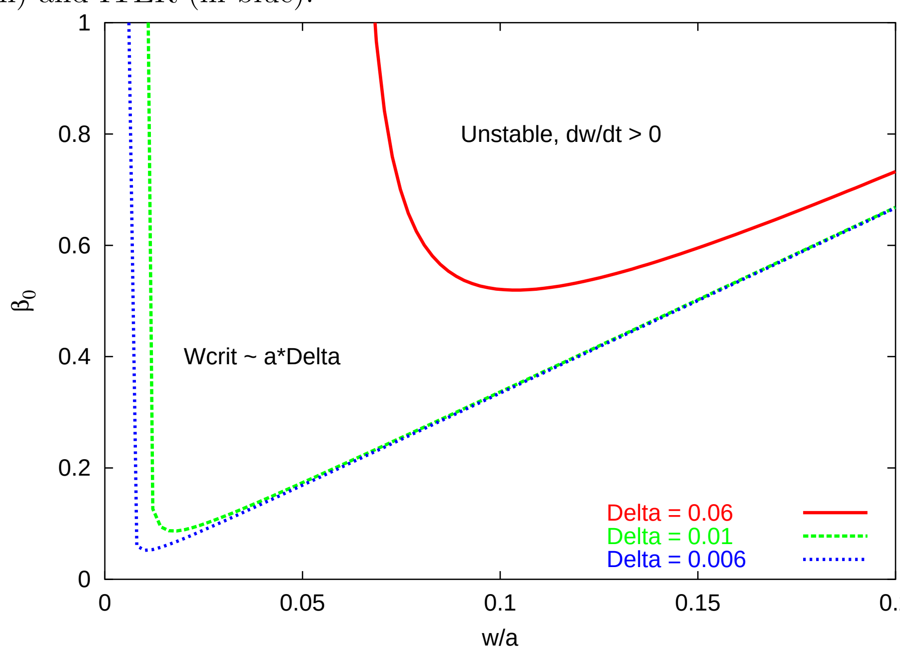
Neoclassical tearing modes in burning plasma experiments

C. C. Hegna, University of Wisconsin, Madison, WI 53706-1687

Neoclassical tearing instabilities are susceptible to excitation at sufficiently high β [1-5]. For plasma parameters characteristic of those envisioned for FIRE and ITER, a large enough seed perturbation will excite neoclassical tearing modes (NTMs). The critical β for NTM excitation scales empirically with $\rho*$ [6-8]. For parameters of interest the characteristic $\rho*$ for ITER and FIRE is smaller than present day devices by a factor of order 3-10 with corresponding threshold islands that are $\leq 1cm$. Conversely, the saturated island widths scale with β_θ and do not depend upon $\rho*$. If uncontrolled, neoclassical tearing modes will produce large saturated islands that can lead to locked modes and disruptions. The following island stability plot indicates the characteristic threshold and saturated island widths as functions of β_θ using parameters of relevance to DIII-D (in red), FIRE, Ignitor (both in green) and ITER (in blue).



The above stability curve is generated from a modified Rutherford equation of the form [9]

$$\frac{dw}{dt} = 1.22\frac{\eta}{\mu_o}(\Delta' + k_1\sqrt{\epsilon}\beta_\theta\frac{L_q}{L_p}[\frac{w}{w^2+w_d^2} - \frac{\Delta^2 a^2}{w^3}])$$

with $\Delta = \sqrt{\epsilon}(\rho_{i\theta}/a)\sqrt{L_q/L_p}\ g(\epsilon, \nu_i/\epsilon\omega*)$. For sufficiently large β_θ, the nonlinear stability curves intersect at two values of the magnetic island width. The lower value indicates the threshold island width. Islands initiated at widths smaller than the threshold width are not excited. The larger value indicates the saturated island width. The island width at which NTMs saturate scales with the local bootstrap current ($w_{sat} \sim \sqrt{\epsilon}\beta_\theta$). The characteristic parameters for the three burning plasmas devices used to generate this plot are calculated in the following.

Since Ignitor is envisioned to operate at low plasma β, it should not be limited by neoclassical tearing modes. However, small neoclassically driven islands may appear. Accordingly, Ignitor will have a limited ability to address neoclassical tearing mode physics.

With the likelihood that neoclassical tearing modes will be excited, methods should be available to control them. Localized electron cyclotron current drive has proven very successful on DIII-D, AUG and JT-60U [10-12]. ITER plans to use this method as well and has allowed for sufficient ECCD to affect neoclassical islands. FIRE has tentatively proposed using LHCD as a method to control magnetic islands. There is as yet no experimental verification that localized LHCD can control neoclassical tearing modes. Additionally, it is problematic to produce and localize LHCD at a desired magnetic surface as part of a feedback stabilization scheme. However, LHCD has been used on COMPASS-D to affect islands by altering the current profile and hence Δ' for the $m/n = 2/1$ mode [13]. It is an open question whether this technique can be effective in a burning plasma experiment.

In the following, we estimate various key neoclassical tearing mode parameters. Estimates for characteristic $m/n = 2/1$ and $m/n = 3/2$ islands are given using analytic theory. For simplicity, we assume the following profiles,

$$q = 1 + 2\frac{r^2}{a^2}$$

$$p = p_o(1 - \frac{r^2}{a^2})$$

which define $q(r/a = \sqrt{0.5}) = 2$ and $q(r/a = 0.5) = 1.5$ and $p_o/\langle p\rangle = 2$. Additionally, the density and temperature profiles are given by $n/n_o = T/T_o = \sqrt{1 - r^2/a^2}$ which yield $n_o/\langle n\rangle = T_o/\langle T\rangle = 1.5$.

Key parameters to be calculated are the saturated island width [14,15]

$$\frac{w_{sat}}{a} = k_1 \frac{\sqrt{\epsilon}\ \beta_\theta}{(-\Delta' a)} \frac{L_q}{L_p},$$

the characteristic length of the threshold island using the polarization model [16]

$$\Delta a = \sqrt{\epsilon}\rho_{i,\theta}\sqrt{\frac{L_q}{L_p}}\ g(\epsilon, \nu_i/\epsilon\omega *),$$

the characteristic amplitude of the threshold island using the diffusion model [17,18] (the "collisionless" free streaming limit is used to estimate the parallel heat conduction)

$$w_d = (\frac{\chi_\perp R q L_q}{m n v_{te}})^{1/3},$$

and the collisionality parameter from the neoclassical polarization model [16]

$$\frac{\nu_i}{\epsilon\omega *},$$

where the factor g in the polarization threshold model asymptotes to unity in the limit $\nu_i/\epsilon\omega * << 1$. The characteristic threshold island width is taken to be the larger of $\sqrt{\epsilon}\ \rho_{i,\theta}\sqrt{L_q/L_p}$ and w_d.

Inserting numbers into these expressions for 2/1 and 3/2 modes accounting for profile effects yields,

$$\Delta|_{q=2} \approx 3.46 \times 10^{-4}\sqrt{\frac{R}{a}}\frac{\sqrt{\langle T\rangle}}{aB}, \quad \Delta|_{q=1.5} \approx 2.42 \times 10^{-4}\sqrt{\frac{R}{a}}\frac{\sqrt{\langle T\rangle}}{aB}$$

$$\frac{w_d}{a}|_{q=2} \approx 7.4 \times 10^{-3}\frac{R^{1/3}}{\tau_E^{1/3}\langle T\rangle^{1/6}}, \quad \frac{w_d}{a}|_{q=1.5} \approx 5.8 \times 10^{-3}\frac{R^{1/3}}{\tau_E^{1/3}\langle T\rangle^{1/6}}$$

$$\frac{\nu_i}{\epsilon\omega *}|_{q=2} \approx 2.38 \times 10^{-13}\frac{\langle n\rangle aRB}{\langle T\rangle^{5/2}}, \quad \frac{\nu_i}{\epsilon\omega *}|_{q=1.5} \approx 2.48 \times 10^{-13}\frac{\langle n\rangle aRB}{\langle T\rangle^{5/2}}$$

where a and R are given in units of meters, B in Tesla, T in eV, n in m^{-3} and confinement time $\tau_E = a^2 n/4\chi_\perp$ in seconds. Additionally, the estimate $k_1 = 2$, $\Delta' r|_{q=2} = -4$, and $\Delta' r|_{q=1.5} = -3$ is used to obtain

$$\frac{w_{sat}}{a}|_{q=2} \approx 0.53(\frac{a}{R})^{0.5}\beta_\theta, \quad \frac{w_{sat}}{a}|_{q=1.5} \approx 0.35(\frac{a}{R})^{0.5}\beta_\theta,$$

where β_θ refers to the volume-averaged pressure relative to the edge poloidal magnetic field.

The following factors are used for each device to estimate the neoclassical tearing mode

Device	$a(m)/R(m)$	$B(T)$	β_θ	$\langle T(keV)\rangle$	$\langle n(10^{20}m^{-3})\rangle$	$\tau_E(s)$
ITER	2.0/6.2	5.3	0.65	10	1.0	3.7
FIRE	0.6/2.14	10	0.80	10	5.0	1.0
Ignitor	0.47/1.32	13	0.20	10	9.5	0.6

Taking these numbers as ballpark figures, one obtains for $m/n = 2/1$

	w_{sat}/a	$w_{sat}(cm)$	Δ	$\Delta a(cm)$	w_d/a	$w_d(cm)$	$\nu_i/\epsilon\omega*$
ITER	0.20	39 cm	0.0057	1.1 cm	0.0019	0.38 cm	0.16
FIRE	0.22	13.5 cm	0.011	0.65 cm	0.0020	0.12 cm	0.15
Ignitor	0.063	3.0 cm	0.0095	0.42 cm	0.0020	0.097 cm	0.18

and for $m/n = 3/2$

	w_{sat}/a	$w_{sat}(cm)$	Δ	$\Delta a(cm)$	w_d/a	$w_d(cm)$	$\nu_i/\epsilon\omega*$
ITER	0.13	26 cm	0.0040	0.80 cm	0.0015	0.30 cm	0.16
FIRE	0.15	8.9 cm	0.0076	0.46 cm	0.0016	0.097 cm	0.16
Ignitor	0.042	2.0 cm	0.0066	0.31 cm	0.0016	0.076 cm	0.19

For all the devices, the polarization model predicts a larger threshold island than the anisotropic transport model. Additionally, all of the devices are in the small collisionality limit with relation to the polarization island threshold model. Growth times for NTMs can be estimated by $\tau_g \approx (w^2/k_1 D_{nc})(\mu_o/\eta) \approx 0.2(w/a)\tau_R$ where τ_R is the resistive diffusion time. This yields $\tau_g/\tau_R = 0.02 - 0.03$ as the timescales of interest.

An isolated magnetic island chain degrades the global energy confinement by decreasing the effective volume of the plasma which exhibits normal thermal transport. The Chang-Callen model [19] quantifies the confinement reduction through the equation $\tau_E \approx \tau_{Eo}(1 - 4r_s^3 w/a^4)$ where τ_{Eo} is the energy confinement time in the absence of the island and τ_E is the energy confinement time when an island of width w is present at the rational surface r_s. Using the previously calculated numbers, estimates for the confinement time reductions resulting from single island chains are given by

	$\tau_E/\tau_{Eo}\|_{q=2}$	$\tau_E/\tau_{Eo}\|_{q=1.5}$
ITER	0.71	0.935
FIRE	0.69	0.925
Ignitor	0.91	0.98

In addition to confinement reduction, large magnetic islands can cause mode locking to external magnetic structures, loss of H-mode and disruptions.

There is not sufficient understanding of the seed island formation physics to make quantitative predictions for FIRE and ITER. Sawteeth are known to excite NTMs. Previous calculations modeled the seed island formation process as a forced reconnection problem due to a dynamically growing external source [20]. For fast growing sources, a shielding factor due to plasma resistivity in the resistive layer reduces the amplitude of the seed island. This suggests that neoclassical tearing modes may be more difficult to excite in high temperature tokamaks since the shielding factor depends on the Lundquist number. Noting that the typical Lundquist numbers for the three burning plasma experiments is expected to be larger ($S \sim 10^8 - 10^9$) than in present devices ($S \sim 10^6 - 10^7$), this would be a beneficial effect. However, there is not sufficient experimental evidence to support such a model [21] and as such, it is difficult to assess the seed island formation problem. Also, at such high values of Lundquist number, kinetic effects might become important and yield different scalings.

Another important aspect of the seed island formation problem is the nature of the source for the seed island. An alpha particle population can provide a stabilizing mechanism for sawteeth. This effect will likely lead to larger sawteeth when they are excited, and hence produce larger magnetic island seeds. However, there has been some success at controlling seed island formation on JET through manipulation of the current profile near q = 1 [22]. Additionally, the appearance of relatively benign MHD modes near the axis (fishbones on AUG, m/n = 3/2 modes on DIII-D [23]) have prevented large sawteeth by altering the evolution of the q-profile. The effectiveness of using techniques to control neoclassical tearing mode stability via current profile control in burning plasma experiments is an open question.

This discussion has focused on the growth, effects and control of a single NTM. For plasmas that plan to operate at β values well in excess of the critical β for more than one rational surface, it may be that multiple NTMs will be exited. In present experiments, sometimes a couple of NTMs are present simultaneoulsy, usually transiently. There is some evidence that one mode (usually that with the lowest mode number) dominates ultimately. Since FIRE and ITER propose to operate at high β, it may be desirable to have control systems that can handle multiple NTMs.

An additional physics feature of the ECCD stabilization scheme proposed for ITER which differs from the methods used on AUG and DIII-D is the role of the stabilizing neoclassical polarization effect. For parameters relevant to ITER, the characteristic size of the polarization term is small compared to the current localization layer while these two scale sizes are comparable in present day devices. Consequently, the polarization stabilization allows for complete stabilization using ECCD in present devices but will play little role in ITER. Therefore the islands in ITER can only be reduced to a level comparable to the current drive width if steady state ECCD it used [24]. However, it may be possible to completely suppress NTMs in ITER if modulated ECCD is used [25].

In summary, NTMs should not be much of a concern in Ignitor, because of its low β_θ. In contrast, FIRE and ITER plasmas are predicted to be susceptible to NTMs because of their their high $\beta_\theta \sim 0.65 - 0.8$ and low values for island threshold. However, these modes grow slowly and can likely be controlled by ECCD (on ITER) and perhaps current profile control or LHCD (on FIRE).

References

1. Z. Chang et al, Phys. Rev. Lett. **74**, 4663 (1995).
2. H. R. Wilson et al, Plasma Phys. Controlled Fusion **38**, A149 (1996).
3. R. J. La Haye et al, *Plasma Physics and Controlled Nuclear Fusion Research 1996*, Sixteenth Conference Proceedings (IAEA, Vienna, 1997), paper AP1-21.
4. O. Sauter et al, Phys. Plasmas **4** 1654 (1997).
5. D. A. Gates et al, *Plasma Physics and Controlled Nuclear Fusion Research 1996*, Sixteenth Conference Proceedings (IAEA, Vienna, 1997), paper AP1-17.
6. G. T. A. Huysman et al, Nucl. Fusion **39**, 1965 (1999).
7. R. J. La Haye and O. Sauter, Nucl. Fusion **38**, 987 (1998).
8. S. Günter et al, Nucl. Fusion **38**, 1431 (1998).
9. C. C. Hegna, Phys. Plasmas **5**, 1767 (1998) and references therein.
10. G. Gantenbein et al, Phys. Rev. Lett. **85**, 1242 (2000).
11. A. Isayama et al, Plasma Phys. Controlled Fusion **42**, L37 (2001).
12. R. J. La Haye et al, Phys. Plasmas **9**, 2051 (2002).

13. C. D. Warrick et al, Phys. Rev. Lett. **85**, 574 (2000).
14. R. Carrera, R. D. Hazeltine, and M. Kotschenreuther, Phys. Plasmas **29**, 899 (1986).
15. W. X. Qu and J. D. Callen, University of Wisconsin Plasma Report No. UWPR 85-5 (1985).
16. H. R. Wilson et al, Phys. Plasmas **3**, 248 (1996).
17. R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).
18. N. N. Gorolenkov et al, Phys. Plasmas **3**, 3379 (1996).
19. Z. Chang and J. D. Callen, Nucl. Fusion **30**, 219 (1990).
20. C. C. Hegna, J. D. Callen and R. J. LaHaye, Phys. Plasmas **6**, 130 (1999).
21. R. Buttery, to be published in Nucl. Fusion (2002).
22. O. Sauter et al, Phys. Rev. Lett. **88**, 105001 (2002).
23. M. R. Wade et al, Phys. Plasmas **8**, 2208 (2001).
24. C. C. Hegna and J. D. Callen, Phys. Plasmas **4**, 2940 (1997).
25. F. W. Perkins, private communication (2002).