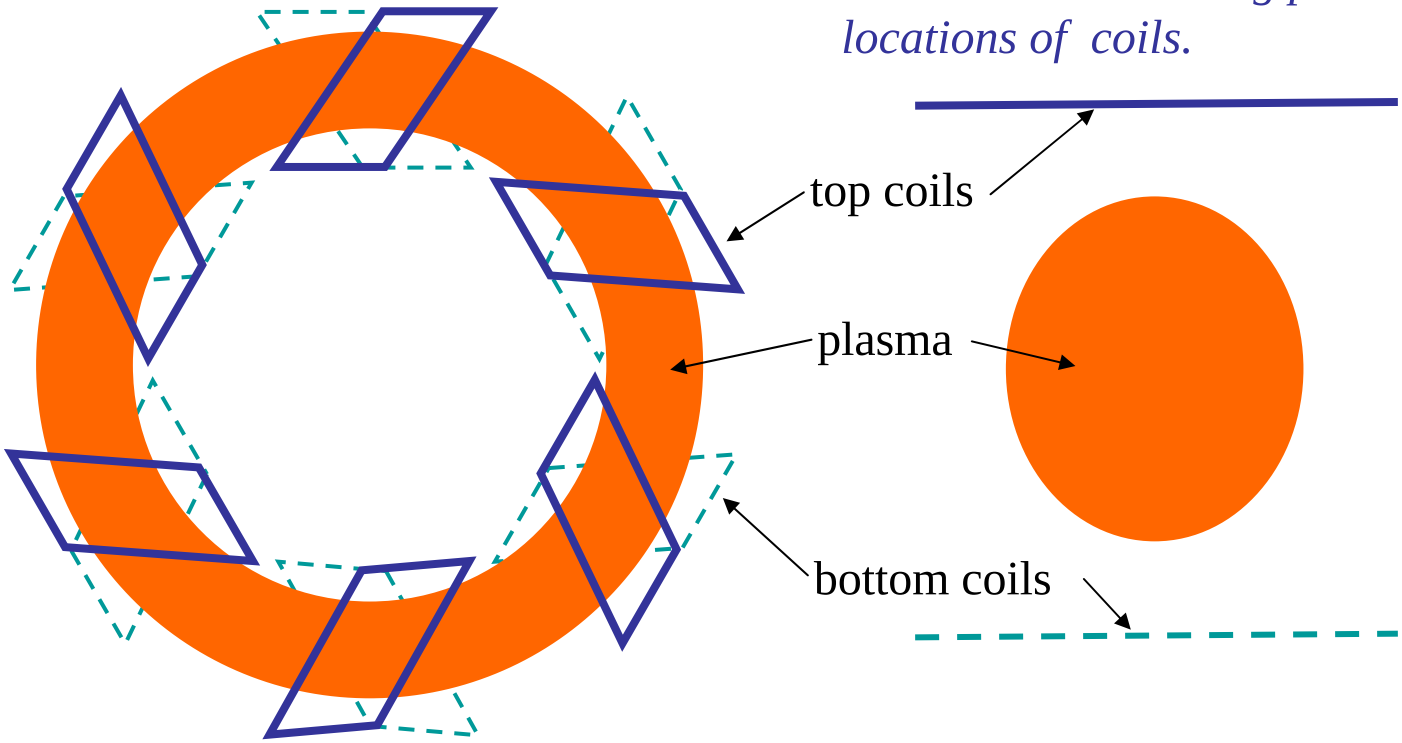
Passive Stabilization of the Vertical Mode in Tokamaks (an analytical calculation)

Allan Reiman, PPPL

View from above.

Cross-section showing plasma and locations of coils.

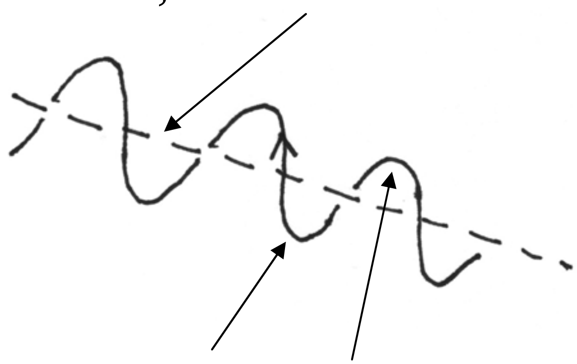


12TH WORKSHOP ON MHD STABILITY CONTROL
COLUMBIA UNIVERSITY, Nov. 18, 2007

Introduction: 3D field provides control over magnetic field line properties not available in 2D

Assume $B_t >> B_c$, where B_t is toroidal field and B_c is field produced by nonaxisymmetric coils.

To 0'th order, field lines follow toroidal field.

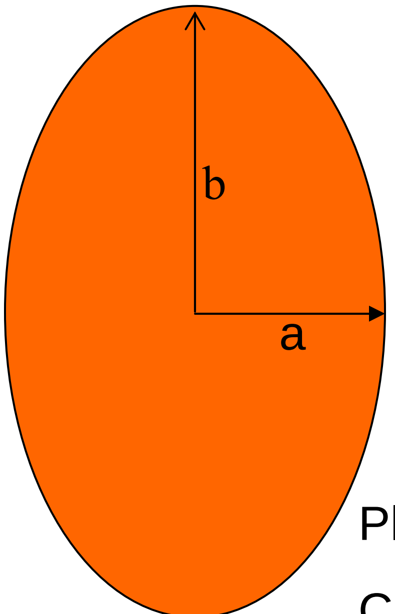


1st order effect of 3D field: spiraling of field lines

2nd order effect: Different magnitudes of fields on inside and outside gives net drift of field lines. (analogous to particle drifts)

Perturbative solution for field line trajectories gives secular drift in 2nd order that can be handled by method of averaging. Method incorporates drift in effective zero order axisymmetric field, $\bar{\mathbf{B}}_c$.

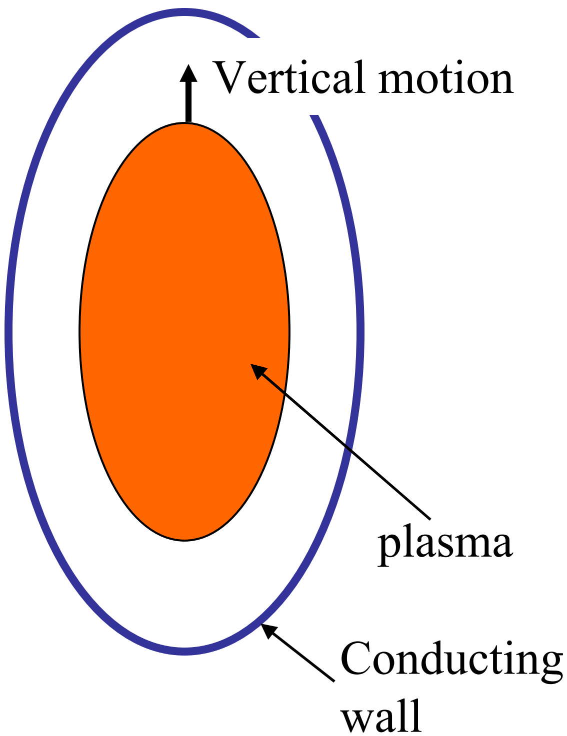
Use 3D field to stabilize vertical mode, which imposes important constraint on tokamak design.



Vertical elongation = b/a ≡ κ

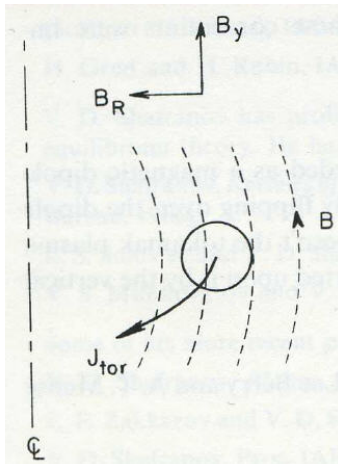
- Free energy driving vertical instability increases with vertical elongation.
- Troyon scaling predicts increase in β limit for ballooning and kink modes with elongation for fixed q: $\beta = CI/aB$, $I \propto (1 + \kappa^2) / (2\kappa)$ for ellipse.
- Global confinement scaling laws find that confinement improves with vertical elongation, $\tau \sim I\ \kappa^{0.5}$

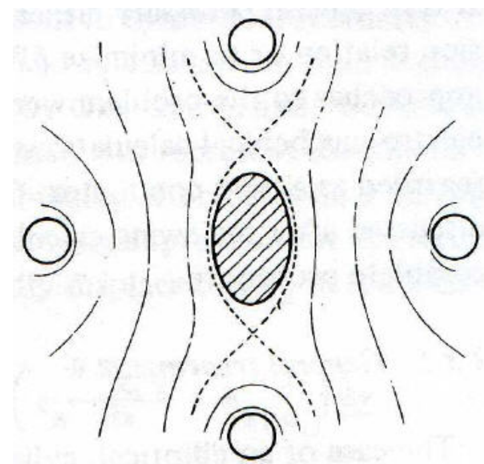
Nonlinear effect of 3D field may also reduce incidence of disruptions caused by vertical instability, called "vertical displacement events" (VDEs).



- Tokamak plasmas typically vertically elongated, with vertical mode stabilized by external conducting elements.
  Instability persists on resistive time scale of conducting elements.
- Feedback stabilization suppresses resistive mode.
- VDEs caused by accidental crossing of ideal stability threshold, failure of feedback system, etc.
- Disruptions initiated by other instabilities often culminate in vertical mode.

Physical mechanism of instability can be understood by considering motion of current carrying ring in quadrupole field.





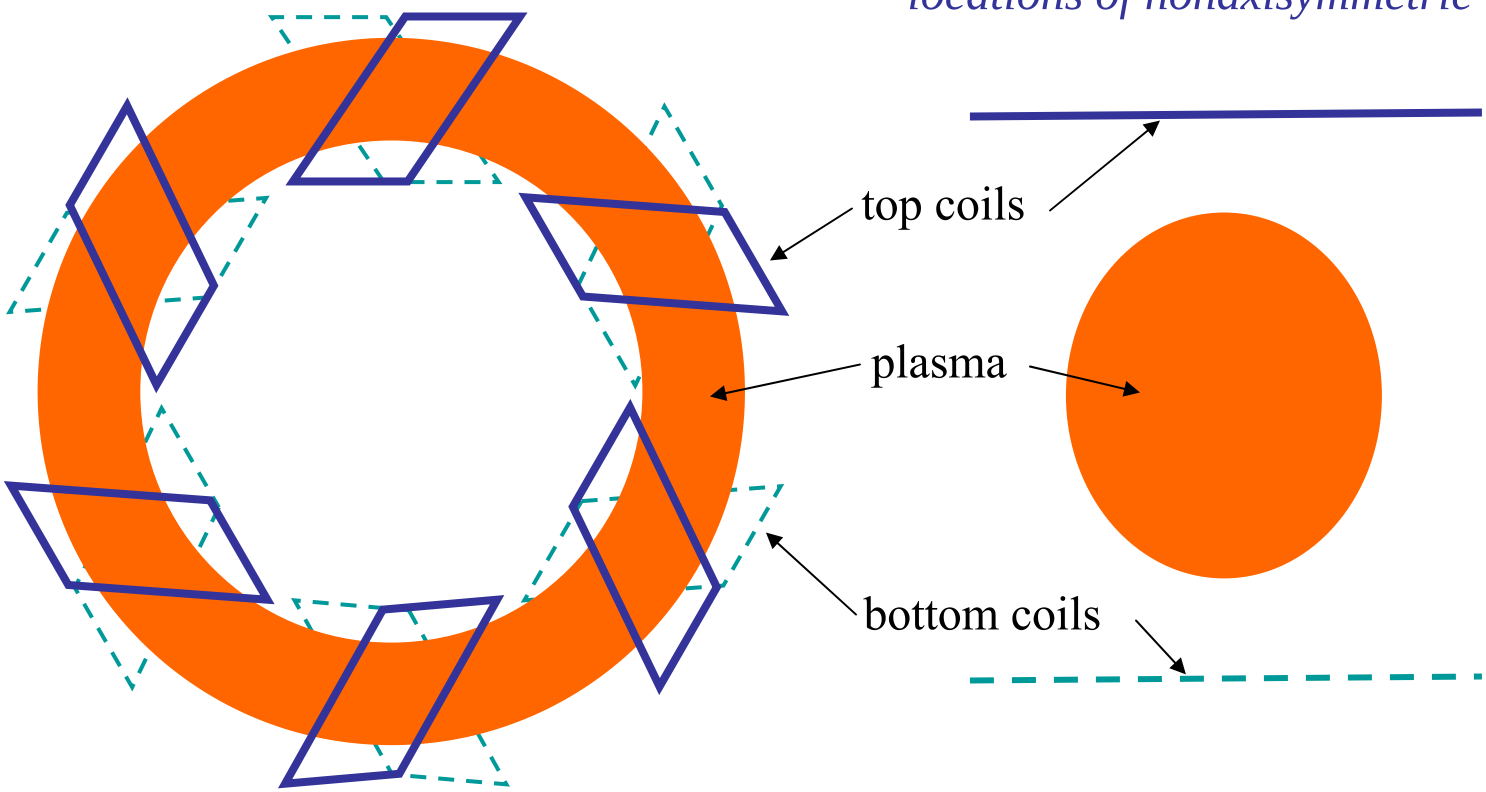
- Force on displaced ring determined by $J_{\mathrm{tor}}\partial B_R/\partial y$
- Quadrupole component of field exerts pressure that controls ellipticity through $\partial B_y/\partial R$
- For axisymmetric quadrupole vacuum field, sign of $\partial B_R/\partial y$ determined by sign of $\partial B_y/\partial R$
- Field that increases vertical elongation produces destabilizing change in $\partial B_R/\partial y$

Allowing magnetic field to be nonaxisymmetric decouples $\partial B_R/\partial y$ from $\partial B_y/\partial R$, allowing stabilization of vertically elongated plasma.

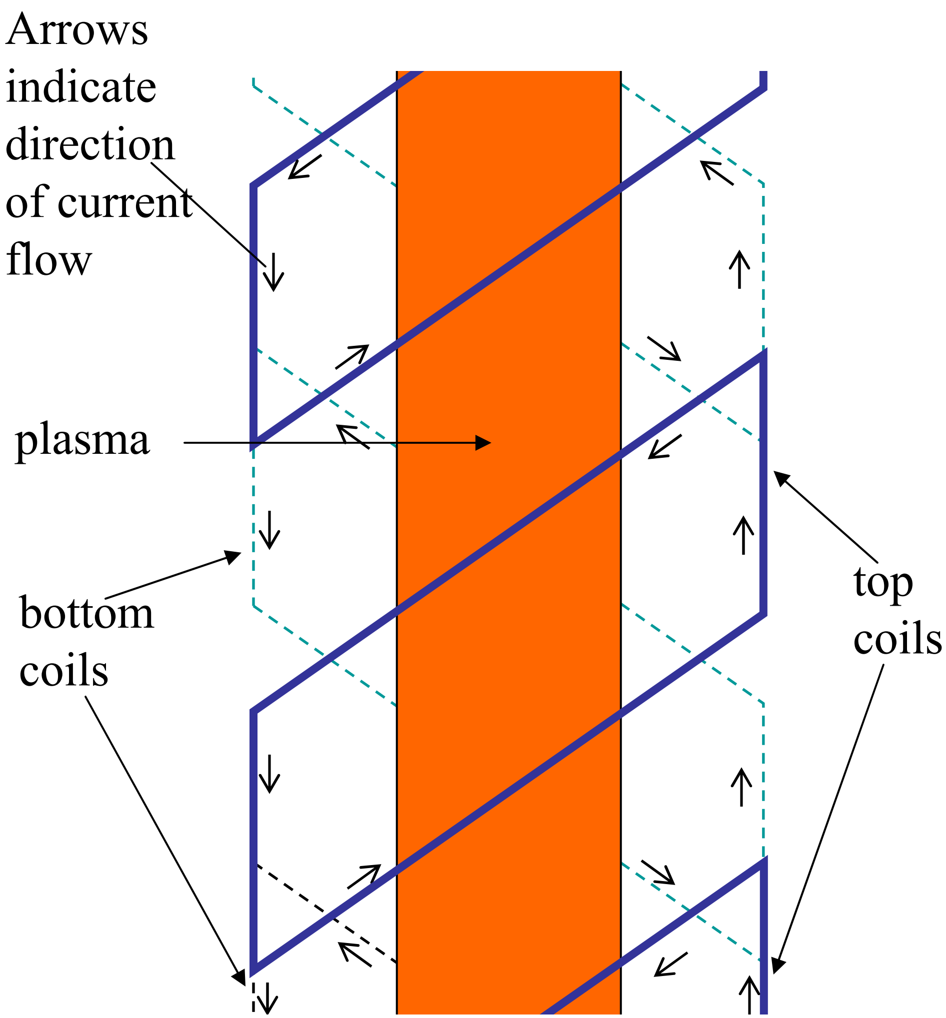
Design simple set of coils to target physics of instability.

View from above.

Cross-section showing plasma and locations of nonaxisymmetric coils.



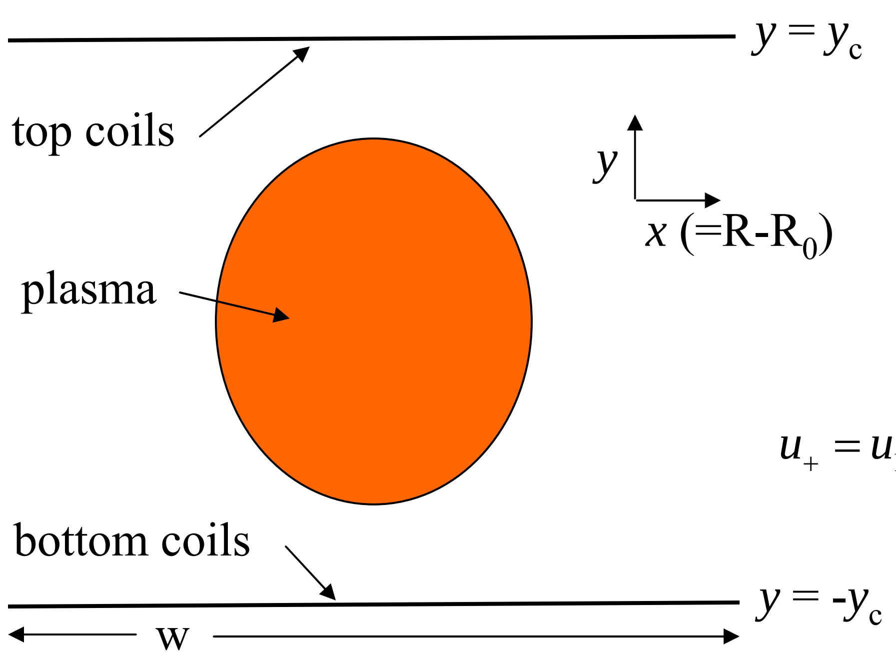
Analytical calculation considers large aspect ratio (cylindrical) plasma. (Phys. Rev. Lett (Sept. 28, 2007))



View from above showing parallelogram-shaped coils above a length of cylindrical plasma, with portions of the coils below the plasma also visible.

Coil currents can be represented as surface currents on two ribbons.

Cross-section showing the plasma and the locations of the nonaxisymmetric coils.



- Write surface current ***K*** in terms of a current potential, ***K*** = ∇ x ($u\hat{y}$), where u finite only for $y = \pm y_c$, $-w/2 \le x \le w/2$.
- Stellarator symmetry relates u on upper and lower ribbons: $u_-(x,z) = u_+(x,-z)$. Focus on u_+.
- Parallelogram-shaped coils correspond to:

$$u_+ = u_1 \left\{ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos[(2n-1)(k_x x + k_z z)]}{2n-1} - \frac{\pi}{4} \right\}$$

Retain only lowest harmonic in vacuum field produced by nonaxisymmetric coils.

- Vacuum field can be expressed in terms of scalar potential, $\mathbf{B}=\nabla\chi$.
- For $y < y_c$, $|x|<w/2$ and $w/2 - |x|$ sufficiently large relative to $|y_c-y|$, nonaxisymmetric part of χ_+ is

$$\sum_{n=1}^{\infty}\chi_{+n}\exp(nky)\cos[n(k_x x + k_z z)],$$

  where $k = (k_x^2+k_z^2)^{1/2}$ and $\chi_{+n} = \mu_0 u_{+n}\exp(-nky_c)/2$. Take w sufficiently large that this can be used throughout plasma.
- Take k sufficiently large that only lowest harmonic needs to be retained in this expression.

For equilibrium, use stellarator expansion.

- Stellarator expansion (Greene and Johnson, Phys. Fluids **4**, (1961)) assumes $B_t >> B_c >> B_p$, where B_p is field produced by plasma current and axisymmetric poloidal field coils in absence of B_c.
- Method of averaging gives effective axisymmetric field produced by upper coils:

$$\bar{\mathbf{B}}_c = \nabla \psi_c \times \hat{\mathbf{z}}, \quad \psi_c = -\exp(2ky)\chi_1^{\,2} k_x k / (2k_z B_t).$$

Condition for validity: $(k_x/k_z)(B_c/B_t) << 1$.

- For $B_t j_z k_x/k_z > 0$, $J_{tor}\partial \bar{B}_{cR} / \partial y$ stabilizing, while $\partial \bar{B}_{cy} / \partial R = 0$.

Use Stellarator Expansion to Evaluate δW

Use energy principle in form $\delta W = \delta W_{\mathrm{p}} + \delta W_{\mathrm{V}}$, where

$\delta W_{\mathrm{p}} = (1/2)\int_{\mathrm{p}} (|Q|^2/\mu_0 - \xi \cdot \mathbf{j} \times \mathbf{Q}) d^3x$ (integral over plasma volume),

$\delta W_{\mathrm{v}} = (1/2)\int_{\mathrm{v}} \left[\left(B_{\mathrm{v}}^{(1)} \right)^2 / \mu_0 \right] d^3x$ (integral over vacuum region),

$\mathbf{Q} \equiv \nabla \times (\xi \times \mathbf{B})$, ξ is plasma displacement, $\mathbf{B}_{\mathrm{v}}^{(1)}$ is field perturbation in vacuum region.

Stellarator Expansion (Johnson & Greene, Phys. Fluids **4** (1961)):

B can be replaced by $\mathbf{B}_{\mathrm{p}} + \bar{\mathbf{B}}_{\mathrm{c}} + B_t \hat{\mathbf{z}}$ in δW_{p}; $B_{\mathrm{v}}^{(1)}$ can be determined from averaged perturbed plasma boundary, which is determined by $\mathbf{B}_{\mathrm{p}} + \bar{\mathbf{B}}_{\mathrm{c}} + B_t \hat{\mathbf{z}}$ and **ξ**.

Calculate stability for cylindrical plasma with uniform current, nearly circular cross-section, with small elliptical and nonaxisymmetric perturbations.

- $\mathbf{B}_\mathrm{p} + \bar{\mathbf{B}}_\mathrm{c} = \nabla\psi \times \hat{\mathbf{z}}$, where $\psi = \psi_0\left\{r^2\left[1-2\varepsilon_e\cos(2\theta)\right] + \varepsilon_c a^2 \exp\left[2k(y-a)\right]\right\}$, ψ_0 a constant, $\psi_0\varepsilon_c = -\exp(2\mathrm{ka})\chi_1{}^2 k_x k/(2k_z B_t a^2)$.
- For circular cylinder, δW minimized by rigid shift, $\xi = \xi_0\hat{\mathbf{y}}$. Retaining this test function for finite ε gives δW to $O(\varepsilon^2)$.
- $\delta W = 4V_\mathrm{p}(\xi_0\psi_0)^2\left\{\varepsilon_c \exp(-2ka)\left[2kaI_1(2ka) - I_0(2ka) - 2I_2(2ka)\right] - \varepsilon_e\right\}$, where V_p is plasma volume, I_j is modified Bessel function of *j*th kind.
- For *ka* large, $\delta W \approx 4V_\mathrm{p}(\xi_0\psi_0)^2[\varepsilon_c(ka/\pi)^{1/2} - \varepsilon_e]$.

Stability Condition

•Express stability condition in terms of max B_c/B in plasma

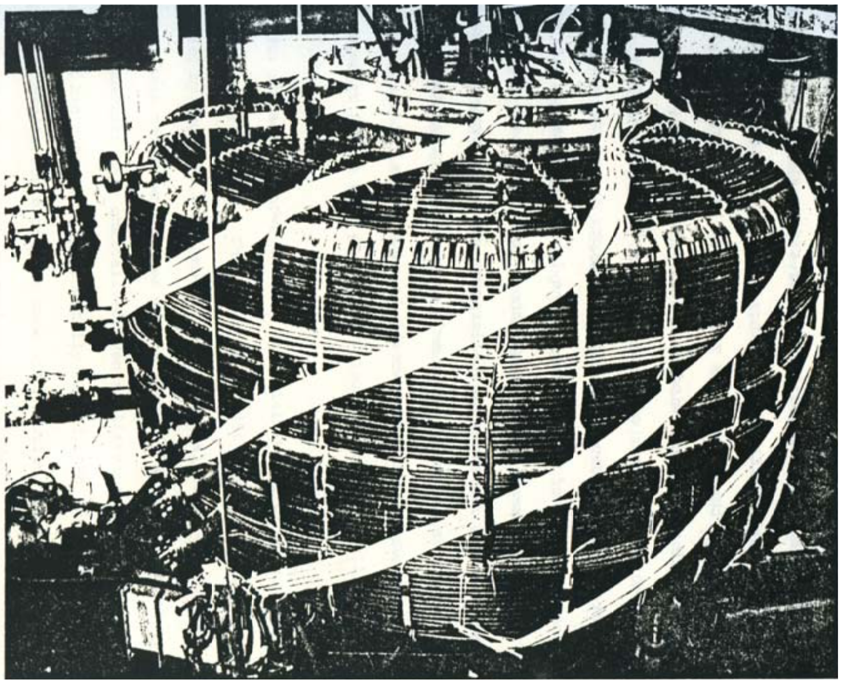
$$\max_{plasma}(B_c/B)^2 > (a/R)(k_z/k_x)(\pi k a)^{1/2}(\kappa\text{-}1)/(2q).$$

Validity of stellarator expansion requires $(k_x/k_z)(B_c/B_t) << 1$.

•For $R/a \approx 3$ and q $\approx$ 3 need $\max_{plasma}(B_c/B_t) \geq 0.1$ to see substantial stabilization.

Will need to assess effect of 3D fields on flux surfaces and drift trajectories, but nonaxisymmetric fields localized near plasma edge, where some degree of nonaxisymmetry may be desirable for suppressing ELMs.

Stabilization of tokamak vertical mode by nonaxisymmetric field demonstrated experimentally.



From A. Janos Ph.D. thesis, 1980.

- "Semi-stellarator" or "Furth-Hartman" coils added to MIT Rector tokamak.
- Current on side calculated to have little effect on vertical mode.
- Current on top and bottom calculated to have approximately the same stabilization effect as parallelogram-shaped coils.

R_0=.6 m, .7 m x .35 m vacuum chamber, B = 3.5 kG, $T_{e0} \leq 200$ eV, $n_e = 2 \times 10^{19}$ m^{-3}.

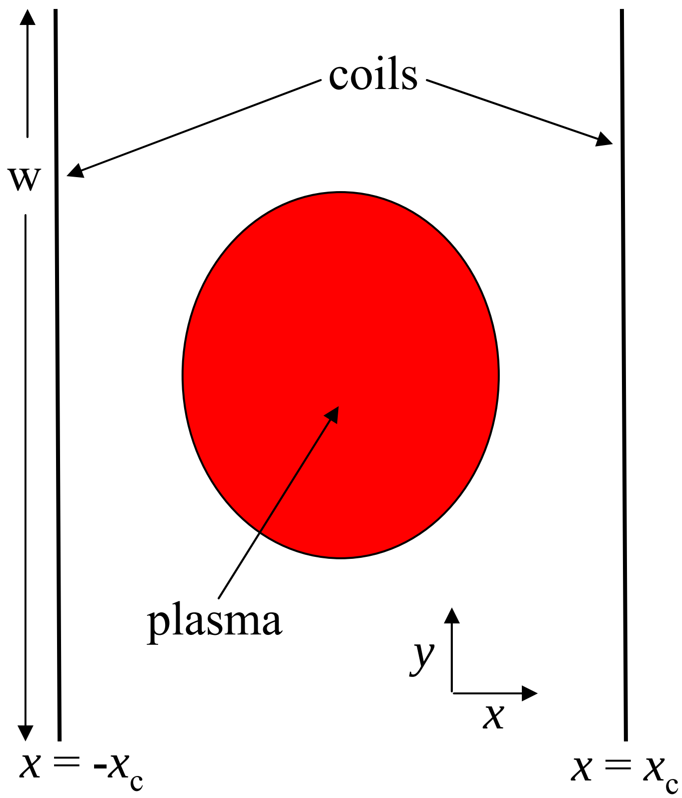
Nonlinear Behavior of the Vertical Instability – Conjecture

- δW analysis calculates response to infinitesimal perturbation. Depends on $\partial \bar{\mathbf{B}}_{cx} / \partial y$.
- Finite vertical excursion of plasma sees exponential increase of $\bar{\mathbf{B}}_{cx}$. Suggests that nonaxisymmetric field can prevent large vertical excursions of plasma even for equilibria linearly unstable to vertical mode.
  - Reduce incidence of VDEs due to accidental crossing of stability boundary?
  - Reduce incidence of VDEs triggered by other instabilities?

Conclusions

- Addition of relatively simple set of parallelogram-shaped nonaxisymmetric coils can improve stability of tokamaks to vertical modes.
  Stable equilibria with more highly elongated cross-sections can potentially lead to devices with improved performance in terms of beta limits and/or confinement.
- Furth-Hartman coils calculated to have essentially the same vertical stabilization effect as the parallelogram-shaped coils.
  Vertical stabilization demonstrated experimentally by Furth-Hartman coils supports feasibility of stabilizing vertical modes by the simpler coils.
- Exponential increase of $\bar{\mathbf{B}}_{cx}$ in vertical direction suggests reduced incidence of disruptions caused by vertical instability.
- Physical picture for stabilization suggests that stability properties do not depend on precise shape of the coils.
  Coil winding surface can be curved to conform to local shape of plasma, or curvature of coils can be introduced to optimize relative to other considerations.

Coils placed on sides of plasma rather than top and bottom have no effect on vertical stability.

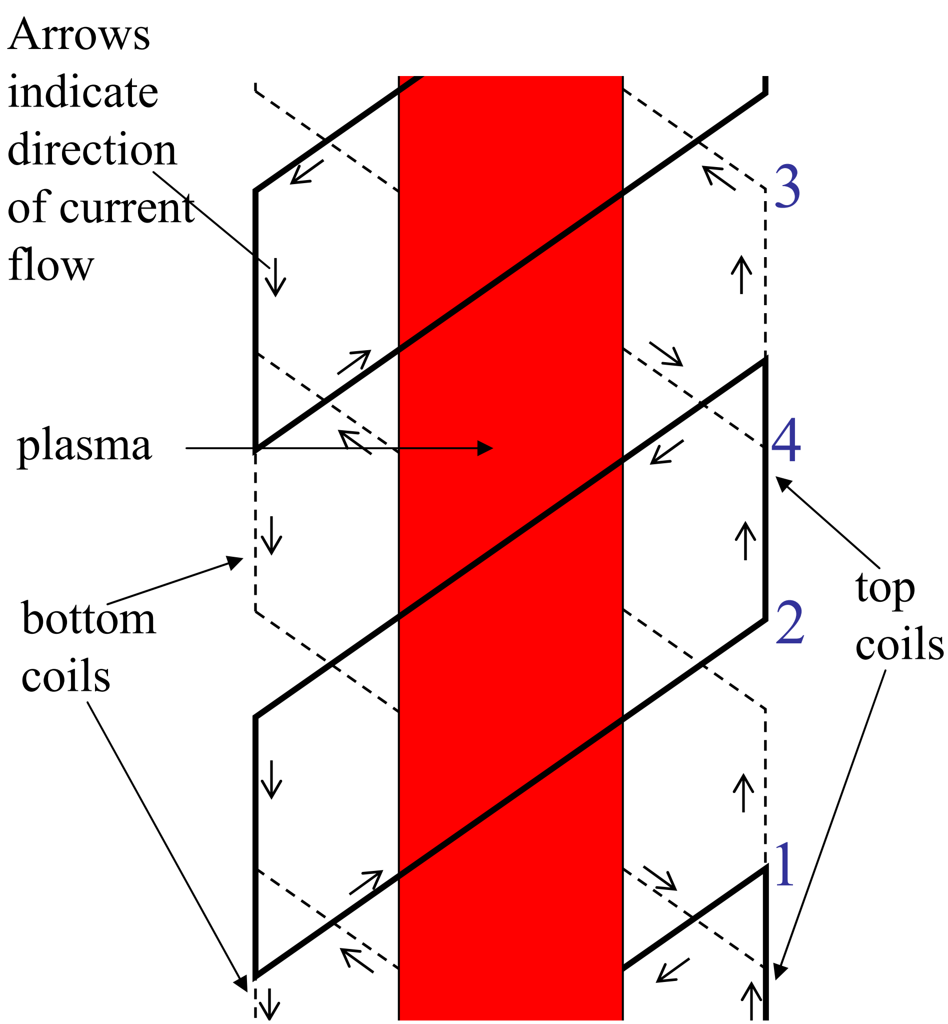


$$\overline{\mathbf{B}}_{\mathrm{c}} = \nabla \psi_c \times \hat{\mathbf{z}}$$

$$\psi_c = \psi_c(x) \propto \exp(\pm 2kx)$$

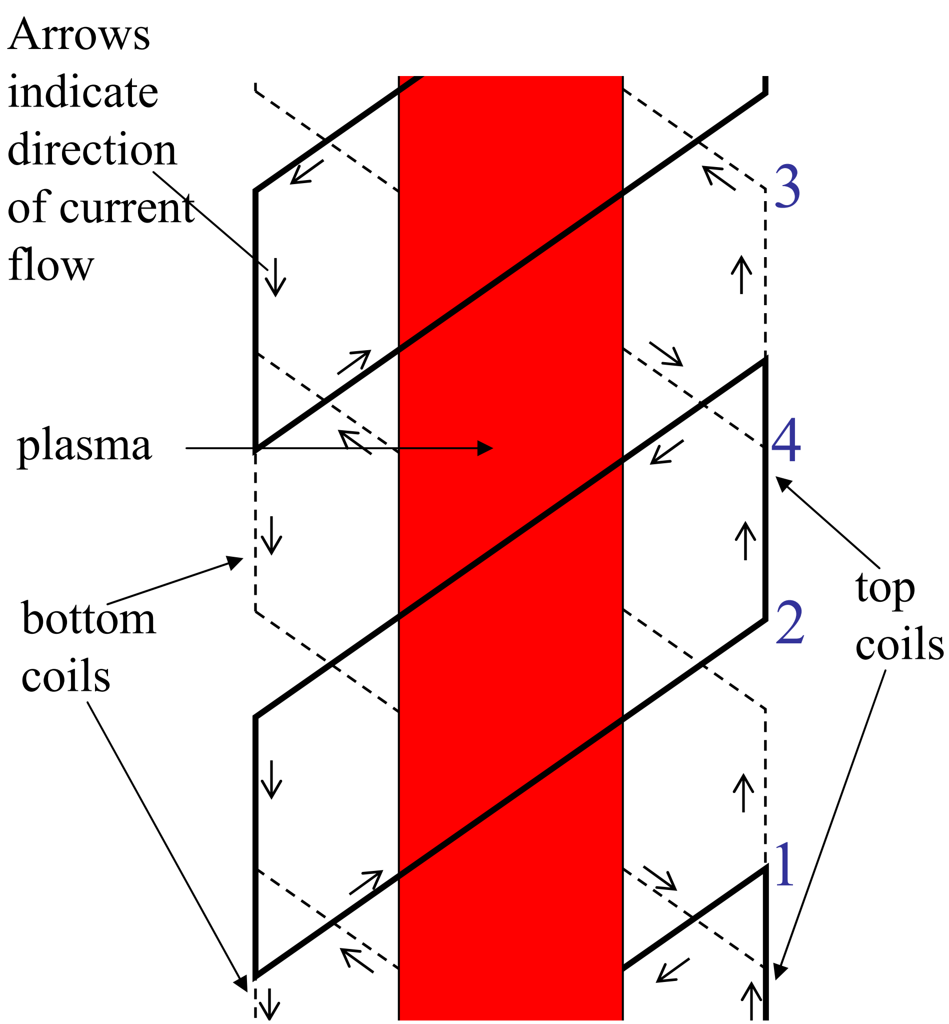
$$\xi \times \overline{\mathbf{B}}_{\mathrm{c}} = \xi_0 \hat{\mathbf{y}} \times \overline{\mathbf{B}}_{\mathrm{c}} = 0$$

Coils on top and bottom have same vertical stabilization properties as equivalent set of Furth-Hartman coils, whose stabilization has been demonstrated experimentally.



- Add coils just on right side to get Furth-Hartman coils wrapped on rectangular prism. (H. Furth and C. Hartman, Phys. Fluids **11**, 408 (1968)) Coils on side do not affect vertical stability.
- Furth-Hartman coils have been demonstrated experimentally to stabilize vertical modes. (H. Ikezi, *et al.*, Phys. Fluids **22** (1979); A. Janos, Ph.D. thesis, M.I.T., (1980).)

Can construct conventional helical stellarator winding, wrapped on rectangular prism, from coils on top, bottom and sides, + axisymmetric filaments.



- Connect numbered vertices, with current flow in direction 1→2→3→4→1.
- Cancel z-directed current on top right with axisymmetric filament.
- Construction similar on left side.
- Unlike coils just on top and bottom, produce vacuum flux surfaces and rotational transform.
- Vertical stability properties same as with coils just on top and bottom.