EXTRAP T2R coils and resistive shell magnetohydrodynamics

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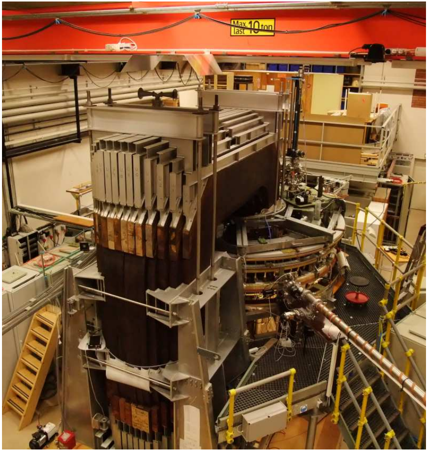
Overview

1. Fast introduction
   - EXTRAP T2R
   - Closed-loop stabilisation & dither injection
2. Experimental modal analysis & *MB*/*BO*/*SSARX*
   - Overview
   - Some signal processing details
   - Multibatch bootstrap extension
   - Results
3. First grated PID experiments
4. Summary
   - Recap

What this talk covers

1. Signal processing methods for MHD modal analysis; **completely empirical results.**
2. Embryonic explorative study of actuation coverage and degrees-of-freedom.

EXTRAP T2R reversed-field pinch (@KTH)



Parameters

- major radius R=1.24 m
- plasma minor radius a=18.3 cm
- shell norm minor radius r/a = 1.08
- shell time constant τ_{shell}=6.3 ms
- plasma current I_p=80-160 kA
- electron temperature T_e=200-400 eV
- pulse length $\tau_{pulse} \leq$ 90 ms

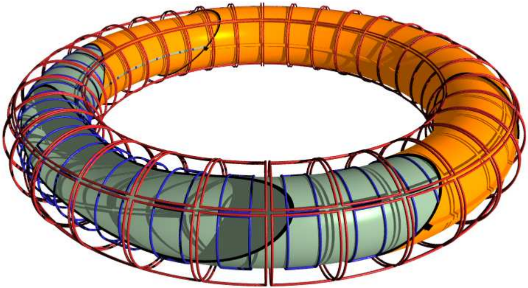
Unstable plant

Without stabilisation plasma terminates after $\sim$ 15 ms.

Resistive shell & saddle coils

Legend

Sensors inside shell.
Actuators outside.
Vacuum vessel innermost.



The plant & some notation

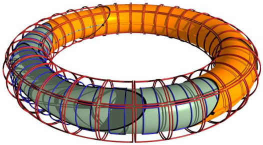
Vector inputs and outputs

The plant

$$\mathbf{y} = G(q)\mathbf{u} \quad (1)$$

Legend

sensors $\mathbf{y}$; shell;
actuators $\mathbf{u}$; vessel



$\mathbf{y}(k) \in \mathbb{R}^{64\times 1}$: time-integrated sensor coil voltages

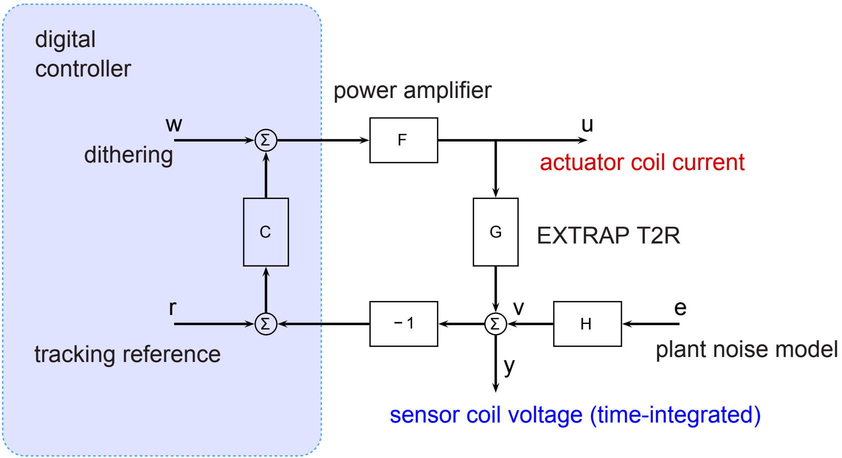
$\mathbf{u}(k) \in \mathbb{R}^{64\times 1}$: active coil currents

k : discrete-time sample index

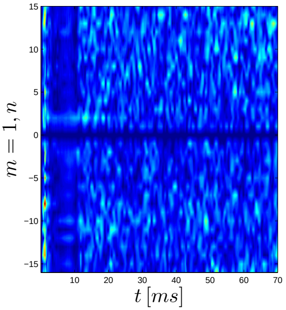
q : shift operator

T2R stabilisation; the generalised control loop

Essentially employing a rack of PIDs (∼ Bishop-style)



T2R dithering; $r = 0, w \neq 0$



System identification

T2R shot #21816: injected pseudo-randomised dithering.

Basic DTLTI concepts

Discrete-time linear time-invariant dynamical systems

Innovations form (IF)

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) + K\mathbf{e}(k) \tag{2}$$

$$\mathbf{y}(k) = C\mathbf{x}(k) + D\mathbf{u}(k) + \mathbf{e}(k) \tag{3}$$

Predictor form (PF)

$$\mathbf{x}(k+1) = A_K\mathbf{x}(k) + B_K\mathbf{z}(k) \tag{4}$$

$$\mathbf{y}(k) = C\mathbf{x}(k) + D\mathbf{u}(k) + \mathbf{e}(k) \tag{5}$$

with $A_K = A - KC$, $B_K = (B - KD, K)$ and $\mathbf{z}^T = (\mathbf{u}^T, \mathbf{y}^T)^T$.

Dataset logistics and computational tools

1. Quite large sizes of input- and output-vectors.
2. $\mathcal{B}^{(b)} = \{\mathbf{y}^{(b)}(k), \mathbf{u}^{(b)}(k)\}_{k=1\ldots N^{(b)}}$ in a bunch of smallish batches $b = 1 \ldots B$, ie. “shots”.
3. It is anticipated that the A-matrix is unstable: $\max_i |\lambda_i(A)| > 1$.

$\Rightarrow$ warrants a multibatch SIM preferrably cast in predictor-form (to stabilise the Markov parameters)

SSARX (Jansson, IFAC Symposium on sysid 2003)

Straightforward rehash for multibatch datasets possible. Predictor-form recast was devised and implemented. Method will not be detailed in this talk. Instead some concepts from statistical learning will be outlined.

Cross validation for state order selection

Meaningful determination of n? The actual n is infinite for PDE-governed systems.

Multifold CV

A quite general recommendation (see e.g. *Elements of statistical learning*, Hastie et al. 2009) is to do 5- or 10-fold randomised cross-validation (if enough data).

The DTLTI residuals are evaluated by recursion:

$$\widehat{\mathbf{x}}(k+1) = A_K \widehat{\mathbf{x}}(k) + B_K \begin{pmatrix} \mathbf{u}(k) \\ \mathbf{y}(k) \end{pmatrix} \quad (6a)$$

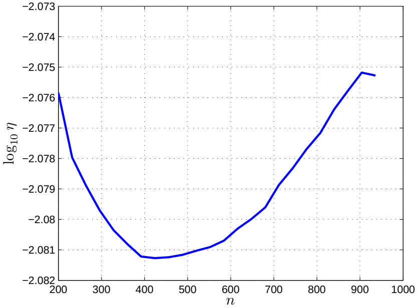
$$\epsilon(k) = -C\widehat{\mathbf{x}}(k) + D_K \begin{pmatrix} \mathbf{u}(k) \\ \mathbf{y}(k) \end{pmatrix} \quad (6b)$$

10-fold randomised CV for T2R

Batch-grouped CV

Available set of 74 plasma data batches. Randomly assign into 10 groups of approximately equal sizes. Estimate plant from 9 groups, evaluate fit on group left-out. Repeat and form average fit.

Do this for a set of trial state orders $\{n\}$; pick best one.



$$\eta(n) = \frac{\mathrm{trace}\ \sum_k \epsilon(k)\epsilon^T(k)}{\mathrm{trace}\ (R_{\mathbf{yy}})} \qquad (7)$$

The modal analysis stage: visualisation

To make sense of MIMO results; relate to MHD stability theory via ocular inspection

What to do with $\left\{\widehat{A},\widehat{B},\widehat{K},\widehat{C}\right\}$? Template approach:

1. Compute eigenvalues $\{\lambda_j\}$ and associated eigenvectors for the system A-matrix; $\widehat{A}$.
2. Visualise these eigenmodes in a "physical" spatial basis (MHD). Prior knowledge; but *not* used for the estimation itself.
3. Specifically zoom in the region close to $z = 1$; the expected MHD action takes place here.

Bootstrap resampled *MB*/*SSARX* (*MBOSS*)

Take uncertainty into account; invoke computational statistics.

One-shot nature of SIMs renders bootstrap resampling easily feasible. Synthetic datasets by sampling the batches with replacement (iid); then repeat MB/SSARX; N_B times.

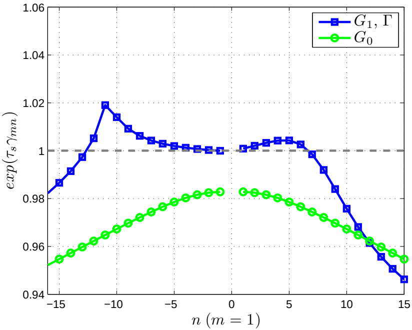
$$G_{nn}^{(i)}(\rho) = \sum_{j=1}^{\dim \mathbf{x}} \frac{1}{h} e^{-\left(\phi_j - \phi_0\right)^2/(2h^2)} \frac{\mathbf{w}_n^* M_j^{(i)} \mathbf{w}_n}{\rho e^{\iota \phi_j} - \lambda_j^{(i)}} \tag{8}$$

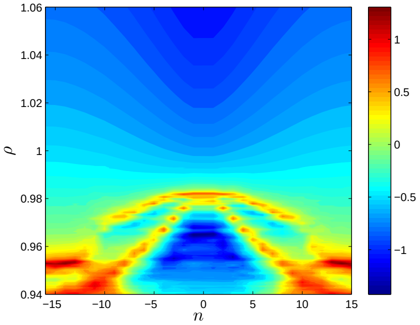
$$G_{nn}^{B}(\rho) = \frac{1}{N_B} \sum_{j=1}^{N_B} G_{nn}^{(j)}(\rho) \tag{9}$$

$\Rightarrow$ basically produces a statistically smoothed histogram $\log_{10} \left| G_{nn}^{B}(\rho) \right|$ of the spatio-temporal spectrum for T2R.

Comparison of theory & dither experiment analysis

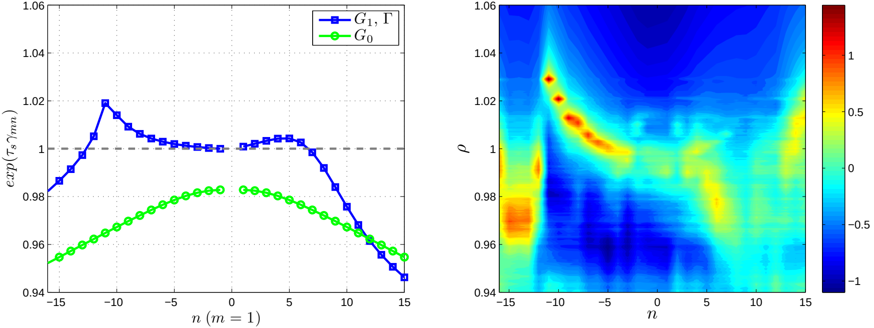
Cylindrical resistive shell theory (left); MBOSS (right); vacuum diffusion





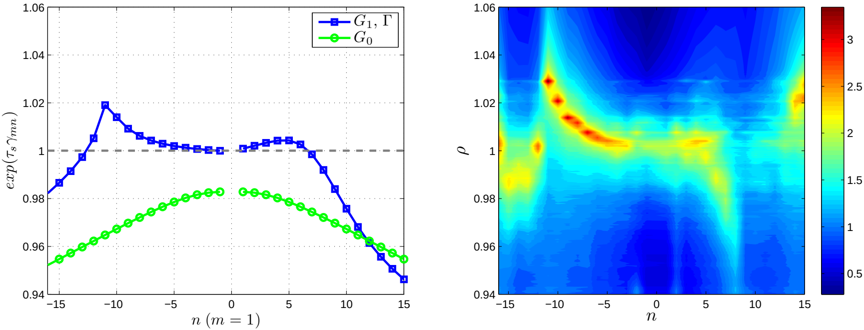
Comparison of theory & dither experiment analysis

Cylindrical resistive shell theory (left); MBOSS (right)



Comparison of theory & dither experiment analysis

Cylindrical resistive shell theory (left); MBOSS (right); vacuum “subtraction”



Masking the intelligent-shell

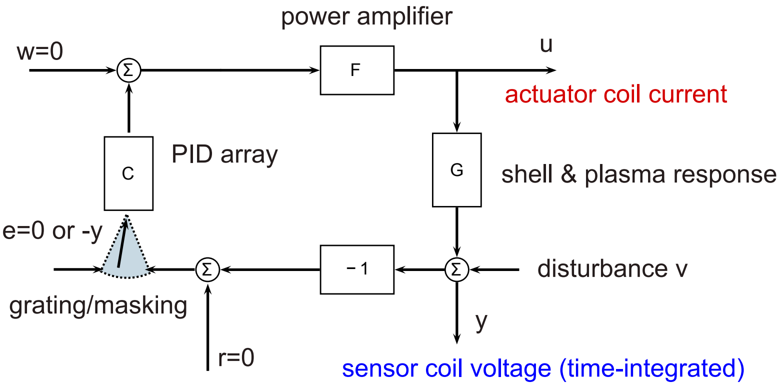
Preliminary “grated PID” experiments

- Ambition is to quantify a baseline requirement of control coverage and number of actuators.
- This may somewhat explore the concept of mode-rigidity for RFPs.
- It is known that full coverage works well for RFPs, but that half may be truly problematic.
- It is also not clear how many actuators actually are needed, in practice.

T2R coils and “ruggedness” seems a good platform to shine some light here.

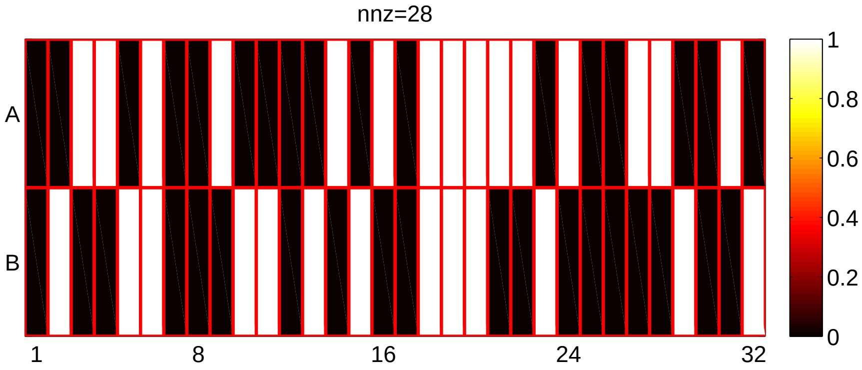
Reduced coverage and reduced number of actuators

Disconnect randomised subsets of localised feedback coils.



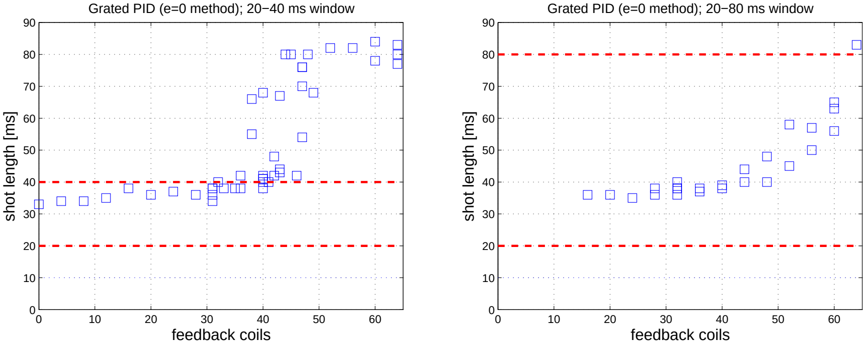
Masking the intelligent-shell

Example grating with 28 nonzeros (feedback coils)



First explorative masked PID array results

Sustained shot-length versus number of active coils (randomised distributions)



What was done

1. Signal processing toolkit developed for multibatch subspace system identification.
2. No induced “regularisation from prior” by invoking MHD parameterisation. *Completely empirical results.*
3. Resistive shell MHD characteristics clearly detected from safe closed-loop operation.
4. Method very generic and appears easy to try out also in tokamak analyses.
5. Tentative first step towards explorative study of baseline requirements for actuation count and coverage for RFPs.

Some references

Thank you for your attendance. Questions & suggestions please.

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