Ideal MHD Study of Snowmass Burning Plasma Candidates

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As a contribution to a uniform assessment of the three burning plasma candidates, Ignitor, FIRE, and ITER, this is a study of sample equilibrium files for each of them. Table 1 gives a summary of the shape and profile properties of the equilibria, while table 2 gives results of ideal MHD stability analyses with the DCON code. In addition to the tables, graphic results are given for sensitivity of some of the ITER equilibria to small changes in the profiles.

Figure 1 shows flux surfaces for the three candidate, uniformly distributed in $\rho = \sqrt{\psi}$, with ψ the normalized poloidal flux. While all figures are the same size, the scales, in meters, vary considerably. While the shapes are quite similar, Ignitor and FIRE are up-down symmetric while ITER has a single-null divertor.

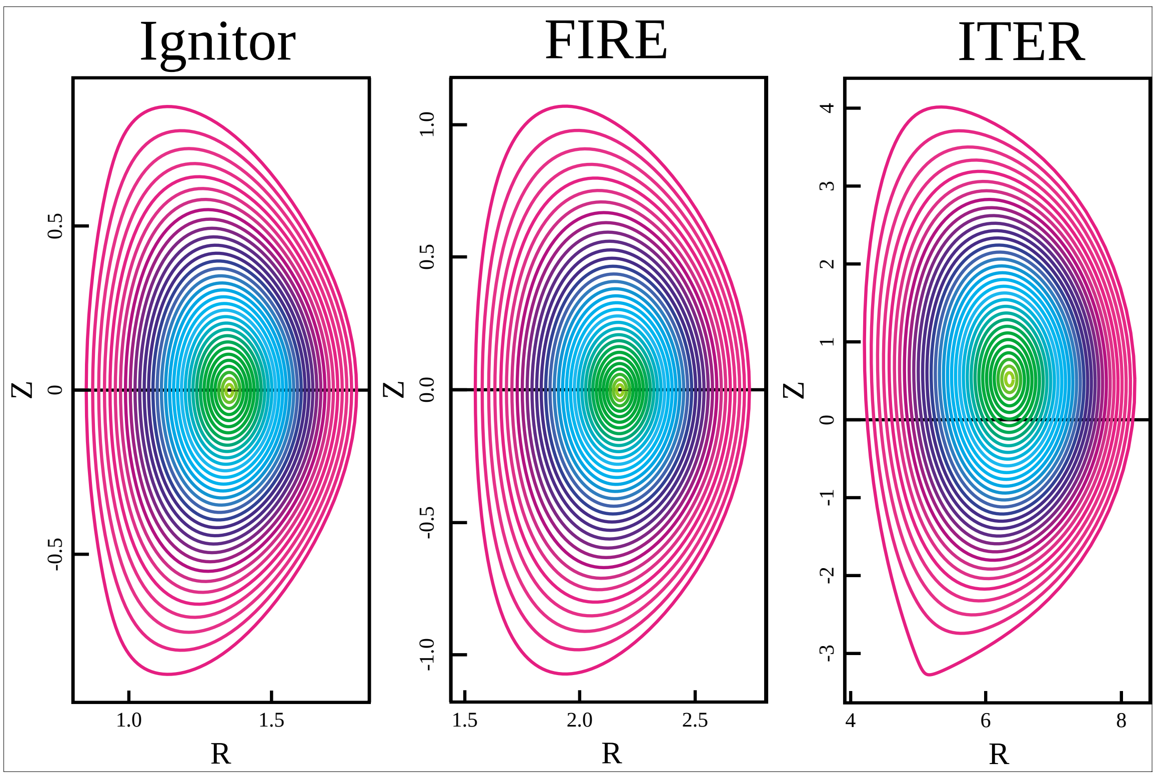

Figure 1. Flux surfaces for the three candidates.

The first column of Table 1 is a reference number. The next 3 columns contain identification information for each equilibrium file: the device it describes, listed in order of increasing size; the equilibrium filename; and the equilibrium code with which it was generated. Of the 3 equilibrium codes used, JSOLVER is Steve Jardin's inverse solver; CORSICA_I refers to the inverse solver in that code, a modified version of Vladimir Drozdov's POLAR1; CORSICA_D refers to the direct solver (TEQ) in that code, a substantial rewrite by LLNL of an original from Dennis Strickler; EFIT is Lang Lao’s direct solver. For the inverse solvers, the last closed flux surface (LCFS) was taken to be $\psi = 1$, with ψ the normalized poloidal flux, while for the direct solver it was taken as .99. The quantity $\rho = \sqrt{\psi}$ is used as an effective radial coordinate.

The next 9 columns contain the following shape parameters:

a	minor radius, m
R	major radius, m
R/a	aspect ratio
Vol	volume, m^3
B_0	magnetic field at midpoint, T
I_P	plasma current, MA
κ	vertical elongation
δ_1	upper triangularity
δ_2	lower triangularity

The next 9 columns contain the following profile parameters:

$\langle\beta\rangle$	volume-averaged beta
β_P	poloidal beta
β_N	normalized beta
l_i	internal inductance
Pfac	ratio of central pressure to volume-average pressure
q_0	safety factor, axis
q_{min}	safety factor, minimum
q_{95}	safety factor at $\psi = 0.95$
ρ_{q1}	ρ at q = 1

	Identification			Shape Parameters							Profile Parameters										
#	**Device**	**Filename**	**Code**	**a**	**R**	**R/a**	**Vol**	**κ**	**δ_1**	**δ_2**	**B_0**	**I_P**	**$\langle\beta\rangle$**	**β_P**	**β_N**	**Pfac**	**l_i**	**q_0**	**q_{min}**	**q_{95}**	**ρ_{q1}**
1	Ignitor	i02008.00000_inv_teq	CORSICA_I	0.47	1.32	2.80	10.0	1.83	0.391	0.391	12.98	11.0	1.170%	0.190	0.653	3.699	0.766	0.900	0.900	2.772	0.440
2	Ignitor	i02008.00001_inv_teq	CORSICA_I	0.47	1.32	2.80	10.0	1.83	0.391	0.391	12.98	11.0	1.170%	0.190	0.652	3.698	0.762	0.950	0.950	2.772	0.366
3	Ignitor	i02008.00002_inv_teq	CORSICA_I	0.47	1.32	2.80	10.0	1.83	0.391	0.391	12.98	11.0	1.170%	0.190	0.652	3.697	0.759	1.000	1.000	2.772	0.000
4	Ignitor	i02008.00003_inv_teq	CORSICA_I	0.47	1.32	2.80	10.0	1.83	0.391	0.391	12.98	11.0	1.170%	0.190	0.651	3.696	0.755	1.050	1.050	2.772	$q_{min} > 1$
5	Ignitor	i02008.00004_inv_teq	CORSICA_I	0.47	1.32	2.80	10.0	1.83	0.391	0.391	12.98	11.0	1.170%	0.190	0.650	3.695	0.751	1.101	1.101	2.772	$q_{min} > 1$
6	Ignitor	i02008.00005_inv_teq	CORSICA_I	0.47	1.32	2.80	10.0	1.83	0.391	0.391	12.98	11.0	1.170%	0.190	0.650	3.693	0.747	1.151	1.151	2.772	$q_{min} > 1$
7	Ignitor	i02008.00006_inv_teq	CORSICA_I	0.47	1.32	2.80	10.0	1.83	0.391	0.391	12.98	11.0	1.170%	0.190	0.649	3.692	0.743	1.201	1.201	2.772	$q_{min} > 1$
8	FIRE	feq041101	JSOLVER	0.60	2.14	3.60	25.9	1.80	0.389	0.389	10.00	7.7	3.305%	1.056	2.553	1.815	0.636	1.116	1.044	2.947	$q_{min} > 1$
9	FIRE	feq041102	JSOLVER	0.60	2.14	3.60	25.9	1.80	0.389	0.389	10.00	7.7	3.029%	0.967	2.340	1.816	0.637	1.119	1.046	2.935	$q_{min} > 1$
10	FIRE	feq041103	JSOLVER	0.60	2.14	3.60	25.9	1.80	0.389	0.389	10.00	7.7	2.753%	0.879	2.127	1.816	0.638	1.122	1.049	2.923	$q_{min} > 1$
11	FIRE	feq041104	JSOLVER	0.60	2.14	3.60	25.9	1.80	0.389	0.389	10.00	7.7	2.477%	0.791	1.914	1.816	0.640	1.125	1.052	2.911	$q_{min} > 1$
12	FIRE	feq060802	JSOLVER	0.60	2.14	3.60	25.9	1.80	0.343	0.343	10.00	7.7	2.371%	0.724	1.828	1.856	0.713	0.873	0.873	3.237	0.287
13	FIRE	feq060803	JSOLVER	0.60	2.14	3.60	25.9	1.80	0.343	0.343	10.00	7.7	2.307%	0.705	1.779	1.907	0.778	0.750	0.750	3.328	0.462
14	FIRE	feq060804	JSOLVER	0.60	2.14	3.60	25.9	1.80	0.343	0.343	10.00	7.7	2.244%	0.685	1.730	1.961	0.845	0.646	0.646	2.704	0.571
15	FIRE	geqdsk_fireAT_042302	EFIT	0.59	2.14	3.62	26.8	1.93	0.589	0.589	6.50	5.3	4.825%	1.471	3.531	2.222	0.437	2.931	2.065	3.167	$q_{min} > 1$
16	ITER	i02002.00150_inv_teq	CORSICA_I	1.98	6.21	3.13	810.9	1.79	0.391	0.442	5.30	15.0	0.329%	0.097	0.274	2.130	0.854	0.979	0.979	2.788	0.305
17	ITER	i02002.00200_inv_teq	CORSICA_I	1.98	6.21	3.13	812.3	1.79	0.439	0.442	5.30	15.0	2.546%	0.633	1.781	1.851	0.853	0.979	0.979	2.862	0.339
18	ITER	i02002.00600_inv_teq	CORSICA_I	1.98	6.21	3.13	809.0	1.78	0.437	0.427	5.30	15.0	2.568%	0.633	1.798	1.846	0.853	0.979	0.979	2.821	0.341
19	ITER	iter_base_129_gfile	CORSICA_D	1.98	6.21	3.13	829.6	1.78	0.412	0.444	5.29	15.0	2.488%	0.614	1.739	2.229	0.992	1.116	1.116	2.973	$q_{min} > 1$
20	ITER	iter_at_129_gfile	CORSICA_D	1.86	6.34	3.41	783.3	1.87	0.410	0.500	5.19	10.0	3.295%	1.670	3.173	3.257	0.669	2.396	1.941	4.298	$q_{min} > 1$
21	ITER	iter_d3d_129.sgfile	CORSICA_D	1.84	6.36	3.45	773.3	1.90	0.373	0.586	5.16	10.0	3.585%	1.840	3.428	2.679	0.566	2.192	1.604	4.652	$q_{min} > 1$
22	ITER	iter_Ped_ref_129.gfile	CORSICA_D	1.98	6.21	3.13	795.8	1.80	0.439	0.470	5.29	14.8	2.610%	0.654	1.854	3.349	0.764	0.934	0.934	3.124	0.537
23	ITER	iter_Ped_AT_129.gfile	CORSICA_D	1.84	6.35	3.45	793.8	1.93	0.442	0.574	5.18	9.0	2.401%	1.566	2.555	2.628	0.629	3.611	2.380	5.228	$q_{min} > 1$

Table 1. Equilibrium Properties

	Identification			Mercier	n = 1			n = 2			n = 3		
#	Device	Filename	Code	ρ_{Mer}	plasma	vacuum	total	plasma	vacuum	total	plasma	vacuum	total
1	Ignitor	i02008.00000_inv_teq	CORSICA_I	0.332	internal instability			1.98E+00	6.72E-01	2.65E+00	1.39E-01	1.20E-01	2.58E-01
2	Ignitor	i02008.00001_inv_teq	CORSICA_I	0.351	internal instability			2.00E+00	6.54E-01	2.66E+00	1.43E-01	1.19E-01	2.62E-01
3	Ignitor	i02008.00002_inv_teq	CORSICA_I	0.354	7.72E-01	1.67E+00	2.33E+00	1.99E+00	6.62E+00	2.65E+00	1.48E-01	1.20E-01	2.67E-01
4	Ignitor	i02008.00003_inv_teq	CORSICA_I	0.335	7.96E-01	1.68E+00	2.38E+00	1.99E+00	6.54E-01	2.64E+00	1.53E-01	1.20E-01	2.73E-01
5	Ignitor	i02008.00004_inv_teq	CORSICA_I	stable	8.17E-01	1.61E+00	2.43E+00	1.99E+00	6.51E-01	2.64E+00	1.58E-01	1.21E-01	2.78E-01
6	Ignitor	i02008.00005_inv_teq	CORSICA_I	stable	8.15E-01	1.65E+00	2.47E+00	1.98E+00	6.47E-01	2.63E+00	1.62E-01	1.22E-01	2.84E-01
7	Ignitor	i02008.00006_inv_teq	CORSICA_I	stable	7.77E-01	1.71E+00	2.49E+00	1.98E+00	6.45E-01	2.62E+00	1.66E-01	1.22E-01	2.88E-01
8	FIRE	feq041101	JSOLVER	stable	-2.96E+00	2.90E+00	-5.87E-02	-2.58E-03	1.17E-04	-2.46E-03	-2.69E-01	7.61E-01	4.92E-01
9	FIRE	feq041102	JSOLVER	stable	-2.61E+00	2.77E+00	1.57E-01	-4.33E-02	1.30E-02	-3.03E-02	-1.77E-01	8.50E-01	6.73E-01
10	FIRE	feq041103	JSOLVER	stable	-2.30E+00	2.67E+00	3.75E-01	-7.73E-02	3.66E-02	-4.08E-02	-6.26E-02	9.16E-01	8.53E-01
11	FIRE	feq041104	JSOLVER	stable	-2.01E+00	2.60E+00	5.93E-01	-9.98E-02	6.77E-02	-3.22E-02	9.96E-02	9.06E-01	1.01E+00
12	FIRE	feq060802	JSOLVER	stable	-1.42E+00	3.42E+00	2.00E+00	7.25E-02	9.87E-01	1.06E+00	5.40E-02	2.02E-01	2.56E-01
13	FIRE	feq060803	JSOLVER	stable	-4.16E+00	5.08E+00	9.20E-01	1.61E-01	4.15E-01	5.76E-01	-2.41E-03	6.63E-04	-1.74E-03
14	FIRE	feq060804	JSOLVER	stable	-1.17E+02	3.07E+01	-8.63E+01	4.46E-02	8.37E-02	1.28E-01	8.01E-01	4.66E-01	1.27E+00
15	FIRE	geqdsk_fireAT_042302	EFIT	stable	-1.13E+01	1.02E+01	-1.12E+00	-5.56E+00	5.87E+00	3.02E-01	-3.05E+00	3.61E+00	5.61E-01
16	ITER	i02002.00150_inv_teq	CORSICA_I	stable	6.09E-02	5.06E-02	1.11E-01	1.48E-01	1.02E-01	2.49E-01	1.89E-01	1.43E-01	3.32E-01
17	ITER	i02002.00200_inv_teq	CORSICA_I	stable	3.55E-01	6.09E-02	4.16E-01	3.19E-01	8.68E-02	4.06E-01	2.70E-01	1.13E-01	3.83E-01
18	ITER	i02002.00600_inv_teq	CORSICA_I	stable	1.59E-02	8.68E-05	1.60E-02	1.59E-02	1.34E-04	1.60E-02	1.58E-02	1.86E-04	1.60E-02
19	ITER	iter_base_129_gfile	CORSICA_D	stable	4.13E-04	2.61E-04	6.73E-04	1.89E-01	1.43E-01	3.32E-01	5.34E-02	4.19E-02	9.53E-02
20	ITER	iter_at_129_gfile	CORSICA_D	stable	-7.74E+00	5.54E+00	-2.20E+00	2.70E-01	1.13E-01	3.83E-01	1.43E-03	7.45E-04	2.17E-03
21	ITER	iter_d3d_129.sgfile	CORSICA_D	stable	-3.44E+00	2.71E+00	-7.38E-01	1.58E-02	1.86E-04	1.60E-02	-3.09E-01	2.18E-01	-9.10E-02
22	ITER	iter_Ped_ref_129.gfile	CORSICA_D	0.515	internal instability			internal instability			3.07E-02	1.35E-01	1.66E-01
23	ITER	iter_Ped_AT_129.gfile	CORSICA_D	stable	-1.53E+00	1.52E+00	-9.71E-03	8.37E-02	4.33E-02	1.27E-01	1.95E-02	6.79E-02	8.75E-02

Table 2. Stability Results

The first four columns of Table 2 repeat the same columns of Table 1 for ease of cross-reference. The remaining columns contain ideal MHD stability results from DCON. ρ_{Mer} gives the value of ρ out to which the equilibrium is Mercier unstable, or indicates that it is everywhere stable. For each toroidal mode number n = 1, 2, and 3, there are 3 columns giving stability results with no conducting wall. In cases with an internal instability, only that information is indicated. In cases for which there is no internal instability, the plasma, vacuum, and total potential energies are given. There is a free-boundary instability if and only if the total energy is negative. The relationship between the potentially negative and therefore destabilizing plasma energy and the positive-definite vacuum energy can be used to estimate proximity to marginal stability.

While DCON treats ideal high-n ballooning stability, and all cases in the table have been tested for it, this is not specified in the table. Mercier instability implies ballooning instability. The only instances of ballooning instability which are Mercier stable for these cases are very narrow regions (penumbra) surrounding regions of Mercier instability (umbra).

It should be understood that these results refer strictly to ideal MHD, not the final word on plasma stability. It omits resistivity and other dissipation, FLR, neoclassical, and nonlinear effects, among other subtle things, which may be either stabilizing or destabilizing. Many instabilities can be stabilized by small changes to the profiles, and some weakly-unstable free-boundary modes can be stabilized with a nearby conducting wall.

Nevertheless, an equilibrium which is strongly ideal MHD unstable is not likely to survive; hence ideal MHD stability is useful as a zeroth-order guide to operating limits. We should carefully distinguish several issues. While some ideal MHD instabilities may saturate benignly without causing any a catastrophic disruption, the mere fact of linear instability implies that an equilibrium cannot persist as such. It will be forced by the linear instability to depart to a neighboring configuration. . In other cases, there may not be a neighboring accessible equilibrium. In that case, the ideal instability can either be benign or cause a disruption of the plasma. The conditions under which the ideal instabilities are benign or not are not well understood. In any case, however, the plasma cannot remain in the unstable confuguration. This is the most general sense in which ideal MHD stability constitutes a hard constraint. The only alternative is if the instability is sufficiently weak as to compete with nonideal or transport effects which could stabilize it.

Equilibrium files were solicited from and contributed by proponents of the three candidate experiments and represent good, though perhaps not final, examples of their plasma equilibria.

Cases 1 - 7 show results for Ignitor, contributed by Bruno Coppi, Paolo Detragiache and Dick Bulmer. Ignitor has the smallest size and lowest β, but the highest field of the three candidates. This is a sequence of cases with increasing values of q_0. Cases 1 – 4 have regions of Mercier instability near the axis, and cases 1 and 2, with $q_0 < 1$, are also unstable to the n = 1 internal kink mode. All of these modes may be susceptible to stabilization by non-ideal effects, beyond the scope of this study. All other modes are stable.

Cases 8 - 15 show results for FIRE, contributed by Steve Jardin and Chuck Kessel. FIRE is somewhat larger than Ignitor, with lower field and current but higher β. The volume of FIRE is about 2.5 times that of Ignitor. Cases 8 – 11 show a sequence decreasing β. All are stable to the Mercier criterion. The first is unstable to an n = 1 free-boundary mode while the others are stable to this mode. All are unstable to an n = 2 free-boundary mode. Cases 12 – 14 show a sequence of cases with decreasing q_0. dropping substantially below 1. They remain stable to Mercier and n = 2 modes, although case 13 is unstable to n = 3 and case 14 is unstable to n = 1. The greater stability of these cases compared to Ignitor cases 1 – 4 is attributable to a much flatter pressure profile in the region q < 1 rather than any intrinsic superiority of FIRE over Ignitor, illustrating the sensitivity of results to small changes in profiles. Case 15 is an Advanced Tokamak (AT) case with higher β, lower current, and negative central shear. It is very weakly unstable to an n = 1 free-boundary mode with no wall, but this is easily stabilized with a nearby conducting wall. All other modes are stable.

Cases 16 - 23 show results for ITER, contributed by Dick Bulmer, Jim Leuer, and Dylan Brennan. ITER is by far the largest of the candidates. The volume of ITER is about 80 times that of Ignitor, but with much lower field and with a current only slightly higher. Cases 16 – 18 show a sequence of cases from the CALTRANS transport code, at start-of-flattop, start-of-burn, and end-of-burn. They are stable to all modes tested. Cases 19 – 21 are widely-used reference cases from the ITER design community: a base case, an advance-tokamak case, and a DIII-D-like case. The first is stable to all modes tested, but the last two are unstable to an n = 1 free-boundary mode, and the last is also unstable to n = 3. These last two cases are intended to operate with a nearby conducting wall, not treated here. Cases 22 and 23 have an edge pedestal intended for improved H-mode operation. Case 22, with $q_0 < 1$, is unstable to Mercier and n = 1 internal kink modes. Case 23 is very weakly unstable to an n = 1 free-boundary mode but stable to all other modes tested.

To further elucidate the sensitivity of stability results, Fig. 2 show the results of a study varying the q profile for Case 20, which is moderately unstable to the n = 1 free-boundary mode. The q profiles in Fig. 2a, generated by Dylan Brennan, show a large variation in the inner half, with no change in the edge value and therefore the total plasma current. Figure 2b shows that as q_0 falls, plasma and total energies have a negative pole, indicating extreme instability, while vacuum energy has a smaller positive pole. Beyond the pole in the free-boundary energies is a fixed-boundary instability.
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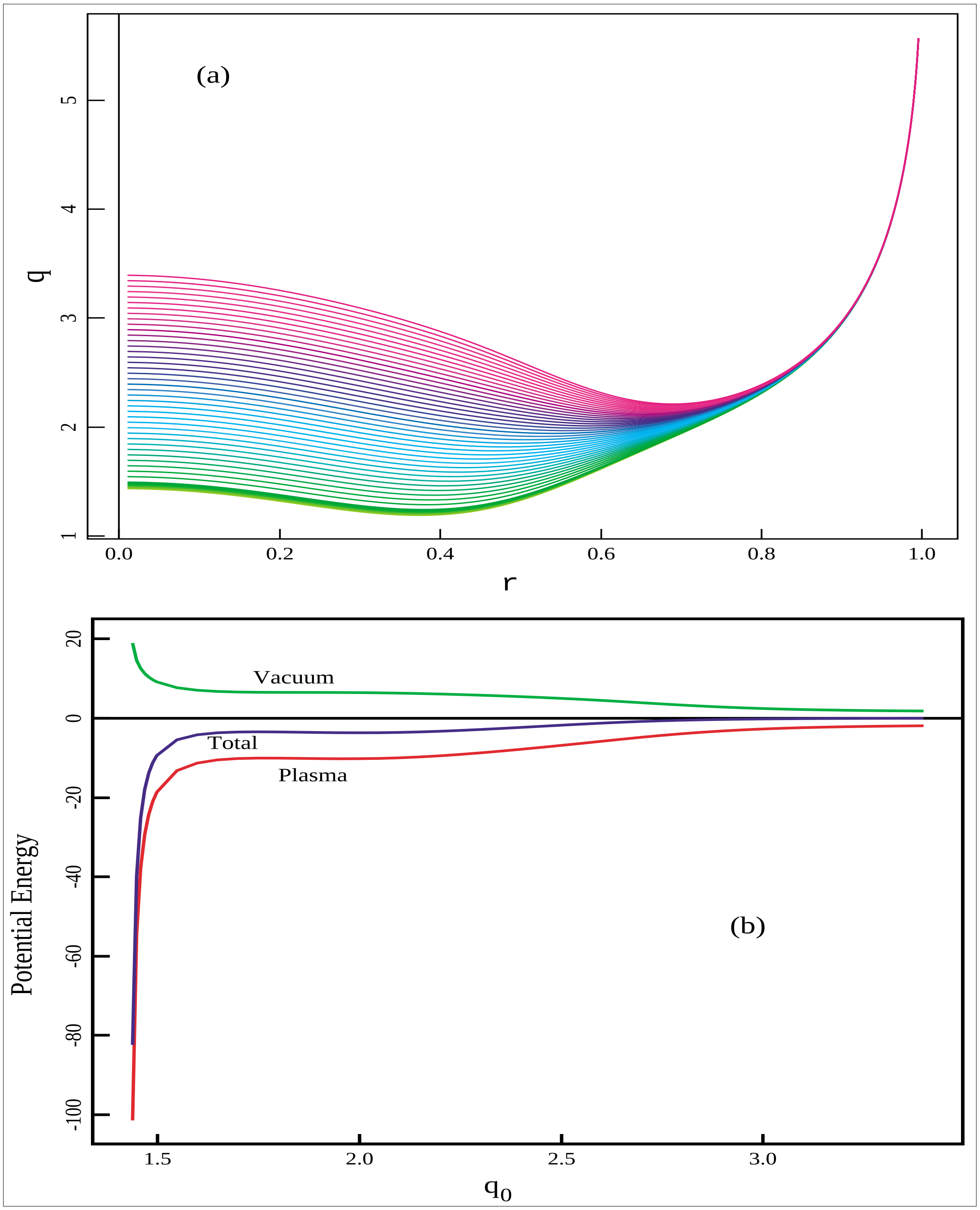

Figure 1. (a) Range of safety factor profiles used to test sensitivity of stability results; (b) variation of plasma, vacuum, and total potential energies vs. q_0.

Discussion of Ideal Stability Results

This study is necessarily limited to certain reference equilibria for each burning plasma candidate. It does not, and cannot, cover the full range of profiles available to each device. Nevertheless, the results are consistent with expectations about ideal stability. All three devices are stable with respect to ideal MHD modes in their "base case" operating mode, with the notable exception of the m=1, n=1 internal kink which is sensitive to the central q value. Due to its unique character, an additional study of the m=1 mode has been carried out which incorporates non-ideal MHD physics. The findings of that study are presented in a separate section of the Snowmass MHD group's report.

Ignitor is expected to operate at lower normalized beta ($\beta_N \sim 0.65$) than the other candidates, because it is intended to reach, or nearly reach, ignition by Ohmic heating alone, without auxiliary heating, and because the lower β_N allows more stable operation with $q_0 < 1$. Because of its low β_N, it is expected to be robustly stable to ideal MHD modes, with the possible exception of the m = 1, n = 1 internal kink. The Ignitor cases here are found to be unstable to n=1 internal modes when $q_0 < 1$, and stable when $q_0 \geq 1$. Other ideal MHD studies have found Ignitor equilibria to be stable with $q_0 \geq 0.8$. This difference is likely to be a result of different treatments of the q and p profiles. In the present set of equilibria, the central q profile was progressively flattened to raise q_0 while keeping the outer portion of the profile nearly fixed; this led to a flat current density profile and low magnetic shear near the axis. In contrast, in the m=1 stability study by J. Manickam (also part of the Snowmass MHD group's report) the shape of the central current density profile was kept fixed, resulting in greater magnetic shear near the axis and stability down to $q_0 \sim 0.8$ for Ignitor's low β value.

FIRE and ITER operate in similar parameter ranges with respect to ideal MHD stability. Their base cases have moderate normalized beta ($\beta_N \sim 1.8$) and again are expected to be stable to ideal MHD modes, with the possible exception of the m = 1, n = 1 internal kink. With q_0 greater than 1, the FIRE cases analyzed here are found to be stable with respect to n=1 kinks up to a no-wall stability limit of $\beta_N \sim 2.5$, in reasonable agreement with the empirical scaling $\beta_N \sim 4\ l_i$ for the no-wall limit.

ITER base cases with q_0 greater than 1 and even slightly less than 1 were also found to be stable, as expected. However, these cases did not have an H-mode edge pressure pedestal, and thus are not completely realistic with respect to possible edge-driven instabilities. Cases from a subsequent study using profiles provided by the ITER team, based on transport simulations and including an H-mode pedestal, were calculated to be unstable to internal n = 1 and Mercier modes even with q_0 as high as 1.05. However, it should be noted that these cases represent work in progress by the ITER team. The profiles had not been optimized with respect to stability, and it is likely that increasing the central magnetic shear would greatly improve the internal stability, as suggested by the m = 1 study described earlier and by the stability of the earlier ITER cases with $q_0 \sim 1$.

The FIRE and ITER "Advanced Tokamak" cases have higher normalized beta ($\beta_N \sim 2.5$ - 3.5) and lie at or beyond the no-wall stability limit. The no-wall limit is, if anything, likely to be lower than for the base cases because of broader current density profiles. The ITER cases analyzed here included a range of pressure profiles (L-mode edge, H-mode profile based on DIII-D data, and H-mode profile based on the ITER team's transport calculations), and minimum q between 1.6 and 2.4; all were unstable to n = 1 without a wall as expected. Other studies (also part of the Snowmass MHD group's report) show that they can be stabilized with an ideally conducting wall at about 1.4 times the minor radius of the plasma, consistent with ITER's first wall. Similarly, FIRE's advanced tokamak can be stabilized by an ideally conducting wall at 1.6 times the plasma minor radius.