MODEL-BASED OPTIMAL CURRENT PROFILE CONTROL DURING THE RAMP-UP PHASE

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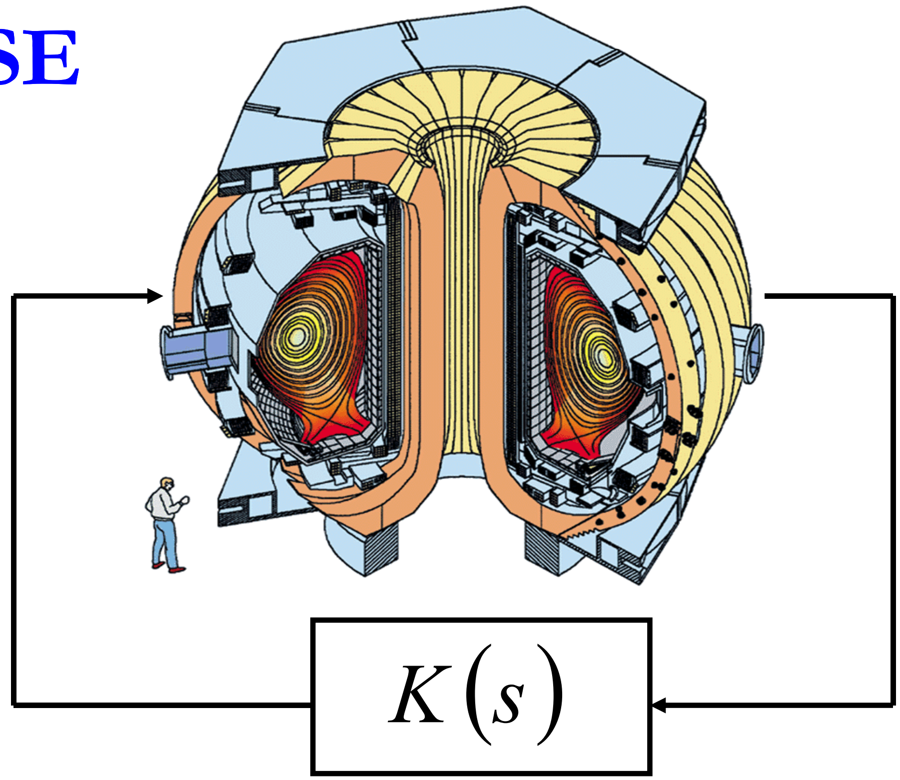


Workshop on Improved MHD Control Configurations
Columbia University, November 18-20, 2007

Collaborators involved in this project:

- ***John Ferron, Tim Luce, MikeWalker, DaveHumphreys***
  General Atomics

- ***Tom Casper, Bill Mayer***

  Lawrence Livermore National Laboratory

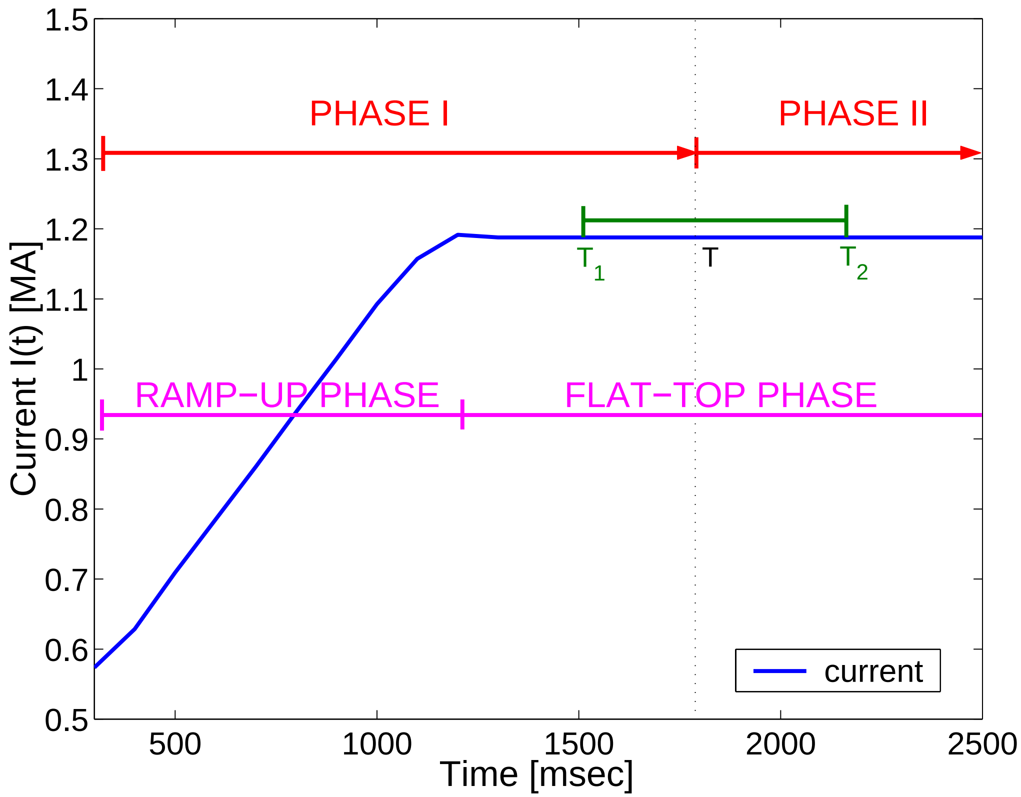
Current Profile Control: Motivation

- A key goal in control of an AT discharge is to maintain safety factor (q) and pressure profiles that are compatible with both MHD stability at high toroidal beta and a high fraction of the self-generated bootstrap current in order to realize a stable, highly efficient, steady-state fusion reactor.

- In present experimental tokamaks, the development of current profile controllers is aimed at saving long trial-and-error periods of time currently spent by fusion experimentalists trying to manually adjust the time evolutions of the actuators to achieve a desired current profile.

- The high dimensionality of the problem and the strong coupling between the different variables describing the current profile evolution of the plasma call for a model-based, multivariable approach to obtain improved closed-loop performance.

Current Profile Control: Objective



During “Phase I” the control goal is to drive the current profile from any arbitrary initial condition to a prescribed target profile at some time $T \in (T_1, T_2)$ in the flat-top phase of the total current $I(t)$ evolution. The prescribed target profile is not an equilibrium profile during “Phase I."

Outline

- **Models for Control Design**
- **Open-loop Optimal Control**
- **Closed-loop Optimal Control**

Current Profile Control: Model

ψ	poloidal magnetic flux
$I(t)$	total plasma current

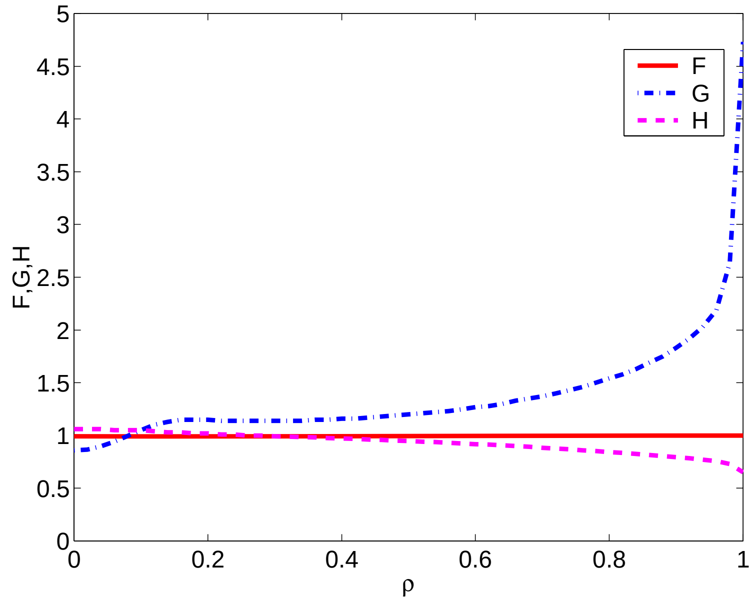
Magnetic diffusion equation:

$$\frac{\partial \psi}{\partial t} = \frac{\eta(T_e)}{\mu_o \rho_b^2 \hat{F}^2} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left(\hat{\rho} \hat{F} \hat{G} \hat{H} \frac{\partial \psi}{\partial \hat{\rho}} \right) - R_o \hat{H} \eta(T_e) \frac{\langle \bar{j}_{NI} \cdot \bar{B} \rangle}{B_{\phi,o}}$$

Boundary conditions:

$$\left. \frac{\partial \psi}{\partial \hat{\rho}} \right|_{\hat{\rho}=0} = 0$$

$$\left. \frac{\partial \psi}{\partial \hat{\rho}} \right|_{\hat{\rho}=1} = \frac{\mu_o}{2\pi} \frac{R_o}{\hat{G}\big|_{\hat{\rho}=1} \hat{H}\big|_{\hat{\rho}=1}} I(t)$$



Current Profile Control: Model

Model for Control Design

GOAL: Develop a model based controller to be used in achieving desirable current profiles during the plasma current ramp-up. A necessary prior task is the development of a dynamic model to use for controller design.

Modeling Alternatives:

•The magnetic diffusion equation is accompanied by transport equations for the density and the temperature .

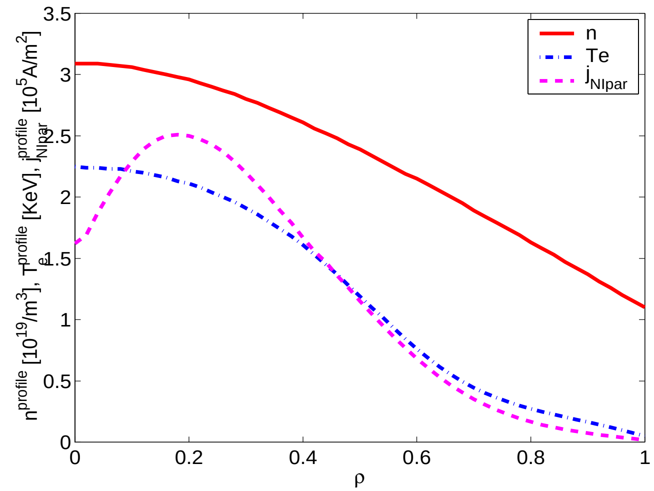
• The magnetic diffusion equation is accompanied by simplified, scenario-oriented, models for the density and temperature.

• The magnetic diffusion equation is evaluated with real-time measurements of the density and the temperature.

Model reduction (PDE → ODE) may be necessary. Particularly for closed-loop control.

Current Profile Control: Model

Highly simplified models for the density and temperature are chosen for the inductive phase (Phase I). The profiles are assumed to remain fixed. The temperature and density responses to the actuators are simply scalar multiples of the reference profiles. These reference profiles are taken from a DIII-D tokamak discharge.



Density:

$$n(\hat{\rho},t) = n^{profile}(\hat{\rho})u_n(t)$$

$$\overline{n}(t) = \int_0^1 n(\hat{\rho},t)d\hat{\rho}$$

Temperature:

$$T_e(\hat{\rho},t) = k_{T_e} T_e^{profile}(\hat{\rho})\frac{I(t)\sqrt{P_{tot}(t)}}{\overline{n}(t)}$$

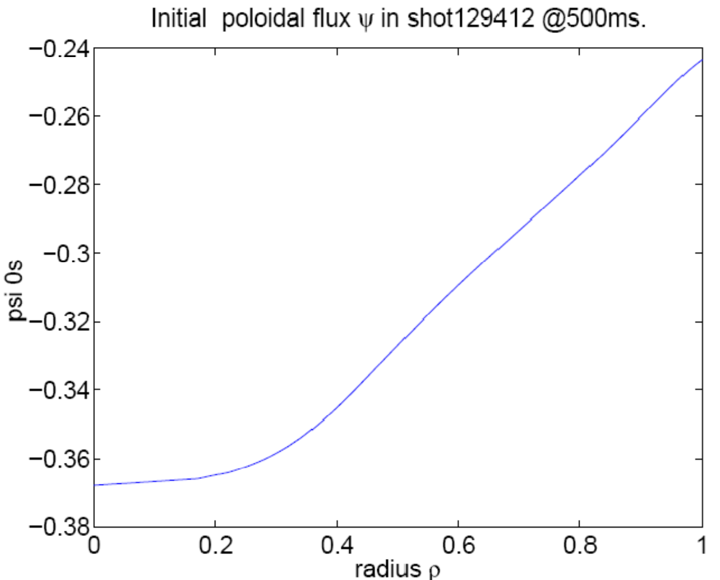
$$\eta(\hat{\rho},t) = \frac{k_{eff}Z_{eff}}{T_e^{3/2}(\hat{\rho},t)}$$

Parallel Current: $$\frac{\langle \bar{j}_{NI} \quad \bar{B} \rangle}{B_{\varphi,o}} = k_{NI_{par}} j_{NI_{par}}^{profile}(\hat{\rho})\frac{I(t)^{1/2}P_{tot}(t)^{5/4}}{\overline{n}(t)^{3/2}}$$

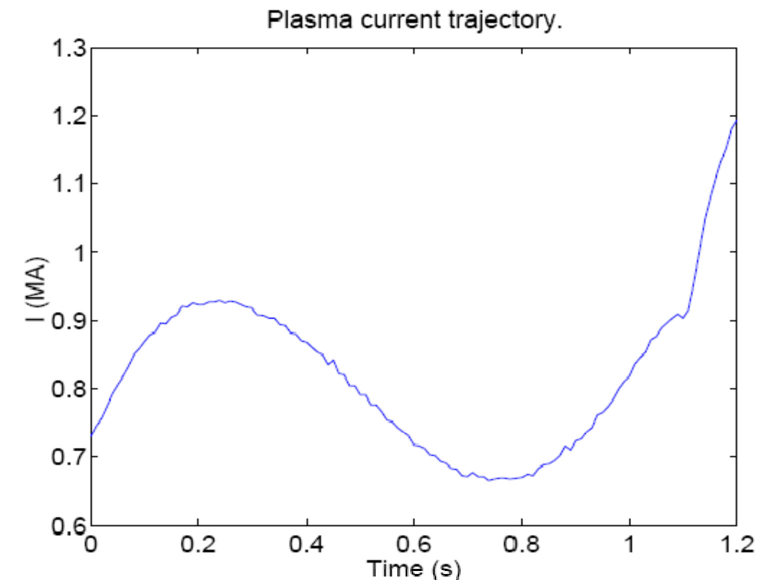
We consider $I(t)$, $\overline{n}(t)$ and $P_{tot}(t)$ the physical actuators of the system.

Current Profile Control: Model Validation

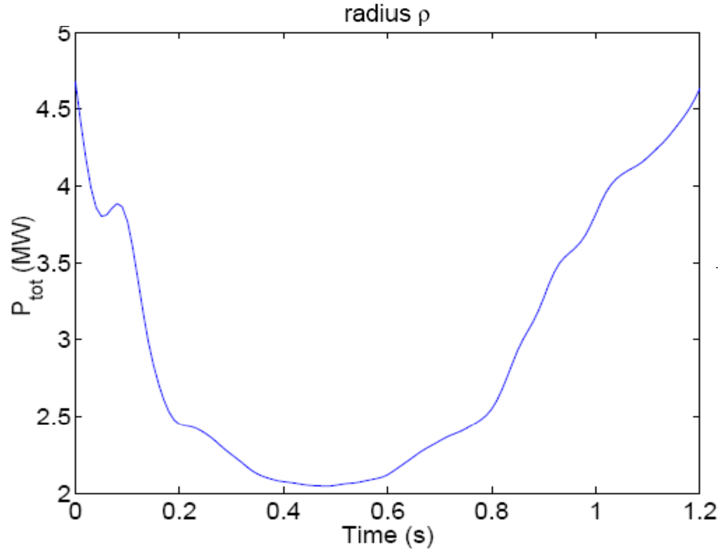
The simplified model has been compared with experiment shot #129412.



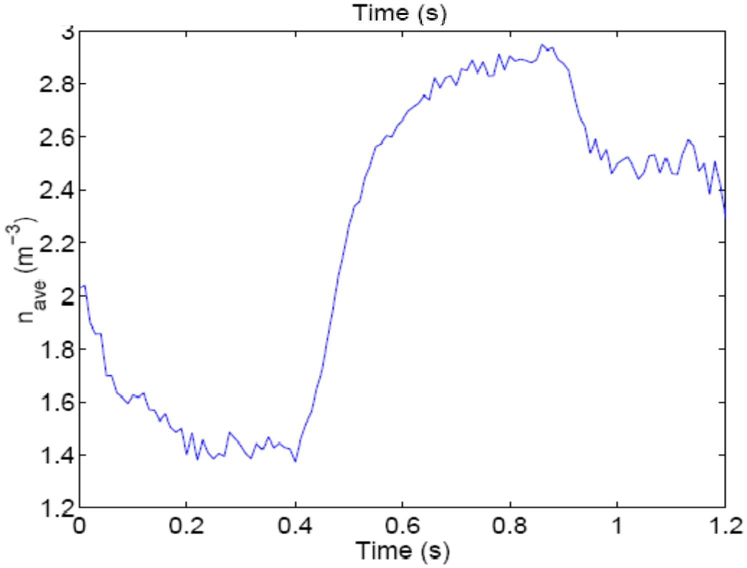
Initial poloidal flux ψ profile



Actuator $I(t)$ trajectory

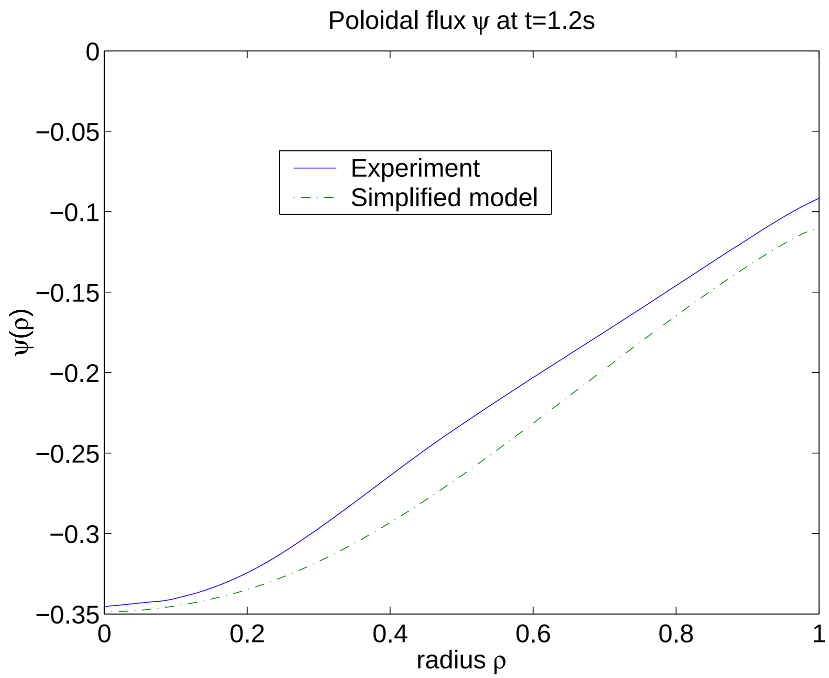


Actuator $P_{tot}(t)$ trajectory

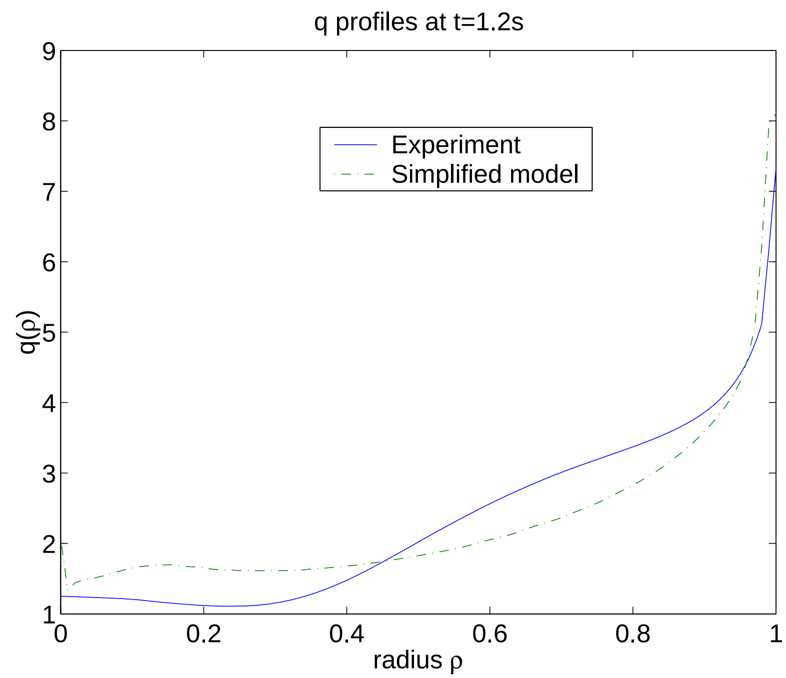


Actuator $\overline{n}(t)$ trajectory

Current Profile Control: Model Validation



Poloidal Flux ψ comparison



q profile comparison

The initial validating results show qualitative agreement between simplified model and experiment. More validation experiments will be carried out during the 2008 campaign.

Outline

- Models for Control Design
- Open-loop Optimal Control
- Closed-loop Optimal Control

Current Profile Control: OL Control

Open-loop Optimal Control

GOAL: During "Phase I" an optimal control problem must be solved, where time evolution for three actuators ($I(t)$, $\bar{n}(t)$ $(u_n(t))$, $P_{tot}(t)$) are sought to minimize the functional.

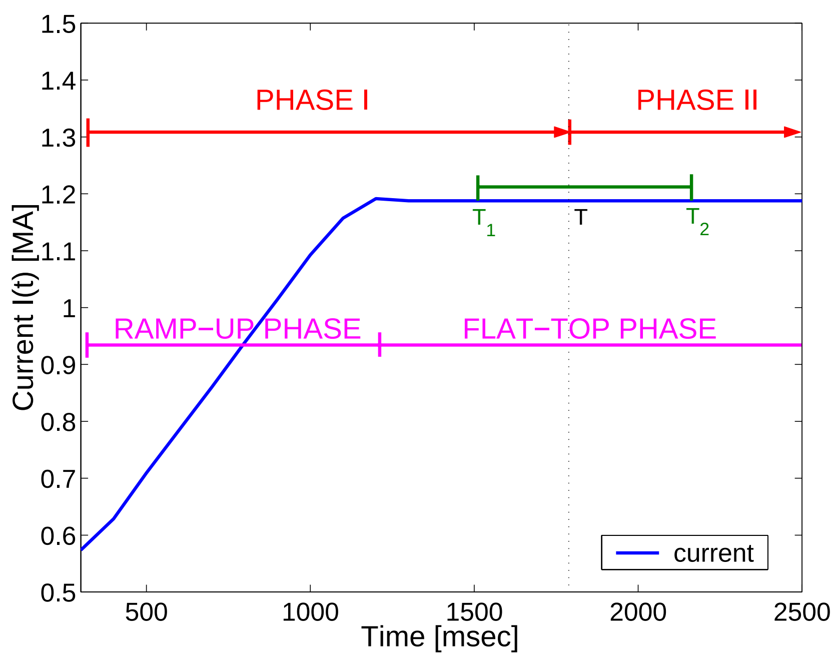
$$J_{\min} = \min_{t\in[T_1,T_2]} \left(J(t)\right)$$

where

$$J(t) = \int_0^1 \left(\iota(\hat{\rho},t) - \iota^{des}(\hat{\rho})\right)^2 d\hat{\rho}$$

or

$$J(t) = \sum_{i=0}^{n} \left(\iota(\hat{\rho}_i,t) - \iota^{des}(\hat{\rho}_i)\right)^2, \quad n = 100$$



Current Profile Control: OL Control

The physical ranges for $I(t)$, $\overline{n}(t)$ and $P_{tot}(t)$, are given by

$$\begin{cases} 0 \le I(t) \le I_{\max} \\ \left|\dfrac{dI(t)}{dt}\right| \le dI_{\max} \end{cases} \qquad I(MA) \le \frac{\overline{n}(t)}{10^{19}} \le 5I(MA)$$

$$0 \le P_{tot}(t) \le P_{\max}$$

To accurately reproduce experimental discharges, we must add constraints for *I(t)* and $\overline{n}(t)$, at the initial time of "Phase I", i.e.,

$$\begin{cases} I(t = 0s) = I_0 \\ \overline{n}(t = 0s) = \overline{n}_0 \end{cases}$$

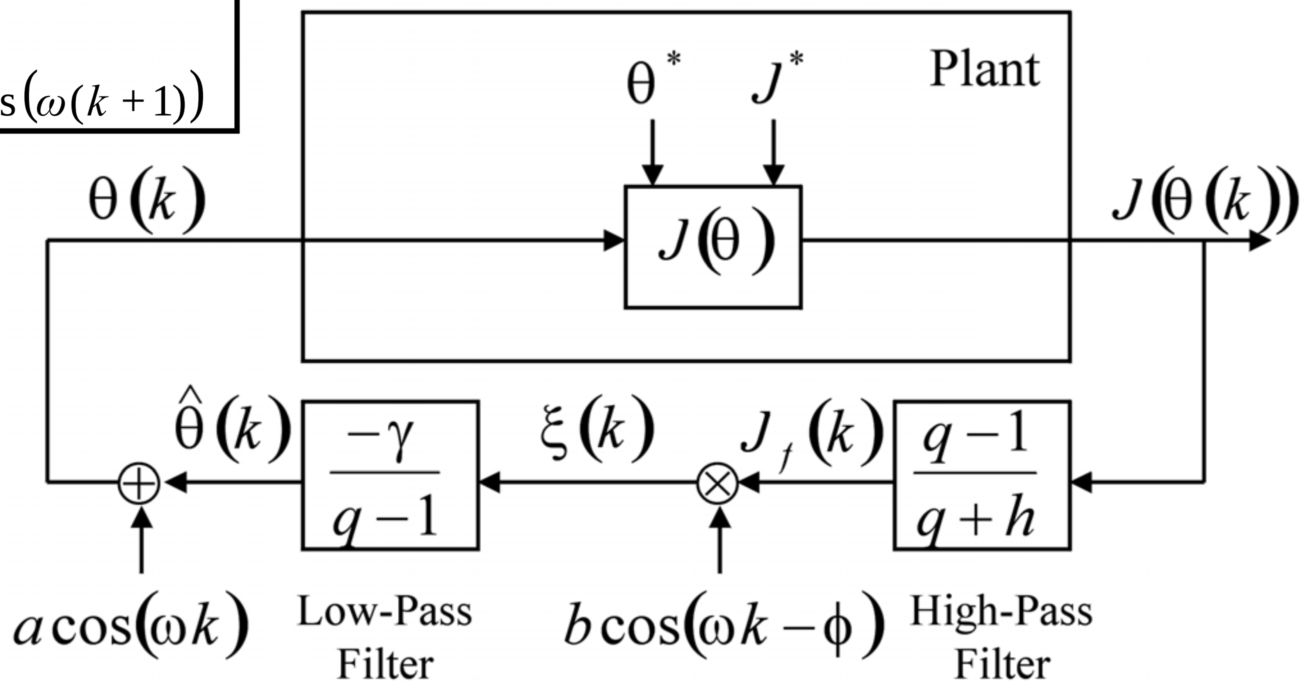
In addition, a value of the total current *I(t)* is prescribed for the flattop phase, i.e.,

$$I(t \ge T_1) = I_{target}$$

Current Profile Control: OL-ES Control

$$J_f(k) = -hJ_f(k-1) + J(k) - J(k-1)$$
$$\xi(k) = J_f(k) b\cos(\omega k - \varphi)$$
$$\hat{\theta}(k+1) = \hat{\theta}(k) - \gamma\xi(k)$$
$$\theta(k+1) = \hat{\theta}(k+1) + a\cos(\omega(k+1))$$

Extremum seeking control scheme



J = quadratic error between ι profile and the prescribed target profile ι_{des}

In each iteration of extremum seeking procedure, θ is used to construct the time evolution of the three physical actuators, $I(t)$, $\bar{n}(t)$ and $P_{tot}(t)$.

Current Profile Control: OL-ES Control

- The vector parameter θ has 10 components

$$\theta = \left\{ \begin{matrix} I(0.4s), I(0.8s), \\ P_{tot}(0s), P_{tot}(0.4s), P_{tot}(0.8s), P_{tot}(1.2s), \\ \bar{n}(0.3s), \bar{n}(0.6s), \bar{n}(0.9s), \bar{n}(1.2s) \end{matrix} \right\}$$

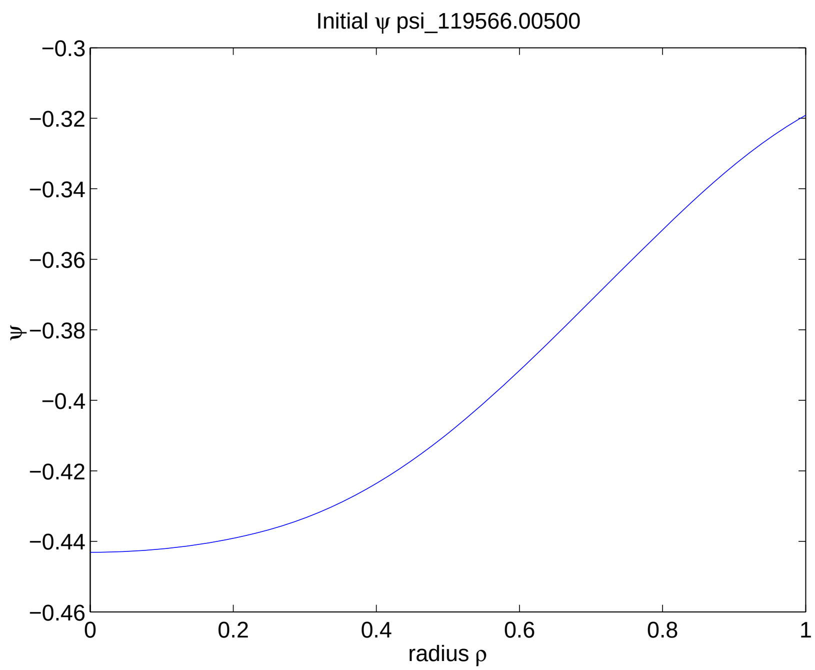
By taking into account that $I(0s) = I_0$ and $I(T_1) = I_{target}$, and using curve fitting for the points $I(0s), I(0.4s), I(0.8s), I(1.2s)$ we can reconstruct the profile for $I(t)$ for $t \in [0, T_1]$. In addition, we make $I(t) = I_{target}$ for $t \in [T_1, T_2]$.

- Following similar procedure, we can construct the law for $P_{tot}(t)$.

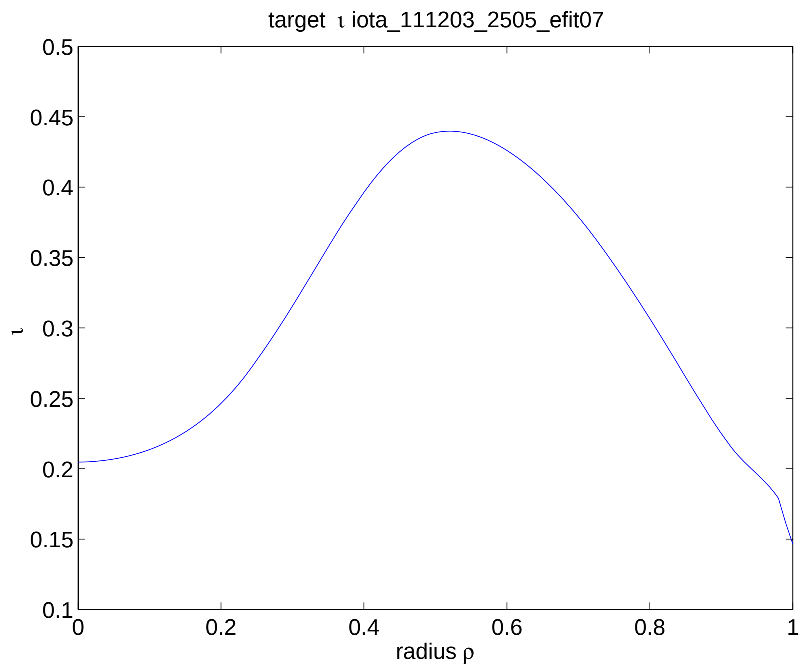
- By considering that $\bar{n}(0s) = \bar{n}_0$, and using linear interpolation, we can define the law for $\bar{n}(t)$.

- The reconstructed control laws are in turn fed into the PDE model. Given initial ψ, the PDE system is integrated to obtain $\psi(\hat{\rho}, t)$, and finally $\iota(\hat{\rho}, t)$, which are necessary to evaluate the cost function

Current Profile Control: OL-ES Control



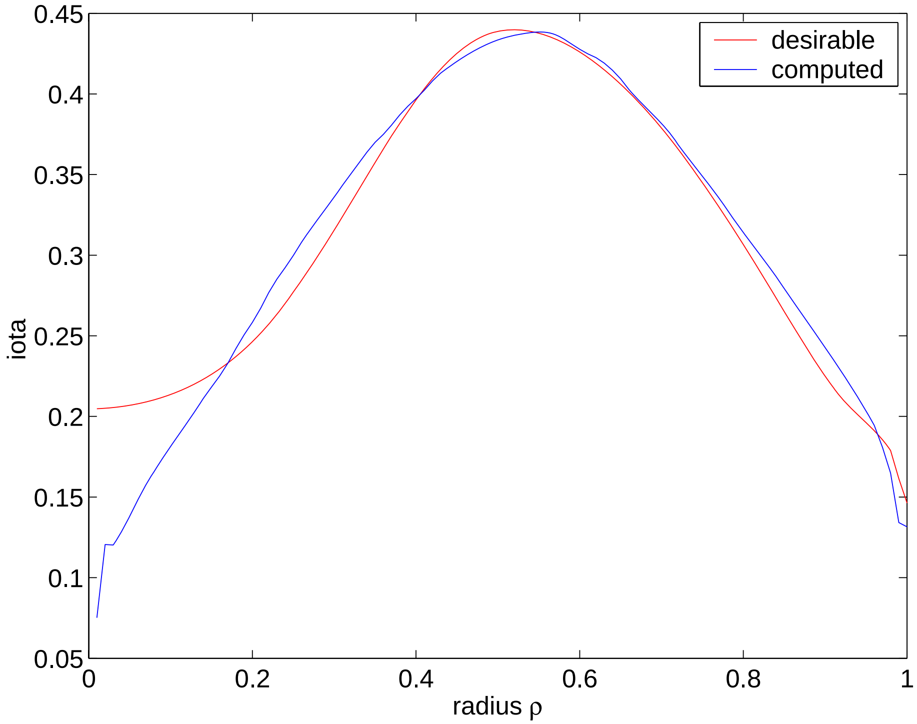
(a)



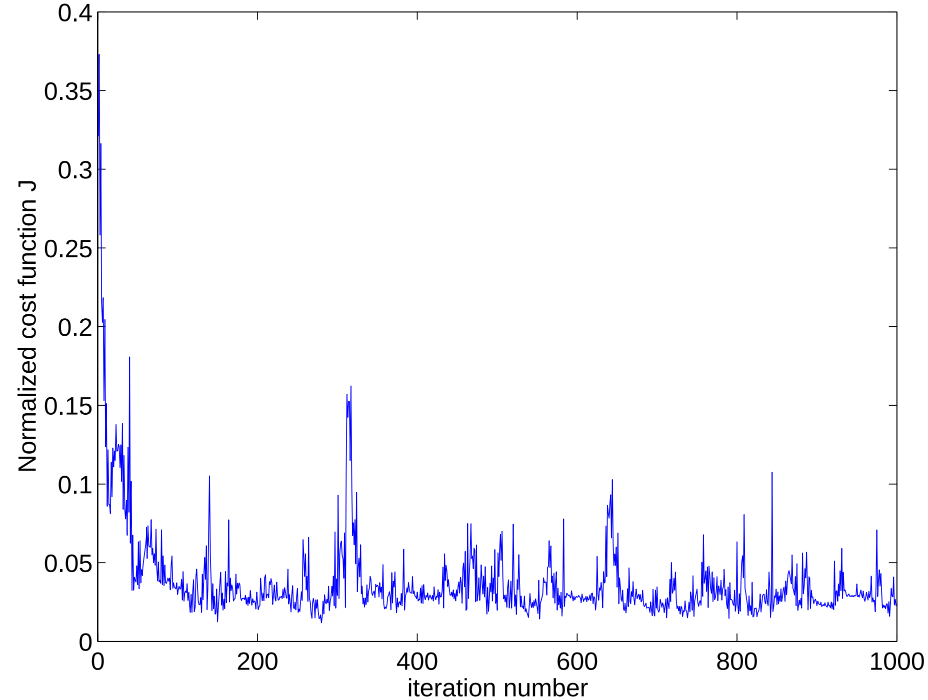
(b)

In this simulation, we consider the time interval [0, 2.4*s*]. The initial poloidal flux ψ is shown in Figure (a) and the target ι profile is shown in Figure (b). The current $I(t)$, average density $n(t)$ and total power $P_{tot}(t)$ are reconstructed as previous description.

Current Profile Control: OL-ES Control

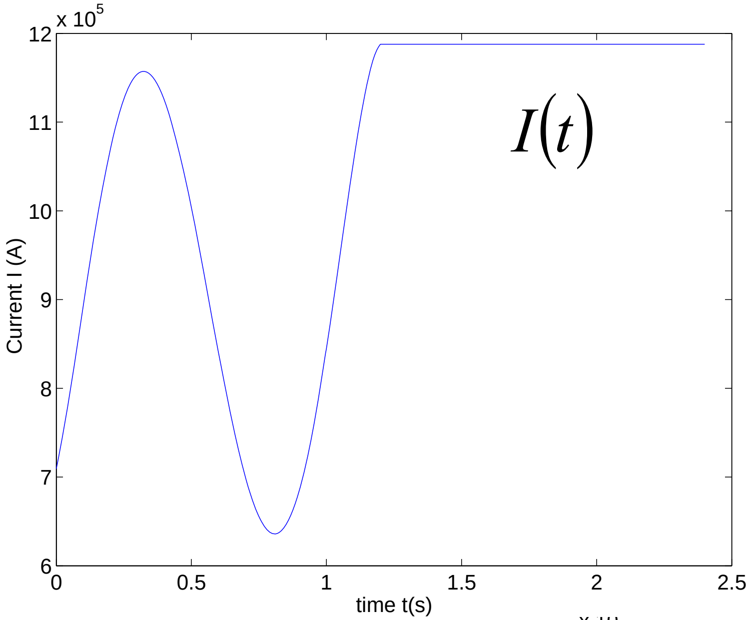


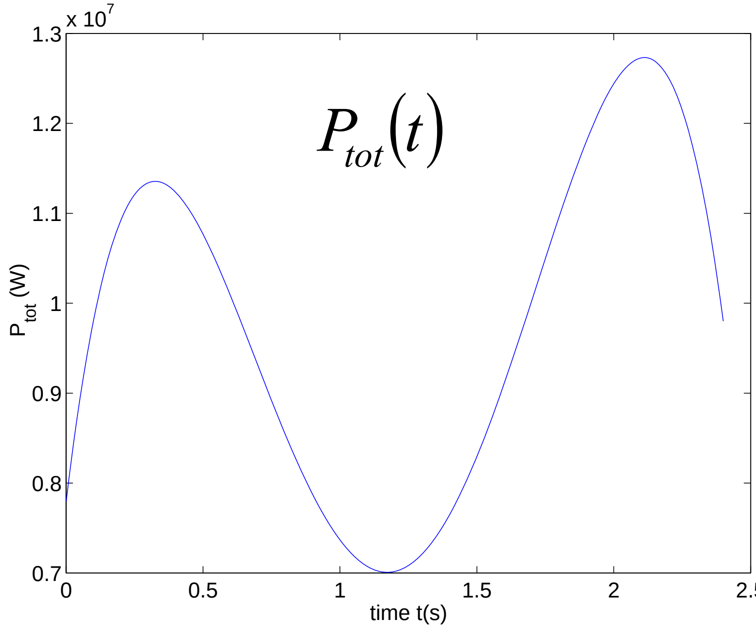
The besting matching for ι profile

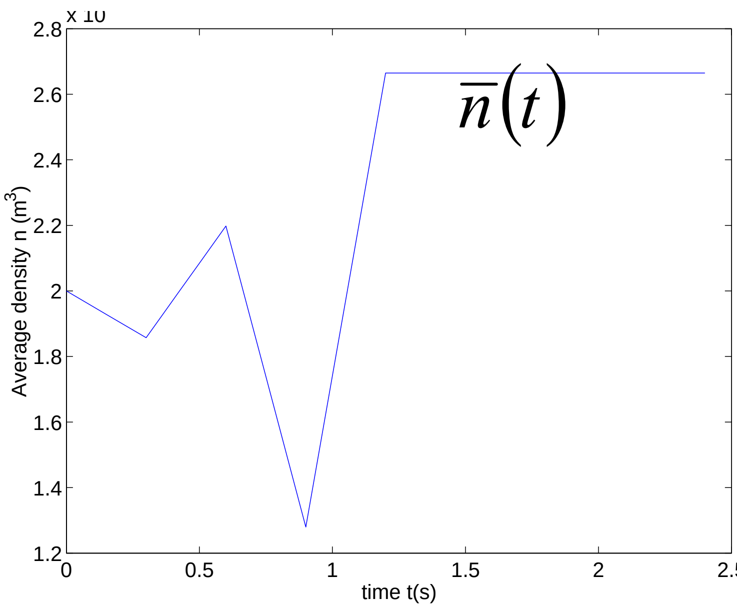


Normalized cost function evolution

Current Profile Control: OL-ES Control



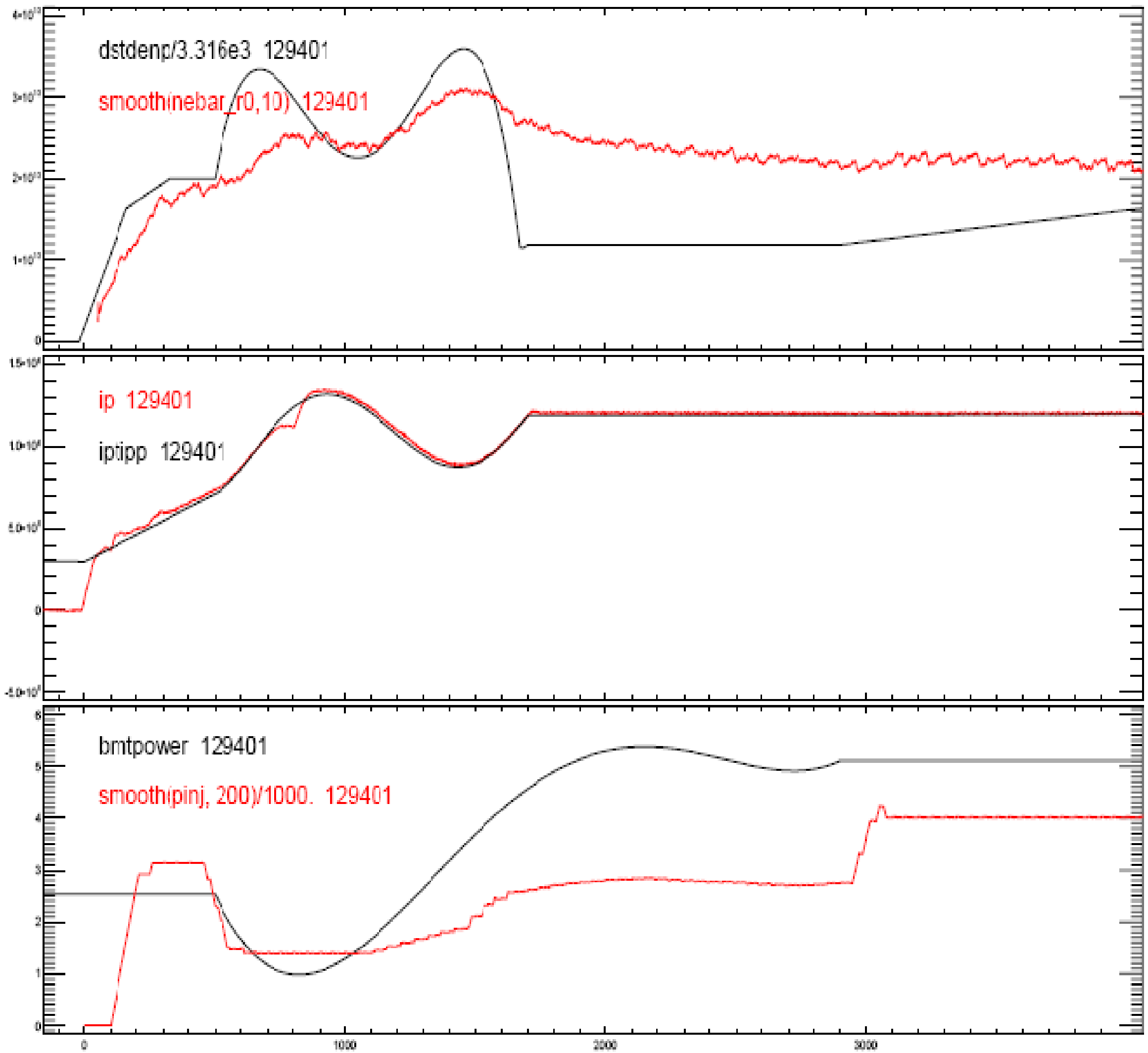




- This is some preliminary work that does not take into account all the dynamics of the actuators.

Current Profile Control: OL-ES Control

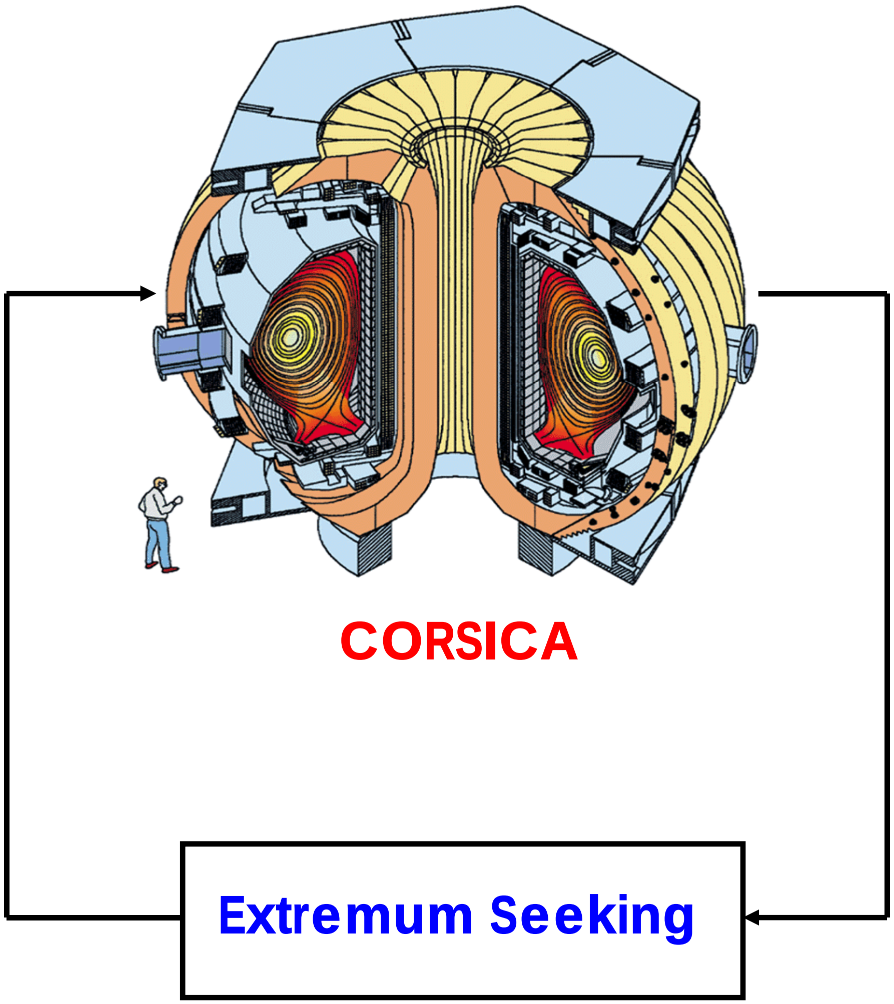
Experiment July 6, 2007



Conclusions:

-Redefine constraints for actuators
- Continue effort on model validation

Current Profile Control: OL-ES Control



What is the potential of ES?

Simple implementation for complex or unknown plants

Integration of CORSICA into MATLAB SIMULINK environment

Outline

- **Models for Control Design**
- **Open-loop Optimal Control**
- **Closed-loop Optimal Control**

Current Profile Control: CL Control

Closed-loop Optimal Control

GOAL: Identical to open-loop control. During "Phase I" an optimal control problem must be solved, where time evolution for three actuators ($I(t),\ \bar{n}(t)\ (u_n(t)),\ P_{tot}(t)$) are sought to minimize the functional. Closed-loop control is expected to be more effective in dealing with model and IC uncertainties, and measurement noise.

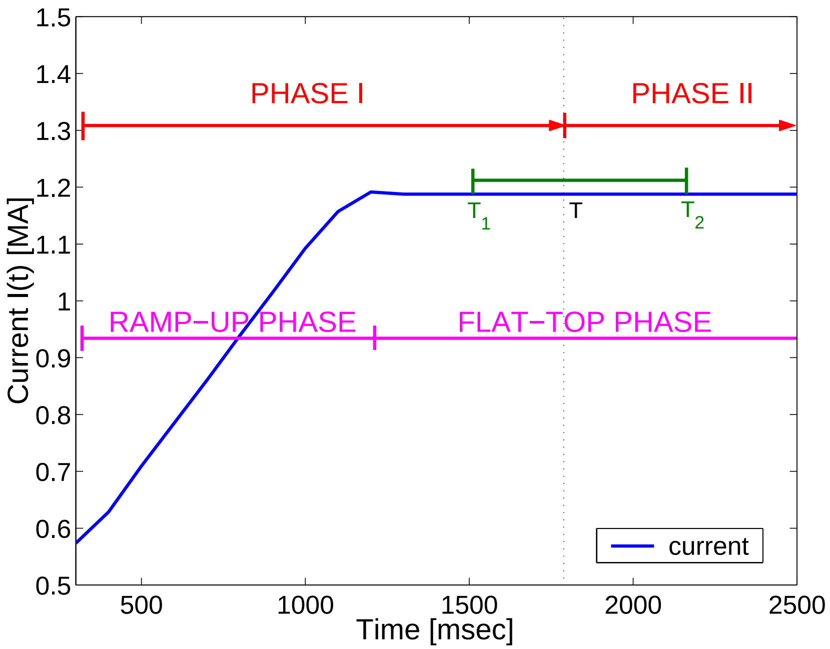
$$J_{\min} = \min_{t \in [T_1, T_2]} \left(J(t) \right)$$

where

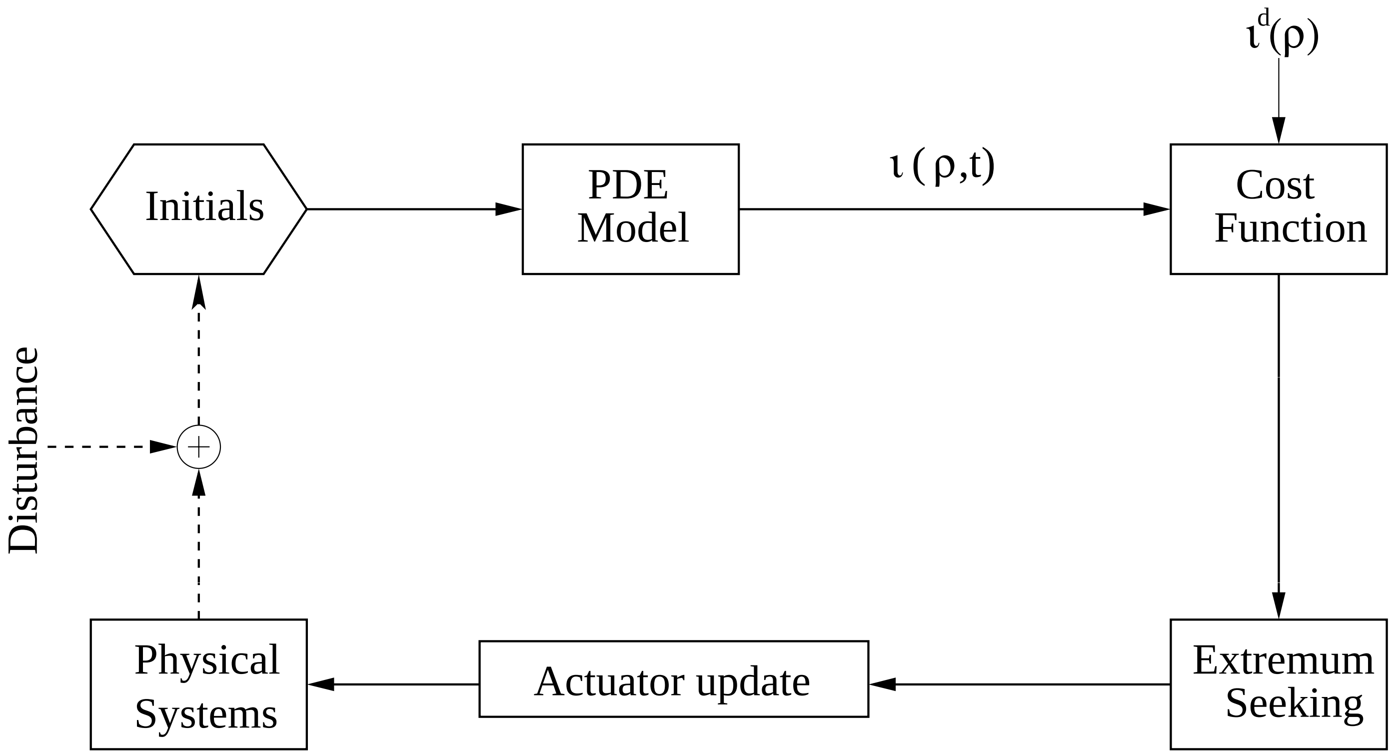
$$J(t) = \int_0^1 \left(\iota(\hat{\rho}, t) - \iota^{des}(\hat{\rho}) \right)^2 d\hat{\rho}$$

or

$$J(t) = \sum_{i=0}^{n} \left(\iota(\hat{\rho}_i, t) - \iota^{des}(\hat{\rho}_i) \right)^2, \quad n = 100$$



Current Profile Control: CL-ES-RH Control



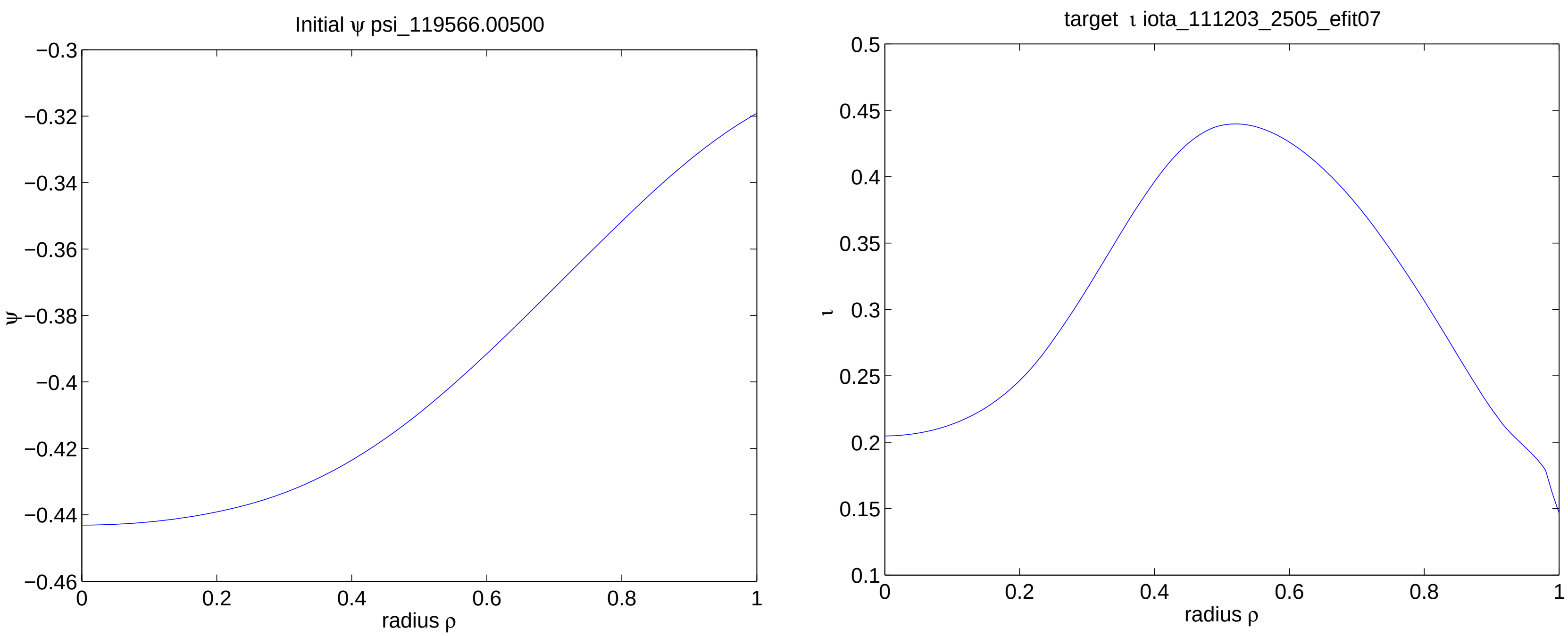
Closed-loop control scheme

This figure shows a closed-loop, receding-horizon, optimal controller based on an extremum-seeking optimization framework.

Current Profile Control: CL-ES-RH Control

1. **Select the tolerance $\varepsilon>0$ and the maximum number of iterations for the extremum seeking control algorithm.**
2. **Define $t_i=t_0$. Provide the off-line actuator trajectories $u(t)$, for $t > t_i=t_0$, and the actual initial poloidal flux profile $\psi(t_0)$ to the PDE model.**
3. **Compute the predicted $\iota(T)$ (control target) from the output sequence $\psi(t)$, for $t > t_i$, obtained from the PDE model.**
4. **Calculate the cost function. If it is less than ε, go to step 6.**
5. **Adjust the parameters θ (or $u(t)$) of the extremum seeking algorithm, until the cost function is less than ε or the maximum number of iteration is reached.**
6. **Implement the calculated actuator trajectories on the actual system for $[t_i + \Delta t, t_i + 2\Delta t]$.**
7. **Move the control horizon one sampling interval Δt ahead, measure the output of the actual system $\psi(t_i + \Delta t)$, make $t_i = t_i + \Delta t$, and go to step 3.**

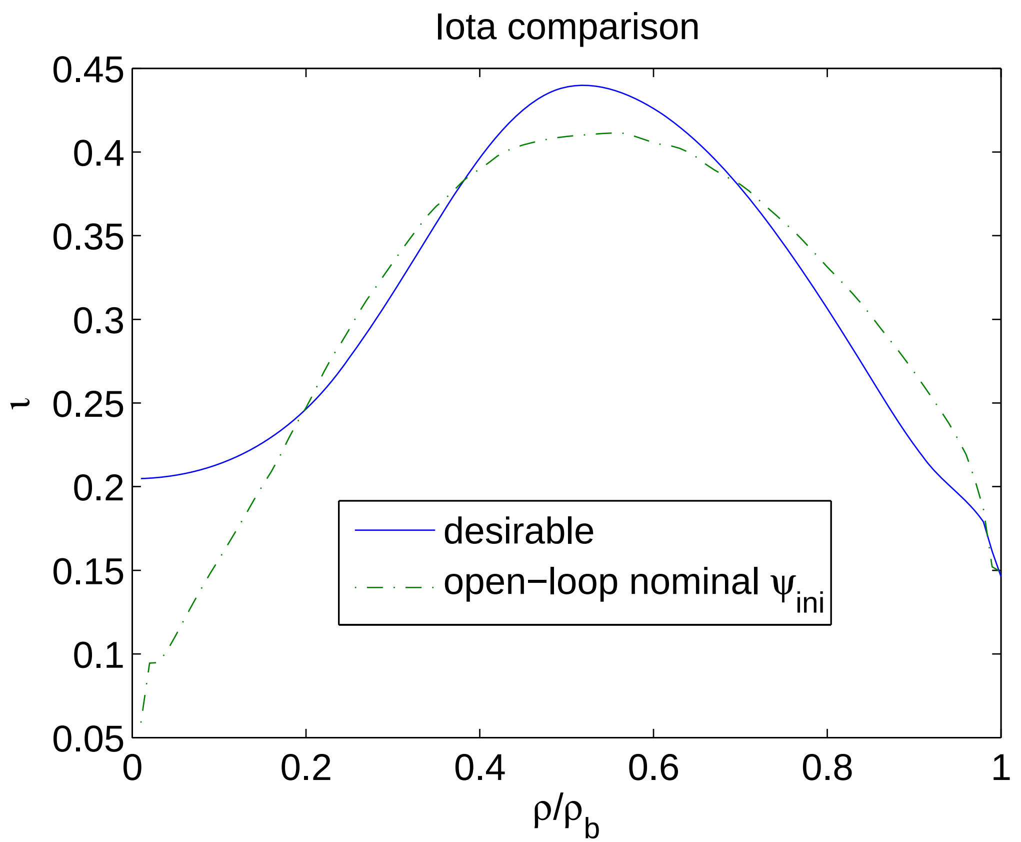
Current Profile Control: CL-ES-RH Control



(a) (b)

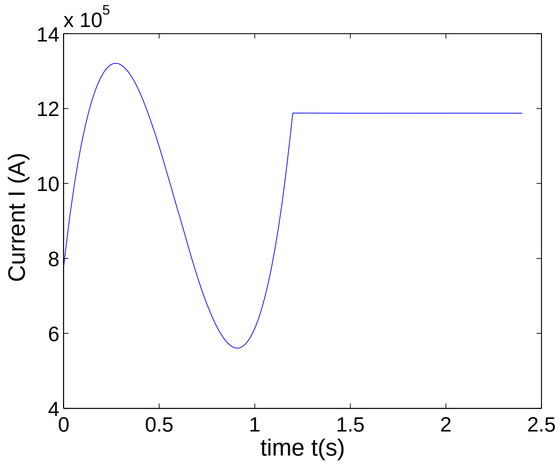
In this simulation, we consider the time interval [0, 2.4*s*]. The initial poloidal flux ψ is shown in Figure (a) and the target ι profile is shown in Figure (b). The current $I(t)$, average density $n(t)$ and total power $P_{tot}(t)$ are reconstructed as previous description.

Current Profile Control: CL-ES-RH Control

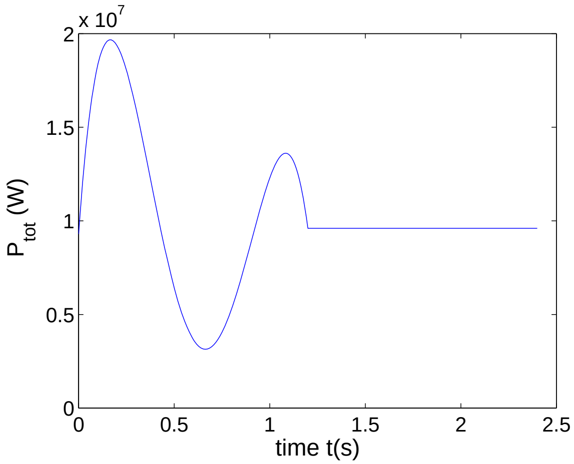


Computed best matching in open-loop control with nominal initial profile.

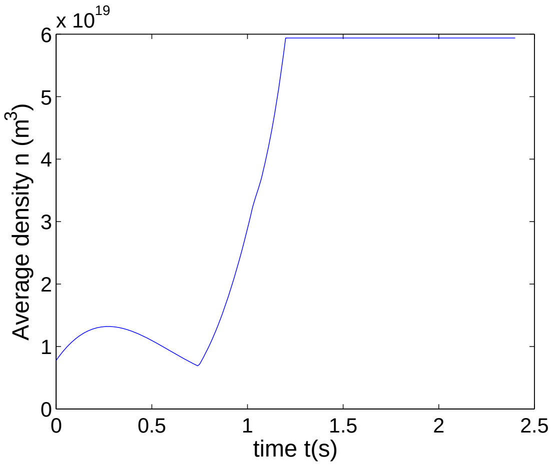
Current Profile Control: CL-ES-RH Control



Time evolution of $I(t)$



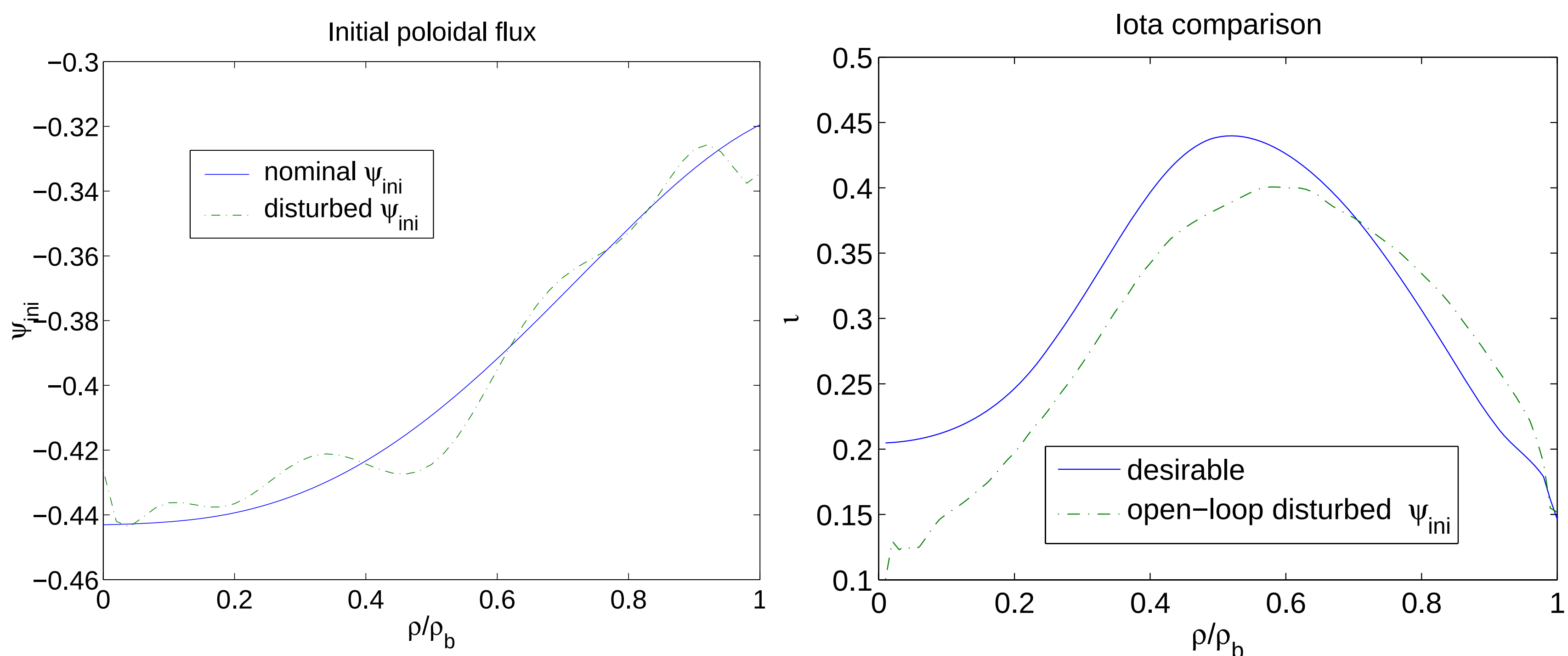
Time evolution of $P_{tot}(t)$



Time evolution of $\overline{n}(t)$

- This is some preliminary work that does not take into account the dynamics of the actuators.

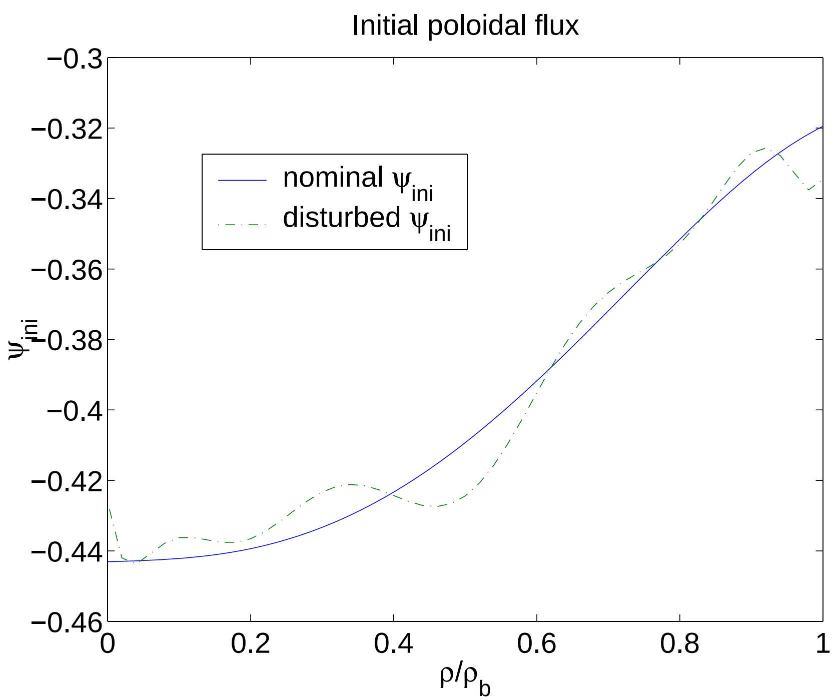
Current Profile Control: CL-ES-RH Control



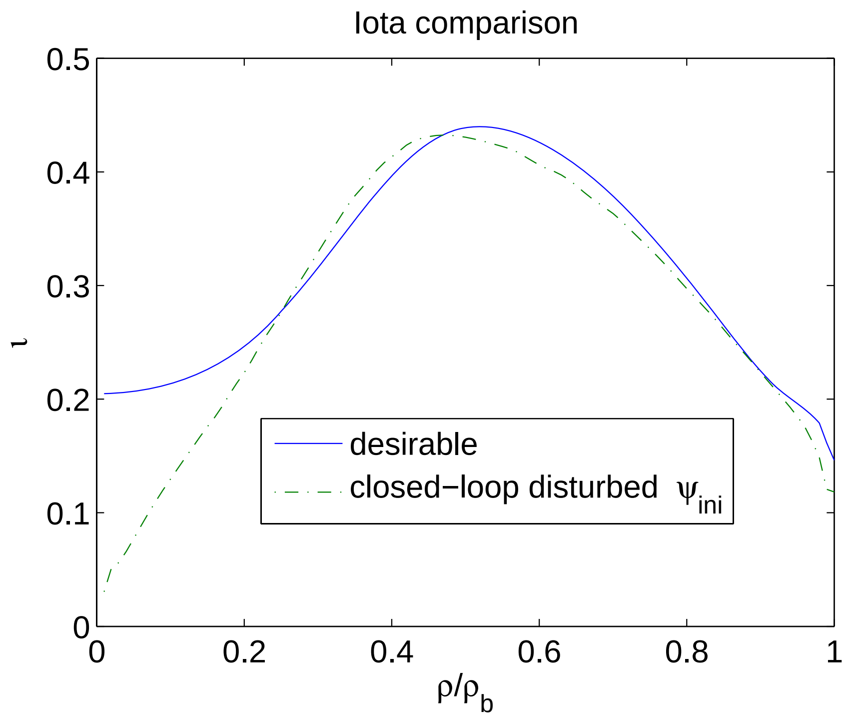
(a) (b)

Figure (a) shows the disturbed initial poloidal flux profile, and compares it with the nominal initial poloidal flux profile. Figure (b) shows the difference between the obtained ι profile and the desirable ι profile. As expected, the matching is worsen due to the disturbance in the initial poloidal flux profile.

Current Profile Control: CL-ES-RH Control



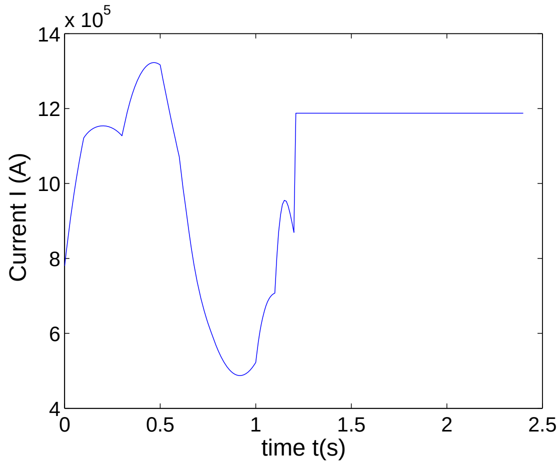
(a)



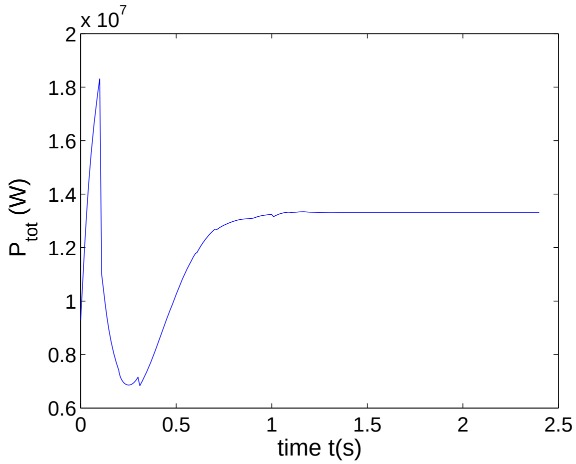
(b)

Figure (a) shows the disturbed initial poloidal flux profile. Figure (b) shows the difference between the obtained ι profile and the desirable ι profile. The closed-loop approach provides a better matching than the open-loop control.

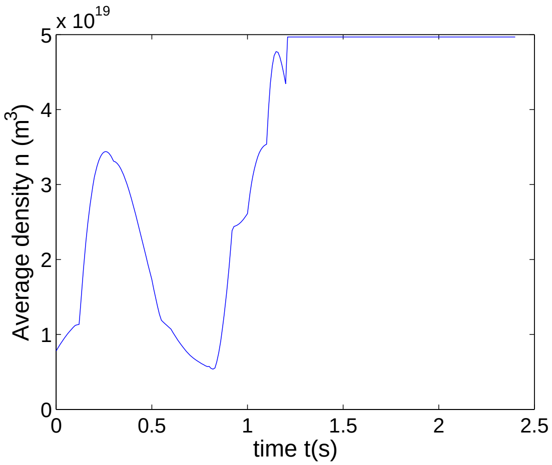
Current Profile Control: CL-ES-RH Control



Time evolution of *I(t)*



Time evolution of $P_{tot}(t)$



Time evolution of $\overline{n}(t)$

- This is some preliminary work that does not take into account the dynamics of the actuators.

Limitation: Computational demand

Conclusions and Future Work

- **We have demonstrated the existence of model-based solutions to the problem of defining actuator trajectories that achieve desired current profiles during the plasma current ramp.**
- **Simplified model validation**
  - Absolutely necessary for closed-loop control
  - The simpler the model, the faster the convergence for open-loop control
- **Incorporation of predictive codes (CORSICA, CRONOS) for model validation, controller validation, and controller design.**
  - CORSICA in the loop for open-loop control design (ES, NLP)
- **Reduced order modeling**
  - Necessary for closed-loop control (Bilinear Opt. Cont., Receding Hor. Cont.)
  - Useful for open-loop control design (ES, NLP)
- **Incorporation of transport equations for density and temperature**
  - Exploit time-scale separation $\rightarrow$ Algebraic-Differential Equations
- **Magnetic diffusion equation with density and temperature measurements**
  - This defines a different control problem (probably more difficult)
- **Extension to Phase II of the discharge (flat-top)**