RWM Modeling with Full Drift Kinetic Damping

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Outline

- Self-consistent kinetic formulation
- Test calculations for Soloviev equilibrium
- Preliminary results for DIII-D plasmas
- Conclusion

Major features of the new formulation

- **Self-consistency (non-perturbative)** [vs. Hu&Betti's approach]

- **Kinetic integration in full toroidal geometry** [vs. semi-kinetic damping model in MARS-F]

- **Included kinetic effects due to**
  - particle bounce resonance[1] and precession drift resonance[2],
  - both transit and trapped particles
  - both ions and electrons (where appropriate)

- **Consider bulk thermal particle resonances (Maxwellian equilibrium distribution),** can be extended to include energetic particles

[1] F. Porcelli, *et al.*, Phys. Plasmas 1, 470(1994).

[2] T.M. Antonsen Jr. and Y.C. Lee, Phys. Fluids 25, 132(1982).

Single fluid MHD equations with kinetic terms

$$
\begin{aligned}
(\gamma+in\Omega)\xi &= \mathbf{v}+(\xi\cdot\nabla\Omega)R^2\nabla\phi & \textbf{(1)}\\
\rho(\gamma+in\Omega)\mathbf{v} &= -\nabla\cdot\mathbf{p}+\mathbf{j}\times\mathbf{B}+\mathbf{J}\times\mathbf{Q} & \textbf{(2)}\\
&\quad -\rho\left[2\Omega\hat{\mathbf{Z}}\times\mathbf{v}+(\mathbf{v}\cdot\nabla\Omega)R^2\nabla\phi\right] & \textbf{(3)}\\
&\quad -\rho\kappa_{\parallel}|k_{\parallel}|v_{th,i}[\mathbf{v}\cdot\hat{\mathbf{b}}+(\xi\cdot\nabla)\mathbf{V}_0\cdot\hat{\mathbf{b}}]\hat{\mathbf{b}} & \textbf{(4)}\\
(\gamma+in\Omega)\mathbf{Q} &= \nabla\times(\mathbf{v}\times\mathbf{B})+(\mathbf{Q}\cdot\nabla\Omega)R^2\nabla\phi-\nabla\times(\eta\mathbf{j}) & \textbf{(5)}\\
\mathbf{j} &= \nabla\times\mathbf{Q} & \textbf{(6)}\\
(\gamma+in\Omega)p &= -\mathbf{v}\cdot\nabla P-\Gamma P\nabla\cdot\mathbf{v}, \quad \Gamma=0! & \textbf{(7)}\\
\mathbf{p} &= p\mathbf{I}+p_{\parallel}^{\text{kinetic}}(\xi_{\perp})\hat{\mathbf{b}}\hat{\mathbf{b}}+p_{\perp}^{\text{kinetic}}(\xi_{\perp})(\mathbf{I}-\hat{\mathbf{b}}\hat{\mathbf{b}}) & \textbf{(8)}
\end{aligned}
$$

- **Formulate MHD equations in Eulerian frame (for resistive plasma), can re-cast in Lagrangian frame for ideal plasma**
- **Assumptions made in this formulation:**
  - **Neglected anisotropy of equilibrium pressure**
  - **Kinetic pressure calculated (Antonsen&Porcelli) for ideal plasma without flow, but used here for plasma with flow, where $\xi_{\perp}$ couples to $\xi_{\parallel}$ due to rotation**
  - **Neglected perturbed electrostatic potential, but kept effect of equilibrium electrostatic potential**
  - **FLR effect neglected**
  - **Neglected radial excursion of particle trajectory (as Hu&Betti)**

Perturbed kinetic pressure for low and finite mode frequency

Perturbed kinetic pressure is calculated from perturbed particle distribution function f_L^1

$$p_\parallel^{\text{kinetic}} e^{-i\omega t + in\phi} = \sum_{e,i} \int d\Gamma M v_\parallel^2 f_L^1 \quad (9)$$

$$p_\perp^{\text{kinetic}} e^{-i\omega t + in\phi} = \sum_{e,i} \int d\Gamma \mu B f_L^1 \quad (10)$$

where f_L^1 satisfies

$$\frac{d f_L^1}{dt} = f_\varepsilon^0 \frac{\partial H^1}{\partial t} - f_{P_\phi}^0 \frac{\partial H^1}{\partial \phi} - \nu_{\text{eff}} f_L^1 \quad (11)$$

where H^1 is the perturbed Lagrangian of particle energy

$$H^1(\xi_\perp, Q_\parallel, t) = [M v_\parallel^2 \kappa \cdot \xi_\perp + \mu (Q_\parallel + \nabla B \cdot \xi_\perp)] e^{-i\omega t + in\phi} \quad (12)$$

NB: $\xi_\perp$ and $Q_\parallel$ are unknown (solution) functions.

At high-frequency limit, kinetic pressures become fluid-like Kruskal-Oberman terms, replacing the MHD term $-\Gamma P(\nabla \cdot \xi)$.

$$p_\parallel^{\text{kinetic}} = -P(\nabla \cdot \xi_\perp - 2\xi_\perp \cdot \kappa) \quad (13)$$

$$p_\perp^{\text{kinetic}} = -P(2\nabla \cdot \xi_\perp + \xi_\perp \cdot \kappa) \quad (14)$$

Kinetic pressures are transformed to facilitate code implementation

A few steps are undertaken:

1. **Decompose solution functions (e.g. $\xi_\perp, Q_\parallel$) in Fourier harmonics along poloidal angle**
2. **Decompose periodic part of coefficients in H^1 in Fourier series of bouncing orbit**
3. **Integrate equation (11) from $-\infty$ to current time t**
4. **Project perturbed pressure into Fourier space along poloidal angle**

We obtain final equations that relate the Fourier harmonics of solution functions $\xi_\perp$ and $Q_\parallel$ to Fourier harmonics of solution function $p_\parallel^{\text{kinetic}}$ and $p_\parallel^{\text{kinetic}}$:

$$(Jp_\parallel^{\text{kinetic}})_k = \frac{1}{\sqrt{\pi}} \sum_{e,i} \sum_{m,l} \frac{P_{e,i}}{B_0} \int d\Lambda I_{ml} H_{ml} G_{kml}^{\parallel} X_m \tag{15}$$

$$(Jp_\perp^{\text{kinetic}})_k = \frac{1}{\sqrt{\pi}} \sum_{e,i} \sum_{m,l} \frac{P_{e,i}}{B_0} \int d\Lambda I_{ml} H_{ml} G_{kml}^{\perp} X_m \tag{16}$$

Kinetic pressures are transformed to facilitate code implementation

$$(Jp_{\parallel}^{\text{kinetic}})_k = \frac{1}{\sqrt{\pi}} \sum_{e,i} \sum_{m,l} \frac{P_{e,i}}{B_0} \int d\Lambda I_{ml} H_{ml} G_{kml}^{\parallel} X_m$$

$$(Jp_{\perp}^{\text{kinetic}})_k = \frac{1}{\sqrt{\pi}} \sum_{e,i} \sum_{m,l} \frac{P_{e,i}}{B_0} \int d\Lambda I_{ml} H_{ml} G_{kml}^{\perp} X_m$$

- $H_{ml}(\psi,\Lambda)$ **= 'geometrical factor' associated with Fourier projection in particle bounce orbit**
- $G_{kml}(\psi,\Lambda)$ **= 'geometrical factor' associated with Fourier projection along poloidal angle**
- $I_{ml}(\psi,\Lambda)$ **= integral over particle energy**

We can prove that the kinetic pressure yields the same kinetic energy as Hu&Betti

$$\delta W_K = \frac{\sqrt{\pi}}{2} \frac{\nu}{B_0} \sum_{e,i} \int d\psi P_{e,i} \int d\hat{\varepsilon}_k \sum_{l} \sum_{\sigma} \hat{\varepsilon}_k^{5/2} e^{-\hat{\varepsilon}_k} \hat{\lambda}_l \int d\Lambda \hat{\tau}_b \left| \left\langle e^{-il\omega_b t + in\tilde{\phi}(t)} H_L \right\rangle \right|^2 \quad (17)$$

Perform analytical integration over particle energy for general geometry

Distinguish three cases:

- **Transit resonance of passing particles**

$$I_{ml} = \sum_{\sigma} \int_0^{\infty} d\hat{\varepsilon}_k \hat{\varepsilon}_k^{5/2} e^{-\hat{\varepsilon}_k} \frac{n[\omega_{*N} + (\hat{\varepsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n\omega_E + \sigma(m + nq + l)\sqrt{\frac{2\hat{\varepsilon}_k T}{M}}\frac{2\pi}{\hat{\tau}_b} - i\nu_{\text{eff}} - \omega} \quad \textbf{(18)}$$

- **Bounce resonance of trapped particles**

$$I_{l\neq 0} = \sum_{\sigma} \int_0^{\infty} d\hat{\varepsilon}_k \hat{\varepsilon}_k^{5/2} e^{-\hat{\varepsilon}_k} \frac{n[\omega_{*N} + (\hat{\varepsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n\omega_E + l\sqrt{\frac{2\hat{\varepsilon}_k T}{M}}\frac{2\pi}{\hat{\tau}_b} - i\nu_{\text{eff}} - \omega} \quad \textbf{(19)}$$

- **Precession drift resonance of trapped particles**

$$I_{l=0} = \sum_{\sigma} \int_0^{\infty} d\hat{\varepsilon}_k \hat{\varepsilon}_k^{5/2} e^{-\hat{\varepsilon}_k} \frac{n[\omega_{*N} + (\hat{\varepsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n\omega_E + n\omega_D - i\nu_{\text{eff}} - \omega} \quad \textbf{(20)}$$

- **These integrals eventually involve plasma dispersion function**
- **At high frequency limit $\omega \to \infty$, $I_{ml}(\psi, \Lambda)$ becomes a constant $\Longrightarrow$ K-O limit**
- **Since H_{ml} and G_{ml} independent of ω, benchmarking kinetic implementation at $\omega = \omega_f = \infty$, with fluid implementation of K-O terms, checks all the kinetic integrals except integrals I_{ml} over particle energy**

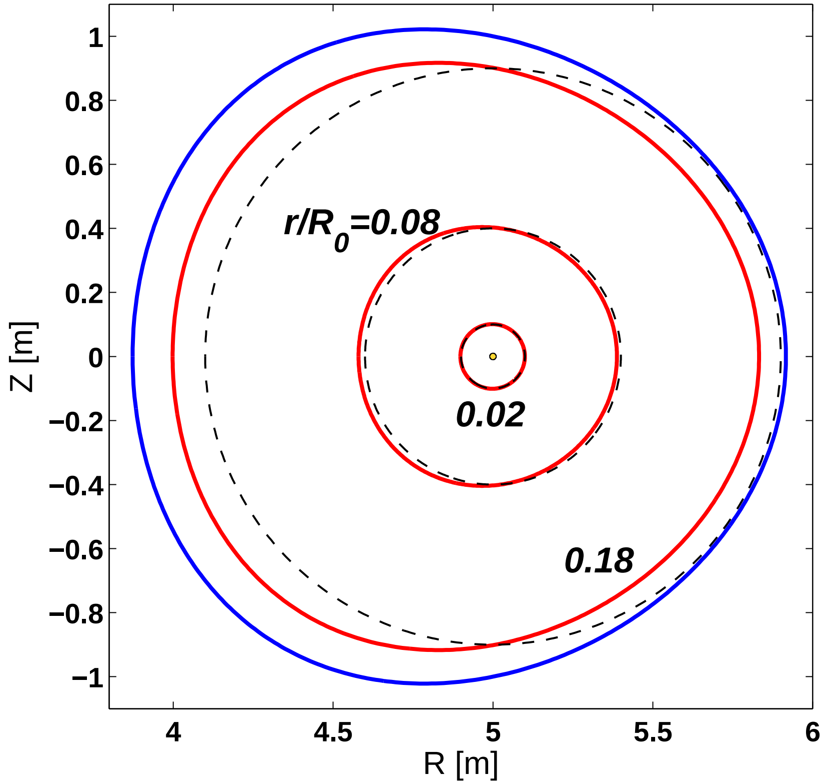
Choose analytical Soloviev equilibrium as test case

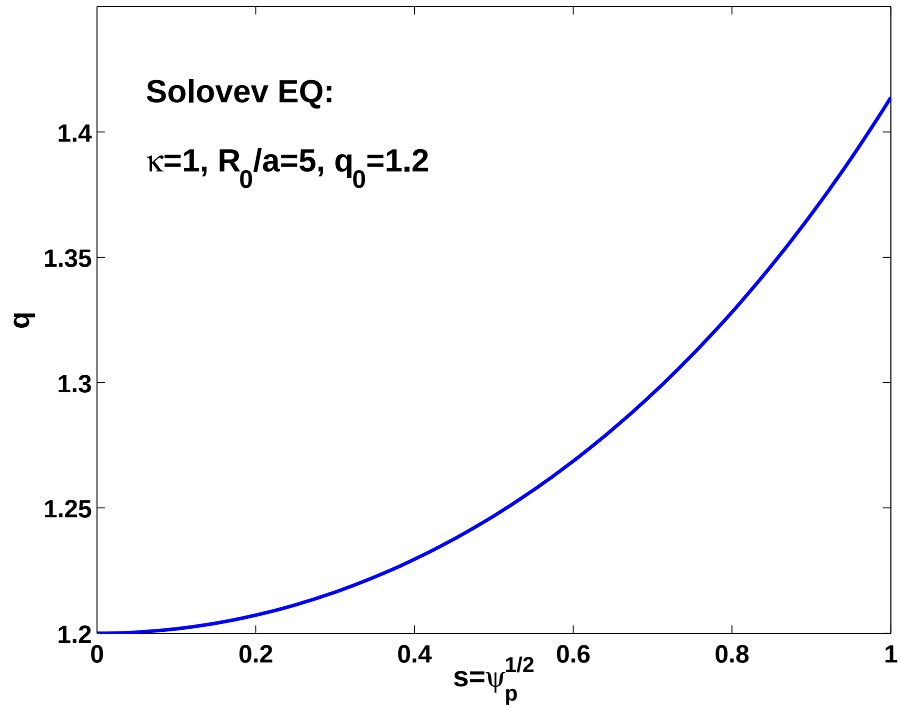
$$P(\psi) = -\frac{1+\kappa^2}{\kappa R_0^3 q_0}\psi, \qquad F(\psi) = 1 \quad \textbf{(21)}$$

$$\psi(R,Z) = \frac{\kappa}{2R_0^3 q_0}\left(\frac{R^2Z^2}{\kappa^2} + \frac{1}{4}(R^2-R_0^2)^2 - a^2R_0^2\right) \quad \textbf{(22)}$$

$$\text{plasma boundary}: R = R_0(1+2\varepsilon_a\cos\theta)^{1/2}, \quad Z = \frac{R_0\varepsilon_a\kappa\sin\theta}{(1+2\varepsilon_a\cos\theta)^{1/2}} \quad \textbf{(23)}$$

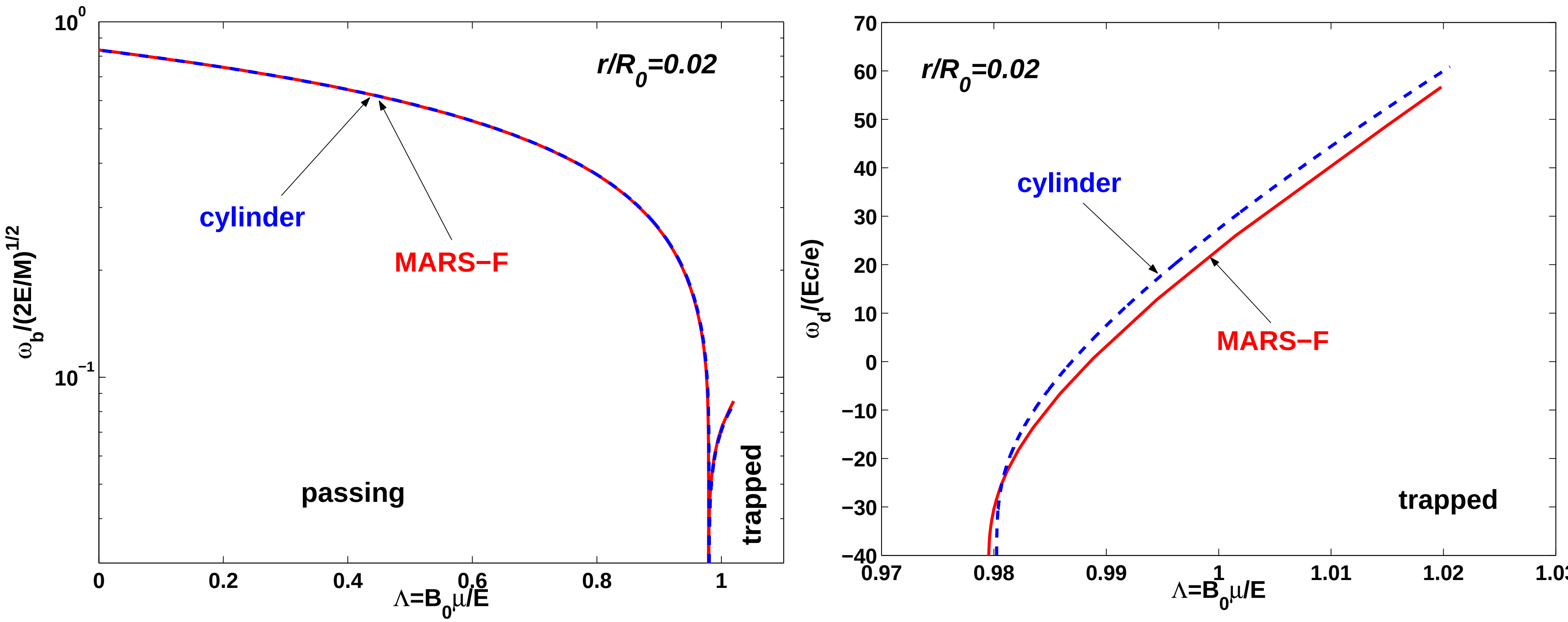
$$\text{test case}: R_0/a = 5, \kappa = 1, q_0 = 1.2 \quad \textbf{(24)}$$





Benchmark bounce/drift frequency calculations

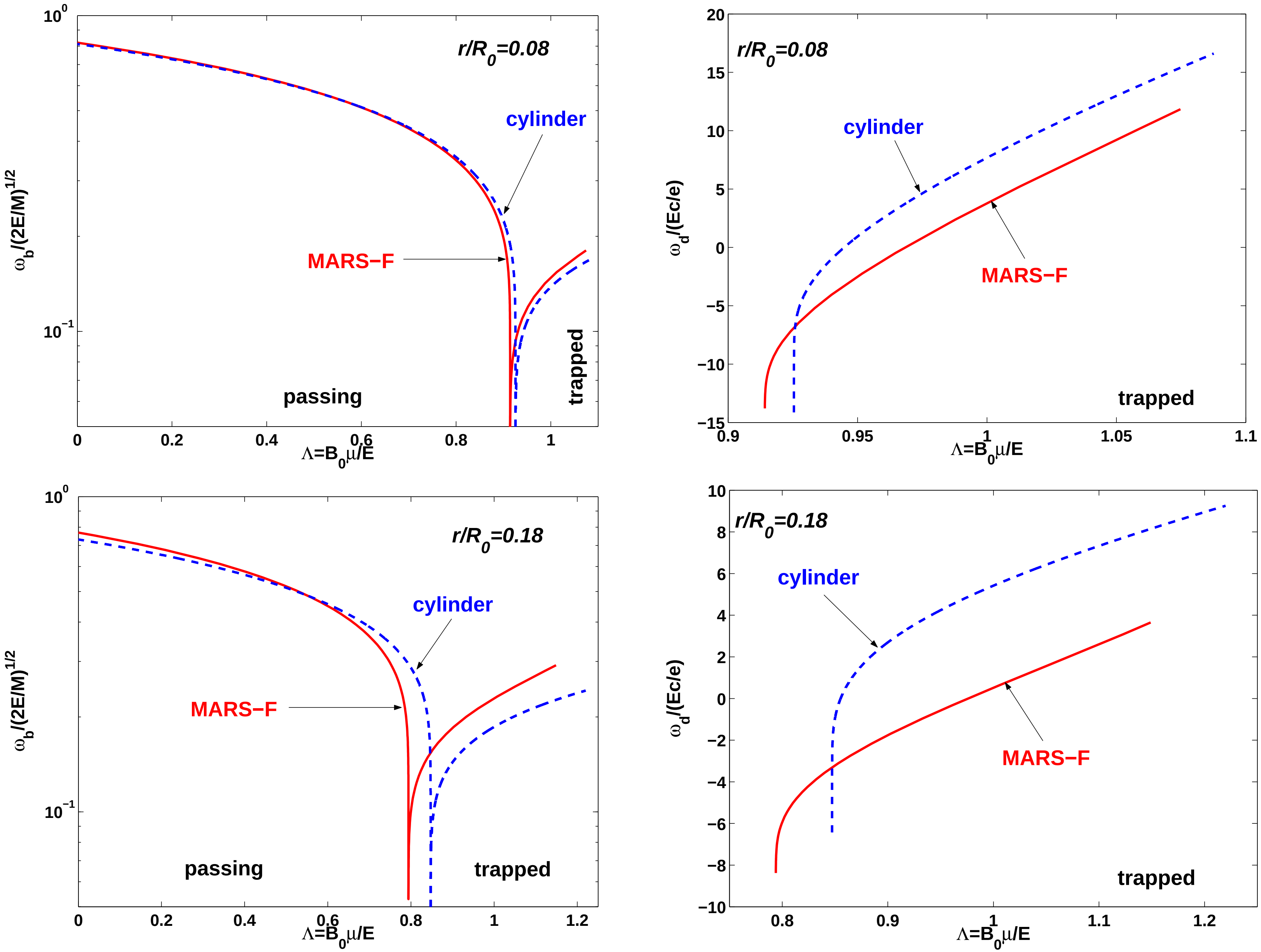
Compare with large aspect ratio circular plasma approximation



$$\frac{\omega_b^{\text{cylinder}}}{\sqrt{2E/M}} = \begin{cases} \frac{(2\Lambda\varepsilon_r)^{1/2}}{4qR_0}\frac{\pi}{K(k_t)}, & \text{(trapped particle)} \\ \frac{(1-\Lambda+\Lambda\varepsilon_r)^{1/2}}{2qR_0}\frac{\pi}{K(k_c)}, & \text{(passing particle)} \end{cases} \qquad \textbf{(25)}$$

$$\frac{\omega_d^{\text{cylinder}}}{\sqrt{Ec/e}} = \frac{2q\Lambda}{R_0^2 B_0 \varepsilon_r}\left[(2s+1)\frac{E(k_t)}{K(k_t)} + 2s(k_t^2-1) - \frac{1}{2}\right], \quad \text{(trapped particle)} \qquad \textbf{(26)}$$

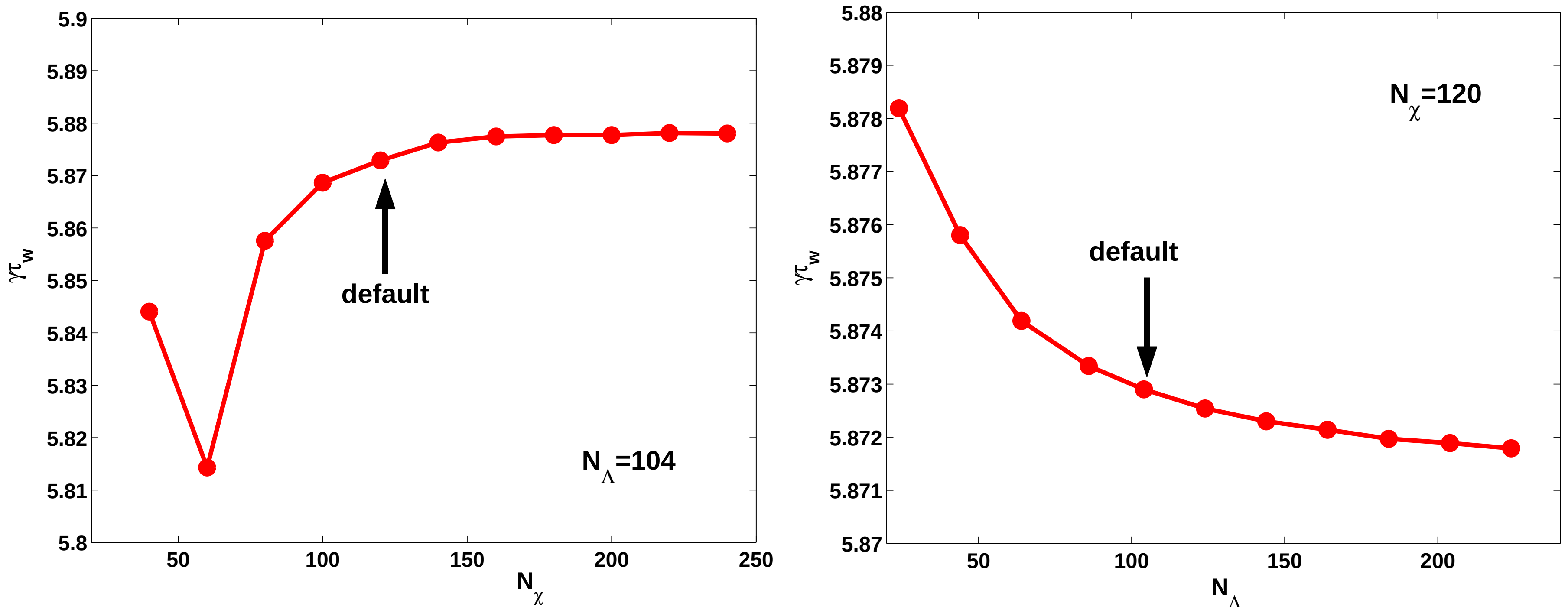
Benchmark bounce/drift frequency calculations



Performed numerical convergence test of kinetic integration

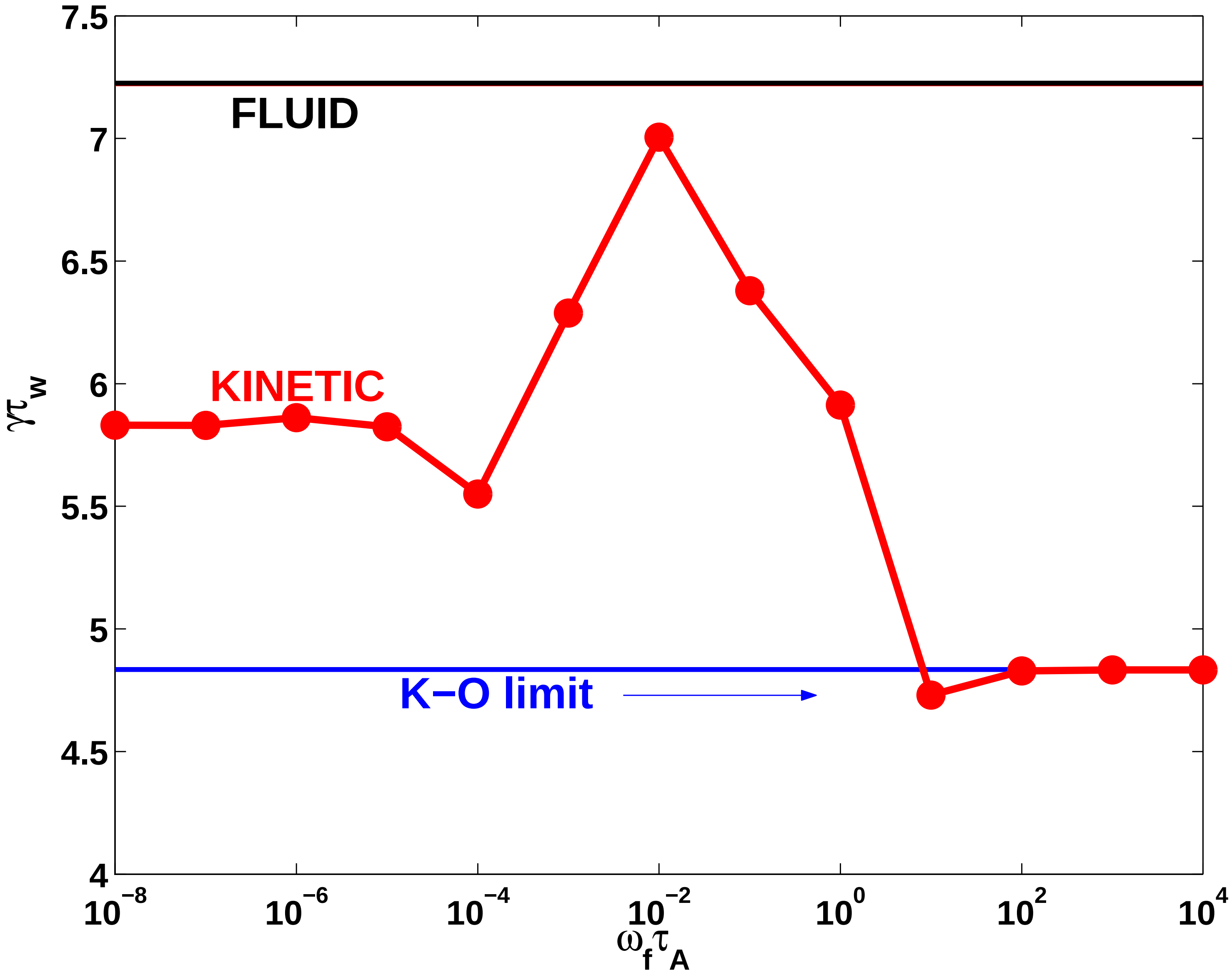
Two important numerical parameters in kinetic integration:

- N_χ = number of integration points along poloidal angle χ
- N_Λ = number of integration points along pitch angle Λ

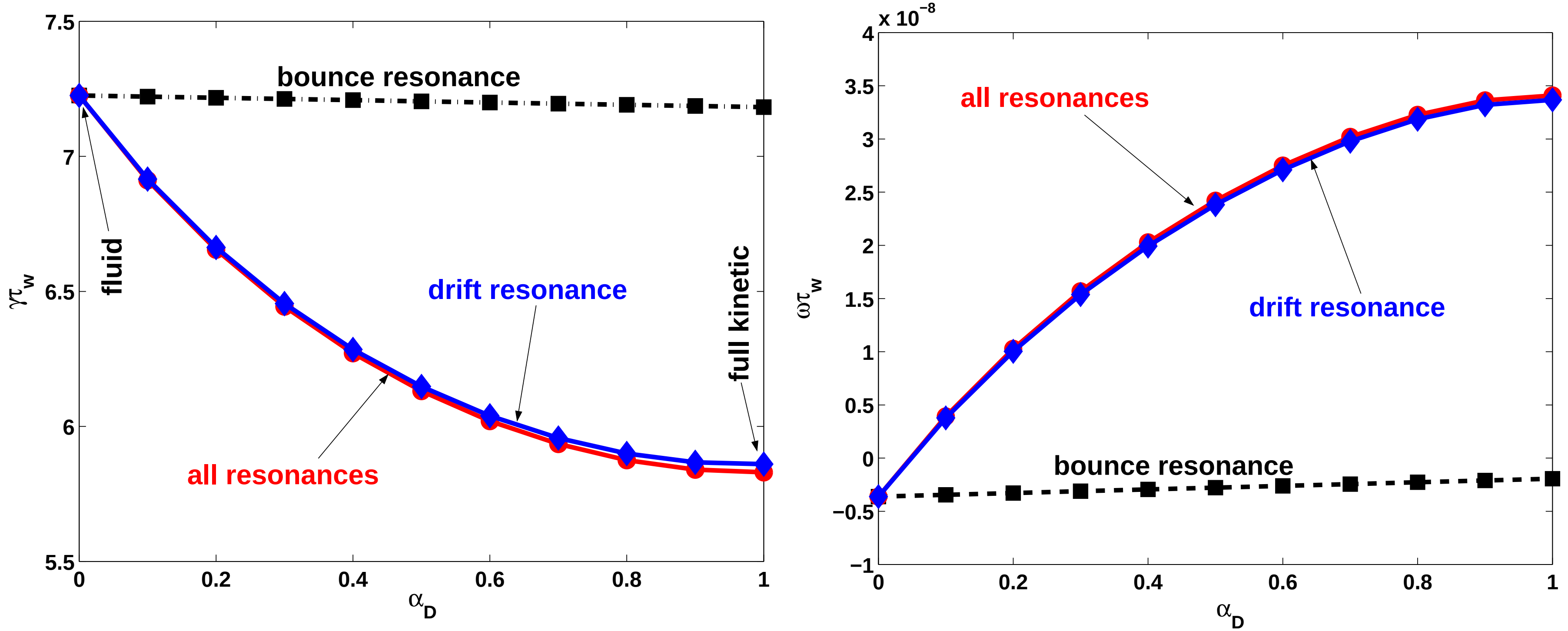


NB: Singularity is extracted analytically in numerical integrations

Benchmark with Kruskal-Oberman limit

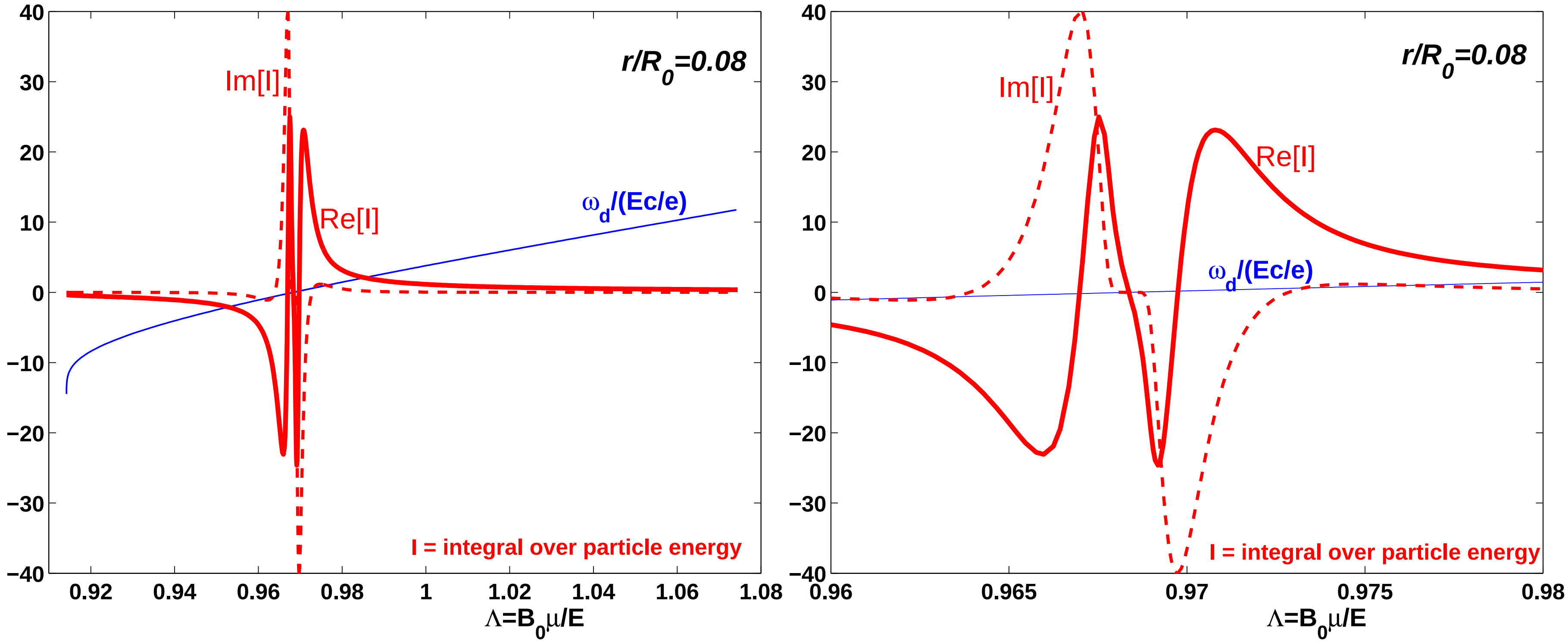


RWM stabilization due to kinetic effects



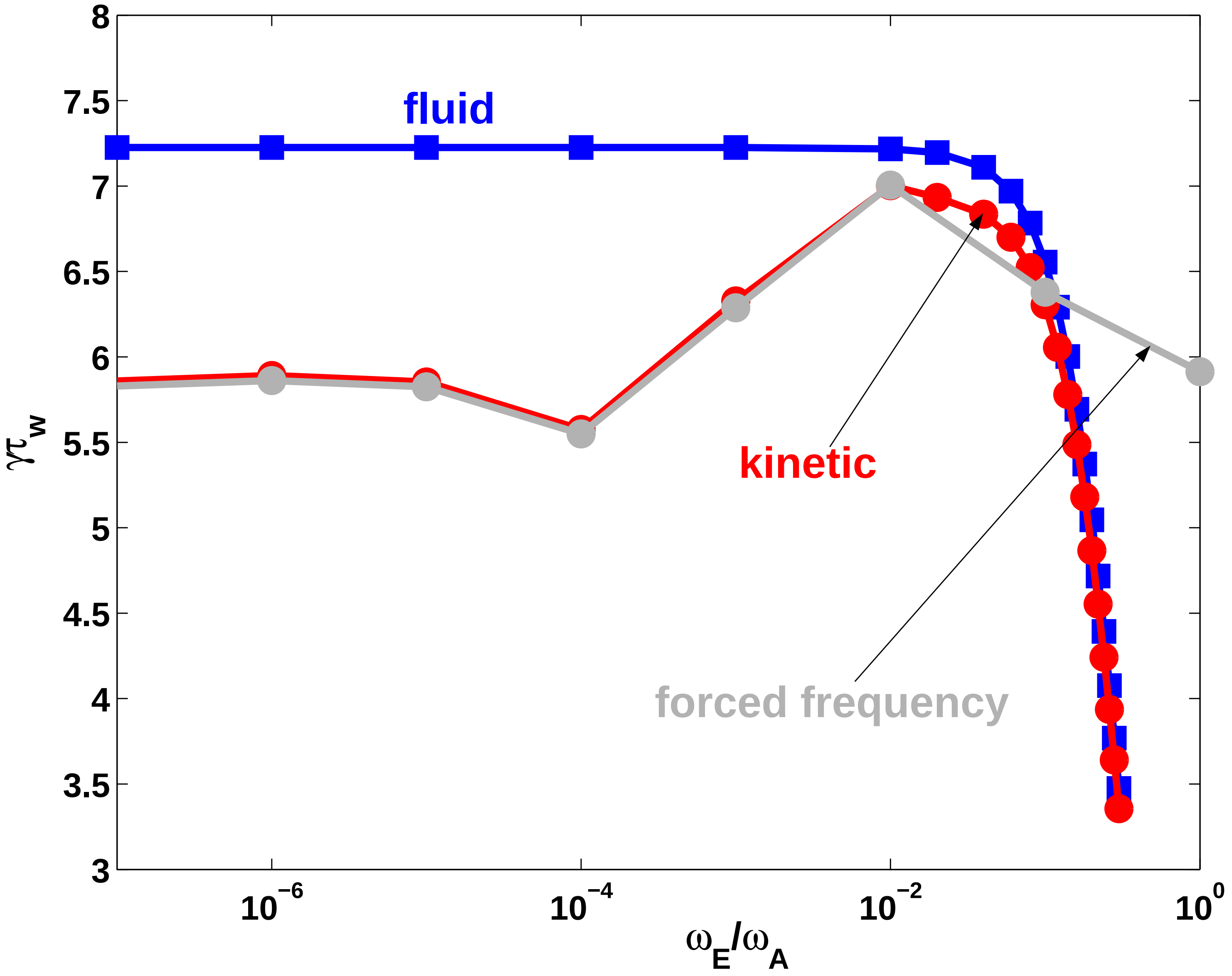
- α_D = **multiplier for the kinetic coefficients:** $\alpha_D = 0 \Leftrightarrow$ **fluid limit (with** $\Gamma = 0$), $\alpha_D = 1 \Leftrightarrow$ **full kinetic limit**
- **For the case considered here, bounce resonance has very weak stabilizing effect, most stabilization comes from precession drift resonance damping by ions and electrons**
- **Not achieved complete stabilization of the mode**

Precession drift resonance of trapped particles



Most drift resonance damping seems come from particles with nearly vanishing magnetic drift frequency.

Three damping regimes with kinetic effects and plasma rotation



Computed growth rates roughly match that from energy principle

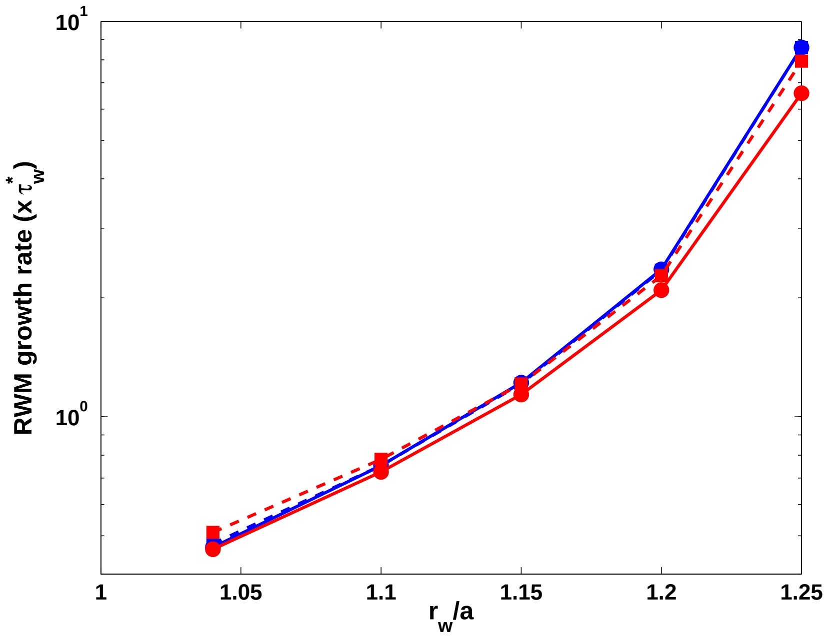
Without kinetic terms,

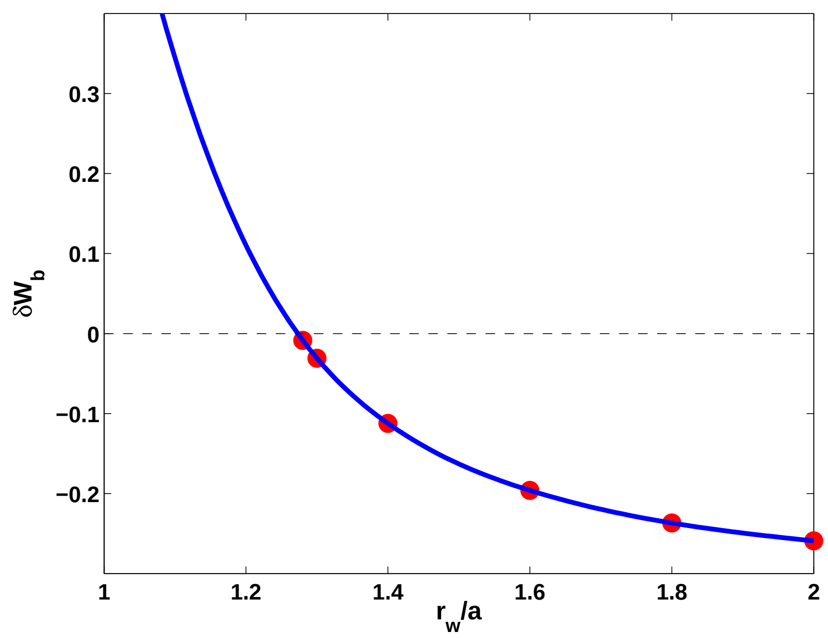
$$\gamma\tau_w^* \approx -\frac{\delta W_\infty}{\delta W_b}$$

With kinetic terms,

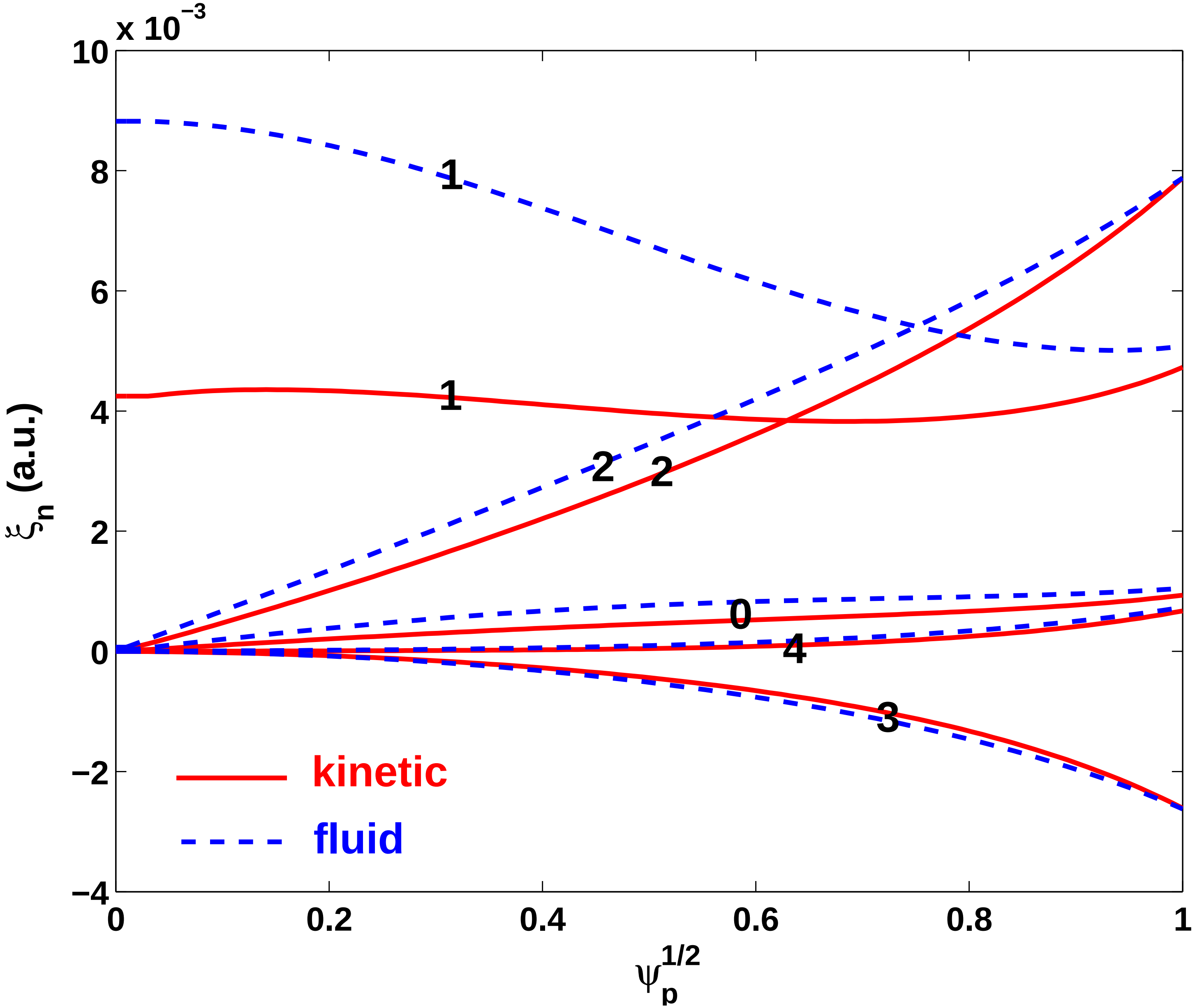
$$\gamma\tau_w^* \approx -\frac{\delta W_\infty + \delta W_k}{\delta W_b + \delta W_k}$$

- **Solid: directly computed by MARS-F and converged after nonlinear iteration over eigenvalue!**
- **Dashed: approximation from energy principle**

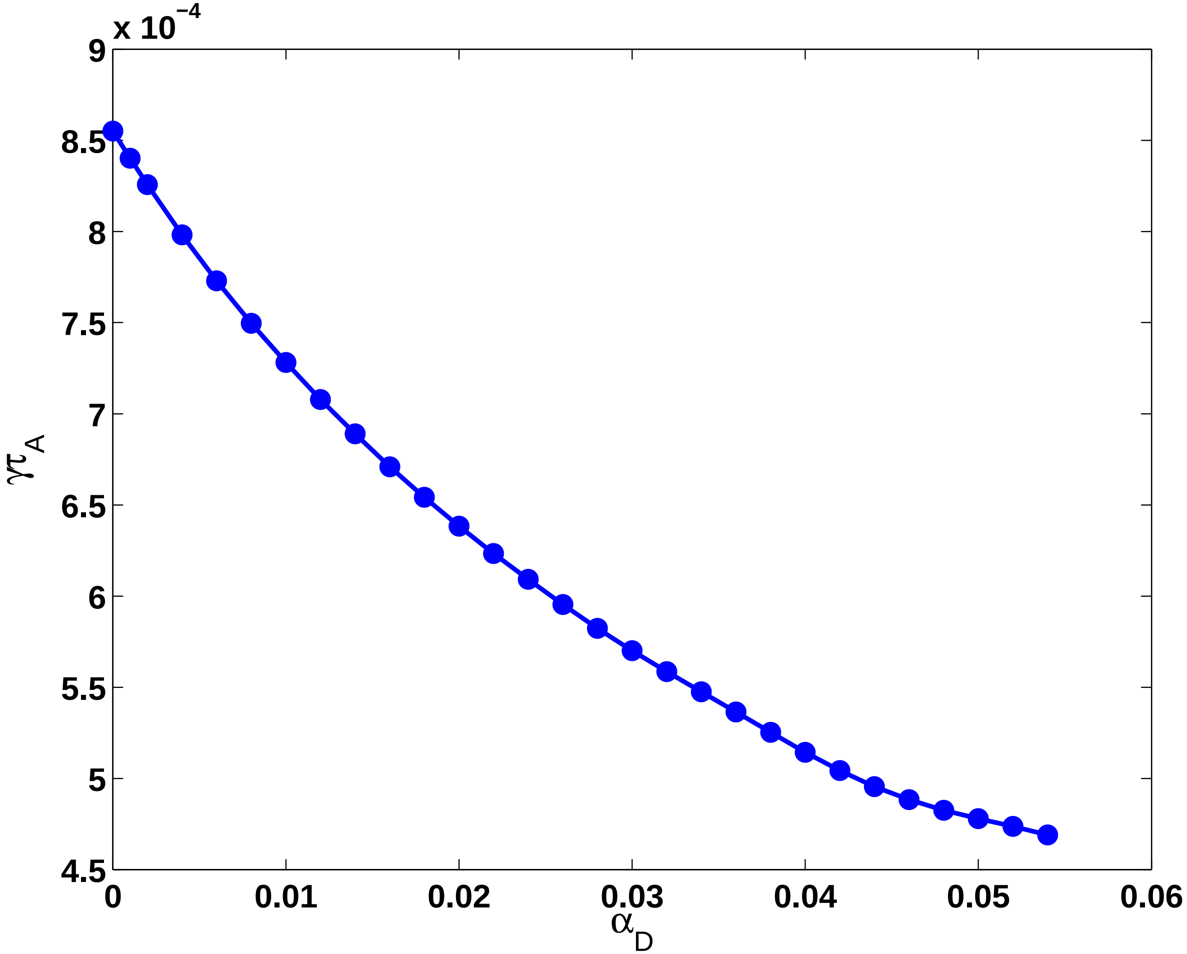




Observe kinetic modification of RWM eigenmode structure



Strong kinetic stabilization of RWM for DIII-D plasma



- For an equilibrium from DIII-D shot #125701. Results preliminary (obtained during APS)!
- Reduction of growth rate by a factor of two after applying only 5% of kinetic damping
- Solution does not converge for $\alpha_D > 5.4\%$. seems that eigenfunction changes too much to follow global mode structure for displacement. Reason still under investigation.

Conclusion

- **We have developed a full drift kinetic version of MARS-F, where kinetic integrals are evaluated in a general toroidal geometry, and self-consistently incorporated into the MHD formulation**

- **The new code is tested on a Soloviev analytical equilibrium. It is observed that most of the kinetic damping comes from the particle precession drift resonances, from particles with nearly vanishing drift frequency**

- **The RWM eigenmode structure is modified by kinetic terms**

- **Kinetic terms may provide strong stabilization for high-pressure plasmas, as those from DIII-D**

- **Future work: more detailed and systematic modeling of DIII-D and ITER plasmas**