ROBUST CONTROL OF RESISTIVE WALL MODES IN TOKAMAK PLASMAS

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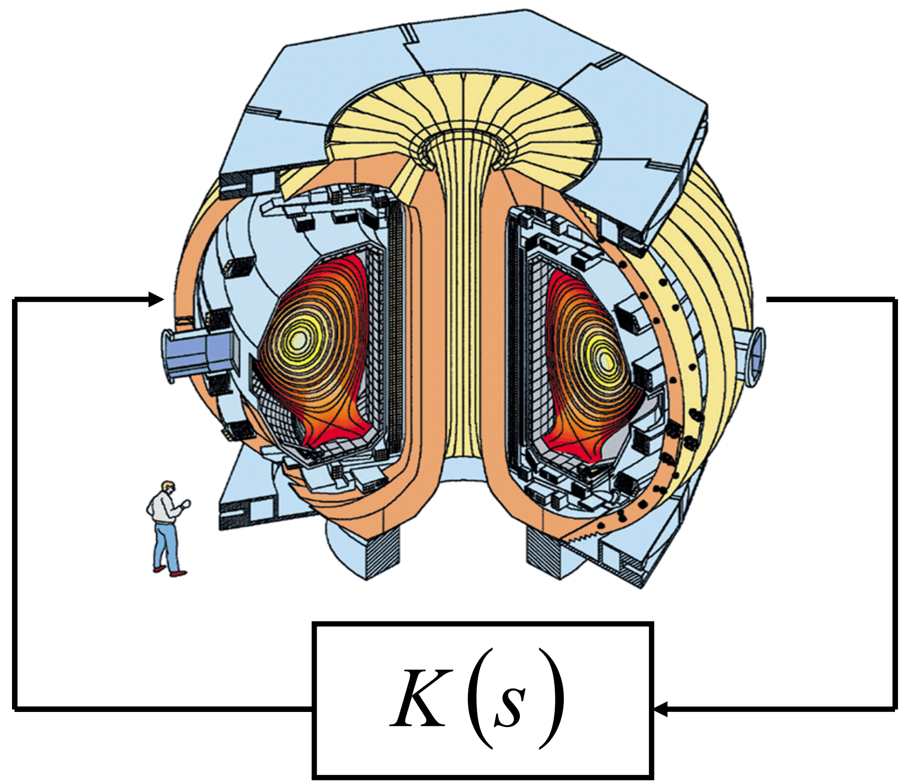

Workshop on Improved MHD Control Configurations
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Collaborators involved in this project:

- ***Mike Walker, Dave Humphreys***
  General Atomics, San Diego, CA

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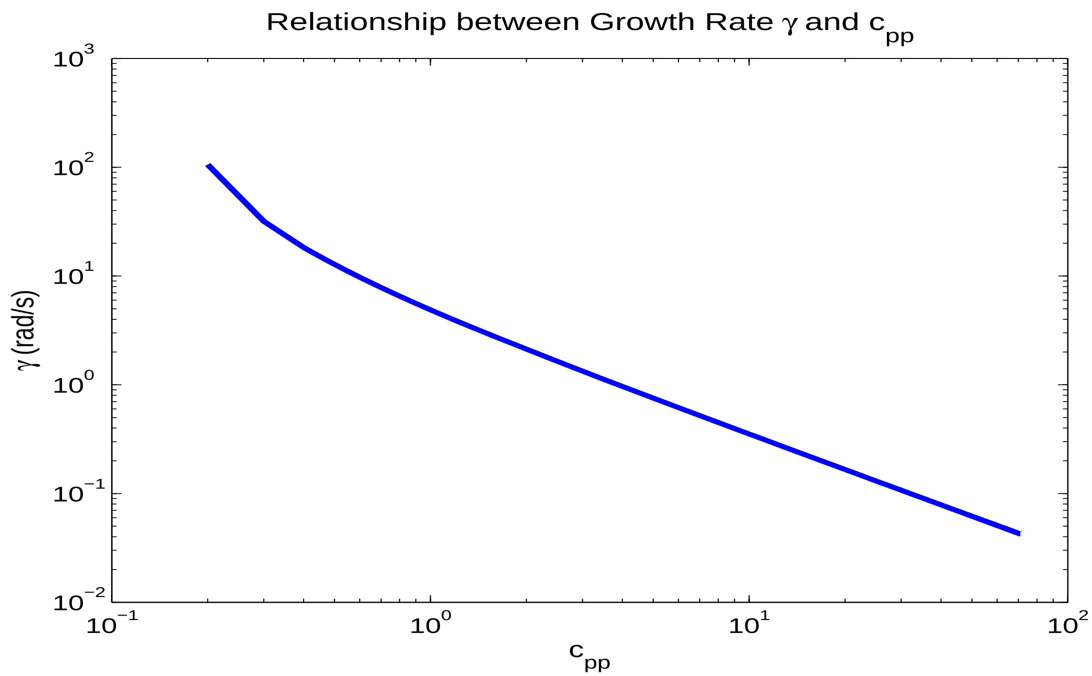
  FAR-TECH Inc., San Diego, CA

Introduction

- Resistive Wall Mode (RWM) is a plasma kink instability whose growth rate is moderated by the influence of a resistive wall.
- FAR-TECH DIII-D/RWM model
- Plasma surface modeled as a toroidal current sheet and the wall modeled with an eigenmode approach.
- State-space model of the plant, whose states are the surrounding wall current and the external control coil currents.

Introduction

- 22 poloidal field probes and saddle loops.
- 12 in-vessel coils are used to oppose the deformation.
- State space model is parameterized with a scalar coupling coefficient c_{pp}, which is directly related to the growth rate γ



- 12 input, 22 output reduced to 3 input, 2 output using a typical quartet configuration for the I-coils and matched filter on the field probes.

System Model

The state matrices are given below, where each matrix has a physical correlation to a parameter in DIII-D which is well known, except for the uncertain parameter c_{pp}.

$$\dot{x} = Ax + Bu$$

$$A = \left(M_{ss} - M_{sp} c_{pp} M_{ps}\right)^{-1} R_{ss}$$

$$B = \left(M_{ss} - M_{sp} c_{pp} M_{ps}\right)^{-1}$$

$$y = Cx$$

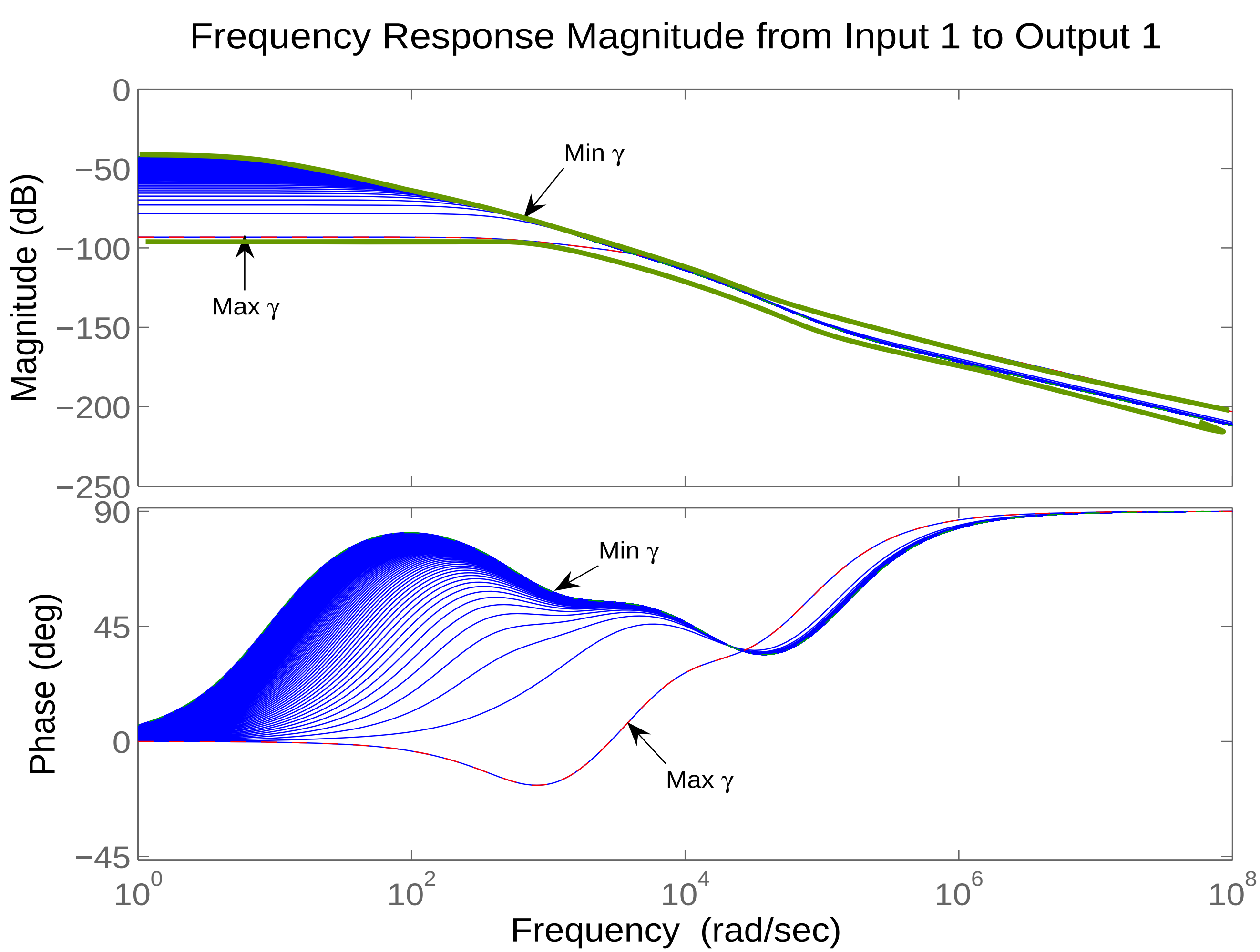
$$C = C_{ss} - C_{yp} c_{pp} M_{ps}$$

The transfer function representation of the state matrices

$$\frac{Y(s)}{U(s)} = G(s) = D + C\left(sI - A\right)^{-1} B = G\left(s, c_{pp}\right)$$

How does the dynamic response change as c_{pp} changes?

Frequency Response for Varying c_{pp}



We are interested in designing <u>one</u> controller that stabilizes <u>all</u> the systems → ROBUST CONTROL

Design Goal

- To design a model-based controller that stabilizes the system and meets performance criteria over a large range of growth rate values.

Condition	Target Value	Maximum Constraint
Rise Time	1.0 ms	5.0 ms
Settling Time	5.0 ms	10 ms
Overshoot	15 %	50 %
Input Voltage	N/A	± 100 V

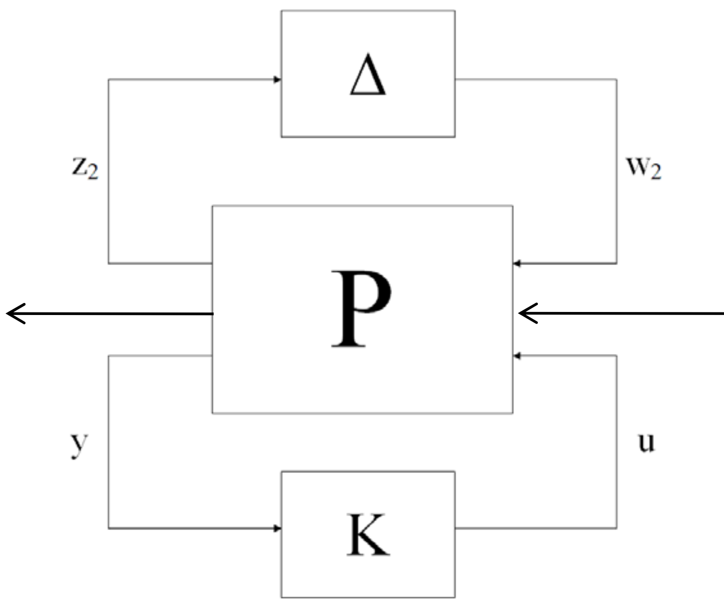
- Physical growth rate (γ) range: 10-5,000 rad/sec
- Corresponding c_{pp} range: 0.3325-71
- The growth rate relationship through c_{pp} is treated as an uncertain parameter that acts as a disturbance to a nominal system.
- Robust control tools are applied to the model to stabilize the system over the design range.
- Results in a single controller the ensures stability and performance within the desired growth rate range.

Controller Design

The goal is to design a feedback controller K that robustly stabilize the system for the applicable range of Δ. Defining

$$N = F_l(P, K) \qquad \mu(N_{11}) = \frac{1}{\min\{k_m \mid \det(I - k_m N_{11}\Delta) = 0\}} \qquad \bar{\sigma}(\Delta) \leq 1$$

where μ is the structured singular value and the term N_{11} isolates the uncertainty from the input and output of the system.



Assuming N and Δ are stable robust stability is given by

$$\mu(N_{11}(j\omega)) < 1, \forall \omega$$

and robust performance is given by

$$\mu(N(j\omega)) < 1, \forall \omega$$

System Space Model

The state matrices are given below, where each matrix has a physical correlation to a parameter in DIII-D which is well known, except for the uncertain parameter c_{pp}.

$$\dot{x} = Ax + Bu \qquad y = Cx$$

$$A = \left(M_{ss} - M_{sp} c_{pp} M_{ps}\right)^{-1} R_{ss} \qquad C = C_{ss} - C_{yp} c_{pp} M_{ps}$$

$$B = \left(M_{ss} - M_{sp} c_{pp} M_{ps}\right)^{-1}$$

Using the Sherman-Morrison formula

$$\left(A_T - b_T C_T D_T\right)^{-1} = A_T^{-1} + \frac{b_T \left(A_T^{-1} C_T\right)\left(D_T A_T^{-1}\right)}{1 - b_T D_T A_T^{-1} C_T}$$

The state matrices can be rewritten as

$$A = A_0 + \sum_{i=1}^{4} \alpha_i A_i \qquad B = B_0 + \sum_{i=1}^{4} \alpha_i B_i \qquad C = C_0 + \alpha_5 C_5$$

where each α term is a nonlinear function of the uncertain parameter c_{pp} and every other term is constant.

Linear Fractional Transformation

Using the linear fractional transformation, which is defined by the upper and lower transform for a matrix M as:

$$F_l(M,\Delta_l) = M_{11} + M_{12}\Delta_l\left(I - M_{22}\Delta_l\right)^{-1}M_{21} \qquad F_u(M,\Delta_u) = M_{22} + M_{21}\Delta_u\left(I - M_{11}\Delta_u\right)^{-1}M_{12}$$

The transfer function representation of the state matrices

$$G(s) = D + C\left(sI - A\right)^{-1}B$$

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

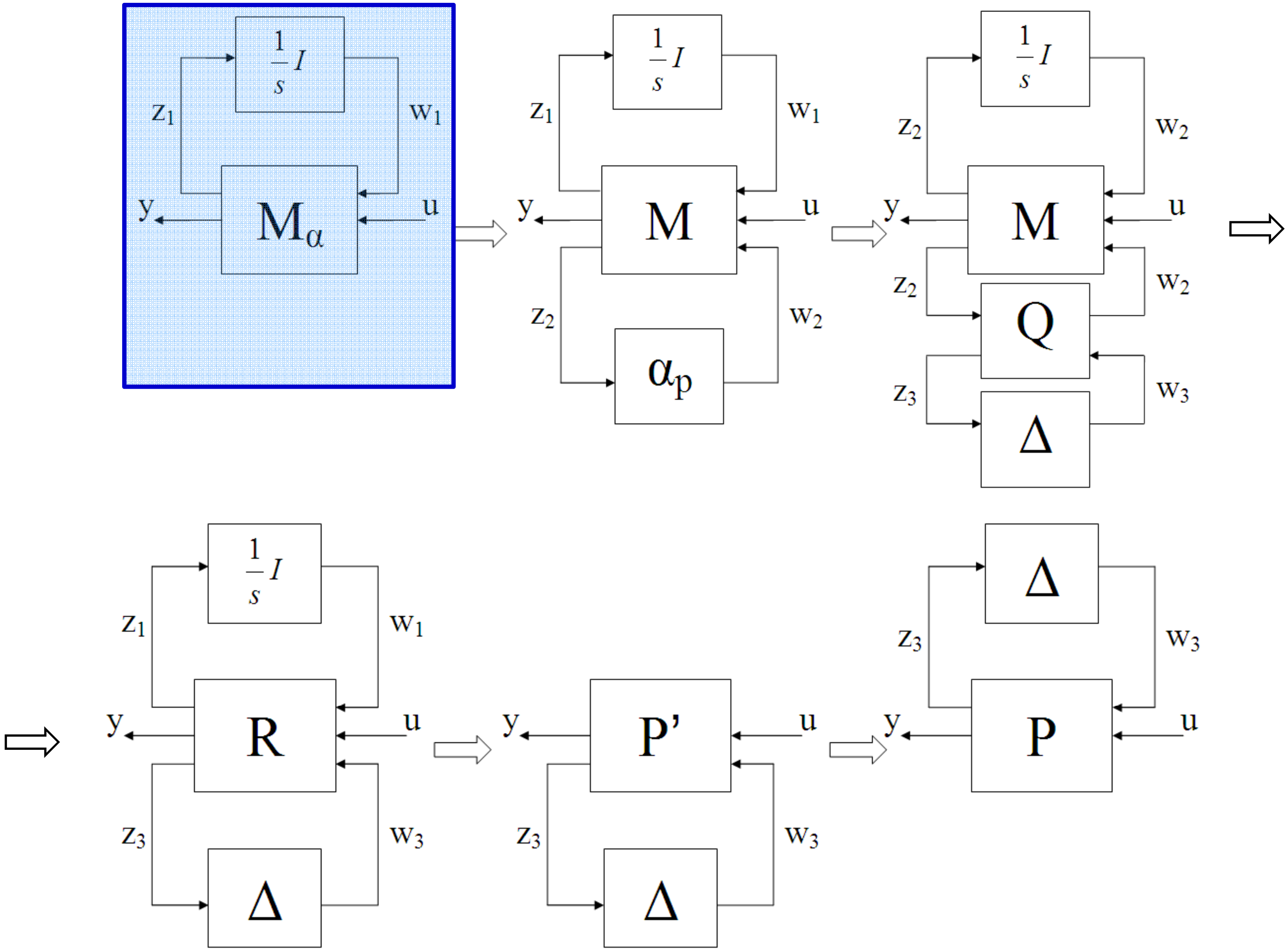
can be written as

$$G(s) = F_u\left(M_\alpha, \frac{1}{s}I\right) = M_{\alpha_{22}} + M_{\alpha_{21}}\frac{1}{s}\left(I - M_{\alpha_{11}}\frac{1}{s}\right)^{-1}M_{\alpha_{12}} = M_{\alpha_{22}} + M_{\alpha_{21}}\left(sI - M_{\alpha_{11}}\right)^{-1}M_{\alpha_{12}}$$

where

$$M_\alpha = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_0 + \sum_{i=1}^{4}\alpha_i A_i & B_0 + \sum_{i=1}^{4}\alpha_i B_i \\ C_0 + \alpha_5 C_5 & 0 \end{bmatrix}$$

Model Progression



Singular Value Decomposition

The uncertainty α can be formulated into a linear fractional transform by achieving the smallest possible repeated blocks.

$$J_i = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = U_i \Sigma_i V_i^* = \left(U_i \sqrt{\Sigma_i}\right)\left(\sqrt{\Sigma_i} V_i^*\right) = \begin{bmatrix} L_i \\ W_i \end{bmatrix} \begin{bmatrix} R_i \\ Z_i \end{bmatrix}^*$$

Denoting q_i as the rank of each matrix J_i, introducing the uncertainty

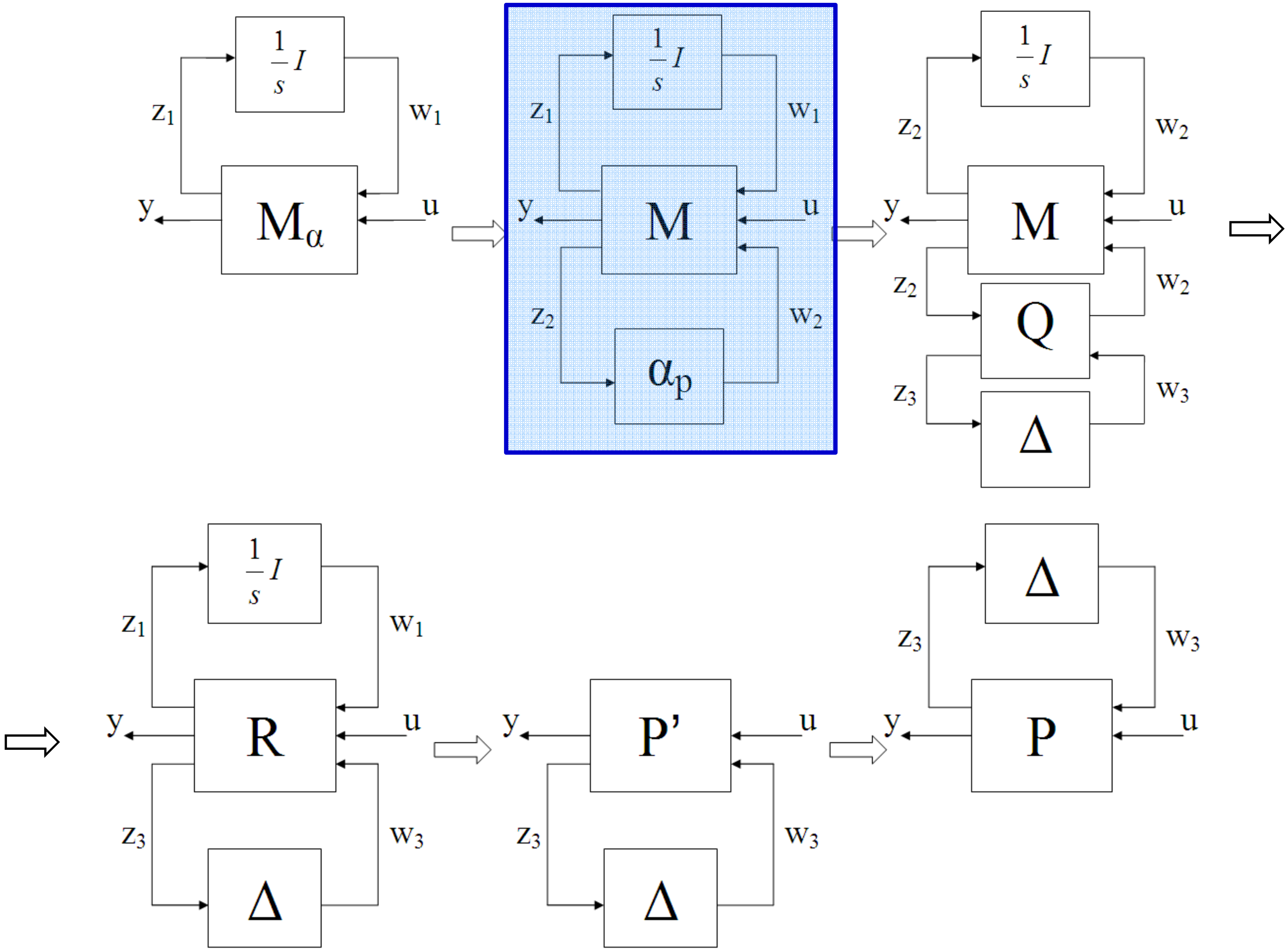
$$\alpha_i J_i = \begin{bmatrix} L_i \\ W_i \end{bmatrix} \left[\alpha_i I_{q_i}\right] \begin{bmatrix} R_i \\ Z_i \end{bmatrix}^*$$

$$M_\alpha = M_{11} + M_{12} \alpha_p M_{21} = F_l\left(M, \alpha_p\right)$$

$$M_{11} = \begin{bmatrix} A_0 & B_0 \\ C_0 & 0 \end{bmatrix} \quad M_{12} = \begin{bmatrix} L_1 & \cdots & L_5 \\ W_1 & \cdots & W_5 \end{bmatrix} \quad M_{21} = \begin{bmatrix} R_1^* & Z_1^* \\ \vdots & \vdots \\ R_5^* & Z_5^* \end{bmatrix} \quad \alpha_p = \begin{bmatrix} \alpha_1 I_{q_1} & & 0 \\ & \ddots & \\ 0 & & \alpha_5 I_{q_5} \end{bmatrix}$$

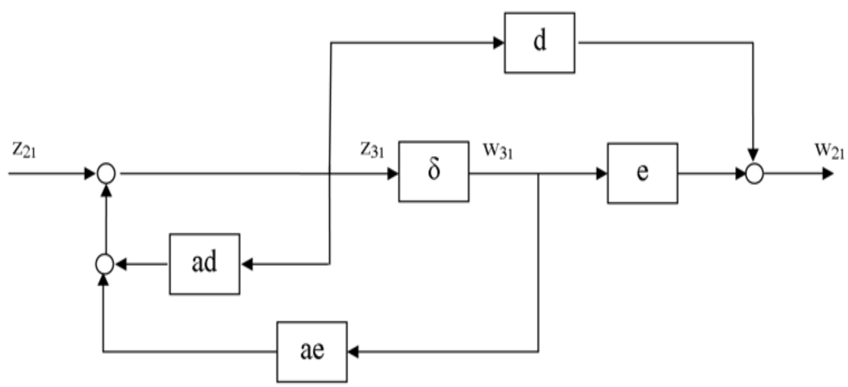
$$G(s) = F_u\left(M_\alpha, \frac{1}{s} I\right) = F_u\left(F_l\left(M_\alpha, \alpha_p\right), \frac{1}{s} I\right)$$

Model Progression



Pulling out the "δ"

The uncertainty α_p is still a nonlinear matrix based on the uncertain parameter c_{pp}. This parameter is extracted using by "pulling out the δ", where δ is the normalized c_{pp}.



$$\alpha_1 = \frac{d + \delta e}{1 - a(d + \delta e)}$$

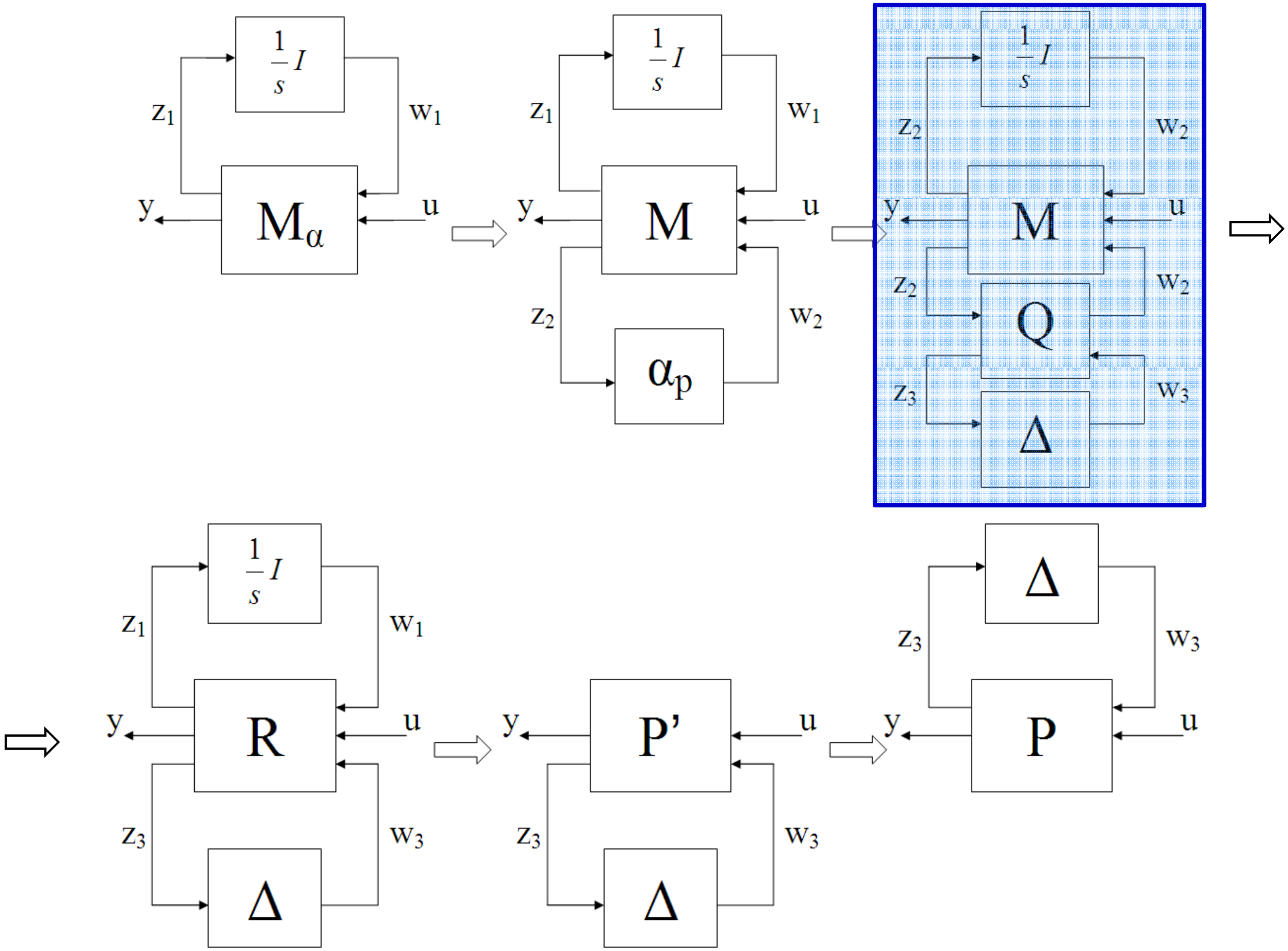
$$\begin{bmatrix} w_{2_1} \\ z_{3_1} \end{bmatrix} = Q_1 \begin{bmatrix} z_{2_1} \\ w_{3_1} \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} \dfrac{d}{1-ad} & e\left(1 + \dfrac{ad}{1-ad}\right) \\ \dfrac{1}{1-ad} & \dfrac{ae}{1-ad} \end{bmatrix} \qquad \alpha_1 = F_l(Q_1, \delta)$$

The final result is a linear fractional transform

$$\alpha_p = F_l(Q, \Delta) \qquad \Delta = \delta I \qquad |\delta| \le 1$$

Model Progression



Pulling out the "δ"

Finally this can be substituted into the plant and simplified using several properties of the linear fractional transform

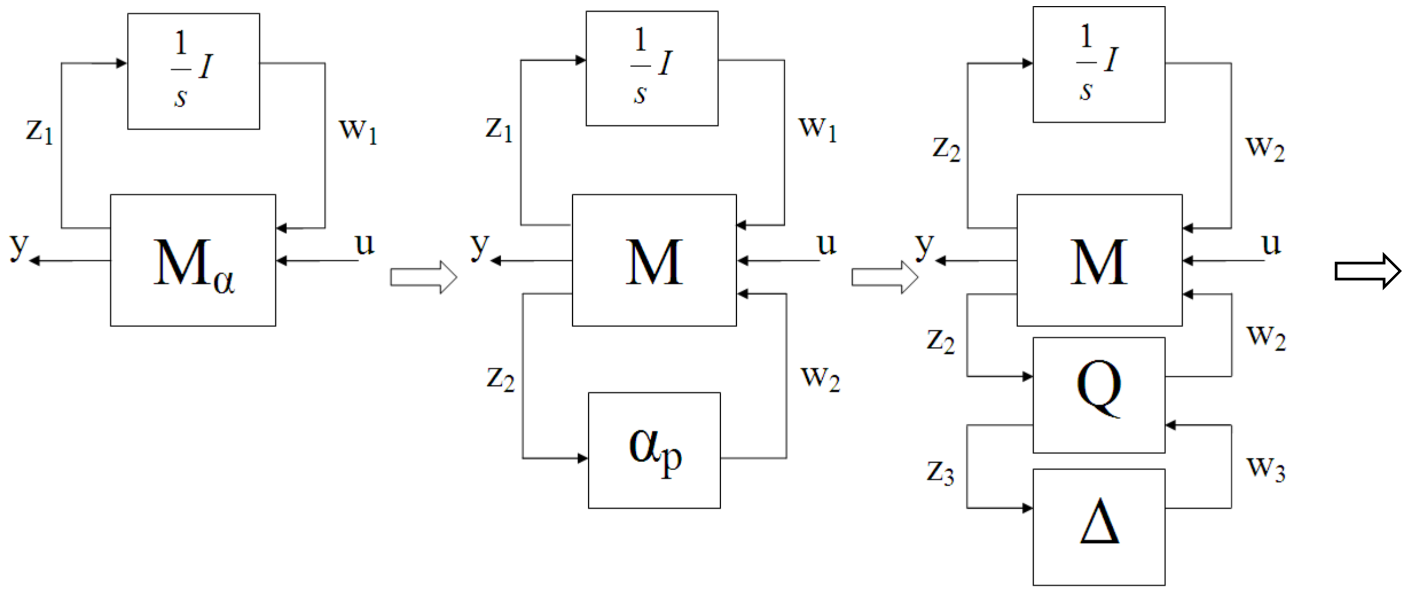
$$G(s) = F_u\left(M_\alpha, \frac{1}{s}I\right) = F_u\left(F_l(M_\alpha, \alpha_p), \frac{1}{s}I\right) = F_u\left(F_l(M_\alpha, F_l(Q, \Delta)), \frac{1}{s}I\right)$$

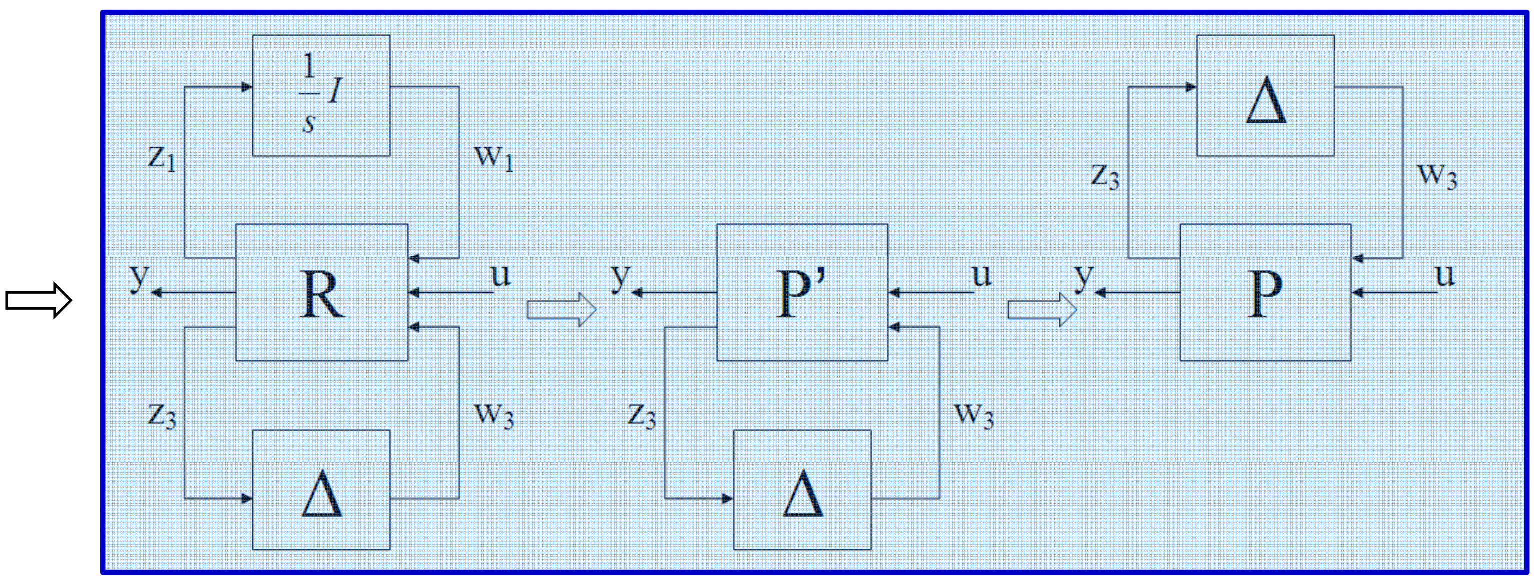
$$G(s) = F_u\left(F_l(R, \Delta), \frac{1}{s}I\right) = F_l\left(F_u\left(R, \frac{1}{s}I\right), \Delta\right) = F_l(P', \Delta) = F_u(P, \Delta)$$

where

$$R = \begin{bmatrix} M_{11} + M_{12}Q_{11}M_{21} & M_{12}Q_{12} \\ Q_{21}M_{21} & Q_{22} \end{bmatrix} \qquad P' = F_u\left(R, \frac{1}{s}I\right) = \begin{bmatrix} P'_{11} & P'_{12} \\ P'_{21} & P'_{22} \end{bmatrix}$$

Model Progression





D-K Iteration

- No direct method to synthesize a μ-optimal controller.
- DK-iteration combines $\boldsymbol{H}_\infty$ synthesis and μ-analysis.
- This method starts with the upper bound on μ in terms of the scaled singular value

$$\mu(N) \leq \min_{D \in \wp} \bar{\sigma}\left(DND^{-1}\right)$$

where $\wp$ is the set of matrices D which commute with Δ, i.e., $D\Delta = \Delta D$.

- Then, the controller that minimizes the peak value over frequency of this upper bound is found, namely

$$\min_K \left(\min_{D \in \wp} \left\| DN(K)D^{-1} \right\|_\infty \right)$$

D-K Iteration

- The controller is designed by alternating between the two minimization problems until reasonable performance is achieved.
- Follow the steps until $\left\| DN(K)D^{-1} \right\|_\infty < 1$, or $\boldsymbol{H}_\infty$ norm doesn't decrease.

  1. *K*-**step**. Design an $\boldsymbol{H}_\infty$ controller for the scaled problem with fixed D(s).

  2. *D*-**step**. Find D(*j*ω) to minimize upper bound at each frequency with fixed N.

  3. Fit the magnitude of each element of D(*j*ω) to a stable and minimum-phase transfer function D(s) and go to step 1.

Controller Simulation Results

- High rotating plasma, the growth rate γ ranges from 10 *rad/s* to 5,000 *rad/s*. This results in a range for the uncertain parameter c_{pp} that goes from 71 to 0.3325.
- Controllers designed with a smaller, more unstable nominal c_{pp} value produce the widest range of stability for c_{pp}.

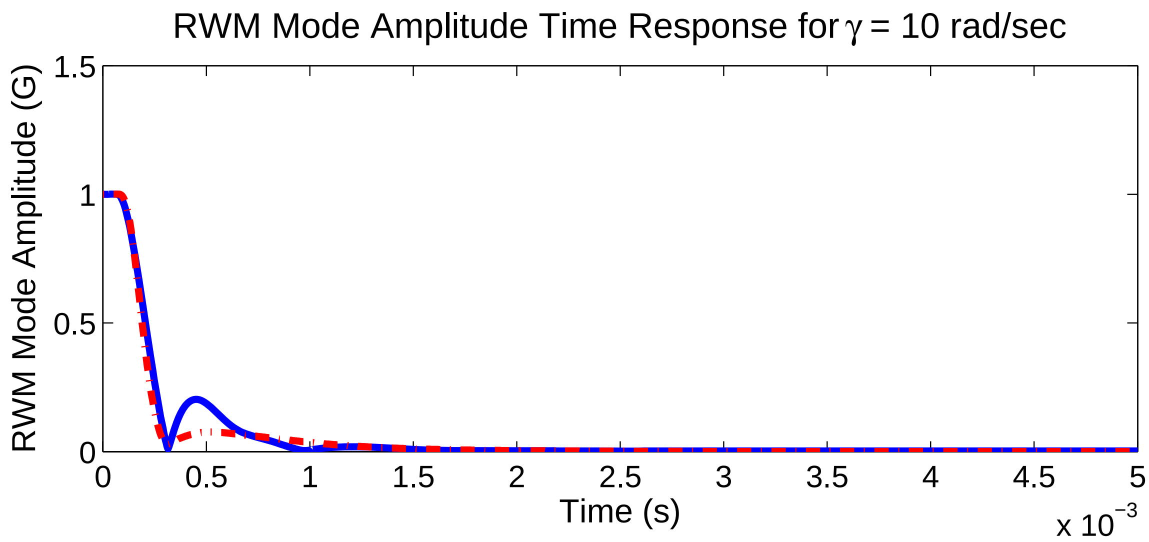
Condition	Target Value	Maximum Constraint
Rise Time	1.0 ms	5.0 ms
Settling Time	5.0 ms	10 ms
Overshoot	15 %	50 %
Input Voltage	N/A	± 100 V

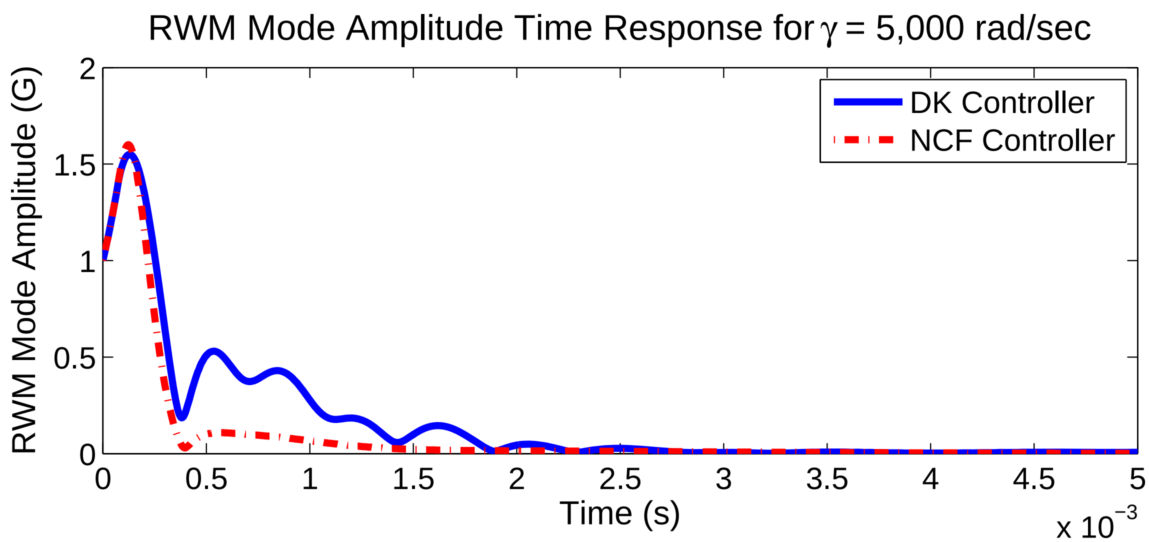
Performance Constraints

Controller	DK
Stability Range	0 - 7,437 rad/s
Perf. Range (Step)	0 - 7,254 rad/s
Perf. Range (Initial)	0 - 6,459 rad/s

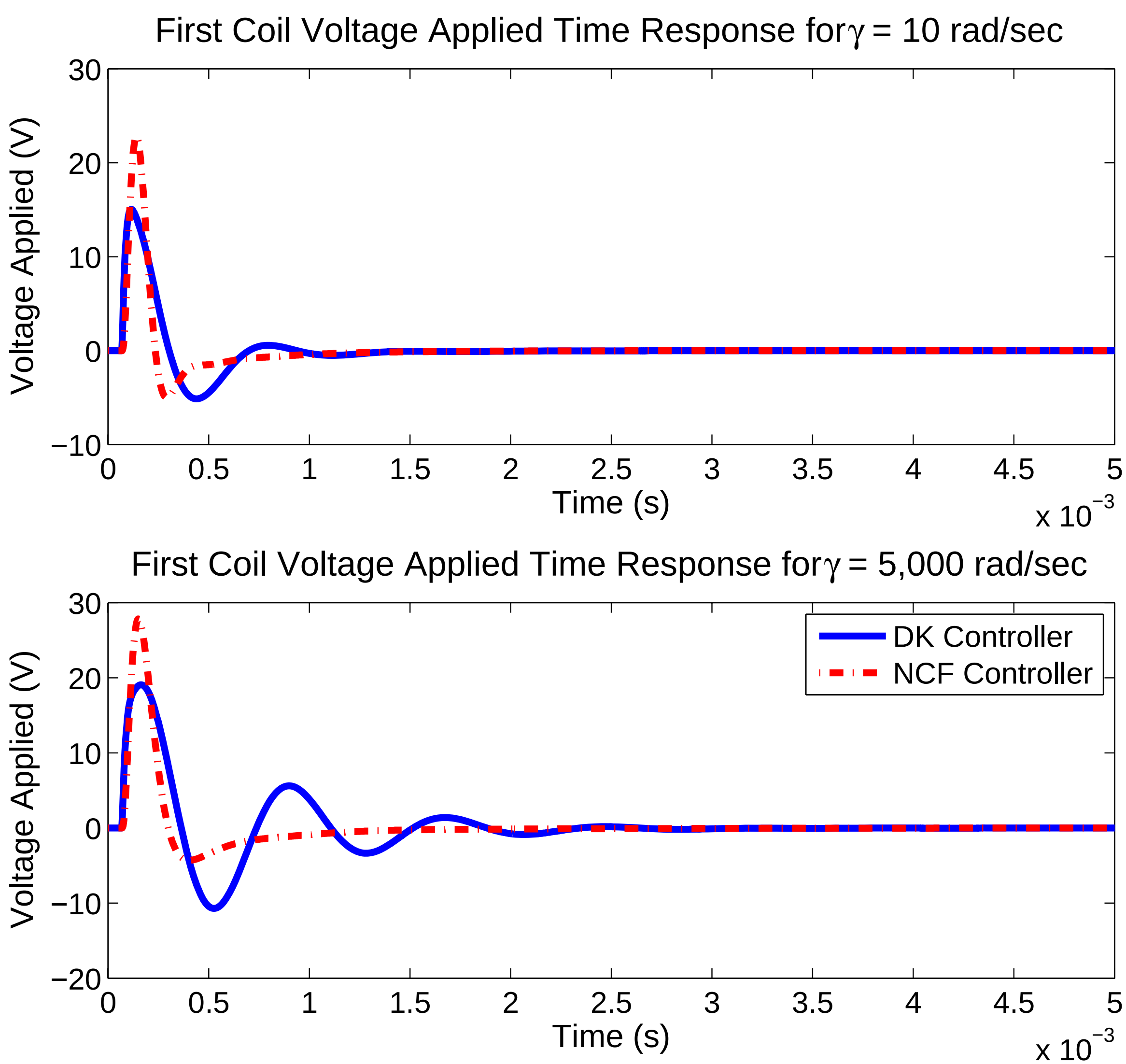
Stability\Performance Ranges

Initial Condition Response

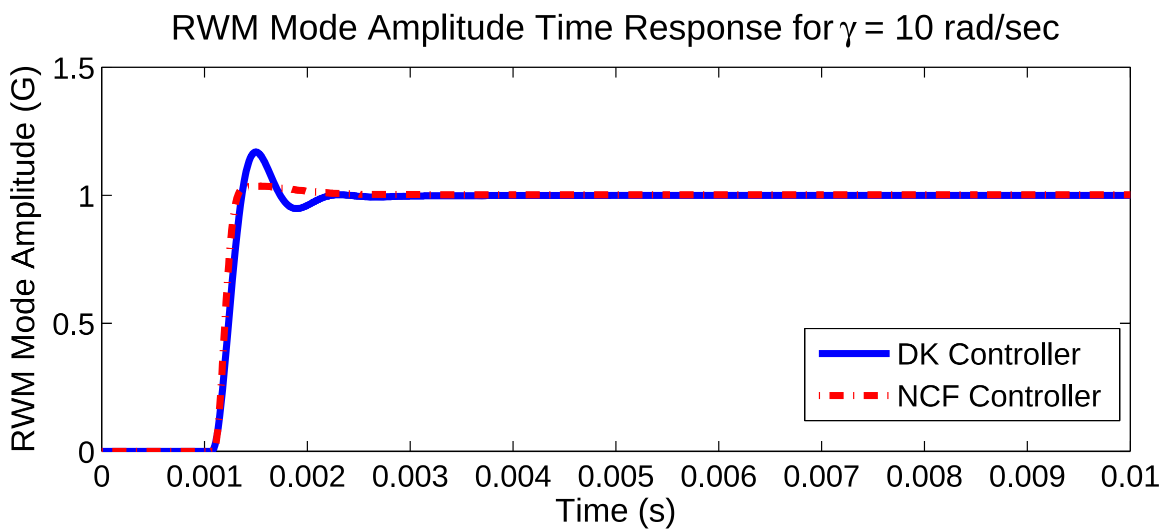


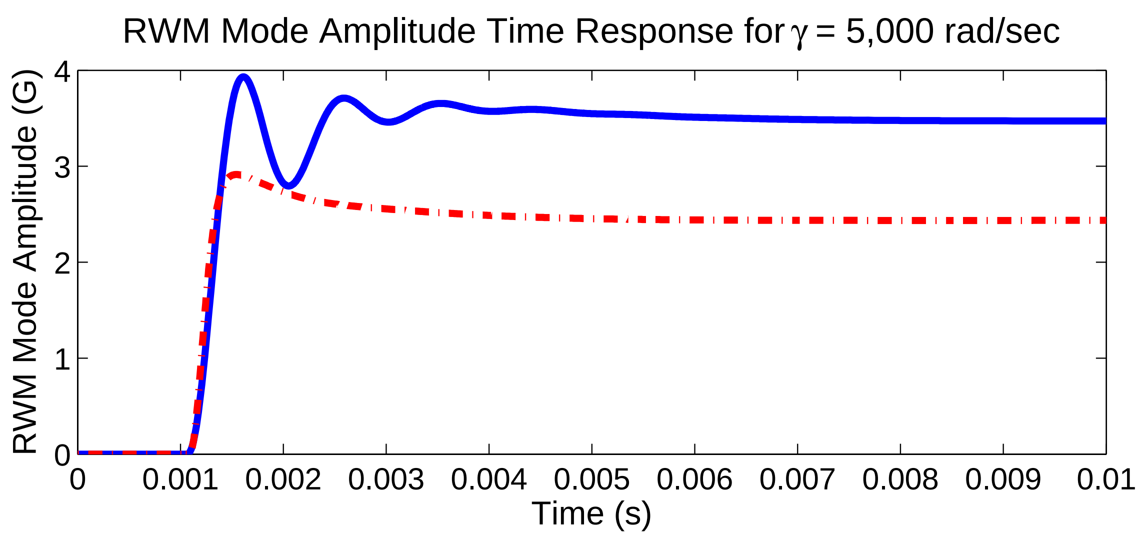


Initial Condition Response

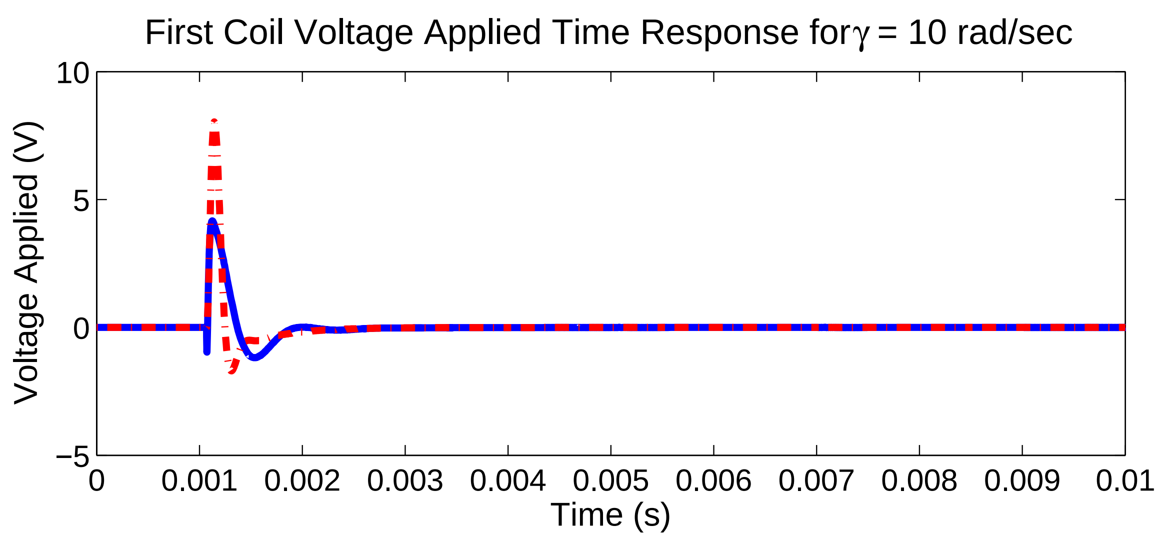


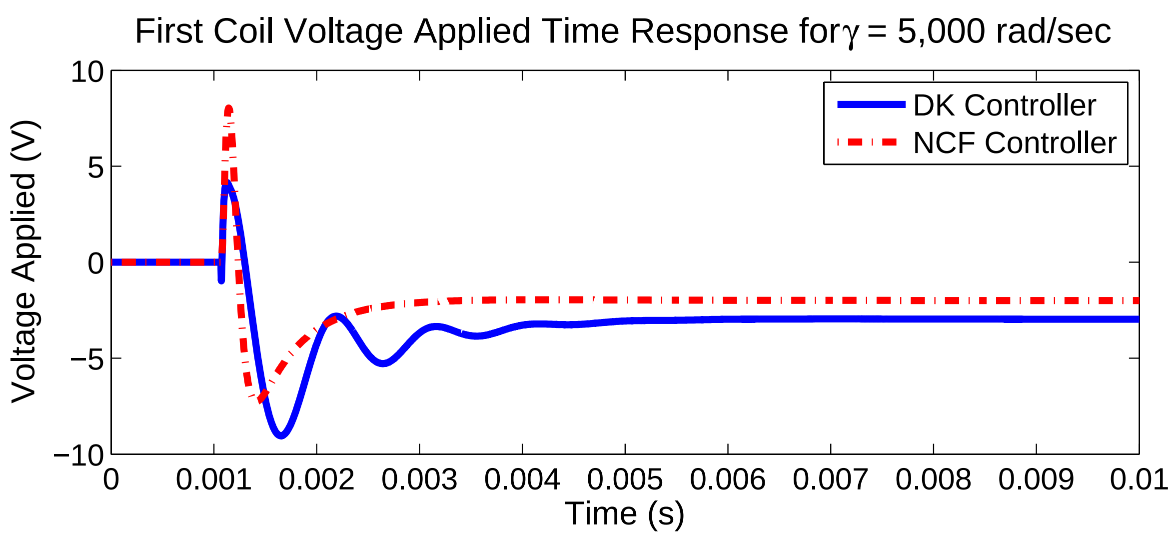
Step Response



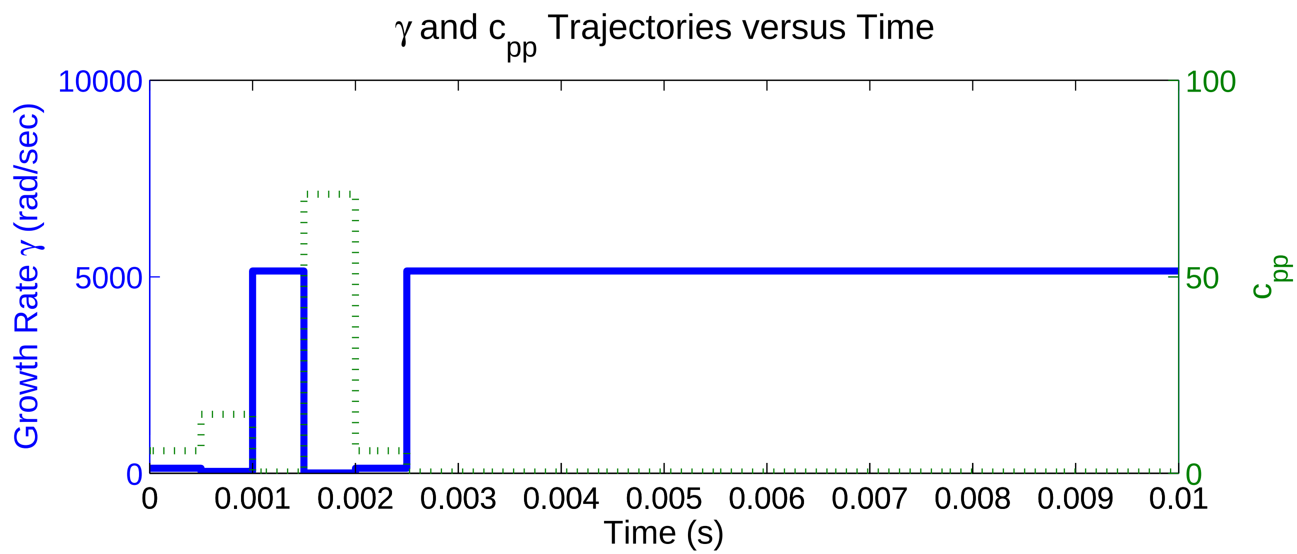


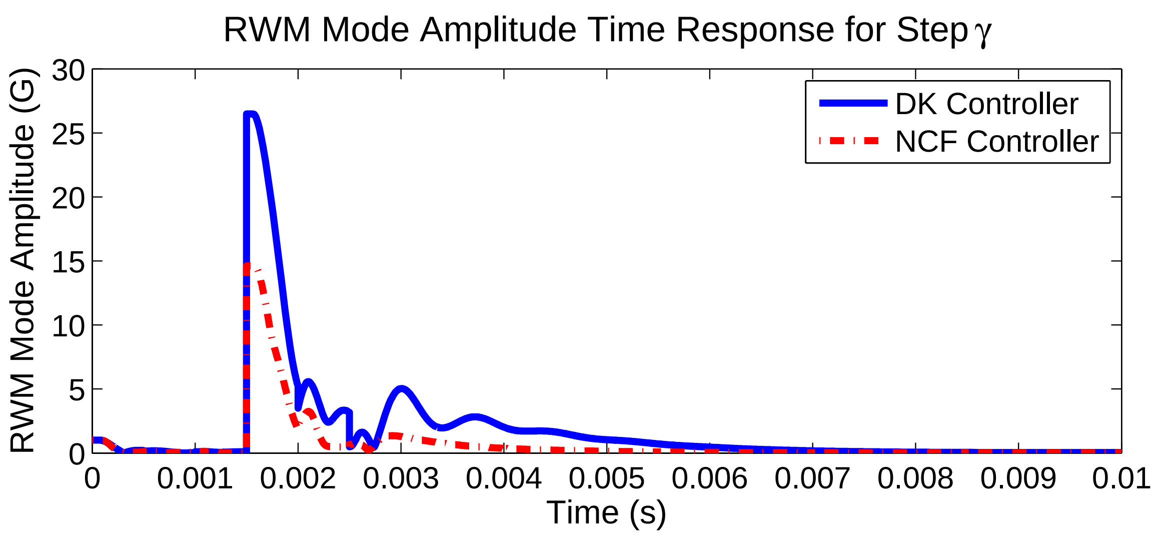
Step Response



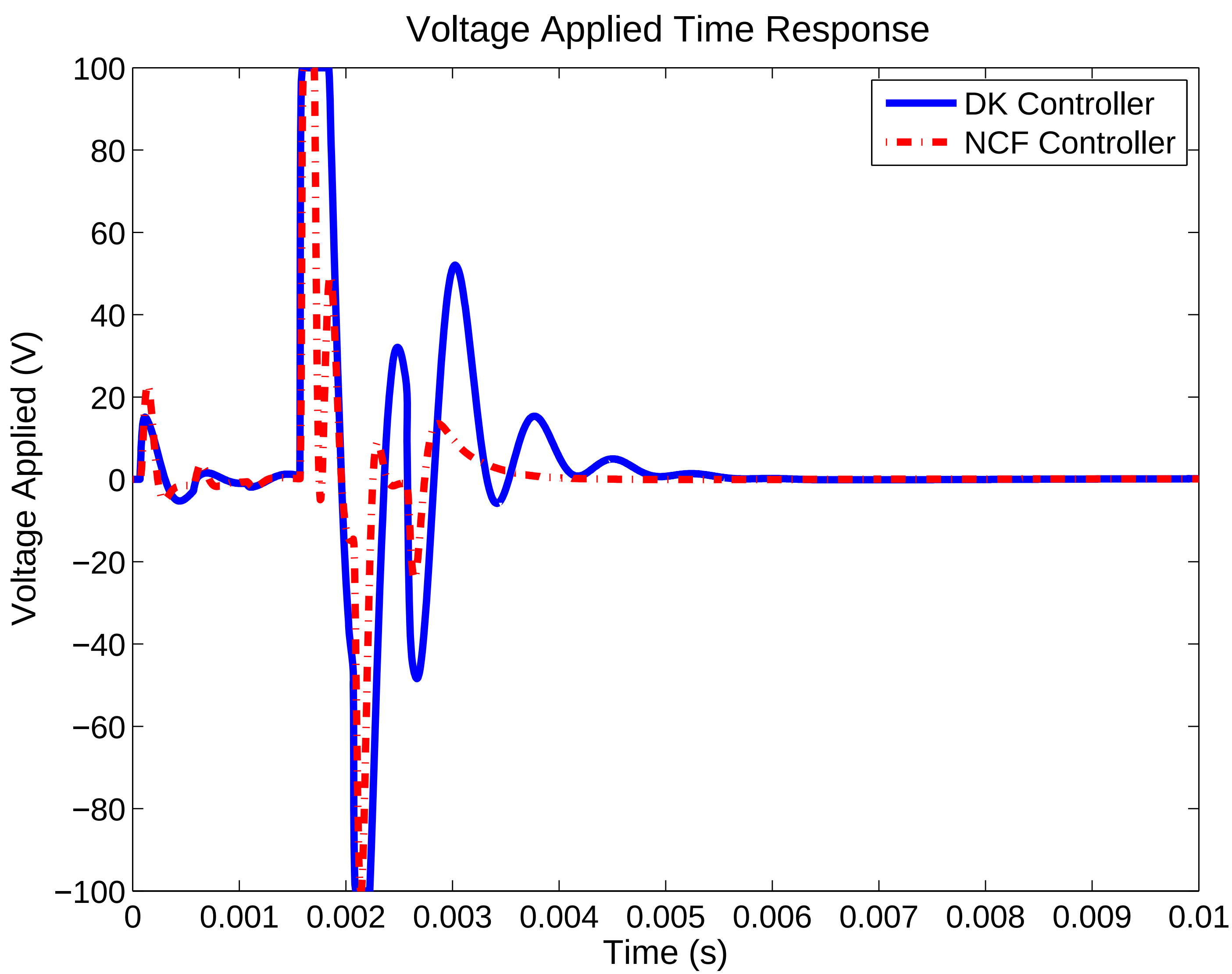


Time-varying Growth Rate Response

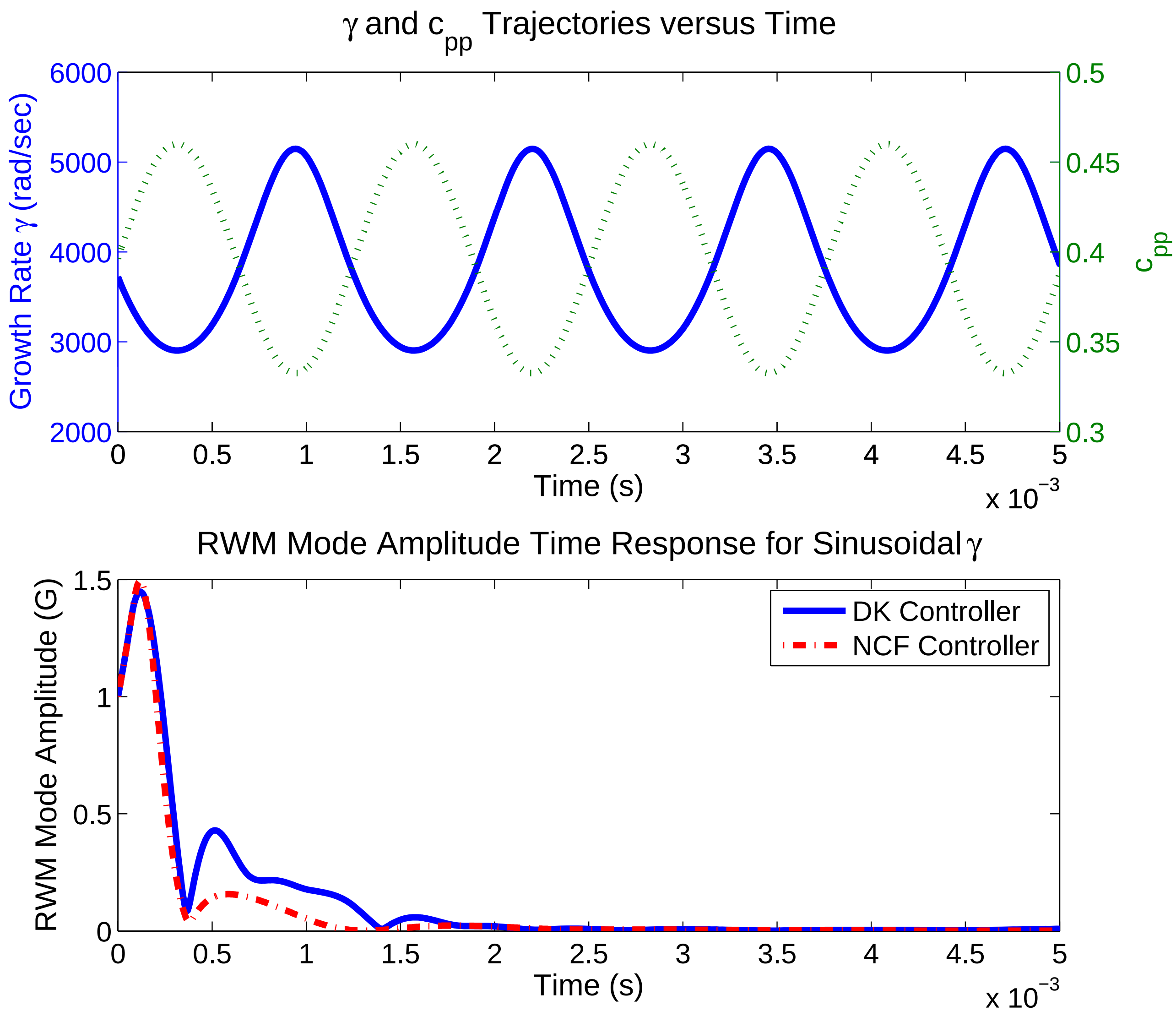




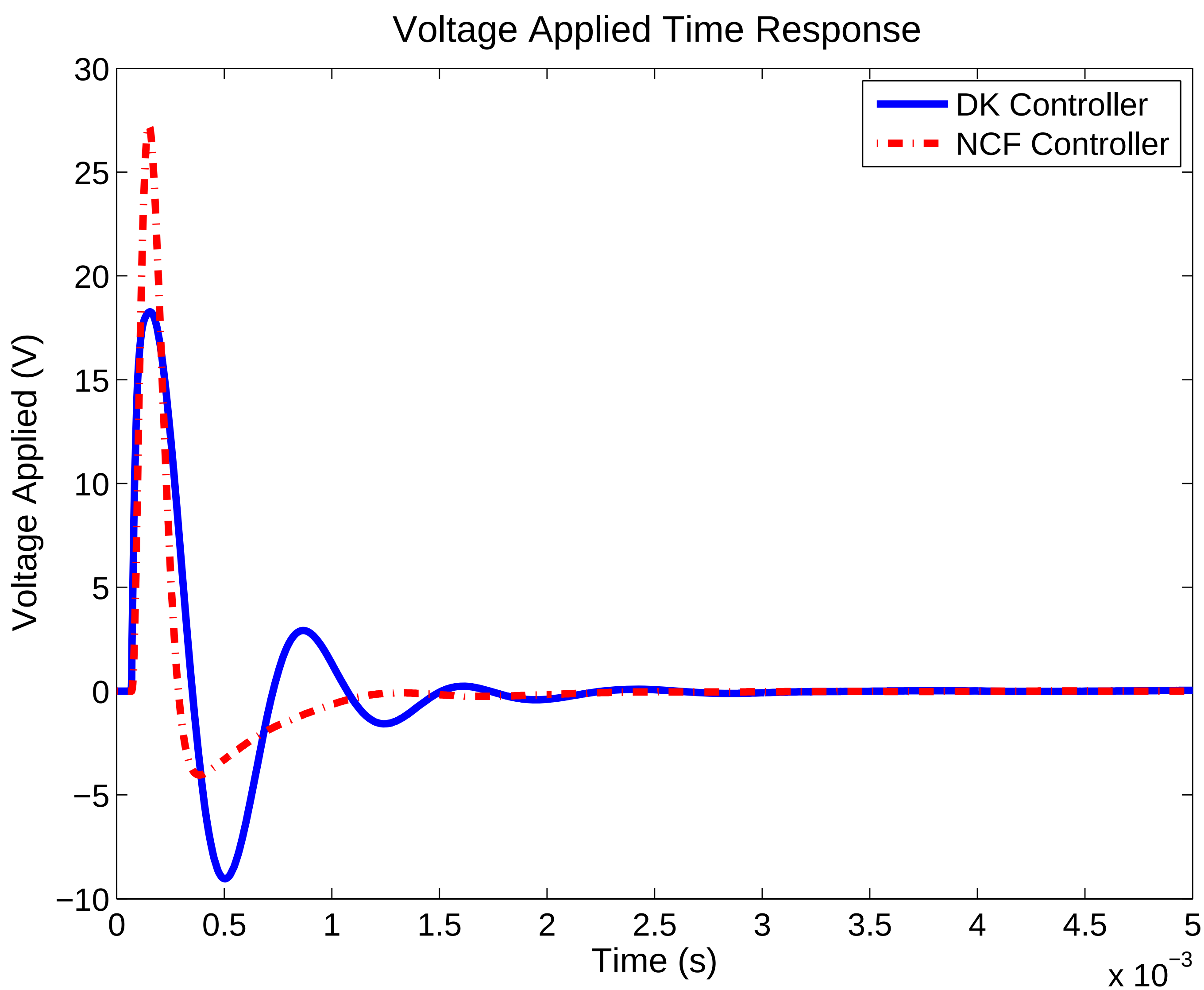
Time-varying Growth Rate Response



Sinusoidal Growth Rate Response



Sinusoidal Growth Rate Response



Conclusions

- Toroidal current sheet model for the DIII-D tokamak plasma was restructured into a robust control framework, isolating the RWM growth rate γ (c_{pp}), the key term of RWM instability.

- With the system model in this framework, the parametric *DK*-iteration method was applied to develop a structured-singular-value based robust controller for a pre-determined range of γ.

- Since the plasma RWM growth rate can vary throughout the operation of the DIII-D tokamak, the design of a controller that can stabilize the system over the entire physical range of γ is critical.

- In terms of robust stability, this method eliminates the need of online identification and controller scheduling.