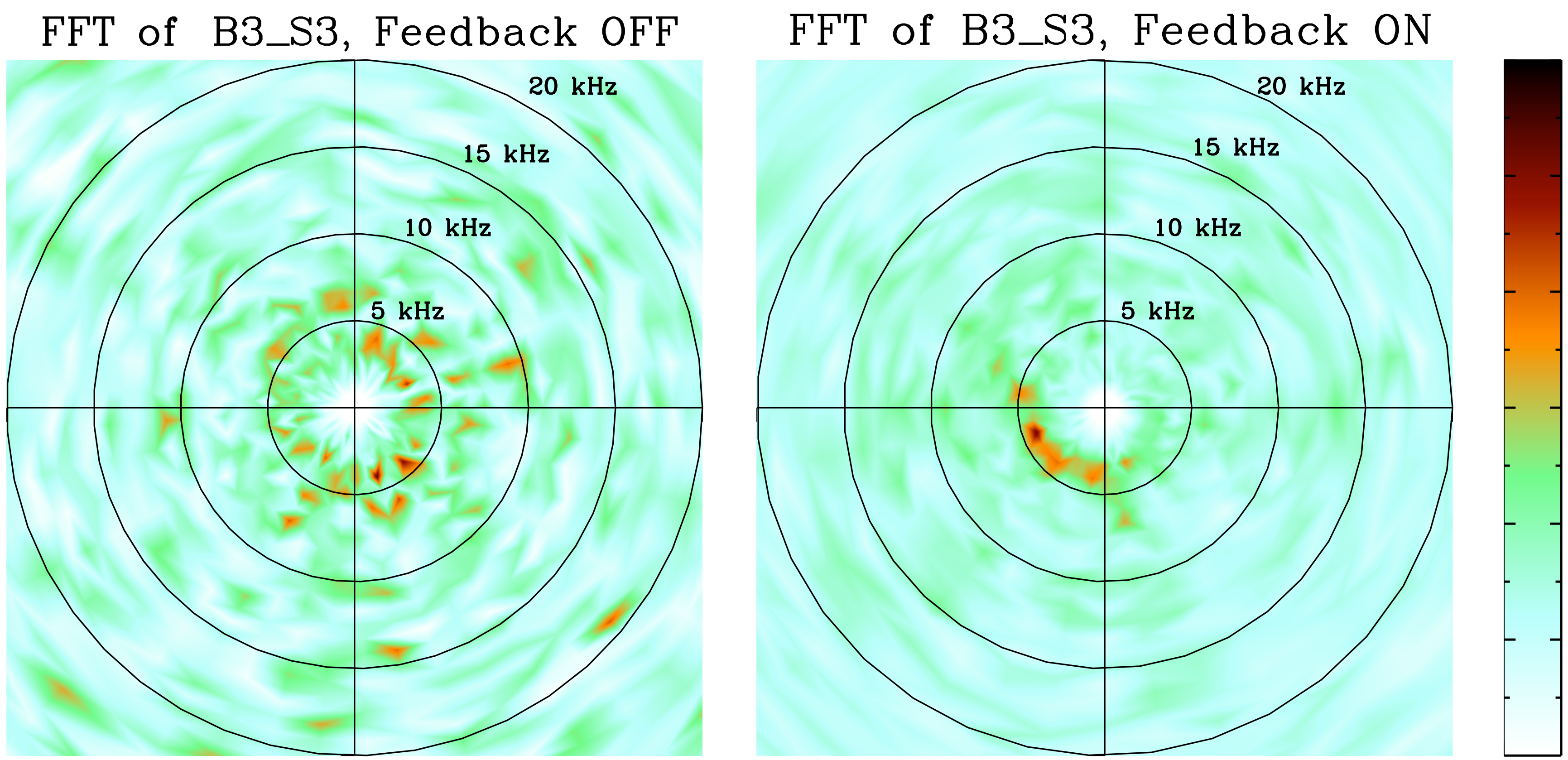


We can phase feedback to either excite or suppress the kink mode



- Feedback off case, 4–5 kHz $m = 3$ fluctuations.
- With positive feedback, we can drive the mode to higher amplitudes.
- With negative feedback, we can suppress the $\sim$5 kHz mode, but some higher frequency activity remains.

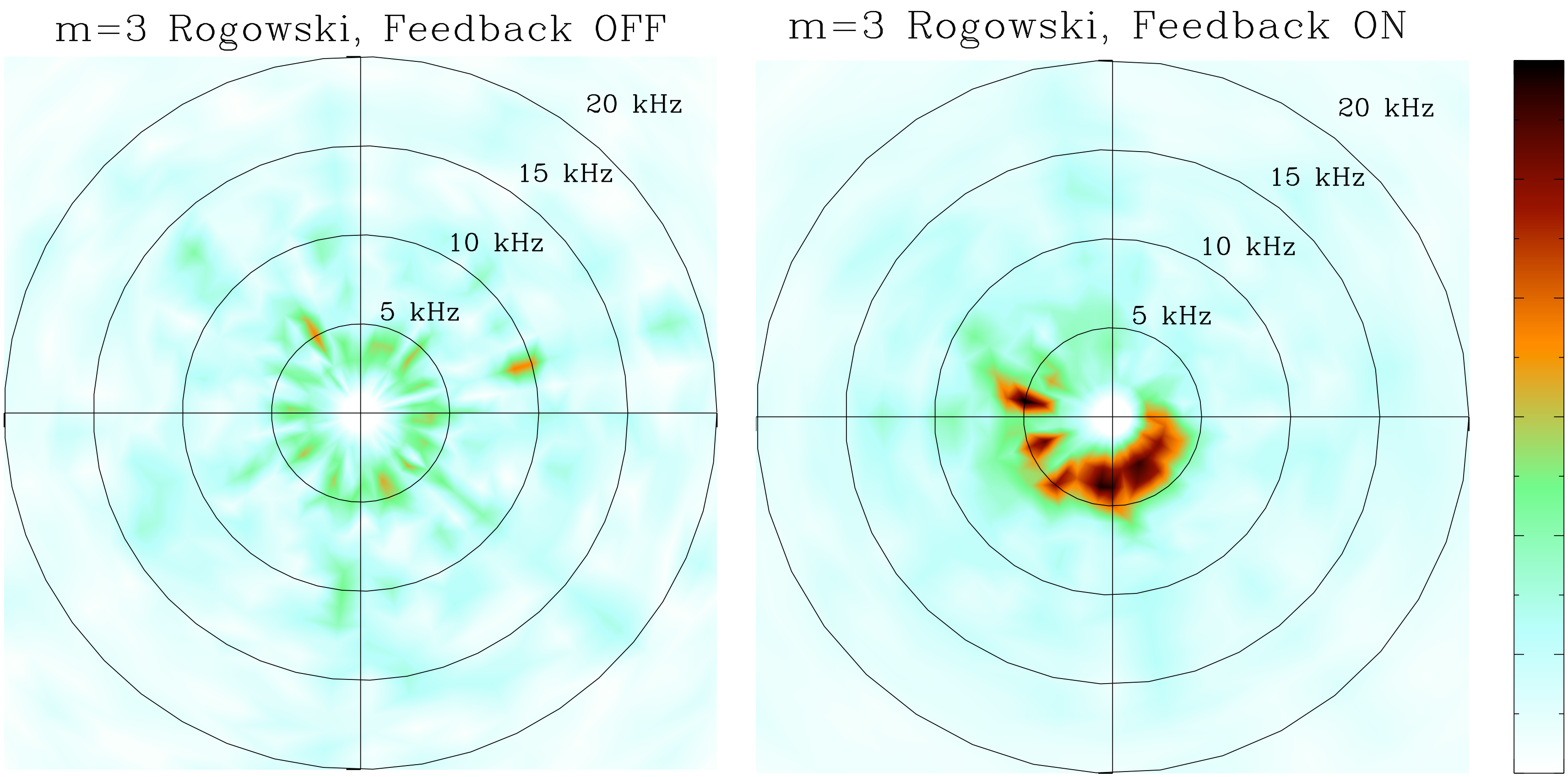
Scanning the feedback phase angle reveals clear evidence of kink excitation and suppression in the sensor coils



(polar angle is arbitrary)

(polar angle is the feedback phase)

Data from the $m = 3$ Rogowski is consistent with that of the sensor coils



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Summary

Conclusions

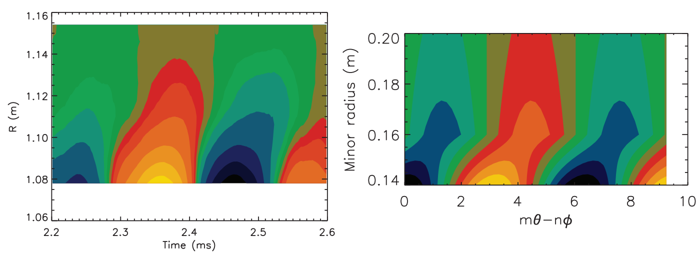
- The Kalman filter demonstrated here uses the simplest possible model to produce an estimate for a growing, rotating mode.
- It has been implemented on a set of low-latency FPGA controllers and used to demonstrate feedback control of $(m, n) = (3, 1)$ external kink modes on HBT-EP.
- The phasing of the Kalman filter algorithm can be adjusted to either suppress or excite kink modes near 5kHz, but there is little excitation of higher frequency activity.

Future Work

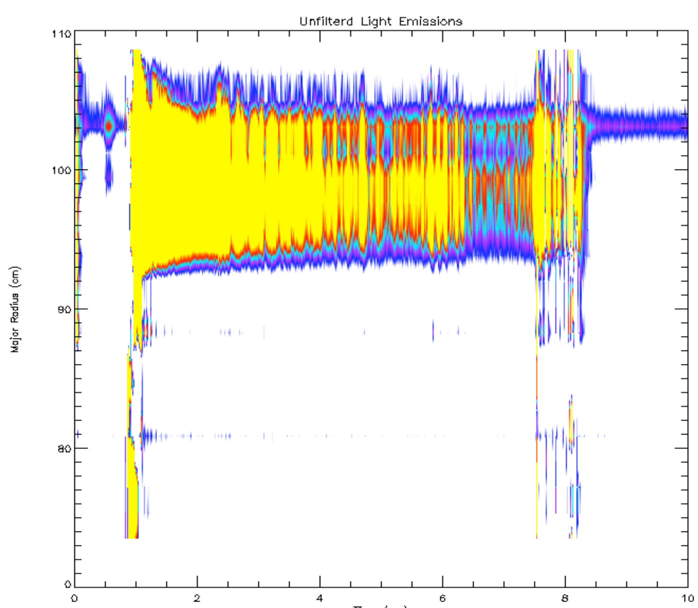
- Partial coil coverage studies are ongoing and will answer questions about mode rigidity.
- Investigation of Kalman filter parameters: growth and rotation rates, covariance matrices.
- Addition extra noise to the system, both white noise and simulated ELMs.

There's a lot of interesting work being done on HBT-EP

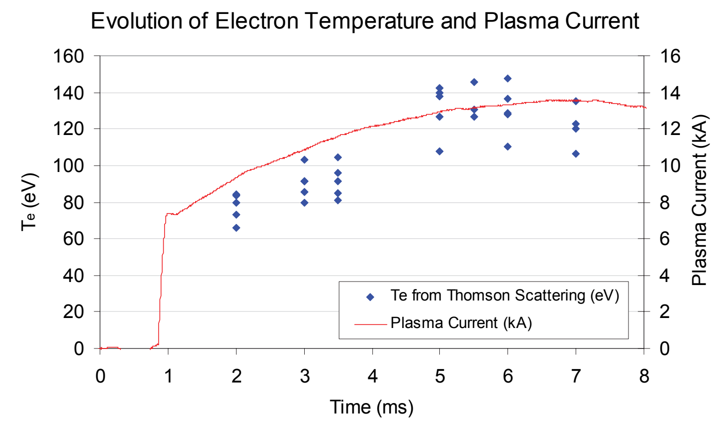
- Measurements of the kink mode's radial structure using a Hall probe array show good agreement with cylindrical theory.



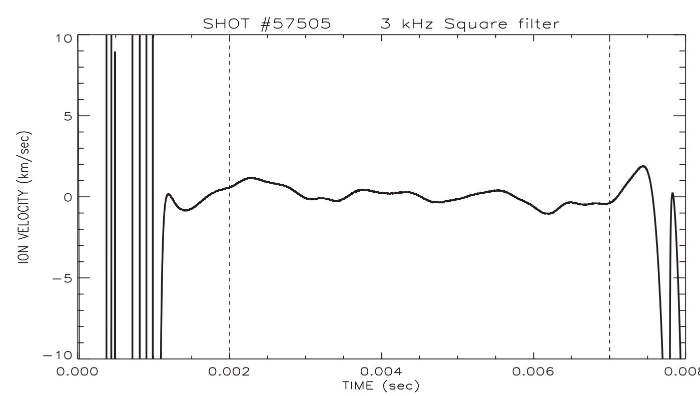
- Measurements D_α emission using a photodiode array show population of neutrals at the plasma edge.



- Thompson scattering diagnostic data shows $T_e \sim 100$ eV in the core.



- Doppler ion rotation diagnostic is up and working.



Extra Slides

A Simple Control Coil Model

The control coils are modeled by[3]

$$\frac{\mathrm{d}\psi_c}{\mathrm{d}t} + \frac{R_c}{L_c}\psi_c = \frac{M_c}{L_c}V_c$$

or, equivalently

$$\psi_{c\,n} = \epsilon\psi_{c\,n-1} + \frac{M_c}{R_c}(1-\epsilon)V_{c\,n}$$

with $\epsilon = \exp(-R_c/L_c\,\delta t)$.

[3] M. E. Mauel, *et. al.*, Bull. Amer. Phys. Soc. Paper BP1.00007

The Time-varying Kalman Filter uses a simple, internal model

$$\begin{aligned} \vec{x}_n^* &= \hat{A}\vec{x}_{n-1} + B\vec{u}_n \\ \vec{x}_n &= \vec{x}_n^* + K_n(\vec{z}_n - H\vec{x}_n^*) \\ \hline P_n^* &= \hat{A}P_{n-1}\hat{A}' + Q \\ K_n &= P_n^* H'(HP_n^* H' + R)^{-1} \\ P_n &= (I - K_n H)P_n^* \end{aligned}$$

The state vector is

$$\vec{x}(n) = (B_p^{\cos}(n), B_p^{\sin}(n), B_p^{\cos}(n-1), B_p^{\sin}(n-1)).$$

The system model A comes from the dynamics of a growing, rotating mode.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} B_p^{\cos} \\ B_p^{\sin} \end{pmatrix} = \begin{pmatrix} \mathrm{Re}\gamma_k & -\mathrm{Im}\gamma_k \\ \mathrm{Im}\gamma_k & \mathrm{Re}\gamma_k \end{pmatrix} \begin{pmatrix} B_p^{\cos} \\ B_p^{\sin} \end{pmatrix}.$$

with $\gamma_k = 1.27 + 4.26i$ kHz.

Time-varying Kalman Filter

The $B\vec{u}$ term must give the response of the kink mode to a control flux.

$$B\vec{u} = 2\delta t \left(\frac{3}{1-c}\right) \mathrm{Re}\left[((2\sqrt{c}, -(1+c)) \cdot \vec{\xi}_k)(\Xi^{-1} \cdot \vec{R}) \cdot \hat{e}_k \begin{pmatrix} 1 \\ \mathrm{e}^{-i\pi/2} \end{pmatrix} \psi_c\right]$$

Here, Ξ contains the eigenvectors of the reduced Fitzpatrick-Aydemir system matrix in its columns, and k the index of the unstable eigenvalue.[4]

Note: to solve this equation, we must measure the flux in the control coils.

[4] M. E. Mauel, *et. al.*, Bull. Amer. Phys. Soc. Paper BP1.00007

The Steady-State Kalman Filter is easy to implement

The time-varying Kalman Filter is probably too large to implement on HBT-EP's present mode control system.

Take the limit of the time-varying Kalman filter in which $n \to \infty$.

This filter has a simple form – the controller does not need to compute a matrix inverse.

$$\vec{x}_n = \Phi \vec{x}_{n-1} + K \vec{z}_n$$

The matrices can be calculated in advance.

$$\Phi = (I - KH)(\hat{\hat{A}} + BGH)$$
$$K = (\hat{\hat{A}} P \hat{\hat{A}}' + Q) H' (H(\hat{\hat{A}} P \hat{\hat{A}}' + Q) H' + R)^{-1}$$

The control flux ψ_c must be added to the state vector.

$$\vec{x}(n) = (\psi_c^{\cos}(n), \psi_c^{\sin}(n), B_p^{\cos}(n), B_p^{\sin}(n))$$

Steady-State Kalman Filter

There is a subtlety: if the control flux is not measured, it must be computed from the control voltage.

$$G = \frac{M_c}{R_c}(1-\epsilon)\left(\begin{array}{cc|cc} \frac{\epsilon R_c}{M_c(1-\epsilon)} & 0 & g_p & i g_p \\ 0 & \frac{\epsilon R_c}{M_c(1-\epsilon)} & -i g_p & g_p \\ \hline \multicolumn{2}{c|}{0_{22}} & \multicolumn{2}{c}{0_{22}} \end{array}\right)$$

The response of the system to the control flux must be calculated, too.

$$B = \left(\begin{array}{cc|c} \sigma & 0 & \multirow{2}{*}{0_{22}} \\ 0 & \sigma & \\ \hline \multicolumn{2}{c|}{0_{22}} & 0_{22} \end{array}\right)$$

Here, $\sigma = 2\delta t \frac{3}{1-c}((2\sqrt{c}, -(1+c)) \cdot \vec{\xi}_i)(\Xi^{-1} \cdot \vec{R}) \cdot \hat{e}_i$.

When calculating the filter parameters, the real part of the product BG is used.

Comparing Filter Performance

After the mode has been suppressed, RMS averages of real (oscillating) quantities are calculated.

Percent reductions are computed with respect to the unfiltered, proportional gain case.

Negative values indicate percent *increases*.

	Low-pass	TV Kalman	SS Kalman
ψ_a	-92	26	5
ψ_w	-16	54	50
ψ_c	-11	56	41
V_c	39	74	60