Example designs: spectrum reference

# Spectrum reference for RFP RWMs and MIMO controller synthesis A control perspective on MHD modes in toroidal configurations

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### Overview



#### Background

- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
  - Physical modeling
  - Generic properties
- Example designs: spectrum reference
  - Signal norms and measures of performance
  - Dexterous control
  - Classic LQG
  - $H_{\infty}$ -type loopshaping

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Example designs: spectrum reference

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Summary

**BEP BWM stabilization** 

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The unstable plant

Example designs: spectrum reference

Summary

Sac

**RFP RWM stabilization** 

#### Successful experiments



#### Multiple RWMs stabilized in EXTRAP-T2R

 Modeling of feedback: in qualitative agreement

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Summary

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**RFP RWM stabilization** 

#### Successful experiments



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Summary

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**RFP RWM stabilization** 

## Typical methods



- Sensor-to-coil pairwise: local response
- Sensors-to-coils modewise: distributed response (FFT)

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Summary

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**RFP RWM stabilization** 

#### Enter a control perspective

- stabilization vs. control
- decentralized / centralized input/output pairing
- decoupled control (SVD methods)

Intelligent shell: decentralized stabilization Mode control: decoupled stabilization Both approaches in general suboptimal.



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The unstable plant

Example designs: spectrum reference

Summary

Sac

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The unstable plant

Example designs: spectrum reference

Summary

Sac

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Example designs: spectrum reference

Summary

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**RFP RWM stabilization** 

# More control lingo

#### Robustness, nominal system

- 1-DOF/2-DOF regulator
- Cascade systems

• Sigma-plot 
$$Y(s) = G(s)U(s)$$
:

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$$\sigma_i(\jmath\omega) \equiv \sqrt{\lambda_i(G^*(\jmath\omega)G(\jmath\omega))}$$
$$\underline{\sigma} \{G(\jmath\omega)\} \leq \frac{|Y(\jmath\omega)|}{|U(\jmath\omega)|} \leq \overline{\sigma} \{G(\jmath\omega)\} \equiv |G(\jmath\omega)|$$

The unstable plant

Example designs: spectrum reference

Summary

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The unstable plant

Example designs: spectrum reference

Summary

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The unstable plant

Example designs: spectrum reference

1

Sac

Summary

MHD mode trajectoria

# Overview



- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
  - Physical modeling
  - Generic properties
- 3 Example designs: spectrum reference
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The unstable plant

Example designs: spectrum reference

Summary

590

MHD mode trajectoria

# RWM perturbations: cylinder-model



Spatially periodic perturbations of magnetic structure:

$$\tau_{m,n}\frac{\partial B_{m,n}}{\partial t} - \gamma_{m,n}\tau_{m,n}B_{m,n} = M_{m,n}I_{m,n}$$

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where  $B_{m,n}^{ext} = M_{m,n}I_{m,n}$ .

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Summary

MHD mode trajectoria

#### Spectrum



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The unstable plant

Example designs: spectrum reference

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Sac

Summary

Physical modeling

#### Overview

#### Background

- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
  - Physical modeling
  - Generic properties
- 3 Example designs: spectrum reference
  - Signal norms and measures of performance
  - Dexterous control
  - Classic LQG
  - $H_{\infty}$ -type loopshaping

The unstable plant

Example designs: spectrum reference

Summary

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Physical modeling

#### State-space representation

#### **Basic representation**

$$\begin{cases} \dot{x} = Ax + Bu + Nv_1 \\ z = Mx \\ y = Cx + v_2 \end{cases}$$
(1)

#### where

$$\begin{array}{lcl} \mathbf{A}_{mn,m'n'} & \sim & \gamma_{mn} \delta_{mn,m'n'} \\ \mathbf{B}_{mn,ij} & \sim & \tau_{mn}^{-1} \int_{\Omega} \left( \hat{\mathbf{r}}(\theta,\phi) \cdot \oint_{I_{ij}} \frac{\mathbf{d}I_{ij} \times \left(\mathbf{r}(\theta,\phi) - \mathbf{r}_{ij}\right)}{|\mathbf{r}(\theta,\phi) - \mathbf{r}_{ij}|^{3}} \right) \, e^{-\iota(m\theta + n\phi)} d\theta d\phi \\ C_{pq,mn} & \sim & \int_{\Omega} f_{pq}(\theta,\phi) \mathbf{A}_{pq}(\theta,\phi) e^{+\iota(m\theta + n\phi)} d\theta d\phi \end{array}$$

$$(2)$$

for cylinder-model with perfectly symmetric shell conductivity.

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Example designs: spectrum reference

Summary

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Physical modeling

#### Actuators and measurements

# Note: (1)-(2) directly obtained from linear model with geometric consideration of sensors and active coils (actuators)

**1** RWM dynamics:  $\tau_{m,n}\dot{B}_{m,n}^w = \tau_{m,n}\gamma_{m,n}B_{m,n}^w + B_{m,n}^{ext}$ 

2 Actuator: 
$$\mathbf{b}(\mathbf{r}) = \frac{\mu_0 n_c i_c}{4\pi} \oint_{l_c} \frac{\mathbf{d} |\mathbf{x}(\mathbf{r} - \mathbf{r}_c)|^2}{|\mathbf{r} - \mathbf{r}_c|^3}$$

**(a)** Sensor: 
$$u_s = -n_s \frac{\partial}{\partial t} \int_{S_s} \mathbf{b} \cdot \mathbf{dS}$$

i.e laws of *Biot-Savart* and *Faraday*.

The unstable plant

Example designs: spectrum reference

Summary

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Physical modeling

# Actuators and measurements

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The unstable plant

Example designs: spectrum reference

Summary

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Physical modeling

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Summary

Physical modeling

# **Peripheral dynamics**

Dynamics of actuators (amplifier characteristics and coil L/R-times) *not* ignorable. A cascade-type PID-system employed.



cascade current-control subsystem



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Summary

Physical modeling

# Compensated SISO lumped amplifier & coil



Key-point: isolate RWM-dynamics.



900

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1

Sac

Generic properties

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#### Background

- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
  - Physical modeling
  - Generic properties
  - Example designs: spectrum reference
    - Signal norms and measures of performance
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The unstable plant

Example designs: spectrum reference

Summary

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Generic properties

# Control and observation *aliasing*

Rectangular schema of equal-size coils; typical complication of actuator and sensor: *aliasing*. Planar  $M \times N$ -DFT pair

$$F_{k,l} = \frac{1}{\sqrt{MN}} \sum_{m,n} f_{m,n} e^{-2\pi i (mk/M+nl/N)}$$
  
$$f_{m,n} = \frac{1}{\sqrt{MN}} \sum_{k,l} F_{k,l} e^{+2\pi i (mk/M+nl/N)}$$

mapping  $f \leftrightarrow F$ . Periodicity<sup>1</sup>

$$F_{k+pM,l+qN} = F_{k,l}$$
 and  $f_{m+pM,n+qN} = f_{m,n} \quad \forall p, q \in \mathbb{Z}$ 

is  $M \times N$ .

<sup>1</sup>also referred to as *sidebands* 

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Generic properties

#### Control and observation aliasing: SVD diagnose



SVDdemonstration of aliasing. Diagonalizationattempt (decoupling approach) *BB*<sup>+</sup>.

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Summary

Signal norms and measures of performance

#### Overview

#### Background

- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
  - Physical modeling
  - Generic properties
- 3 Example designs: spectrum reference
  - Signal norms and measures of performance
  - Dexterous control
  - Classic LQG
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The unstable plant

Example designs: spectrum reference

Summary

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Signal norms and measures of performance

 $H_2$  and  $H_\infty$ 

#### Typically

# *H*<sub>2</sub>: $||\mathbf{x}(t)||_{Q,2} \equiv \sqrt{\int_{-\infty}^{\infty} \mathbf{x}^{T}(\tau) Q \mathbf{x}(\tau) d\tau}$ Normally good for disturbance rejection requirements.

 $H_{\infty}$ :  $||\mathbf{x}(t)||_{\infty} \equiv \max_{i,\tau} |x_i(\tau)|$ Often used for enforcing robustness performance Performance channel: Denoted  $\mathbf{z}$ ,  $z_i$ , a mathematically

constructed output for design synthesis and comparison.

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Example designs: spectrum reference

Summary

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Signal norms and measures of performance

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Summary

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Signal norms and measures of performance

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The unstable plant

Example designs: spectrum reference

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3

Sac

Dexterous control

### Overview

#### Background

- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
  - Physical modeling
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#### Example designs: spectrum reference

- Signal norms and measures of performance
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The unstable plant

Dexterous control

#### **Objective formulation**

Use admissible inputs (active coil currents  $\mathbf{u}(t)$ ) in the best way possible to keep a subset  $\mathbf{z}(t)$  of MHD-mode trajectoria  $\mathbf{x}(t)$  at a specified set-point (reference-spectrum vector  $\mathbf{r}(t)$ ), by responding to plant output  $\mathbf{y}(t)$  (sensor coil voltages).

Example designs: spectrum reference

Summary

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Example designs: spectrum reference

Summary

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**Dexterous control** 

# Linear estimation and optimal filtering

#### Estimation: Given an interval of observations $\mathbf{y}(t)$ , $t \in [t_A, t_B]$ , form a clever guess of interesting quantity $\mathbf{z}(t)$ . Filtering: Do this in a real-time causal fashion.

Where a linear model is applicable: model-based filtering a la *Kalman* is standard procedure.

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Example designs: spectrum reference

Summary

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Dexterous control

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Example designs: spectrum reference

Summary

Dexterous control

# Employing model reduction

For rapid development (e.g. highly iterative controller synthesis), and real-time capabilities of synthesized controller-dynamics: reduction techniques. Definition of *Hankel* singular values

$$AP + PA^{T} + BB^{T} = 0$$
  
 $A^{T}Q + QA + C^{T}C = 0$   
 $\sigma_{Hankel,i} \equiv \sqrt{\lambda_{i}(PQ)}$ 



Example designs: spectrum reference

Summary

Dexterous control

# A word on Linear Matrix Inequalities

- Central concepts: Lyapunov stability, convex programming.
- Paradigm: multi-objective, selective channel convex optimization and conservatism.
- Objectives:  $H_2$ ,  $H_\infty$ , CL pole-region, etc.
- Unified formulation: LMI (Linear Matrix Inequality) constraints.
- Example (Lyapunov stability)  $\dot{\mathbf{x}} = A\mathbf{x}$  stable iff

 $\exists$  symmetric *P* s.t. *P* > 0, *A<sup>T</sup>P* + *PA* < 0

meaning  $M < 0 \Leftrightarrow \mathbf{q}^T M \mathbf{q} < 0 \forall \mathbf{q} \in \mathbb{R}^n \Leftrightarrow \lambda_{max}(M) < 0$ , i.e. LMI feasibility problem.

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Summary

Dexterous control

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Dexterous control

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**Dexterous control** 

#### Test structure for EXTRAP-T2R spectrum control



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**Classic LQG** 

#### **Overview**

#### Background

- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
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#### Example designs: spectrum reference

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#### Classic LQG

•  $H_{\infty}$ -type loopshaping

**Classic LQG** 

#### Theory

The unstable plant

Example designs: spectrum reference

Reconstructed-state feedback. Solves a quadratic cost functional minimization:

 $J_{Q_1Q_2} = \mathbb{E}\left\{\int (z^T Q_1 z + u^T Q_2 u) dt\right\}$ . Design involves tuning of both KF and LQ. Controller:

$$\begin{cases} u = -L\hat{x} \\ \dot{x} = A\hat{x} + Bu + K(y - C\hat{x}) \end{cases}$$

where

$$\begin{array}{l} AP + PA^T + NR_1N^T - (PC^T + NR_{12})R_2^{-1}(PC^T + NR_{12})^T = 0 \\ K = (PC^T + NR_{12})R_2^{-1} \\ \text{and} \quad \begin{array}{l} A^TS + SA + M^TQ_1M - SBQ_2^{-1}B^TS = 0 \\ L = Q_2^{-1}B^TS \end{array} . \end{array}$$

Modified plant for field-error compensation:

$$\frac{d}{dt} \begin{pmatrix} x \\ z_s \end{pmatrix} = \begin{pmatrix} A & NM^T \\ 0 & -\delta I \end{pmatrix} \begin{pmatrix} x \\ z_s \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + \begin{pmatrix} N \\ I \end{pmatrix} v_1$$
$$z = (M \quad 0) \begin{pmatrix} x \\ z_s \end{pmatrix}$$
$$y = (C \quad 0) \begin{pmatrix} x \\ z_s \end{pmatrix} + v_2$$

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Example designs: spectrum reference

Summary

#### Classic LQG

#### Theory: keeping it compact

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Starting from nominal plant (1), formally augment output:

$$\dot{x} = Ax + Bu + Nv_1, \ \begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} M \\ C \end{pmatrix} x + \begin{pmatrix} 0 \\ l \end{pmatrix} v_2$$
 (3)



$$\dot{x} = \bar{A}x + \bar{B}u, \ \begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} x + \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} u$$
 (4)

emerged: direct-terms C<sub>i</sub>, D<sub>i</sub>.

Augment (4): static field  $x_s$ ,  $\dot{x}_s = 0$  thus

$$\frac{d}{dt}\begin{pmatrix}x\\x_s\end{pmatrix} = \begin{pmatrix}\bar{A} & \bar{N}\\0 & 0\end{pmatrix}\begin{pmatrix}x\\x_s\end{pmatrix} + \begin{pmatrix}\bar{B}\\0\end{pmatrix}u, \ \begin{pmatrix}z\\y\end{pmatrix} = \begin{pmatrix}C_1 & 0\\C_2 & 0\end{pmatrix}\begin{pmatrix}x\\x_s\end{pmatrix} + \begin{pmatrix}D_1\\D_2\end{pmatrix}u$$
(5)

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Example designs: spectrum reference

Summary

#### **Classic LQG**

#### Theory: keeping it compact, contd.



Compute KF for (5) hence form observer

$$\frac{d}{dt}\begin{pmatrix}\hat{x}\\\hat{x}_{s}\end{pmatrix} = \begin{pmatrix}\bar{A} & \bar{N}\\0 & 0\end{pmatrix}\begin{pmatrix}\hat{x}\\\hat{x}_{s}\end{pmatrix} + \begin{pmatrix}\bar{B}\\0\end{pmatrix}u + K(y - C_{2}\hat{x} - D_{2}u)$$
(6)

This step ignores  $(C_1, D_1)$ .

Compute LQ state-feedback gain L for (4) involving || · ||2-cross-terms

$$J_{O_1,O_2} = \mathbf{E} \int \begin{pmatrix} \mathbf{x} \\ u \end{pmatrix}^T \begin{pmatrix} C_1^T Q_1 C_1 & C_1^T Q_1 D_1 \\ \bigstar & D_1^T Q_1 D_1 + Q_2 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ u \end{pmatrix} dt$$
(7)

This step ignores  $(C_2, D_2)$ .

Finding static-gains and sum up control u:

$$L_r = (-\bar{B}^{-1}\bar{A} - L)(C_1 - D_1\bar{B}^{-1}\bar{A})^{-1}$$
(8a)

$$L_s = -\bar{B}^{-1} \tag{8b}$$

$$u = L\hat{x} + L_s\hat{x}_s + L_r r \tag{8c}$$

done. Control system (6) with (8c), inputs: (u, y, r), output  $u_{set}$ .

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The unstable plant

Example designs: spectrum reference

Summary

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**Classic LQG** 

#### Structure



Signal routing for LQG test.

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Classic LQG

#### Result

- 128-state control-system (192 inputs, 64 outputs)
- spectrum sweep-type reference
- model-mismatch, eigenvalue perturbations
- static-field errors, noise everywhere
- fictitious shot: 200ms





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The unstable plant

Example designs: spectrum reference

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Summary

 $H_{\infty}$ -type loopshaping

#### Overview

#### Background

- RFP RWM stabilization
- MHD mode trajectoria
- 2 The unstable plant
  - Physical modeling
  - Generic properties

#### Example designs: spectrum reference

- Signal norms and measures of performance
- Dexterous control
- Classic LQG
- $H_{\infty}$ -type loopshaping

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#### loopsyn theory

Example designs: spectrum reference

Method: observer + normalized coprime factorization synthesis; MATLAB: Robust Control Toolbox; doc loopsyn.

$$\gamma \underline{\sigma} \{ G_{z}(j\omega) F(j\omega) \} \geq \overline{\sigma} \{ G_{d}(j\omega) \} \quad \omega < \omega_{0}$$

$$\gamma \overline{\sigma} \{ G_{Z}(j\omega) F(j\omega) \} \leq \underline{\sigma} \{ G_{d}(j\omega) \} \qquad \omega > \omega_{0}$$

where  $\gamma$  is the minimized  $H_{\infty}$ -performance measure.

#### Strategy:

-	•
	Form plant $G_z : u \to z$ from nominal modeling
	Design parameter: desired loop-shape $G_d$ Typically $G_d(s) \sim rac{\omega_{c,des}}{s}$
	Apply <code>loopsyn</code> to $G_{\rm Z},$ yield control-law K with $H_\infty$ -performance $\gamma$
)	Connect K to an observer (here KF) of z: $\hat{z}$

🕨 Technique similar to hinfsyn & h2syn

 NB: KF important by itself (overlooked/blackboxed by direct output-feedback synthesis)

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# loopsyn applied



Sigma-plot:  $\sigma \{G_z(j\omega)F(j\omega)\}\$ with  $H_\infty$ -bound, in decibels as a function of angular frequency  $\omega$ . Example designs: spectrum reference

Summary

objective: internally z=Mx=r



Control structure: observer + loopshaping *F*.

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#### Result



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Example designs: spectrum reference



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#### Comparison

#### Why complicate?

SVD-decoupled PID-control:

$$\mathbf{u}(t)_{set} = \mathbf{B}^{+} \mathbf{M}^{T} \mathbf{F}_{PID}(s) (\mathbf{D}_{r} \mathbf{r}(t) - \eta \mathbf{M} \mathbf{C}^{+} \mathbf{y}(t))$$
(9)

- $F_{PID}(s)$ , diagonal (10)
  - $D_r$ , diagonal: s.t.  $G_c(s) \sim I$  for  $\omega \sim 0$  (11)

Structure:



Static state estimation.

Example designs: spectrum reference

Summary



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Example designs: spectrum reference

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#### Comparison

#### A last example:

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- Precision: Specifically transient precision significantly improved. Generality: More involved geometries. Heavily constrained actuator coverage. Optimal use of available sensors.
- Mode-coupling: Multi-*m* eigenfunctions, dispersive field diffusion: static estimator insufficient.
  - Physics: Experiments need precision. Studies in plasma dynamics.

Yes, complicate.

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#### Comparison

#### A last example:

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Yes, complicate.

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### Comparison

Precision:	Specifically transient precision
	significantly improved.

- Generality: More involved geometries. Heavily constrained actuator coverage. Optimal use of available sensors.
- Mode-coupling: Multi-*m* eigenfunctions, dispersive field diffusion: static estimator insufficient.
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Yes, complicate.

#### A last example:

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#### Last points





$$G_d(s) = rac{\omega_c}{s} rac{\omega_l + s}{s}, \ \omega_l < \omega_c$$

Feedforward:	Reproducible error-fields. Isolate RWM-control problem.
Quick-solution:	Pre-set observer initial state. Trigger.

Summary

Example designs: spectrum reference

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Example designs: spectrum reference

Summary

# Summary



- Using recent methods for control design
- Model-based; validate and harness
- Extend experimental possibilities
- Some things todo:
  - Implement and experiment
  - Improve modeling
  - Apply to specific tokamak geometries

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Example designs: spectrum reference

Summary

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Example designs: spectrum reference

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Example designs: spectrum reference

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Example designs: spectrum reference

Summary

SQA

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Example designs: spectrum reference

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SQA

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Example designs: spectrum reference

Summary

SQA

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Example designs: spectrum reference

Summary

# Summary

#### Soon: a firm grip on magnetic structure?



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