Models for the Pedestal Temperature at the Edge of H-mode Tokamak Plasmas

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Outline of Poster Presentations

- ❑ **A. H. Kritz's presentation**
  - ➢ **Models for T_{ped} using different width scalings**
- ❑ **T. Onjun's presentation**
  - ➢ **Examine effect of magnetic shear models and geometrical factors on T_{ped} predictions**
  - ➢ **Simulate ρ* scans in DIII-D and JET tokamaks using predictive core together with T_{ped} models**
- ❑ **G. Bateman's presentation**
  - ➢ **Simulations using predictive core and pedestal**
  - ➢ **Predictions of T_{ped} for ITER and FIRE**
  - ➢ **Predictions for performance of ITER and FIRE**

H-mode Pedestal

Objective: **Develop model to predict temperature at top of the pedestal for H-mode plasmas**

Motivation:

- **Boundary condition required for integrated predictive modeling codes**
  - **Needed for predicting performance of present day tokamaks, new experiments and fusion reactor designs**
- **Temperature in plasma core depends on $T_{\mathbf{ped}}$**

Experimental Data

- **Models will be used to predict T_{ped} and compared against the experimental data**
  - **533 data points of type I ELMy H-mode obtained from the International Pedestal Database Version 3.1**

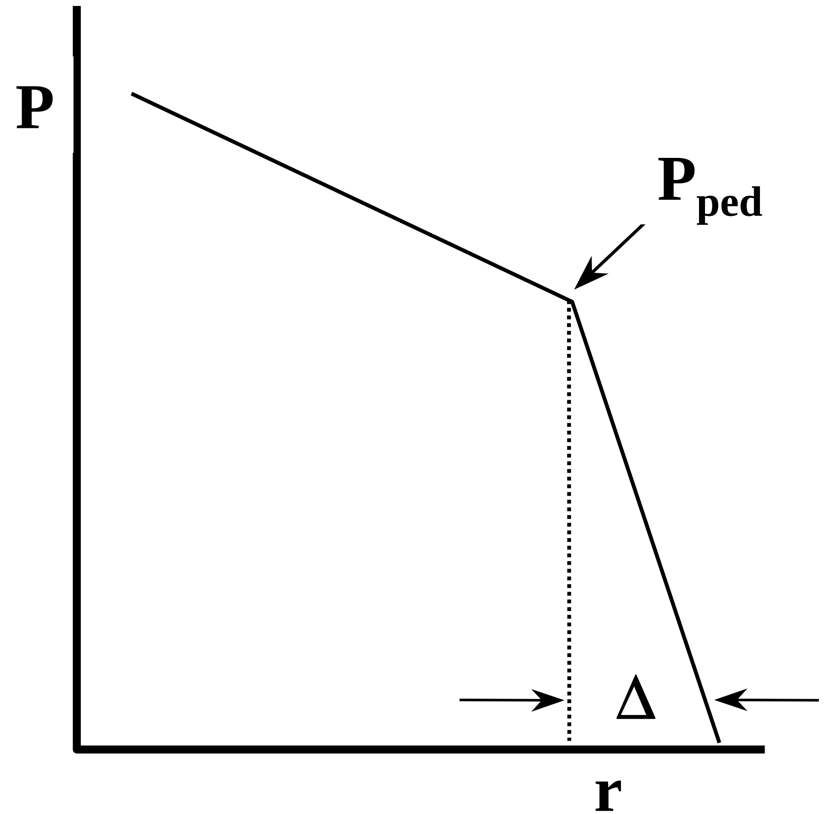
    http://pc-sql-server.ipp.mpg.de/Peddb/

Tokamak	Data points	Pedestal Measurement Method
JT-60U	367	At Psi95
ASDEX-U	105	At 2 cm from separatrix
JET	56	Linear fit
DIII-D	5	Tanh fit

  - **$T_{i,\,ped}$ is used in comparison when it is available**

H-mode Pedestal Temperature

- Assume pressure gradient is constant: $p_{\text{ped}} = (\partial p / \partial r)\Delta$

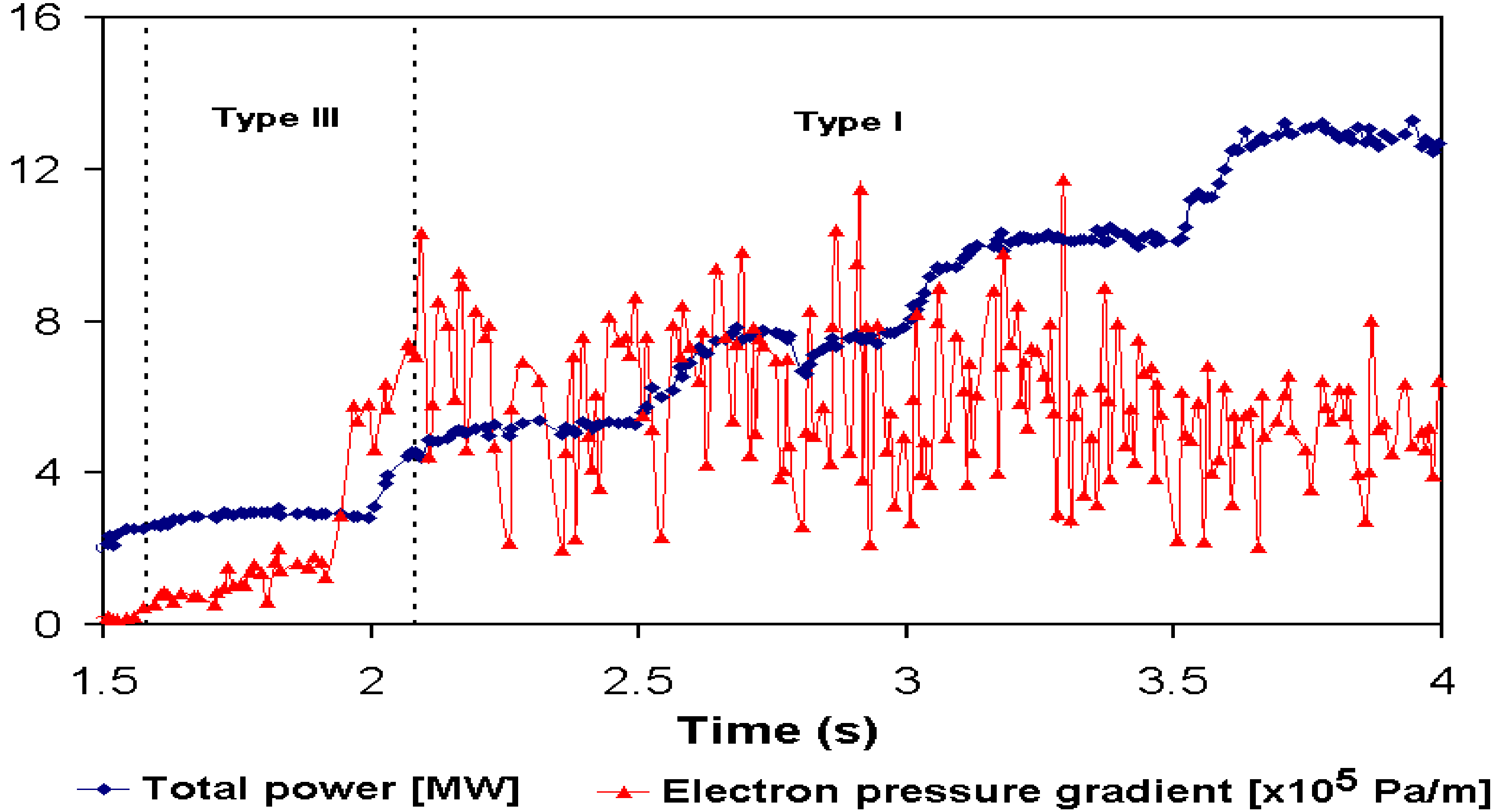


For type I ELMs, the pressure gradient is assumed to be approximately the critical pressure gradient for the ballooning mode instability:

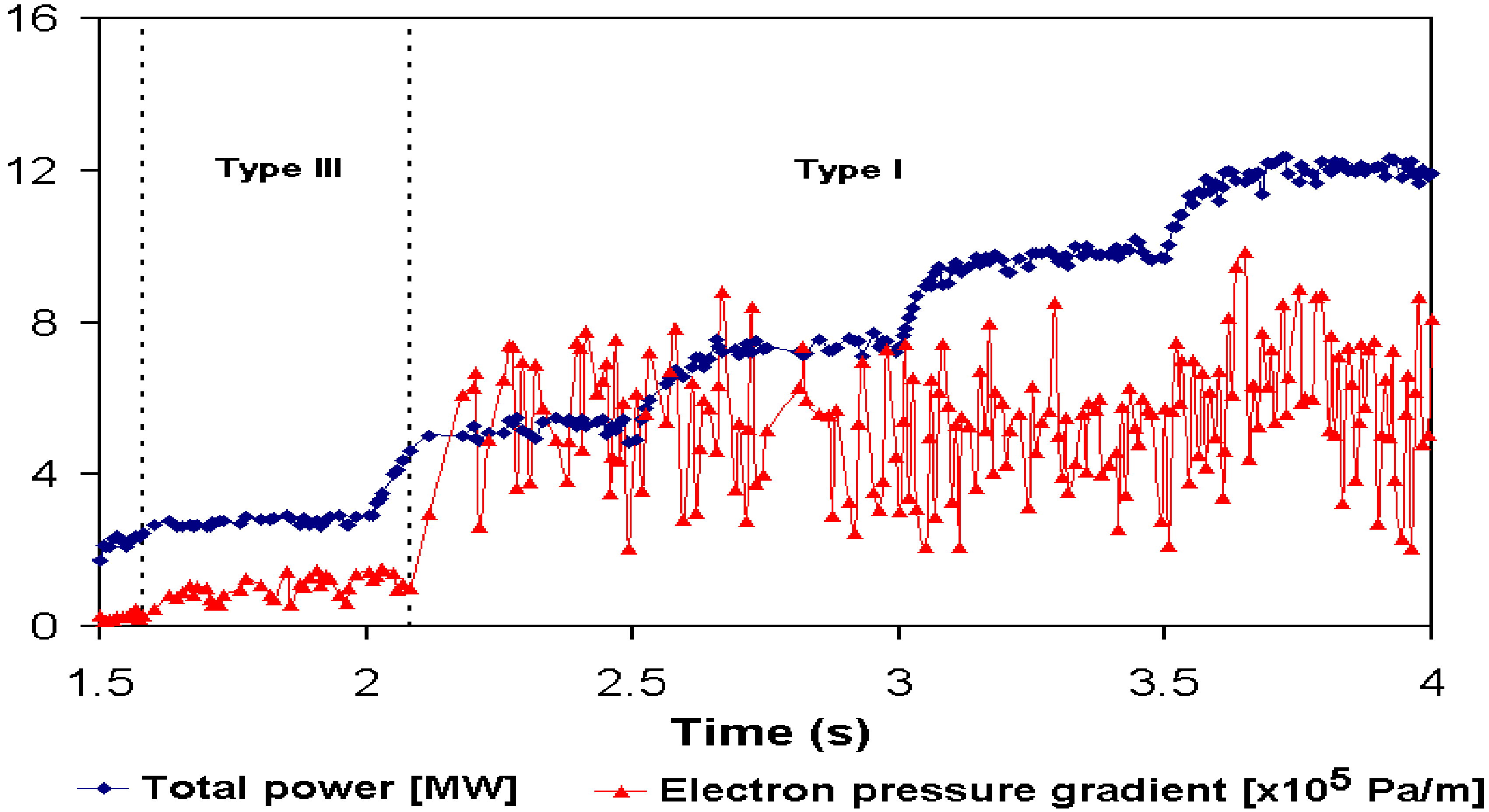
$$(\partial p / \partial r) \approx (\partial p / \partial r)_c$$

$$T_{ped} = \frac{1}{2 n_{ped} k} \Delta \left(\frac{\partial p}{\partial r} \right)_c$$

∇P_{ped} Prior to Type I ELM Crash Does Not Increase with P_{tot} (DIII-D 89733)



∇P_{ped} Prior to Type I ELM Crash Does Not Increase with P_{tot} (DIII-D 90503)



Scaling of Pressure Gradient

- **Critical pressure gradient for the ballooning mode instability is defined as:**

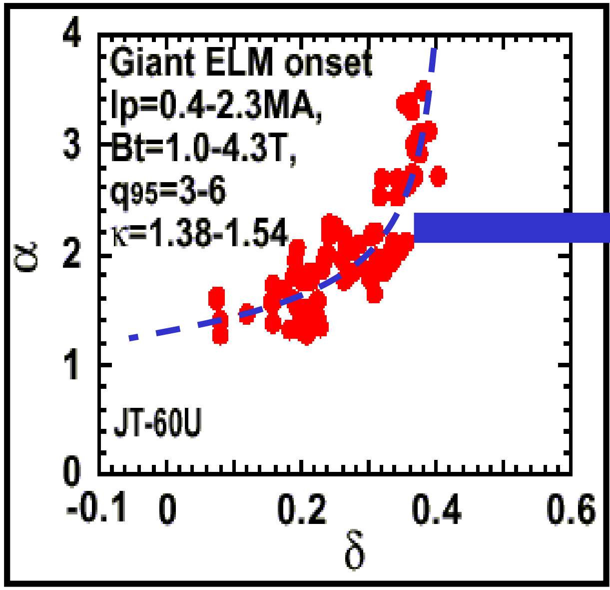
$$\left(\partial p / \partial r\right)_c \equiv -\frac{B^2}{2\mu_0 R q^2}\alpha_c$$

- α_c **is the normalized critical pressure gradient**
  - **usually studied in terms of the** $s-\alpha_c$ **diagram**
  - **limited analytic work on non-circular geometry of local ballooning instability**
  - **plasma more stable with increased elongation and triangularity**

Critical Pressure Gradient

❑ **For circular geometry, the critical pressure gradient is expected to be linearly proportional to magnetic shear in the first stability regime:**

$$\alpha_c = 0.8s$$



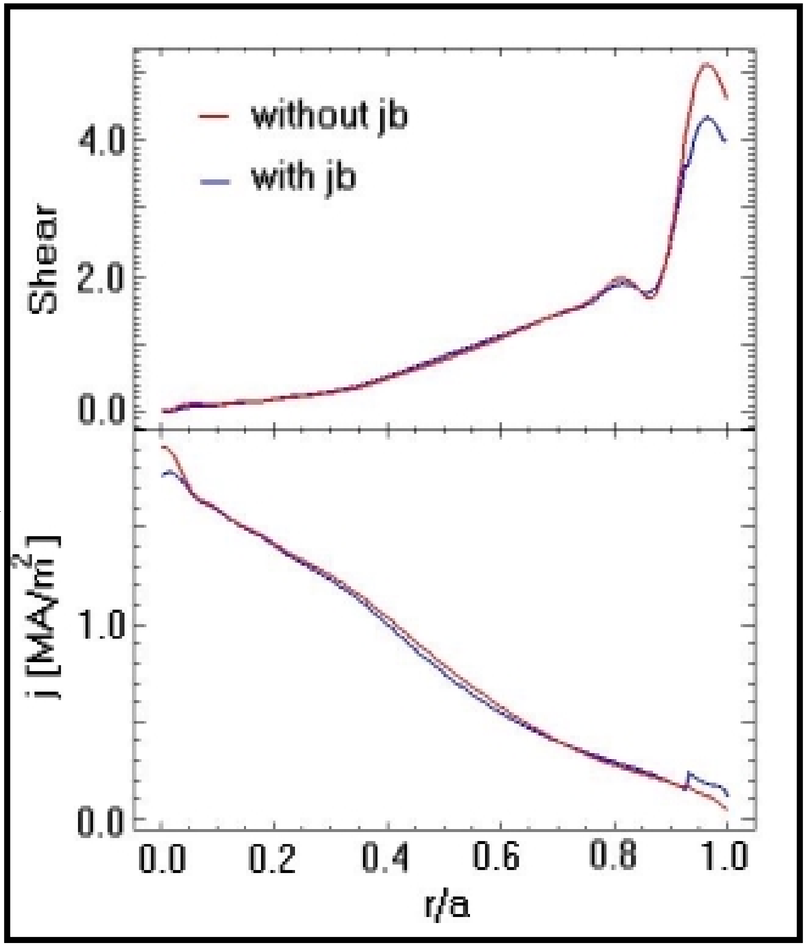
(Y.Kamada, et al., IAEA 1996)

❑ **In terms of magnetic shear and geometric factors, elongation (κ) and triangularity (δ), α_c is given by**

$$\alpha_c = 0.4s(1+\kappa_{95}^{\;2}(1+5\delta_{95}^{\;2}))$$

Additional study of geometrical factor see T. Onjun's poster

Bootstrap Current Effect



- **Bootstrap current is large at the edge due to the steep pressure gradient, which can reduce magnetic shear:**

$j_b \uparrow \quad s \downarrow \quad \alpha_c \downarrow \quad T_{\text{ped}} \downarrow$

- **Magnetic shear, with the bootstrap current included, can be estimated as:**

$$s \equiv \frac{r}{q}\frac{\partial q}{\partial r} \approx s_0\left(1 - \frac{j_b}{j_{tot}}\right)$$

s_0 is shear without j_b

Bootstrap Current Effect on α_c

- **Bootstrap current:**

$$j_b \approx -b(\nu^*, \varepsilon)\frac{\sqrt{\varepsilon}}{B_\theta}\frac{\partial p}{\partial r}$$

- **Modifies magnetic shear, *s*:**

$$s = \frac{s_0\sqrt{\varepsilon}}{\sqrt{\varepsilon} + 0.1 s_0 b(\upsilon^*, \varepsilon)(1+\kappa_{95}^2(1+5\delta_{95}^2))}$$

- **Results in modification of critical pressure gradient:**

$$\alpha_c = \frac{s_0\sqrt{\varepsilon}(1+\kappa_{95}^2(1+5\delta_{95}^2))}{\sqrt{\varepsilon} + 0.1 s_0 b(\upsilon^*, \varepsilon)(1+\kappa_{95}^2(1+5\delta_{95}^2))}$$

Magnetic Shear

- **Magnetic shear is found by solving equations below:**

$$\Delta = a(1-x) = \text{Pedestal width}$$

$$q(x) = \frac{q_{95}(\kappa_{95},\delta_{95},\varepsilon)}{2.933}\left[\left(1+\left[\frac{x}{1.4}\right]^2\right)^2 + 0.267\left|\ln(1-x)\right|\right]$$

$$q_{95} = \frac{5a^2B}{IR}\frac{(1+\kappa_{95}^2(1+2\delta_{95}^2-1.2\delta_{95}^3))(1.17-0.65\varepsilon)}{2(1-\varepsilon^2)^2}$$

$$s = \frac{s_0\sqrt{\varepsilon}}{\sqrt{\varepsilon}+0.1s_0 b(\upsilon^*,\varepsilon)(1+\kappa_{95}^2(1+5\delta_{95}^2))} \qquad s_0 = \frac{x}{q}\frac{\partial q}{\partial x}$$

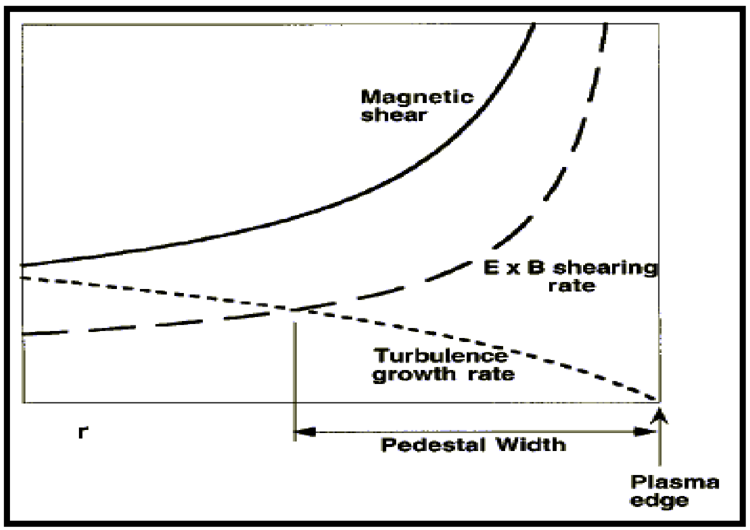
As width increases, magnetic shear decreases

(M. Sugihara, Nuclear Fusion, 40 (2000) 1743)

Width: Flow & Magnetic Shear Stabilization

- **Assume that, at the top of the H- mode pedestal, the shearing rate ($\gamma_{E\times B}$) is equal to the linear growth rate (γ_s). The linear growth rate is chosen to provide a gyro-Bohm type transport and includes the stabilizing effect due to magnetic shear.**

(M. Sugihara, Nuclear Fusion, 40, 1743 (2000))



$$\gamma_{E\times B} \approx \frac{\rho_{\mathrm{tor}} c_s}{\Delta^2} \qquad \gamma_s \approx \frac{c_s}{\Delta}\frac{1}{s^2}$$

$$\Delta \propto \rho_{\mathrm{tor}} s^2$$

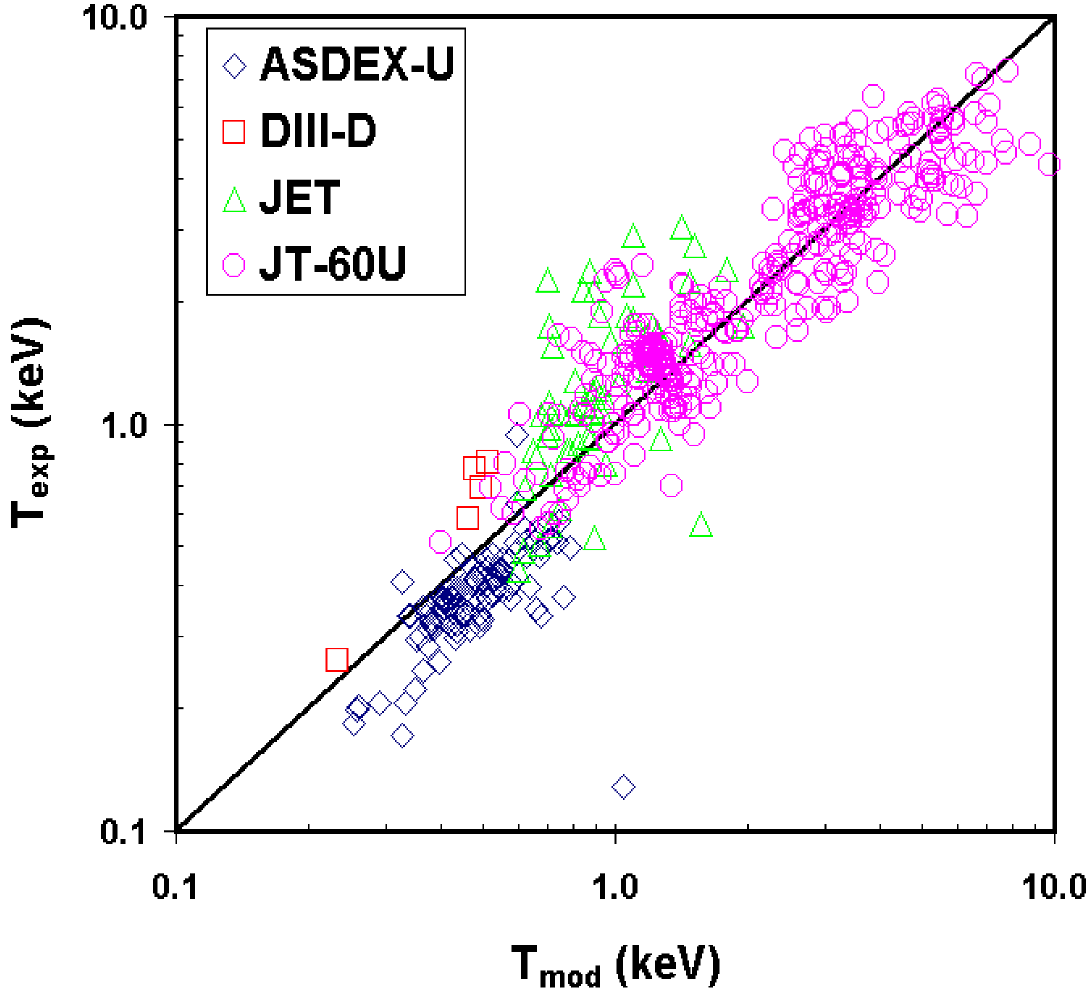
Flow & Magnetic Shear Stabilization

- Using pedestal width based on magnetic and flow shear stabilization ($\Delta \propto \rho s^2$), the pedestal temperature is given by:

$$T_{\text{ped}}[\text{keV}] = C_{\text{w}}^2 (3.231\times 10^{37}) \left(\frac{B}{q(x)^2} \right)^2 \left(\frac{A_H}{R^2} \right) \left(\frac{\alpha_c(x)}{n_{\text{ped}}} \right)^2 s(x)^4$$

- Equation above is a non-linear equation since $q(x)$, $\alpha_c(x)$ and $s(x)$ are functions of the pedestal width Δ which in turn is a function of T_{ped}

H-mode Pedestal Temperature



RMS (%)	
JT-60U	27.7
ASDEX-U	24.9
JET	48.5
DIII-D	39.4
All	32.0
Offset (%)	
JT-60U	3.7
ASDEX-U	-24.4
JET	27.2
DIII-D	32.5
All	0.9

Statistical Analysis

❑ **Use RMSE and offset to quantify agreement between prediction and experimental data**

- **RMSE is defined as**

$$\text{RMSE}(\%) = 100 * \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}[\ln(T_{Exp_i}) - \ln(T_{Mod_i})]^2}$$

- **Offset is defined as**

$$\text{Offset}(\%) = 100 * \frac{1}{N}\sum_{i=1}^{N}[\ln(T_{Exp_i}) - \ln(T_{Mod_i})]$$

Width: Flow Shear Stabilization

- **Assume that ExB suppression of low-*k* modes is relevant for the pedestal. The low-*k* are slower growing but may have large mixing length transport.** **(G. Hammett, Snowmass Meeting 1999)**

$$\gamma_{E\times B} \approx \frac{\rho_{\mathrm{tor}} c_s}{\Delta^2}$$

$$\gamma_{\mathrm{low}\ k} \approx \frac{c_s}{qR}$$

$$\Delta \propto \sqrt{\rho_{\mathrm{tor}} R q}$$

Flow Shear Stabilization

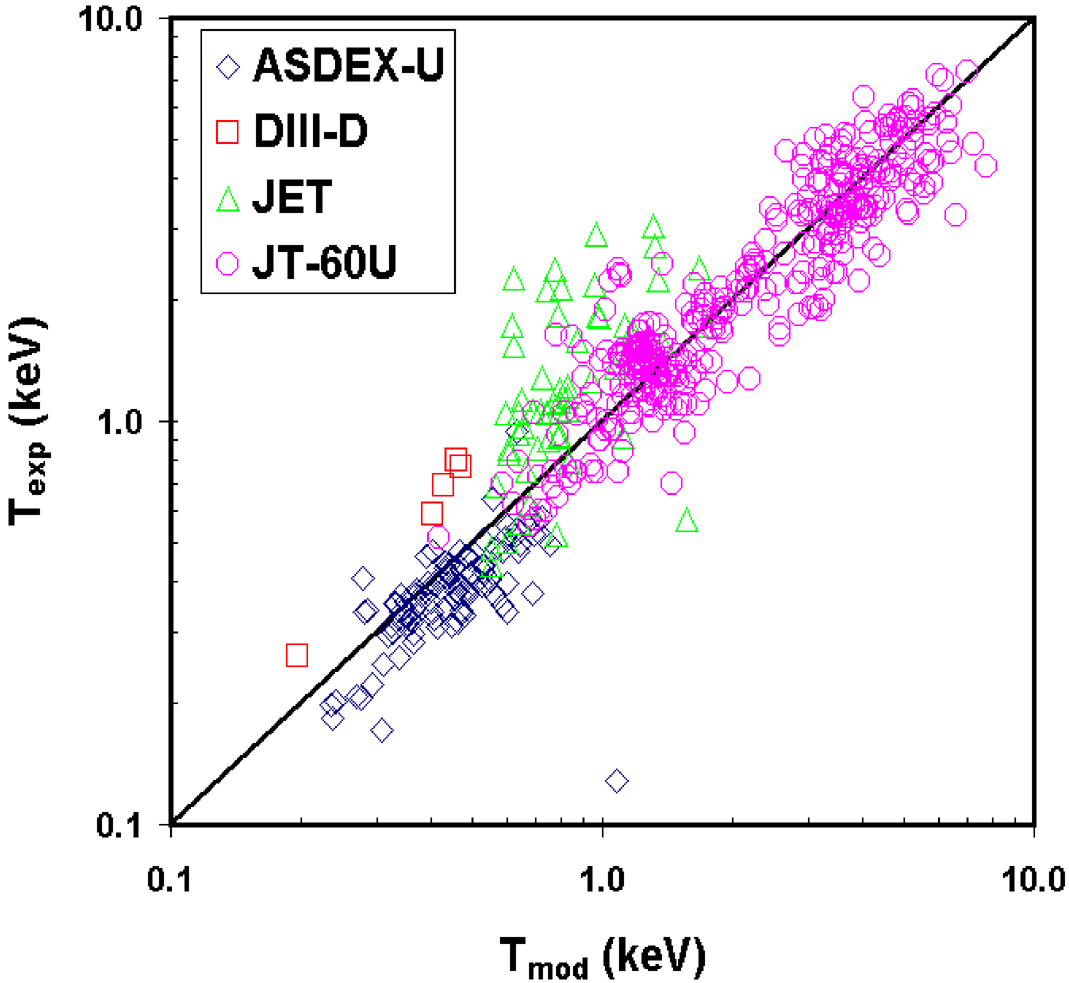
- **Pedestal width based on flow shear stabilization ($\Delta \propto \sqrt{\rho R q}$), yields the nonlinear equation for T_{ped}**

$$T_{\text{ped}} = \frac{B^2}{4\mu_0 n_{\text{ped}} k R q^2} \alpha_c \left[C_{\text{w}} \sqrt{(4.57\times 10^{-3} \frac{\sqrt{A_H T_{\text{ped}}}}{B}) R q} \right]$$

$$T_{\text{ped}} = C_{\text{w}}^{4/3} (3.684\times 10^{26}) \left(\frac{B}{q(x)} \right)^2 \left(\frac{\sqrt{A_H}}{R} \right)^{2/3} \left(\frac{\alpha_c(x)}{n_{\text{ped}}} \right)^{4/3}$$

Non-linear equation: $q(x)$ and $\alpha_c(x)$ are functions of Δ which in turn is a function of T_{ped}

H-mode Pedestal Temperature



RMS (%)	
JT-60U	24.6
ASDEX-U	31.0
JET	56.1
DIII-D	50.5
All	30.8
Offset (%)	
JT-60U	-0.2
ASDEX-U	-16.3
JET	37.3
DIII-D	41.2
All	1.0

Width: Based on Poloidal β

- **Based on DIII-D data, Osborne proposed scaling of the form $\Delta / R \propto (\beta_{\mathrm{POL}}^{\mathrm{PED}})^{1/2}$ based on the assumption that the magnetic well stabilizes edge turbulence**
  (T. H. Osborne, J. Nuclear Materials 1999)

$$\Delta / R \propto (\beta_{\mathrm{POL}}^{\mathrm{PED}})^{1/2} = \left(\frac{4\mu_0 n_{\mathrm{ped}} k T_{\mathrm{ped}}}{< B_\theta >^2}\right)^{1/2}$$

$< B_\theta >$ **is the average poloidal field around flux surface**

$$< B_\theta > \approx \mu_0 I_p / (\pi a (1 + \kappa))$$

T_{ped} Model Based on Poloidal β

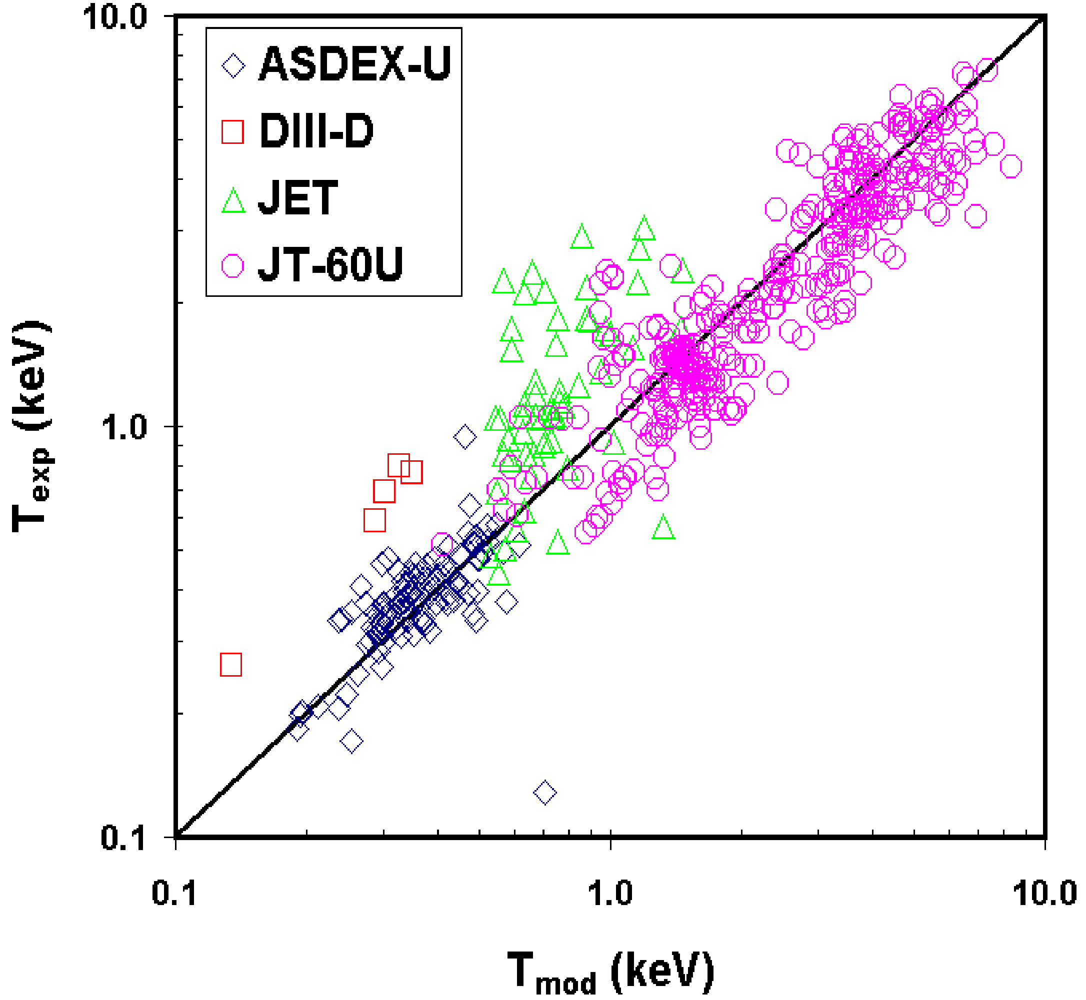
- **Using pedestal width based on a model proposed by Osborne at DIII-D ($\Delta \propto \sqrt{\beta_\theta} R$), T_{ped} is given by**

$$T_{\rm ped} = \frac{B^2}{4\mu_0 n_{\rm ped} k R q^2} \alpha_c \left[C_{\rm w} \sqrt{ (4\mu_0 n_{\rm ped} k T_{\rm ped}) \Big/ \left(\frac{\mu_0 I_p}{\pi a (1+\kappa)} \right)^2 } R \right]$$

$$T_{\rm ped} = C_{\rm w}^2 (1.244 \times 10^{21}) \left(\frac{B}{q(x)} \right)^4 \left(\frac{a\pi(1+\kappa)}{\mu_0 I_p} \right)^2 \left(\frac{\alpha_c(x)^2}{n_{\rm ped}} \right)$$

Non-linear equation for T_{ped}: $q(x)$ and $\alpha_c(x)$ are functions of Δ which in turn is a function of T_{ped}

H-mode Pedestal Temperature



RMS (%)	
JT-60U	27.7
ASDEX-U	25.2
JET	63.2
DIII-D	87.5
All	33.5
Offset (%)	
JT-60U	-8.4
ASDEX-U	4.5
JET	46.4
DIII-D	77.8
All	0.7

Comparison of T_{Mod} and T_{Exp}

Width scaling	RMSE(%)	Physics basis
$\Delta \propto \sqrt{\rho R q}$	30.8	Flow shear stabilization
$\Delta \propto \rho s^2$	32.0	Magnetic and flow shear stabilization
$\Delta \propto \sqrt{\beta_\theta} R$	33.5	Poloidal pressure
$\Delta \propto \rho^{2/3} R^{1/3}$	33.7	Diamagnetic stabilization
$\Delta \propto \sqrt{\varepsilon}\, \rho_\theta$	34.4	Ion orbit loss
$\Delta \propto 1/n_{\text{ped}}$	41.4	Neutral Penetration

Summary-1

- **Developed models for predicting temperature at the top of the pedestal in type I ELMy H-mode plasmas**
  - **Based on models for the width of the pedestal and for the pressure gradient within the pedestal**
  - **Pressure gradient does not depend on power**
- **Considered six theory-motivated scalings for the width of the pedestal**
  - **All the scalings considered yield T_{ped} inversely proportional to the pedestal density**
  - **All models calibrated using 533 experimental data points**
    - **All are type I ELMy H-mode plasma**
    - **From 4 tokamaks (ASDEX, DIII-D, JET and JT-60U)**

Summary-2

- **Pressure gradient within pedestal limited by high-*n* ballooning mode instability**
  - **Magnetic shear and safety factor are calculate at one pedestal away from the separatrix**
  - **Bootstrap current effect is included**
- **Models predict T_{ped} in the range of 30.8% to 41.4% compared with 533 data points**
  - **Model with pedestal with $\Delta \propto \sqrt{\rho R q}$ yields best agreement with experiment (RMSE = 30.8%)**
  - **Five models with different width scalings yield similar agreement (RMSE 30.8% to 34.4%)**
  - **Neutral penetration model can be excluded**