1D Full Wave Analysis of Parametric Decay Instability (PDI) in Alcator C-Mod and NSTX

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PDI and edge ion heating in IBW and ICRF experiments

Pinsker 1993, DIII-D

Thomas 1995, Tore Supra

Rost 2002, CMOD

Biewer 2005, NSTX

Langmuir Probe
Magnetic Probe

NBA ERD

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Motivation

• PDI has been widely observed in tokamak ICRF and IBW heating experiments (NSTX, ALCATOR C-MOD, DIII-D, TORE SUPRA, TST-2, HT-7, etc.).
• PDI in the scrape-off-layer (SOL) can lead to significant power loss, degrading the rf heating efficiency in the bulk plasma (eg. direct IBW).
• A quantitative understanding of PDI is needed.
Outline

• AORSA code, PDI model equations.
• PDI simulations for Alcator C-Mod and NSTX.
• A full wave simulation of edge reflectometry on NSTX.
• Summary.
Our analysis is based on All-ORders Spectral Algorithm (AORSA)

- AORSA solves Maxwell’s equations using a linearized plasma response.
  \[-\nabla \times \nabla \times \mathbf{E} + \frac{\omega^2}{c^2} \left( \mathbf{E} + \frac{i}{\omega \epsilon_0} \mathbf{J}_p \right) = -i\omega \mu_0 \mathbf{J}_{\text{ant}}\]
  
- The plasma current is given by
  \[j_p(r,t) = \int_{-\infty}^{t} dt' \int dr' \sigma(r - r', t - t') \cdot E(r', t')\]

- We use a spectral method with Fourier basis functions.
- Integral equation leads to a dense set of linear equations with \(~500k\) linear complex equations.
An example: High Harmonic Fast Wave heating on NSTX

- An antenna excites fast waves.
- Linear theory predicts that fast waves mainly heat electrons via Landau damping.
- To explain the observed edge ion heating, we may have to apply a non-linear theory.
PDI model equations

- The pump wave is solved independently, and is assumed constant in the edge

\[- \frac{c^2}{\omega_0} (\nabla \times \nabla \times E_0) + K_0 \cdot E_0 = - \frac{i}{\varepsilon_0 \omega_0} J_{\text{antenna}}\]

- Daughter waves are solved by

\[- \frac{c^2}{\omega_1} (\nabla \times \nabla \times E_1) + K_1 \cdot E_1 = C_1(E_0, E_1, E_2) - \frac{i}{\varepsilon_0 \omega_1} J_{\text{noise},1}\]

\[- \frac{c^2}{\omega_2} (\nabla \times \nabla \times E_2) + K_2 \cdot E_2 = C_2(E_0, E_1, E_2) - \frac{i}{\varepsilon_0 \omega_2} J_{\text{noise},2}\]

- Parametric Coupling given a dipole-pump:

\[C_1(E_0, E_1, E_2) = - \sum_{\text{species}} \frac{\mu^*}{2} (\sigma_1 - \sigma_2) \cdot E_2(x) + \sum_{\text{species}} \frac{|\mu|^2}{4} (\sigma_1 - \sigma_2) \cdot E_1(x)\]

- With the pump given, the system of equations is linear and can be solved by a direct solver.

Selection rules:
\[\omega_0 = \omega_1 + \omega_2\]
\[k_0 = k_1 + k_2\]
PDI simulations for Alcator C-Mod

An example.

Exponential growth of the daughter field

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_2$</td>
<td>53.4 MHz</td>
</tr>
<tr>
<td>$f_3$</td>
<td>26.6 MHz</td>
</tr>
<tr>
<td>$T_e$</td>
<td>15 eV</td>
</tr>
<tr>
<td>$T_i$</td>
<td>3 eV</td>
</tr>
<tr>
<td>$n_e$</td>
<td>$1 \times 10^{19}$ m$^{-3}$</td>
</tr>
<tr>
<td>$B_0$</td>
<td>5.3 T</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.9 m</td>
</tr>
<tr>
<td>$k_y$</td>
<td>2000 m$^{-1}$</td>
</tr>
<tr>
<td>$n_\phi$</td>
<td>40</td>
</tr>
</tbody>
</table>
Which waves to simulate?

Daughter waves satisfying the following conditions are to be excited:

1. selection rules

\[ \omega_0 = \omega_1 + \omega_2 \]
\[ k_0 = k_1 + k_2 \]

2. one of the daughter waves is a linear plasma wave. In the ICRF range, fast wave--\( \rightarrow \)IBW+quasimode (Porkolab, 1990).
Spatial evolution of IBW’s in the SOL

- \( k_{\perp} \) (left) \(<\!< k_{\perp} \) (right)
  - Slow convergence of simulation for large \( k_{\perp} \)
  - Larger coupling for large \( k_{\perp} \)
- Damping(left)\(<\!<\) damping(right) \(\rightarrow\) Ion cyclotron heating
Local IBW dispersion relation & decay channels

CMOD Minority heating regime
PDI dispersion calculations

\[ \varepsilon(\omega_1, k_1)\varepsilon(\omega_2, k_2) = -\frac{\mu_D^2 \mu_D^3}{4} (\chi_e(\omega_1, k_1) - \chi_e(\omega_2, k_2))(\chi_i(\omega_1, k_1) - \chi_i(\omega_2, k_2)), \]

\[ \mu_D = \mu_e - \mu_i \quad , \quad \mu = \frac{q}{m} \left[ \frac{E_\perp \cdot k_\perp}{\omega_0^2 - \omega_c^2} + \frac{E_\parallel k_\parallel}{\omega_0^2} \right]^2 + \frac{|E_\perp \times k_\perp| \omega_c^2}{(\omega_0^2 - \omega_c^2)^2 \omega_0^2} \right]^{1/2} \]

Growth rate (s\(^{-1}\))

\( E_\perp = 9\text{ kV/m} \)

\( E_\perp = 27\text{ kV/m} \)
PDI threshold is consistent with experiment

*CMOD Minority heating regime*

**Fast wave → IBW+QM**

- Linear 3D AORSA simulations predict typical fast wave edge amplitude = (5,12) kV/m @0.5MW rf power.
- Scaling trends for the PDI threshold behavior with density, temperature are consistent with Ref. [Rost 2002].
PDI simulations for NSTX

- Daughter IBW amplitudes vs. frequency at a fixed point in the edge.
- Both weak and strong PDI are simulated.
- Peaks are separated by the cyclotron frequency at the probe ($\Omega_i$).

Simulation parameters:
- $T_e = T_i = 50$ eV,
- $n_e = 2 \times 10^{18}$ m$^{-3}$,
- $B_\phi = 0.45$ T
- $k_y = 7000$ m$^{-1}$, $n_\phi = 15$
PDI dispersion calculations for NSTX

Growth rate (s^{-1})

MOST UNSTABLE MODES

pump

$\frac{\omega}{\omega_{cD}}$

$n_\phi$

$\omega / \omega_{cD}$

MOST UNSTABLE MODES
PDI $k_{\phi}$ dependence is consistent with edge ion heating and reflectometer measurements.

- Poloidal ion temperature increases with toroidal wave length (Talyor 2010).
- Predicted power threshold is consistent with previous reflectometer measurements (100 to 300kW), (Wilgen 2005).

$B_{\phi} = 0.5$ T

$P_{RF} = 1.2-1.3$ MW

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Reflectometer and Langmiur probe show similar spectra on NSTX
Full wave cold plasma simulation of X mode reflectometry

\[ \hat{n} = 1 + 0.001 e^{(r-r_p)^2 / \Delta^2} \sin(-\omega_p t + k_p (r - r_p)) \]

Matching condition*:
\[ k_{\text{pertrub}} = 2 (k_x) \]

Typical NSTX wave numbers:
- FAST WAVE: 50-100 m\(^{-1}\)
- IBW: 1000-10000 m\(^{-1}\)

* N. Bretz, Phys. Fluids B 4, 2414 (1992)
Full wave simulation of X mode reflectometry

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Higher order Bragg condition

\[ n (k_{\text{pertrub}}) = 2 (k_x) \]

\[ n = 1, 2 \]

Reflectometer seems insensitive to very short wave length signals.

* N. Bretz, Phys. Fluids B 4, 2414 (1992)
The plasma responses of fast wave and IBW are different

- Plasma density fluctuations caused by rf waves is evaluated by
  \[ \tilde{n}_p = \int \tilde{f}(\omega, n_0, B_0, E) dv \]
- Assume noise level IBW density fluctuation is 0.1% \( n_0 \)

<table>
<thead>
<tr>
<th>PDI strength</th>
<th>Density fluctuation (1/m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pump wave</td>
</tr>
<tr>
<td>Below threshold</td>
<td>8e13</td>
</tr>
<tr>
<td>Above threshold</td>
<td>4e14</td>
</tr>
</tbody>
</table>

- Density fluctuation of IBW above the threshold is about 1000 times larger than that caused by fast wave.
- Given the same level of fluctuations, the reflectometer signal of fast wave is about 10,000 times lower than that of IBW.

Daughter wave signal ≈ 0.1 fast wave signal.
Summary

• We successfully simulated Fast wave-\(\rightarrow\)IBW+QM using the AORSA code.
• The most unstable modes are found to be the waves excited just above an cyclotron harmonic.
• PDI simulations agree with
  – the threshold in a minority heating experiment on Cmod,
  – the edge ion heating measurements on the \(k_\phi\) dependence in NSTX,
  – the threshold measurement by reflectometer in NSTX.
• Full wave reflectometry simulations show that the measurement for short-wave-length daughter waves is non-local.
• Future works include NSTX reflectormeter and probe data analysis, a prediction for NSTX upgrade, and the calculation of PDI power deposition (1D to 2D and 3D).