Report on MHD Stability to Resonant Error Fields for SNOWMASS

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I. SUMMARY

Resonant helical "radial" error fields B_{rmn} arising from inevitable toroidal non-axisymmetries in tokamaks due to coil feeds, slight coil misalignments, etc. induce a helical response plasma current $J_{\theta mn}$. The resulting surface averaged toroidal force $\langle J_{\theta mn} \times B_{rmn} \rangle$ at q = m/n acts as a drag on the plasma toroidal rotation. The consequences in a conventional tokamak (q_{min} ~ 1) are bringing the rotation to a halt, "locking" or error field penetration with confinement degradation particularly in low density ohmic startup [1]. In an advanced tokamak (q_{min} >> 1, beta above the no-wall n=1 ideal kink limit) rotation could be reduced below what is needed for resistive wall stabilization of the n=1 kink, a resistive wall mode (RWM) [2].

For the conventional tokamak operation of IGNITOR, FIRE, and ITER, low density ohmic target limits for locking at 0.2 of the Greenwald density are scaled from a model based on DIII-D and JET experiments. IGNITOR and FIRE are susceptible to locked modes at a relative m/n = 2/1 resonant helicity radial field at q = 2 of $B_{r21}/B_{\phi 0} \approx 0.9\times10^{-4}$ and ITER at 0.3×10^{-4}. Note that ITER is a relatively larger extrapolation. The model used is that of "the induction motor" [3] with $J_{\theta 21}$ about 90 deg out of phase with respect to B_{r21} such that no reconnection (and drag) occur except due to a slight deviation from 90 deg arising from resistive skin effects at q=2 for the resistive "slipping" plasma.

Correction and/or minimization of the 2/1 relative error field of 0.3×10^{-4} is tractable considering that the intrinsic error field in JET is $\approx 0.6\times10^{-4}$ [4,5], the best corrected error field in DIII-D is $\approx 0.3\times10^{-4}$ [2] and with careful design and alignment COMPASS achieved as low as 0.1×10^{-4} [6].

Both FIRE and ITER have extensive n=1 error field correction coils in their designs and ITER has done extensive sensitivity studies of the 2/1 error field arising from given possible misalignments.

For the advanced tokamak, the deviation of the phase shift from 90 deg can be large due to the interaction of the m/n = 2/1 error field with a m/n = 2/1 component of the assumed stable ideal kink, "error field amplification" [7]. This tends to decrease the relative critical error field for locking. The need to maintain sufficient plasma rotation for wall stabilization of the ideal kink, $\omega\tau_A \approx 0.02$ at q = 2 or ≈ 0.01 at q = 3, places an even lower limit on $B_{r21}/B_{\phi 0}$ [9].

Benchmarking the model to DIII-D advanced tokamak experiments [2,8], with $q_{min} \lesssim 2$ and $\beta_N/\beta_{N,no\ wall} \approx 1.5$: (1) IGNITOR has no AT mode so is not evaluated, (2) neither FIRE nor ITER have enough rotation for RWM stabilization at any error field without a tangential beam to drive rotation and (3) FIRE and ITER-FEAT would lock at $B_{r21}/B_{\phi 0} \approx 0.2\times10^{-4}$ and 0.1×10^{-4} respectively. Note that the rotation model used for rf heated plasmas is very uncertain. Using tangential neutral beams of 100 kV in FIRE and 500 kV in ITER-FEAT to drive rotation, in order to avoid locking and provide enough rotation for n=1 RWM stabilization was considered. This could be achieved at a tractable $B_{r21}/B_{\phi 0} = 0.3\times10^{-4}$ with 30 MW in FIRE; however only 8 MW is under consideration. With 30 MW in ITER-FEAT (which is a viable option) locking could be avoided at a similar tractable error field but there would still not be enough rotation for wall stabilization of the n=1 ideal kink, the resistive wall mode (RWM).

II. CONVENTIONAL TOKAMAK

The error field torque in the toroidal direction on a “slipping” plasma at q = m/n is

$$T_\phi \approx -R_0\left(\frac{B_{rmn}^2}{\mu_0}\right)\left(\frac{r_{mn}}{qR_0}\right)\frac{2\pi R 2\pi r_{mn}}{\omega\tau_{rec}} \tag{1}$$

where $\omega\tau_{rec} >> 1$ is the slipping condition [3]. Helical “radial” components on a q = m/n surface scale as $B_{rmn} \propto r^{m-1}$ from external coils and sources where r is the minor radius. Thus, the B_{r21} from m/n = 2/1 gives the greatest drag having the least decay (low m) and largest surface area to act on (larger r_{mn}).

Locking occurs when $\omega/\omega_0 = 1/2$ [3,5] where ω_0 is the angular rotation frequency of the plasma at q = 2 in the absence of resonant error field. A critical condition for locking is given by

$$\frac{B_{r21}}{B_{\phi 0}} \approx \frac{\omega_0\tau_A}{2}\left(\frac{\tau_{rec}}{\tau_v}\right)^{1/2} \tag{2}$$

where τ_A is the Alfvén time and τ_{rec} is the visco-resistive reconnection time (with τ_R the resistive time and τ_v the viscous time). All definitions are given in Appendix A. The rotation frequency is assumed to be $\omega_0 \approx \omega_{*e}$ the electron diamagnetic drift frequency in the absence of rotation drive [1]. The sensitive regime is the ohmic startup at low density. Assumptions are that the line-averaged density is 20% of the Greenwald density, energy confinement time τ_E is neo-Alcator, the viscous time $\tau_v \equiv 4\ \tau_E$ with a profile factor included and the classical tearing stability parameter $\Delta'_0 r_{21} \approx -4$ is assumed.

Evaluations using the model of Eq. (2) are given in Table I. Experimental comparisons of DIII-D and JET are discussed further in Ref. [10]. Note that the phase shift deviation of $\lesssim 1$ deg is very small in all cases so the slipping regime is relevant.

TABLE I: CONVENTIONAL TOKAMAK

Relative critical m/n = 2/1 resonant error field for locking in ohmic "target" discharges in I_p flattop at G = $\bar{n}_{20}\ \pi a^2 / I_p$ = 0.2 with neo-Alcator confinement. All quantities evaluated at q = 2. Note $\Delta'_0\, r_{21} \equiv -2\, m = -4$, parabolic n_e, parabolic squared T_e and deuterium assumed.

	DIII–D*	JET†	IGNITOR‡	FIRE$^\Delta$	ITER§
$\omega_0 \equiv \omega_{*e}$ (10^4 rad/s)	0.83	0.26	0.96	0.72	0.082
τ_A (μs)	0.34	0.32	0.11	0.16	0.35
τ_R (s)	0.54	2.4	6.2	4.0	58
τ_E (s)	0.030	0.091	0.22	0.21	0.96
$\tau_v \equiv 4\ \tau_E$ (s)	0.12	0.36	0.86	0.84	3.8
τ_{rec} (s)	0.0063	0.018	0.024	0.019	0.17
$\tan^{-1} (\omega_0\tau_{rec})^{-1}$ (deg)	1.1	1.2	0.2	0.4	0.4
$B_{r21}/B_{\phi 0}$ (10^{-4})	3.0	0.9	0.9	0.9	0.3

*$B_{\phi o}$ = 1.3 T, I_p = 1.0 MA, R_0 = 1.7 m, a = 0.6 m, LSND, q_{95} = 3.5, $\bar{n}_{20}$ = 0.18.

†$B_{\phi o}$ = 2 T, I_p = 2 MA, R_0 = 3.0 m, a = 1.0 m, LSND, q_{95} = 3.5, $\bar{n}_{20}$ = 0.13.

‡$B_{\phi o}$ = 13 T, I_p = 11 MA, R_0 = 1.32 m, a = 0.47 m, LSND, q_{95} = 3.6, $\bar{n}_{20}$ = 3.2.

$^\Delta B_{\phi o}$ = 8.5 T, I_p = 5.7 MA, R_0 =2.0 m, a = 0.53 m, DND, q_{95} = 3.5, $\bar{n}_{20}$ = 1.3.

§$B_{\phi o}$ = 5.3 T, I_p = 15 MA, R_0 = 6.2 m, a = 2.0 m, LSND, q_{95} = 3.8, $\bar{n}_{20}$ = 0.24.

III. ADVANCED TOKAMAK

The n=1 error field torque is greatly increased near an n=1 ideal stability limit [7,11]. A DIII–D advanced tokamak is studied experimentally above the n=1 ideal kink no-wall beta limit with 1.5 < q_{min} < 2, $q_{95} \approx 3.5$, $\beta_{N,\,ideal\,wall} > \beta_N > \beta_{N,\,no\,wall}$, and $\beta_N/\beta_{N,\,no\,wall} \sim 1.5$ [2]. Empirical fits to Eq. (2) are for $\omega/\omega_0 = 1/2$, with locking at a relative error field of

$$\frac{B_{r21}}{B_{\phi 0}} = 0.009\left(\frac{\omega_0\,\tau_A}{2}\right)\left(\frac{\tau'_{rec}}{\tau_v}\right) \tag{3}$$

with $\tau'_{rec} \equiv \tau_R^{5/6}\,\tau_A^{1/3}/\tau_v^{1/6}$ and $\Delta'_0\,r_{21}/4 \approx -1\times 10^{-4}$ found empirically (compared to ≈–1 in the ohmic conventional tokamak). For RWM stabilization the empirical fit is

$$\frac{B_{r21}}{B_{\phi 0}} = 0.009\left[\omega_0\,\tau_A\,\omega_c\,\tau_A - \left(\omega_c\,\tau_A\right)^2\right]^{1/2}\left(\frac{\tau'_{rec}}{\tau_v}\right)^{1/2} \tag{4}$$

with $\omega_c\,\tau_A$ for n=1 ideal kink RWM stabilization. Additional definitions are given in Appendix B.

Note that the DIII-D AT experiments with $q_{min} > 2.5$ are also very sensitive to rotation decay with $B_{r21}/B_{\phi 0}$ despite $q_{min} > 2$ [Garofalo (2002 DIII-D campaign just completed)] so that the induction motor model should not be relevant. Theoretical evaluation of the drag of the non-resonant error field that has a component of the stable n=1 ideal kink eigenmode needs to be evaluated and is not done here.

Evaluations using the model given in Eq. (3) for locking and (4) for RWM stabilization are given in Table II. The assumption of $\omega_0 \approx \omega_{e*}$ for rf heated discharges in FIRE and ITER-FEAT may be too simplistic.

TABLE II: ADVANCED TOKAMAK

Relative critical m/n = 2/1 resonant error field for locking or RWM in $1.5 < q_{min} < 2$ discharges with $\beta_{N,\ no\ wall} < \beta_N < \beta_{N,\ ideal\ wall}$. High dissipation regime assumed with $\omega_c\ \tau_A$ at q=2 ≡ 0.019 from DIII–D experiments.

	DIII–D*	JET†	IGNITOR‡	FIRE$^{\Delta}$	ITER-FEAT§
ω_0 (10^4 rad/s)	5.0	–	–	3.2	0.5
τ_A (μs)	0.4	–	–	0.34	0.70
$\omega_0\ \tau_A$	0.020 > 0.019	–	–	0.011 < 0.019	0.0035 < 0.019
τ_R (s)	18	–	–	14	2000
τ_E (s)	0.12	–	–	0.30	3.0
τ_v (s)	0.24	–	–	0.60	6.0
τ'_{rec} (s)	0.11	–	–	0.072	3.8
$\tan^{-1}\left[\left\lvert\frac{\Delta'_{0\ 21} r}{4}\right\rvert(\omega_0\ \tau'_{rec})\right]^{-1}$ (deg)	61	–	–	77	28
$B_{r21}/B_{\phi 0}\ (10^{-4})\rvert_{RWM}$	0.3	–	–	No RWM stab.	No RWM stab.
$B_{r21}/B_{\phi 0}\ (10^{-4})\rvert_{locking}$	0.6	–	–	0.2	0.1

*$B_{\phi o}$ = 2.1 T, I_p = 1.6 MA, R_0 = 1.7 m, a = 0.6 m, LSND, q_{95} = 3.6, 9.5 MW tan beams, $\bar{n} = 0.4\ \bar{n}_{GR} = 0.53\times10^{20}\ m^{-3}$.

†Experiments planned for 2003 campaign.

‡No AT operation planned.

$^{\Delta}B_{\phi o}$ = 8.5 T, I_p = 5.4 MA, R_0 =2.0 m, a = 0.53 m, DND, 34 MW rf, $\bar{n} = 0.65\ \bar{n}_{GR} = 4.0\times10^{20}\ m^{-3}$, q_{95} = 3.7, q_{min} = 1.4.

§$B_{\phi o}$ = 5.3 T, I_p = 10 MA, R_0 = 6.2 m, a = 1.86 m, 35 MW rf, q_{95} = 4.6, q_{min} = 1.6, $\beta_N/4\ \ell_i$ = 1.5, $\bar{n} = 1.0\ \bar{n}_{GR} = 0.9\times10^{20}\ m^{-3}$.

Both FIRE and ITER-FEAT are considering the use of tangential beams to drive rotation which could avoid locking and/or n=1 RWM at tractable 2/1 error field levels. The requirement for <u>locking</u> is

$$\omega_0\ \tau_A = 200\left(\frac{B_{r21}}{B_{\phi 0}}\right)\left(\frac{\tau_v}{\tau'_{rec}}\right)^{1/2} \quad (5)$$

and for loss of <u>RWM stabilization is</u>

$$\omega_0\,\tau_A = \frac{\left[100\left(\frac{B_{r21}}{B_{\phi 0}}\right)\left(\frac{\tau_v}{\tau'_{rec}}\right)^{1/2}\right]^2}{\omega_c\,\tau_A} + \omega_c\,\tau_A \quad . \tag{6}$$

Note that $B_{r21}/B_{\phi 0} \approx 6\times10^{-5}$ is the JET intrinsic error field and the DIII–D best corrected is $\approx 3\times10^{-5}$. Note also that $\omega_c\tau_A \equiv 0.019$ at q = 2 from DIII–D [La Haye (2002 campaign just completed)]. The tangential beam requirements to avoid locking and/or to avoid a RWM for a tractable $B_{r21}/B_{\phi 0} \equiv 0.3\times10^{-4}$ are given in Table III. Note that FIRE is considering 8 MW of tangential beams.

TABLE III: ADVANCED TOKAMAK TAN BEAM REQUIREMENTS ASSUMING $B_{r21}/B_{\phi 0} = 3\times10^{-5}$ AND $\omega_c\tau_A = 0.019$

	IGNITOR	FIRE	ITER-FEAT
$\omega_0\,\tau_{A,\,lock}$	—	0.017	0.008
$\omega_0\,\tau_{A,\,RWM}$		0.023	0.020
E_{beam} (kV)	—	100	500
$P_{tan\ beam}$ (MW)	—	33 WM ND OCK	28 OCKING NLY
Actual P (MW) nominally all rf	—	34	35

IV. REFERENCES

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APPENDIX A: DEFINITIONS

All at q = 2 unless global, deuterium assumed, MKS units.

$$\tau_A = \frac{R_0}{B_{\phi 0}} \left(\mu_0 m_i n_e\right)^{1/2}$$

$$\omega_0 = \omega_{*e} = \frac{2\,T_e}{r\,L_{pe}\,B_{\phi 0}}$$

$$L_{pe} = \frac{1}{6}\,\frac{a^2}{r}\left(1 - \frac{r^2}{a^2}\right)$$ for n_e par[1] and T_e par[2] for Ohmic plasma, r/a ≡ 0.75 for q = 2

$$\tau_E = 9.5 \times 10^{-20}\ \bar{n}^{0.90}\ a^{0.90}\ R_0^{1.63}$$

$$T_e\,(\mathrm{eV}) = 4.2 \times 10^6 \left(\frac{\tau_E}{\bar{n}}\right)^{2/5} \left(\frac{I_p}{\kappa\pi a^2}\right)^{4/5} \left(1 - \frac{r^2}{a^2}\right)^2$$

$$\tau_R = \frac{\mu_0 r^2}{\eta},\ \eta = 7.8 \times 10^{-4}\ T_e\,(\mathrm{eV})^{-1.5}$$

$$\tau_v \equiv 4\,\tau_E$$

$$\tau_{rec} = \left(\frac{-\Delta_0' \, r}{4}\right) \tau_R^{5/6}\ \frac{\tau_A^{1/3}}{\tau_v^{1/6}}\ ,\ \left|\frac{-\Delta_0\, r}{4}\right| \equiv 1 \text{ for } \Omega \text{ plasma}$$

APPENDIX B: ADDITIONAL DEFINITIONS FOR AT

$\tau_E = 0.9 * 0.106\, I_p^{1.03}(MA)\, P^{-0.46}(MW)\, R^{1.48}(m)$ is 90% of JET/DIII–D ELM-free H–mode

$\tau_v \equiv 2\,\tau_E$

r/a ≡ 0.60 for q=2 unless provided

ω_0 = as measured for DIII–D with tangential beams driving toroidal rotation

$\omega_0 \equiv \omega_{*e}$ in absence of rotation input

$\omega_0 = \frac{2\,\tau_E}{\bar{n}\, m_i\, R_0} \left\{ \frac{P_b / Vol}{\left(2\, k_B\, E_b / m_D\right)^{1/2}} \left(1 - r^2 / a^2\right) \right\}$ for tan beams of energy E_b, power P_b,

$Vol = 2\pi R_0 \kappa \pi a^2$