Tearing Mode Stability Analyses of ITER, FIRE and IGNITOR

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1 Introduction

This report contains computational tearing stability results pertaining to the comparison of five equilibrium cases, and modifications around those equilibria. Three of these equilibria are ITER, specifically the standard base case, an advanced tokamak scenario, and a DIII-D-like advanced tokamak scenario. The other two equilibria are base cases for FIRE and IGNITOR. Each of these equilibria are analyzed to determine the linear tearing stability index $\triangle'$ at the relavant rational surfaces [1] using the PEST-III code [2, 3], and the results are independently verified [4]. For simplicity, rational surfaces are studied individually for a specific toroidal mode number, and coupling between modes is not addressed. The fluxgrid code from the NIMROD [5] suite is also used to determine the profile of the neoclassical parameter D_{nc} as well as profiles of D_I, D_R and H. The island evolution equation [6, 7] is then evaluated and integrated to determine the neoclassical thresholds and saturated island widths with an assumed polarization term D_{pol}. The dependence of the threshold and saturated island widths on the linear tearing stability index $\triangle'$ and D_{pol} is then discussed, and the different cases compared.

2 The base case comparisons

All $n = 1$ and $n = 2$ modes are studied between $q = 1$ and $q = 3$ where applicable. In Fig. 1 are shown the equilibrium profiles for each device as functions of ψ. For the initial equilibria without modification Table 2 shows the $\triangle'$ results for each device at the important rational surfaces. Here a conformal conducting wall was placed at 1.2*a in each case. These results indicate significantly positive $\triangle'$ values for ITER AT and ITER DIII-D for the 2/1 mode, and for each case the 2/1 mode has the most positive $\triangle'$. These values are normalized by $\psi_s^{2\mu}$ where ψ_s is the poloidal flux value at the rational surface, and $\mu \equiv \sqrt{-D_I}$,

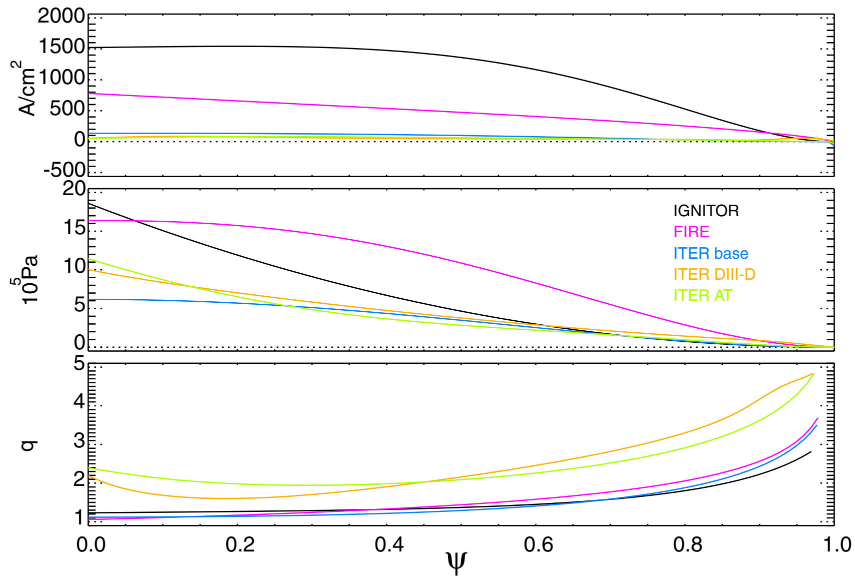

Figure 1: The equilibrium profiles in flux space for the five cases being studied here. The important rational surfaces are 3/2 and 2/1 for the low q_{min} cases, and 2/1 and 5/2 for the higher q_{min} cases.

	ITER BASE	ITER AT	ITER DIII-D	FIRE	IGNITOR
3/2	3.8			−0.5	6.5
2/1	5.8	~ 30	50	1.0	17
5/2	stable	1.1	3.0	−60	stable

Table 1: The linear tearing stability index $\triangle'$ (normalized by $\psi_s^{2\mu}$) from PEST-III for the five cases studied here.

where D_I is the Mercier index. This normalization is within a factor of 2 of the more common $\triangle' r$ normalization from the cylindrical model, but has more relevance in shaped toroidal plasmas. It should be noted that in high β cases such as these it has been shown [8, 9] that $\triangle'$ must exceed at critical value to linearly destabilize the tearing mode. It has also been shown [10] that for agreement with experiment $\triangle' = 0$ is not the stability boundary for these modes, but instead a significantly positive value of $\triangle'$ is needed. For FIRE, the only positive value 2/1 is not significantly positive in this case. In the case of IGNITOR the 3/2 and 2/1 modes have very large and positive $\triangle'$ values which is likely due to the peaked current profile in this device. We next discuss how robust these values are to equilibrium modifications.

3 Equilibrium Modifications

Three ITER equilibria, one FIRE equilibrium and one IGNITOR equilibrium were each modified in two ways. In the Grad-Shafranov equation, given a specified toroidal field strength, the profiles of q, P and F have one redundant member in the sense that only two of the three can be independently specified. Here we modify either the q profile keeping the pressure fixed, or the modify the pressure profile keeping the q profile fixed. The limits of the modifications are chosen to be reasonably consistent with the operational range within the experimental scenarios being discussed. Also, in order to study effects of modifications to the core profiles while maintaining edge profiles, a *tanh* modification profile was used

$$\triangle P = \triangle P_{core}[1 + \frac{1}{2}(tanh[\alpha(\psi_m - \psi)] - 1)] \qquad (1)$$

both on the pressure and q modifications. Here $\psi_m = 0.5$ is the point where the modification gradient is steepest, which is chosen to be in a location away from the location of the rational surface. The results would be highly sensitive to equilibrium modifications if the most strongly varying part were close to the rational surface being studied. The value $\alpha = 10$ determines how steep that slope is, and the modification in the range $\psi_m < \psi < 1$ approaches 0 asymptotically, while the modification in the range $0 < \psi < \psi_m$ approaches P_{core} asymptotically. Fig. 2 shows some typical modifications to ITER base equilibria both in q and P.

4 $\triangle'$ as a Function of Equilibrium Modifications

In cases where the ideal external kink stability limit is far outside of the range of modifications to the equilibria, $\triangle'$ values are relatively insensitive to the modifications in the specified range. This is mainly because the modifications

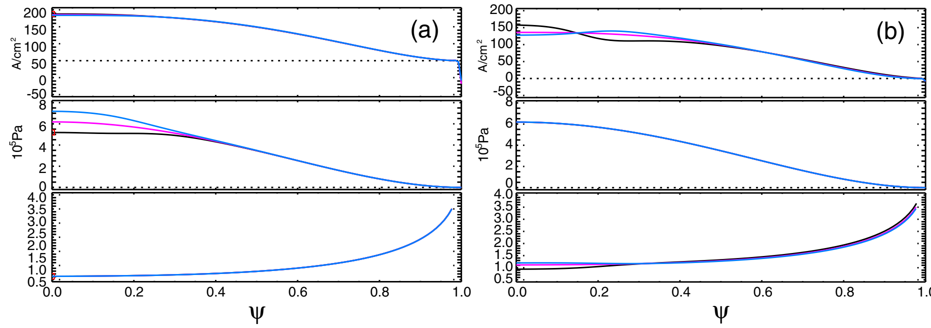

Figure 2: Typical modifications to the ITER BASE equilibrium case in P and q. Shown are the extremes of the range.

were imposed inside of the rational surfaces in question, having little effect on the gradients at the rational surfaces. However, in both the ITER AT and ITER DIII-D cases the approach to the $n = 2$ external kink ideal boundary causes a sharp increase in the $\triangle'$ of the 5/2 mode as q_{min} is decreased, or as the pressure is increased [10], as is seen in Fig. 3. This could mean that 5/2 modes would be spontaneously generated with small increases in the core pressure and/or reduction in q_{min} if this equilibrium scenario were actuated. The 4/2 mode also shows a similar dependence on the pressure and q profile modification. Although the 2/1 modes in these series of equilibria also show ideal unstable regions and sharp increases in $\triangle'$ in this range of equilibria, the substantial movement of the rational surface with the modifications to the equilibrium (due to the flat nature of the core q profile) causes large oscillations in the value of $\triangle'$ and difficulty in diagnosing the results. This is typical of any tearing analyses at rational surfaces that are near q_{min}. We can say that as the q values in the core are reduced, and the rational surface moves out, the $\triangle'$ values at the 2/1 surface are reduced, but remain substantially positive in both cases.

Also note that in the case of the core pressure modification on ITER DIII-D $\beta_N \sim 3.7 \sim 5.1 l_i$ at the pole, while $\beta_N \sim 3.3 \sim 4.5 l_i$ for the initial equilibrium, suggesting that these equilibria are above the $n = 1$ no wall stability limit [11, 12, 13]. The ITER AT case also shows a pole with a similar increase in core pressure, as does the ITER DIII-D case with a similar decrease in q_{min}. The other three base cases for ITER, FIRE and IGNITOR all have substantially lower ratios for β_N/l_i.

Equilibria with lower $q_{min} \sim 1$ are typically susceptible to 3/2 and 2/1 modes, due to the position of these rational surfaces on the profiles. For ITER BASE, FIRE, and IGNITOR the pressure modifications had little effect on the positive $\triangle'$ values for the 2/1 mode listed above, and all three are robustly positive, as shown in Fig. 4. Of the three FIRE shows the smallest $\triangle'$ and is the most likely to escape spontaneous 2/1 modes. It is interesting to note

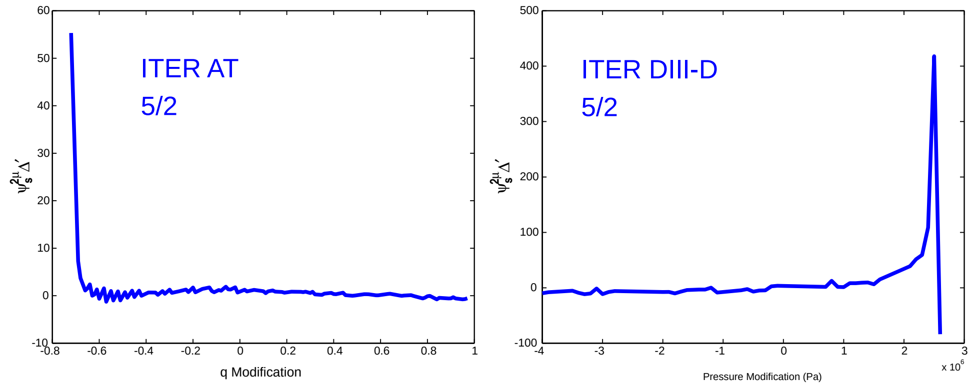

Figure 3: The results from PEST-III for a range of equilibrium modifications where the change in q_{min} (a) or core pressure (b) causes a pole and large positive increases in $\triangle'$ as the $n = 2$ external kink ideal stability boundary is approached. Beyond the pole the equilibrium is ideal unstable.

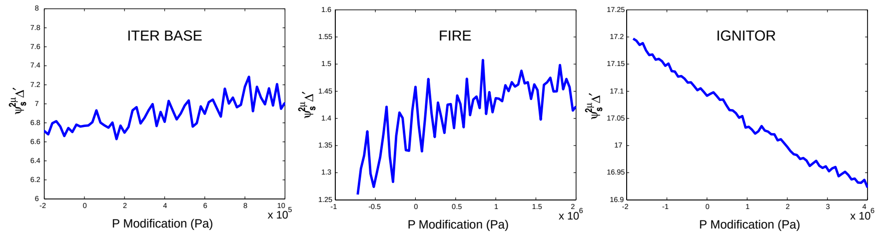

Figure 4: A comparison of the effect of increasing core pressure on the 2/1 linear tearing stability index for base cases in ITER, FIRE and IGNITOR.

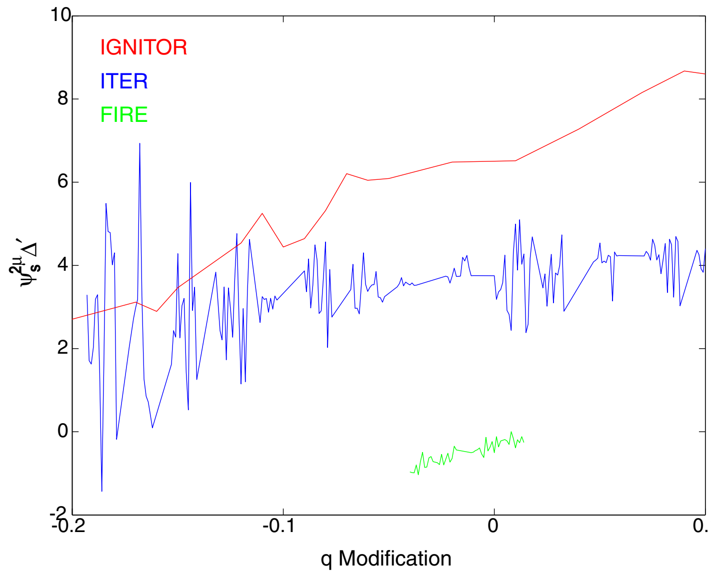

Figure 5: A comparison of the effect of decreasing core q on the 3/2 linear tearing stability index for base cases in ITER, FIRE and IGNITOR.

that in IGNITOR the pressure modifications had the effect of decreasing the $\triangle'$ values with increasing core pressure, which is counter intuitive.

Also interesting is that the modifications to the q profile in all three cases caused the $\triangle'$ values for the 3/2 mode to decrease with lower q_{min} as shown in Fig. 5. Here the modifications to the q profile are small, and the effect is moderate. Not all the analyses were successful in the case of FIRE for numerical reasons, but the trend is clear. It is not likely that this effect comes from changes to the gradients at the rational surface, nor from an increased radius of the 3/2 surface at lower q_{min}, since the 3/2 surface is virtually unaffected locally by the modification. It is interesting to note that FIRE and IGNITOR have similar rates of change in $\triangle'$ with the modification, while the rate of change for ITER is slightly less, with a higher standard deviation in the calculations.

5 Evaluation of the Island Evolution Equation

For each of the equilibria and rational surfaces discussed above, the fluxgrid code was used to determine the profiles of D_I, D_R, D_{nc} and H which appear in the island evolution equation [7]:

$$\begin{aligned}
\frac{dw}{dt} &= \frac{\eta^*}{k_0}[\triangle^* + \frac{k_1}{w}(D_{nc} + \frac{D_R}{\alpha_s - H}) + \frac{D_{pol}}{w^3}] \quad (2)\\
\text{where} & \\
\triangle^* &= \triangle' \| \frac{w}{2} \|^{-2\alpha_l} (-4D_I)^{1/2} \\
\frac{\eta^*}{k_0} &\sim \frac{r_s^2}{\tau_r} \\
k_1 &= \frac{w^2}{w^2 + w_d^2} \\
D_{nc} &= k_2 \sqrt{\epsilon} \beta_\theta \frac{L_q}{L_p} \\
D_{pol} &= D_{nc} \frac{\rho_{i\theta}}{a^2} \frac{L_q}{L_p} g
\end{aligned}$$

and for simplicity reasonable assumptions of $g = 1$ and $k_2 = 2$ are made in the evaluation of D_{nc} and D_{pol}. Sample profiles for the ITER base case are shown in Fig. 6, although the profiles often varied drastically among cases and too often to fully report here. Typically, the D_R values are negative, meaning resistive interchange stability, except near the axis when $q_{min} \sim 1$. The interpolated values for these at the rational surfaces are used in the island evolution equation.

The neoclassical threshold island size is strongly dependent on the polarization term for marginally negative or positive values of $\triangle'$, if we take the form of the polarization term to be valid at small island width $w << \rho_{i\theta}$. For modes that have a positive $\triangle'$ and positive D_{tot}, the dw/dt will be positive at large island width, but can still be negative at small island width due to the polarization term. This causes a seed island threshold in this model even for positive $\triangle'$ and D_{tot}.

The $\triangle'$ values for the 3/2 mode were relatively insensitive to pressure modifications in the given range in all three base cases discussed in Section 4. This is interesting because it is often an increase in core pressure that triggers a 3/2 mode in similar equilibria on DIII-D [10] and other conventional tokamaks. This disparity is likely due to the lower $\beta_N < 4l_i$ in these equilibria as compared to equilibria in present day devices, which effects both the 3/2 and 2/1 sensitivity to equilibrium modifications.

A change in β_θ and/or $\triangle'$ changes both the saturated island width and to some extent the threshold island width in present day devices. However, the effect on the threshold island width in a burning plasma experiment is expected to be less than in present day devices. Were equilibria generated for these devices with $\beta_N \sim 4l_i$ the $\triangle'$ values are expected to be more sensitive to modifications to the core pressure, but the effect on the threshold island width is expected to be small.

It has been shown [10] that the $\triangle'$ value at a rational surface with a saturated

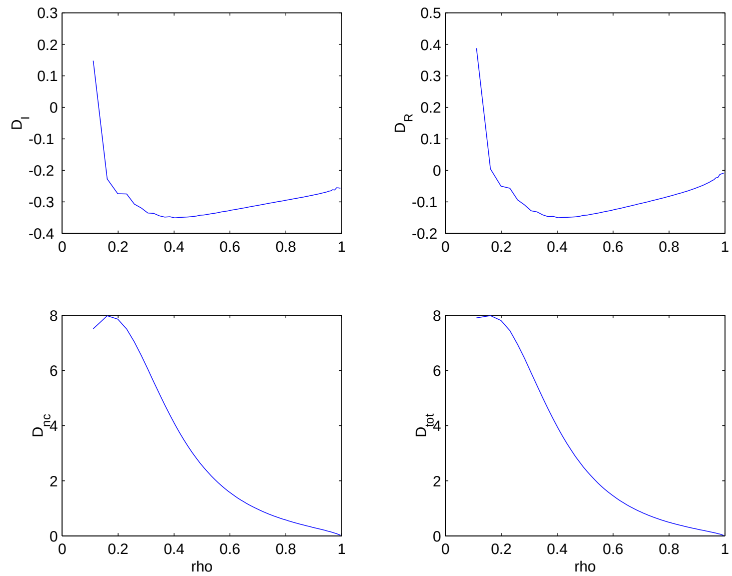

Figure 6: Radial profiles of the neoclassical parameters for the ITER BASE case. Here $D_{tot} = D_{nc} + D_R/(\alpha_s - H)$.

NTM is typically negative, either taking into account the finite w or simply taking the axisymmetric approximation to the equilibrium, where after the mode has saturated $\triangle' \sim -m$. Using the cylindrical model where $\triangle' r_s = -2m$ may over estimate the stability of the mode and underestimate the saturated island width. Therefore, in this study evaluations of the island evolution equation with finite w are made with $\triangle' = -m$. For cases that are driven unstable by a positive $\triangle'$, by the time the mode has saturated we still expect $\triangle' \sim -m$ from previous results [10]. This suggests that the phase space trajectory of dw/dt vs. w will be time dependent in these cases.

For the FIRE equilibrium, with these assumptions, the 2/1 mode has a slightly larger threshold and smaller saturated width than the 3/2, as seen in Fig. 7. Here we demonstrate the effect of imposing the assumption $\triangle' = -m$. This figure suggests that the 3/2 is the dominant mode for this configuration. Although the 2/1 mode has a positive $\triangle'$ while the 3/2 mode has a negative $\triangle'$ in Table 2, using these calculated $\triangle'$ values produces a larger initial positive dw/dt for the 3/2 in Fig. 7(a). At larger w the dw/dt for the 2/1 mode becomes larger where the $\triangle'$ value from Table 2 has a greater effect. For both modes in Fig. 7(a) $dw/dt > 0$ for all w greater than the threshold. However, as the island grows, the $\triangle'$ value will typically decrease and become negative, and Fig. 7(b) will apply for the saturated state. In Fig. 7(b), the 3/2 has the larger growth rates and saturated size even though we're assuming a $\triangle' = -m$, which favors

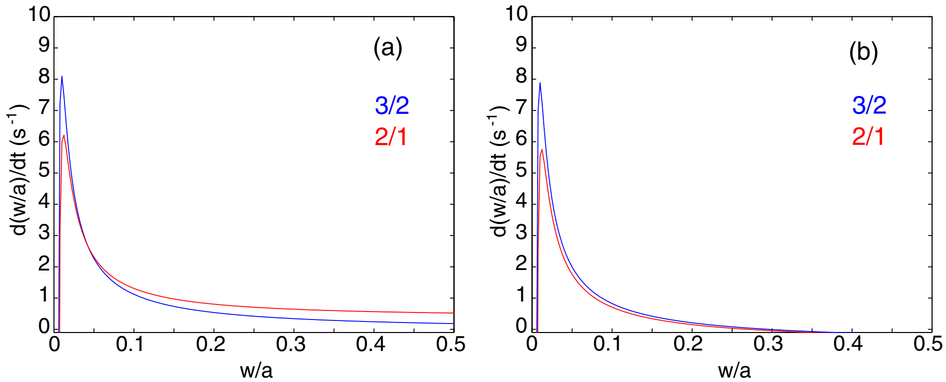

Figure 7: The phase space plots for the FIRE case with the $\triangle'$ values from PEST-III (a), and $\triangle' = -m$ (b).

the 2/1 much like the $\triangle'$ values from PEST-III in Table 2.

In the ITER BASE case, assuming a value of $\triangle' = -m$, we see the dw/dt versus w plot in Fig. 8, showing the 3/2 and 2/1 modes. This indicates that the 3/2 mode has a smaller threshold island size and a larger saturated island size for this case as well. This is interesting because the $\triangle'$ for the 2/1 is larger in Table 2, but the 3/2 mode is still dominant even when assuming $\triangle' = -m$, which favors the 2/1.

For IGNITOR, using the values of $\triangle'$ calculated with PEST-III, the 2/1 and 3/2 modes have positive dw/dt for all w as expected, with the 2/1 having the larger dw/dt. But, with a negative $\triangle' = -m$ the dw/dt curves become negative for all w. Even for the assumption $\triangle' = 0$ the 2/1 mode is still dominant over the 3/2, only taking into account the neoclassical parameters. It is likely that the 2/1 mode will be most unstable in IGNITOR for the equilibrium scenario studied here. However, due to the low β_θ in this device, the islands should saturate at a small size and hence be unimportant.

In the case of the ITER AT and ITER DIII-D equilibria, the 2/1 and 5/2 modes are most unstable. According to previous results [10] the $\triangle'$ values for AT cases tend to be significantly large. Furthermore, a reasonable prediction for values of $\triangle'$ with an island is not available for AT cases. It is therefore difficult to compare the dw/dt vs. w characteristics in the ITER AT and DIII-D cases. It can only be said that the 2/1 mode is more unstable that the 5/2 using the values of $\triangle'$ from PEST-III in Table 2, and even assuming some negative value for $\triangle'$.

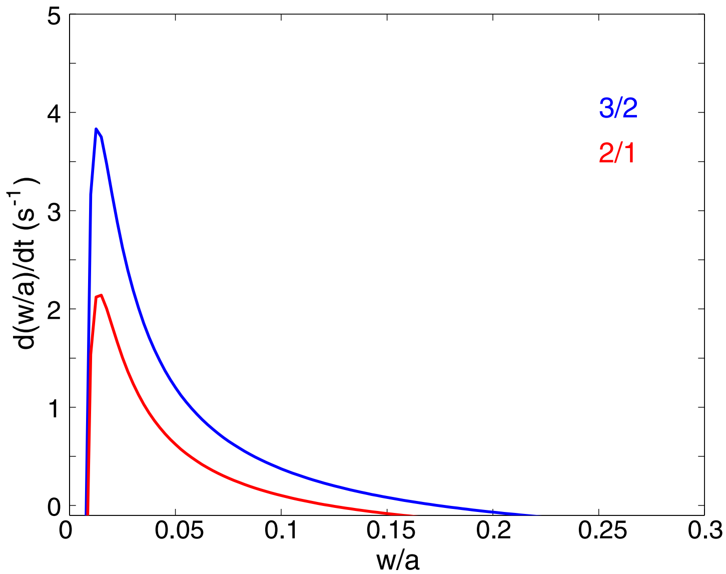

Figure 8: The phase space plot for the 2/1 and 3/2 islands in ITER BASE with the assumption of and $\triangle' = -m$. The 3/2 mode is dominant.

6 Conclusions

Linear tearing stability analyses showed several of the base cases have significantly high $\triangle'$ values at the 3/2, 2/1 and/or 5/2 rational surfaces. However, it may be that in high temperature tokamaks discharges with moderately large $\triangle'$ values will not trigger NTMs. The threshold island width for NTMs will be determined by the polarization term for moderate $\triangle'$. It has been shown recently [10] that a large increase in $\triangle'$ values is responsible for the spontaneous onset of NTMs in an AT discharge in DIII-D. The AT equilibria analyzed for ITER show similar rapid increases in $\triangle'$ at the 5/2 surface when the core pressure is increased. In the same study [10], for an ITER-like sawtoothing discharge an increase in $\triangle'$ at the 3/2 surface causes a reduction in the neoclassical threshold, and a seed island from a sawtooth sets off a 3/2 NTM. In ITER and FIRE the dependence of the neoclassical threshold on $\triangle'$ is much weaker, and this effect should not be observed. The neoclassical threshold will be approximately $w_{thresh}/a \sim 0.01$ in both devices with the ITER threshold being slightly smaller than that of FIRE, which agrees with the expected ρ^* scaling of the threshold. In IGNITOR, the positive $\triangle'$ values may drive tearing modes, but neoclassical effects should be slight since this device operates at low β.

In each of these cases the result is very sensitive to the details of the chosen profiles, and it may be possible to impose a small variation in these profiles which would reduce the $\triangle'$ values. Although modifications to the core were implemented in this study, these were done without significantly affecting the profiles at the rational surfaces, in order to retain some similarity to the given case and to emulate experimental scenarios. Modifications to the gradients at the rational surfaces will have a strong effect on the $\triangle'$ values, but it is difficult to design equilibria that are both fusion relevent and tearing stable. Tearing stability was not considered when generating these original equilibria, and it is sensible that tearing stability be considered when preparing equilibria that are proposed as baseline cases in these devices in the future.

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