

Uncovering RWM stability limits in tokamaks

By

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In collaboration with

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Measurements of plasma response to applied perturbations used to understand, control RWM stability

- **Motivation: need to validate theories of RWM stability for ITER, beyond**
 - Resistive wall modes arise from the interaction between an external kink mode and wall eddy currents. RWMs can be ***global, beta-limiting instabilities***.
 - Recent experiments on several devices have shown complex dependence of stability on plasma rotation, ***lack of a critical rotation threshold***.
- **Compare driven response of stable plasma with theory**
 - $\delta B/B < 10^{-3}$ – refer to this state as a *perturbed equilibrium*
- **Ideal MHD describes perturbed equilibria below no-wall beta limit**
 - Large body of measurements consistent with ideal MHD when $\beta < \beta^{no-wall}$
- **Kinetic modifications to ideal MHD needed above no-wall limit**
 - Evidence for wave–particle interactions uncovered, qualitatively consistent with theory.
 - Off-axis NBI used to probe kinetic damping in recent experiment
- **Direct stability control demonstrated using NBI feedback**
 - Feedback dynamics are linear below no-wall limit.

Outline

1. Making perturbative measurements of RWM stability

- Measure plasma response δB^{plas} to applied fields in stable plasmas

2. Linking plasma response and ideal MHD theory

- Ideal MHD describes experiments for $\beta < \beta^{\text{no-wall}}$, plasma rotation sufficiently high

3. Uncovering kinetic modifications to ideal MHD

- Kinetic modifications important when $\beta > \beta^{\text{no-wall}}$

4. Controlling the proximity to the RWM stability limit

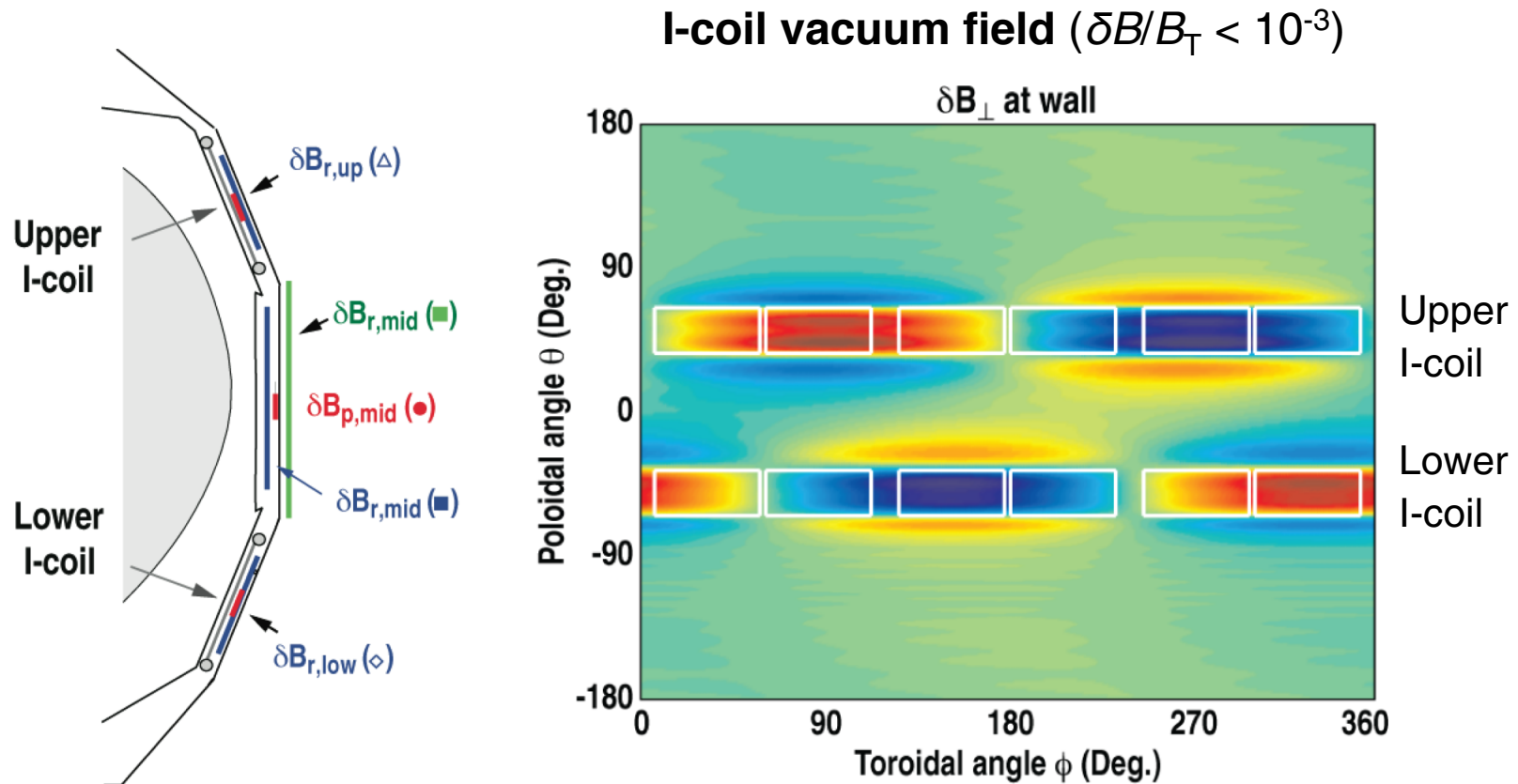
- Direct control of RWM stability margin demonstrated using NBI feedback



Perturbative measurements of RWM stability

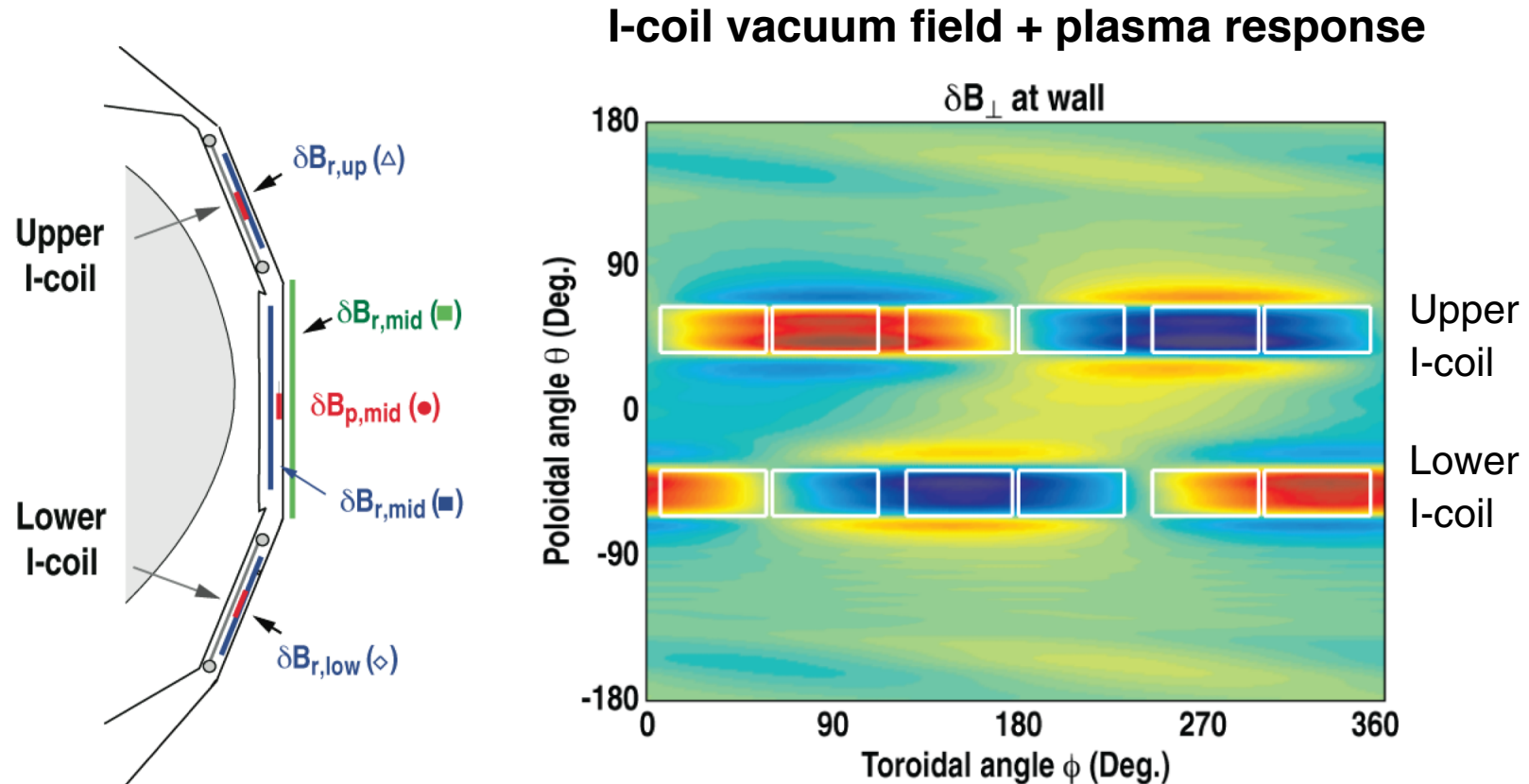


DIII-D tokamak is well equipped to create and measure perturbed equilibria



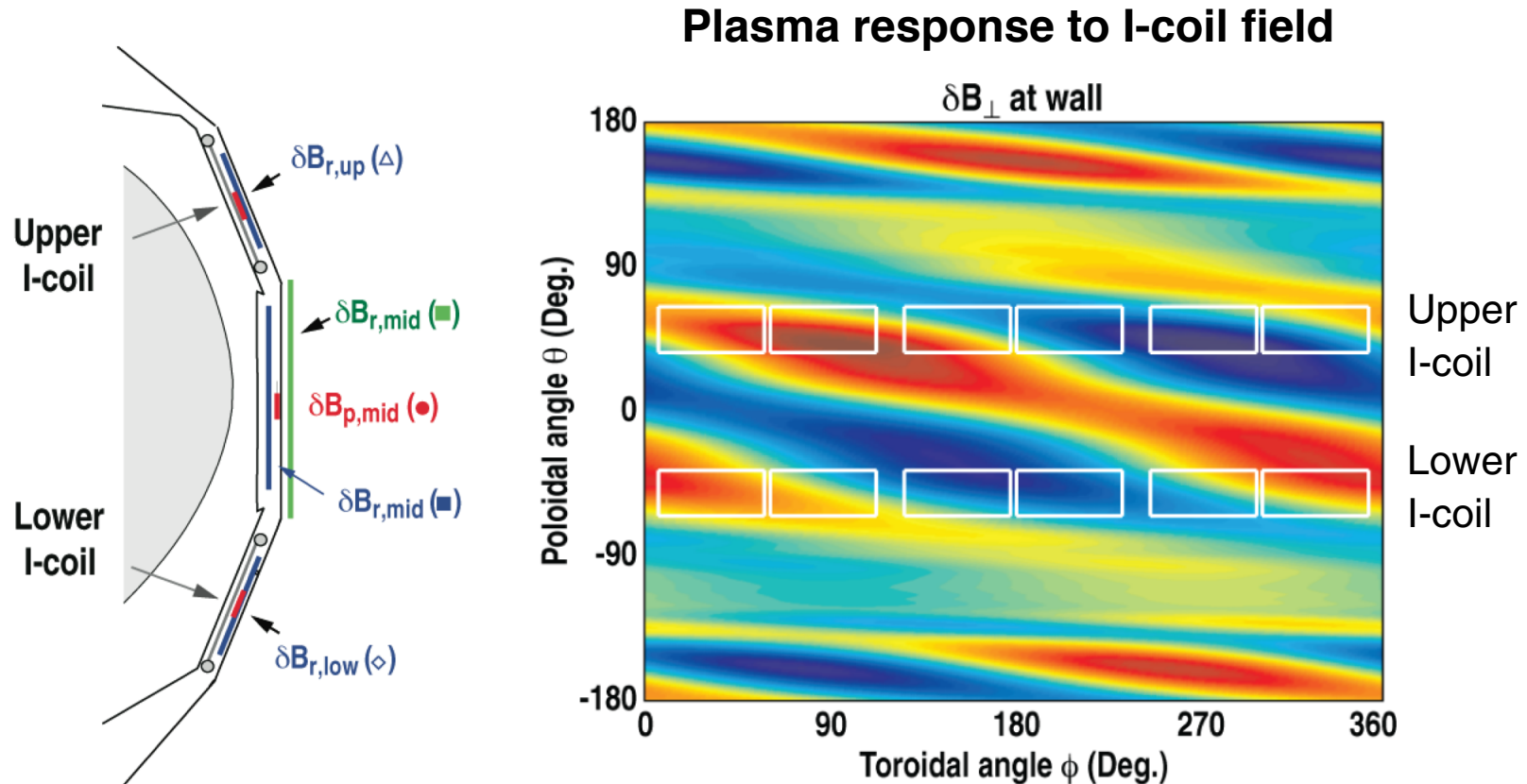
Spectrum of applied I-coil field chosen to resonate with plasma kink mode

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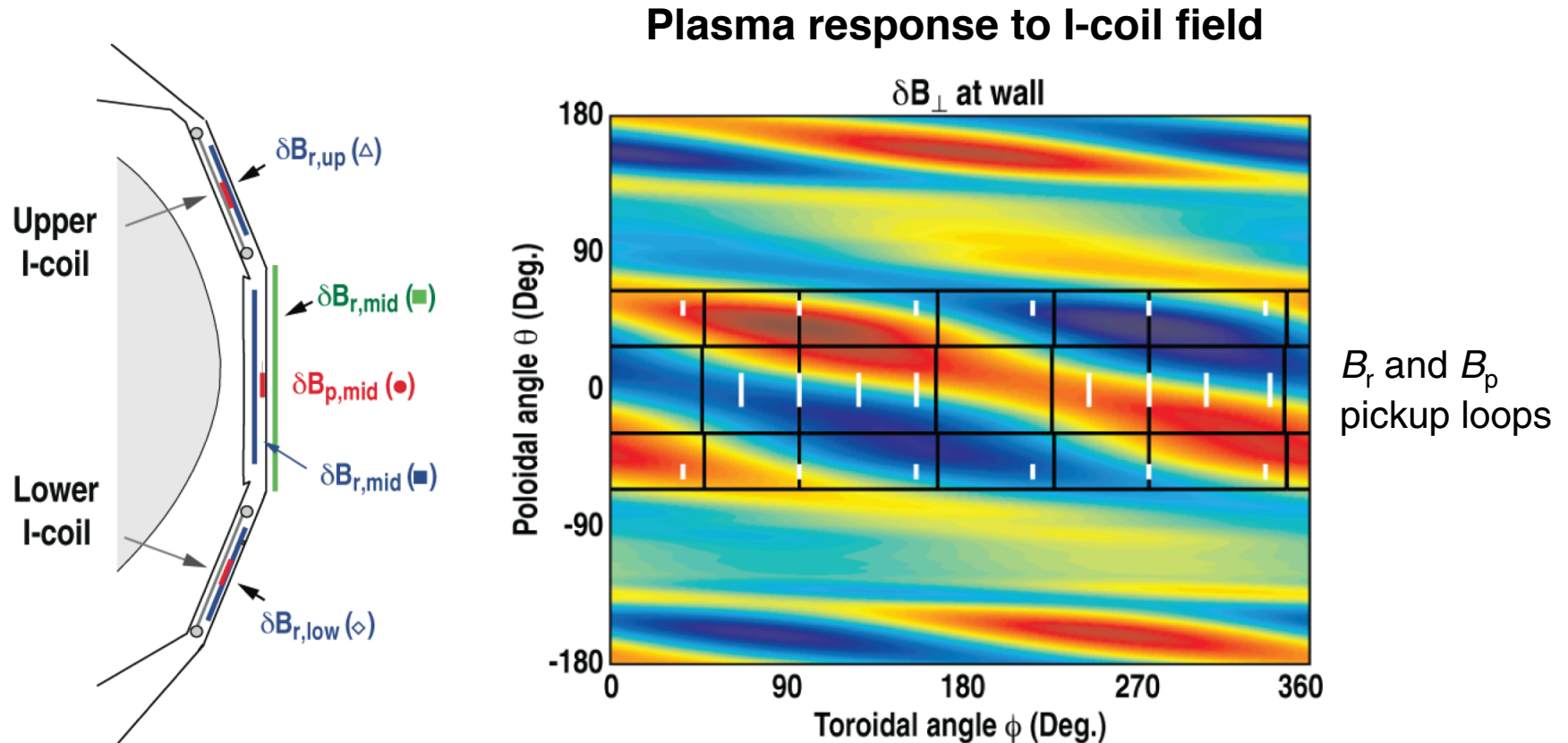
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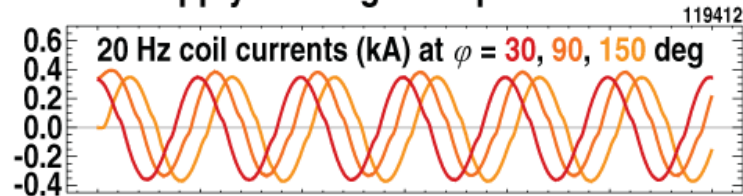
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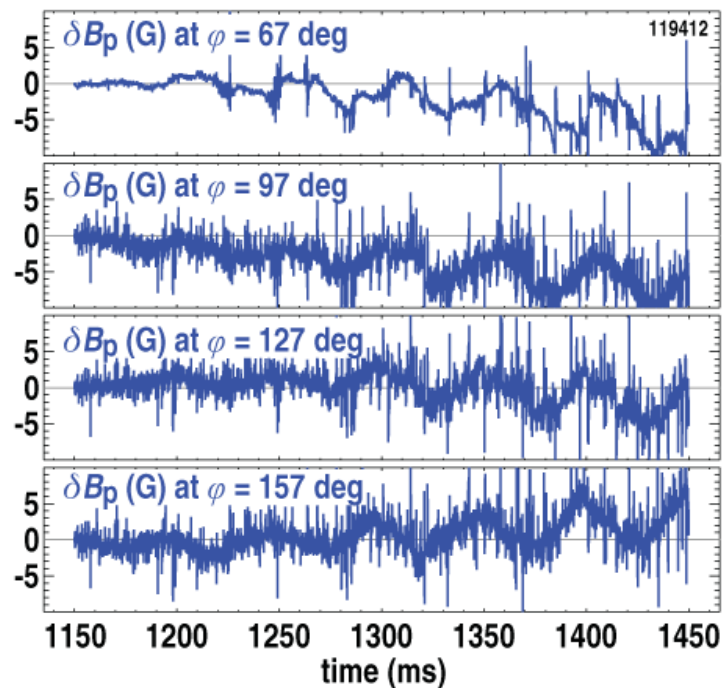
Fits to magnetic measurements yield amplitude and toroidal phase of plasma response

Single-frequency analysis yields plasma response to applied, rotating perturbation

1. Apply rotating $n = 1$ perturbation

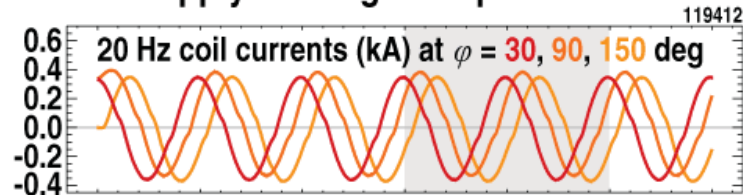


- **Apply rotating perturbation** near natural rotation frequency of RWM, ~ 20 Hz in DIII-D.

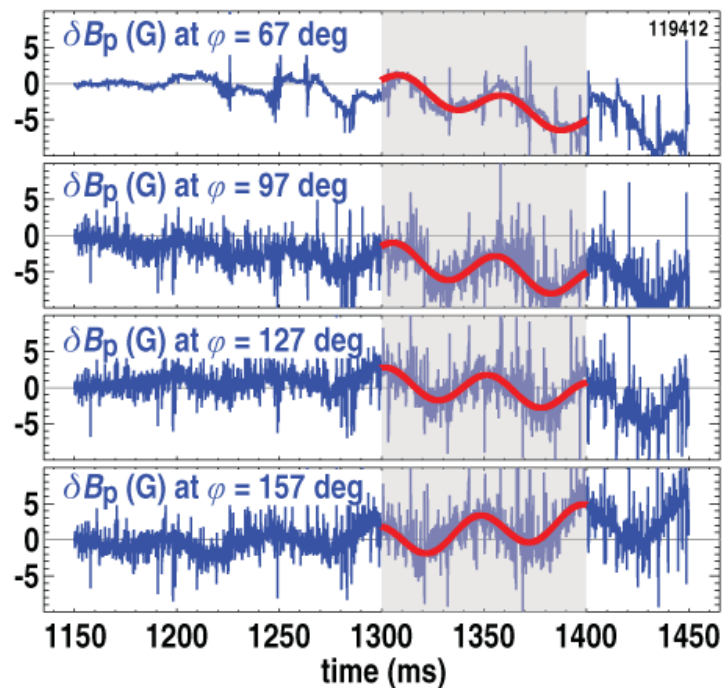


Single-frequency analysis yields plasma response to rotating, 3D perturbation

1. Apply rotating $n = 1$ perturbation



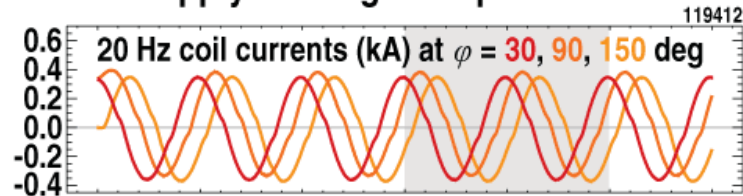
2. Fourier analysis gives resonant response



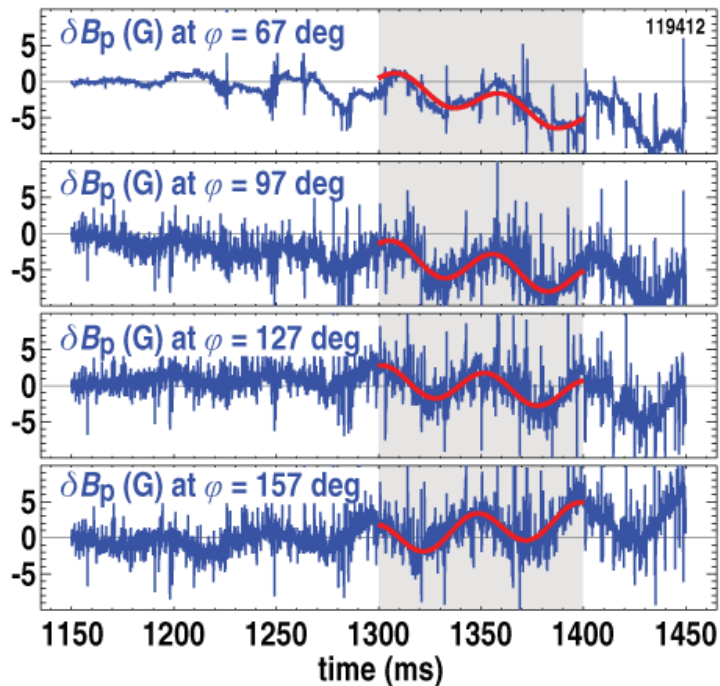
- **Apply rotating perturbation** near natural rotation frequency of RWM, ~ 20 Hz in DIII-D.
- **Measure response using synchronous detection.** Fourier analyze over a sliding window containing several oscillation periods
- In real-time analysis and feedback, sliding averaging window leads to delay of half the window size, $\tau_{\text{lag}} \sim 100$ ms.

Single-frequency analysis yields plasma response to rotating, 3D perturbation

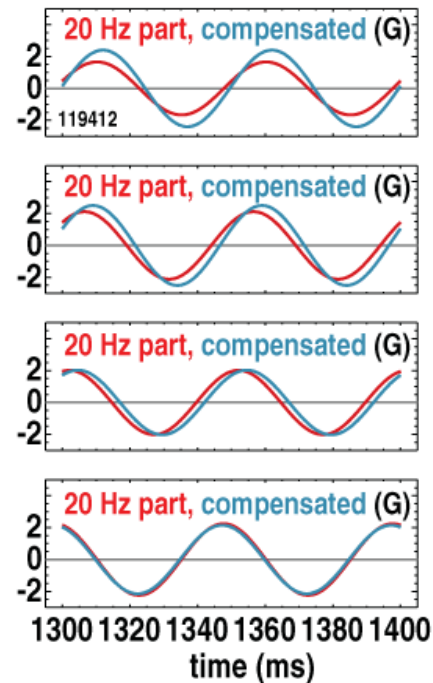
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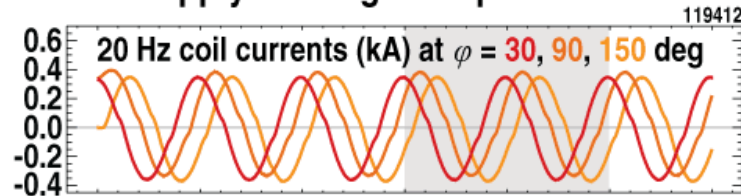


3. Subtract direct ac pickup from coils

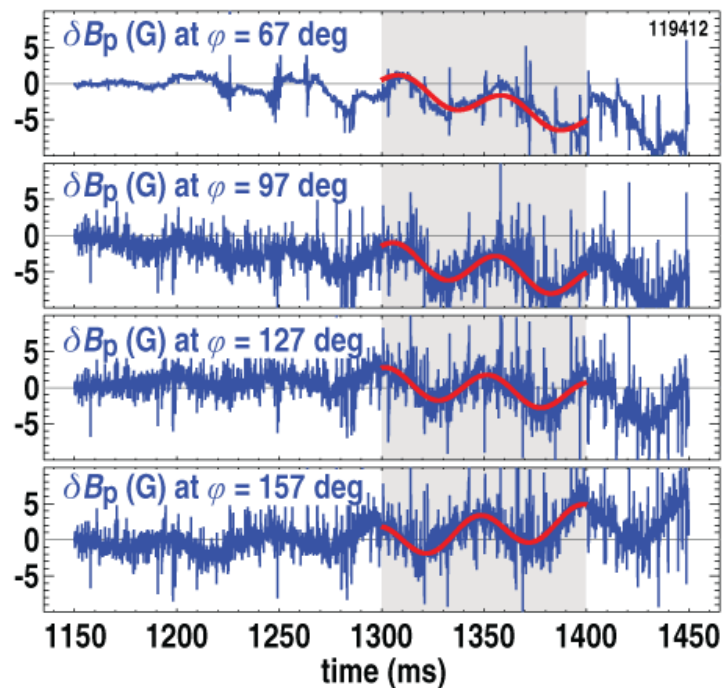


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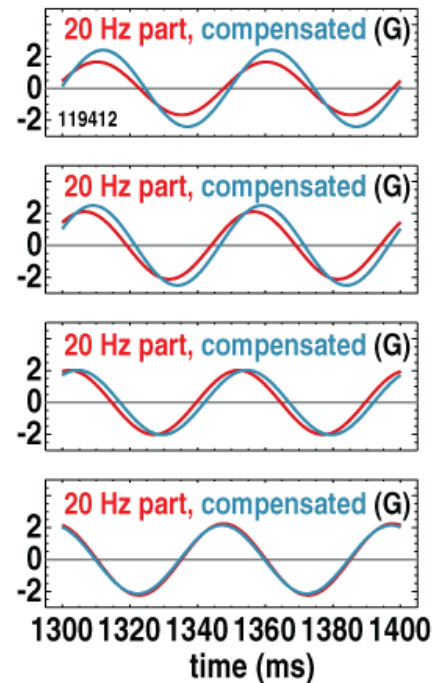
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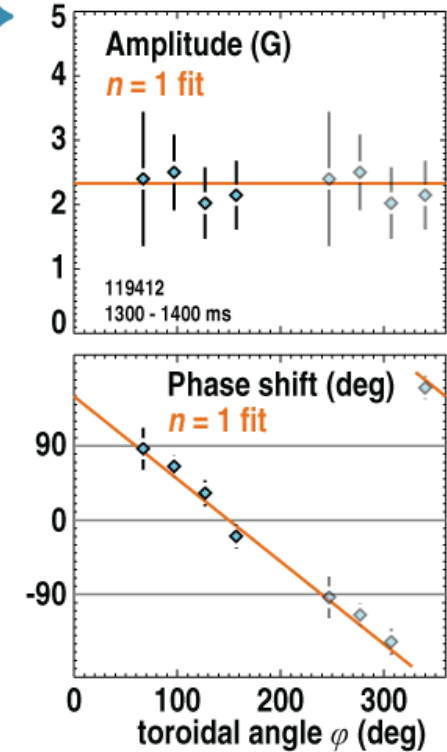
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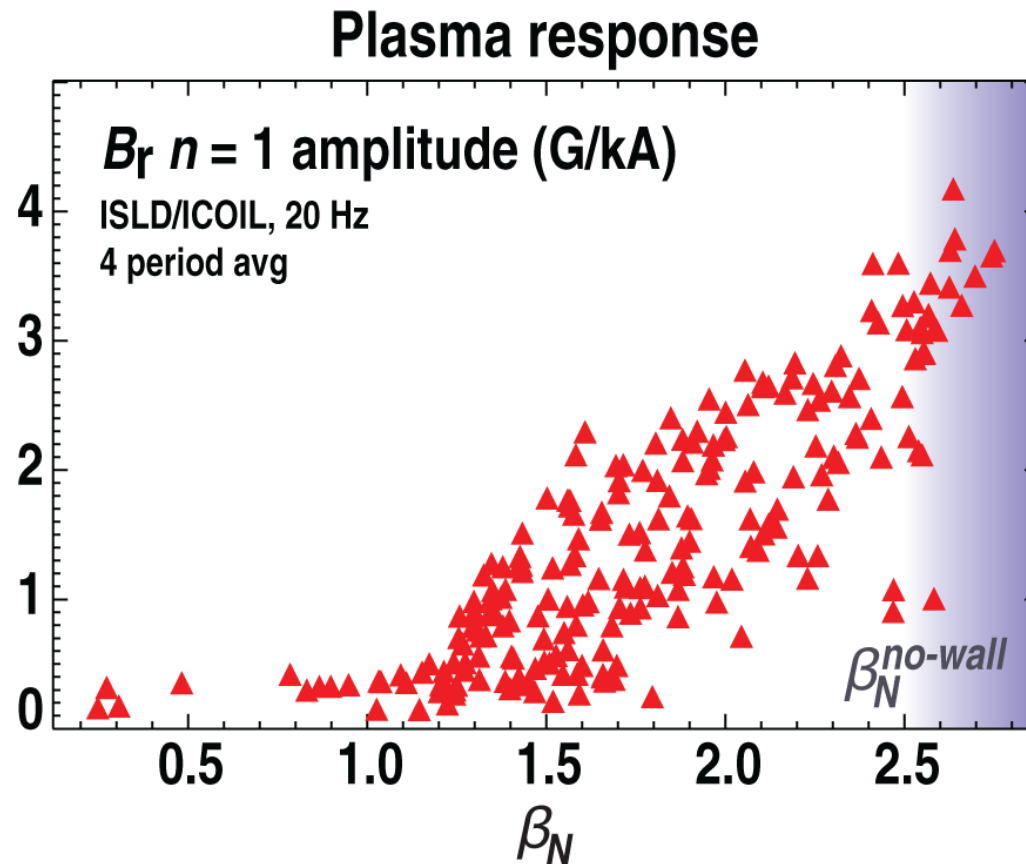


4. Toroidal mode number fit



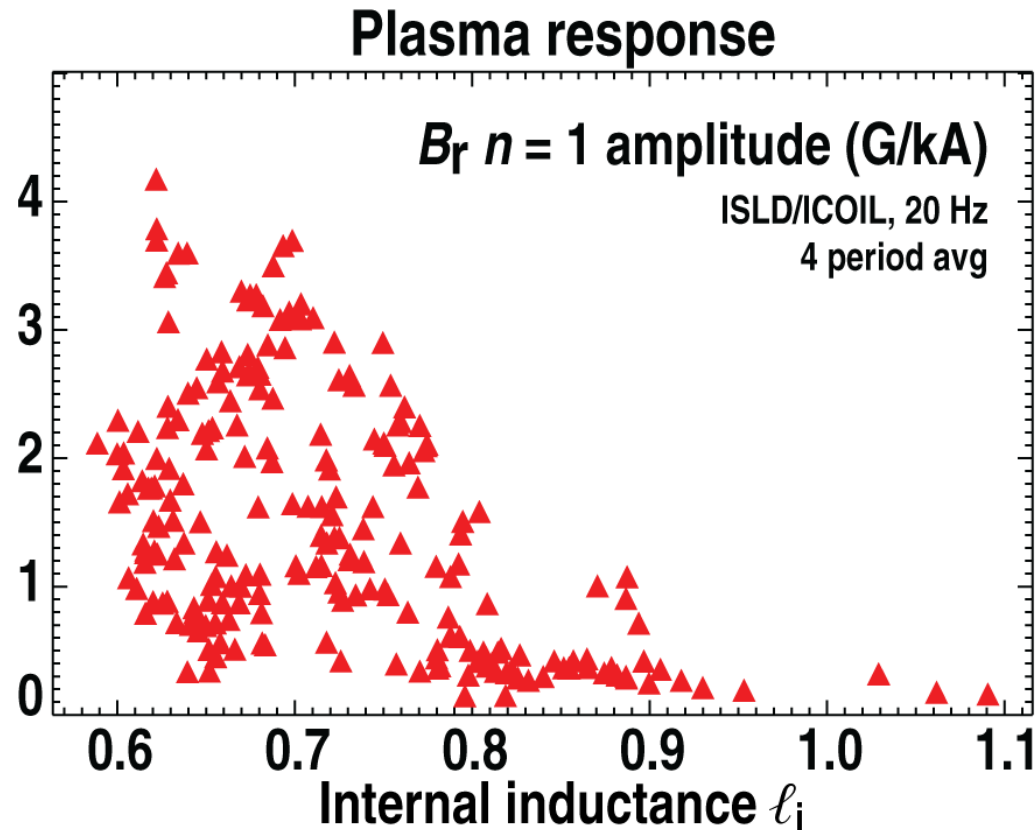
Link between plasma response and ideal MHD theory below no-wall limit

Plasma response dependencies consistent with ideal MHD expectations for RWM stability, below no-wall limit



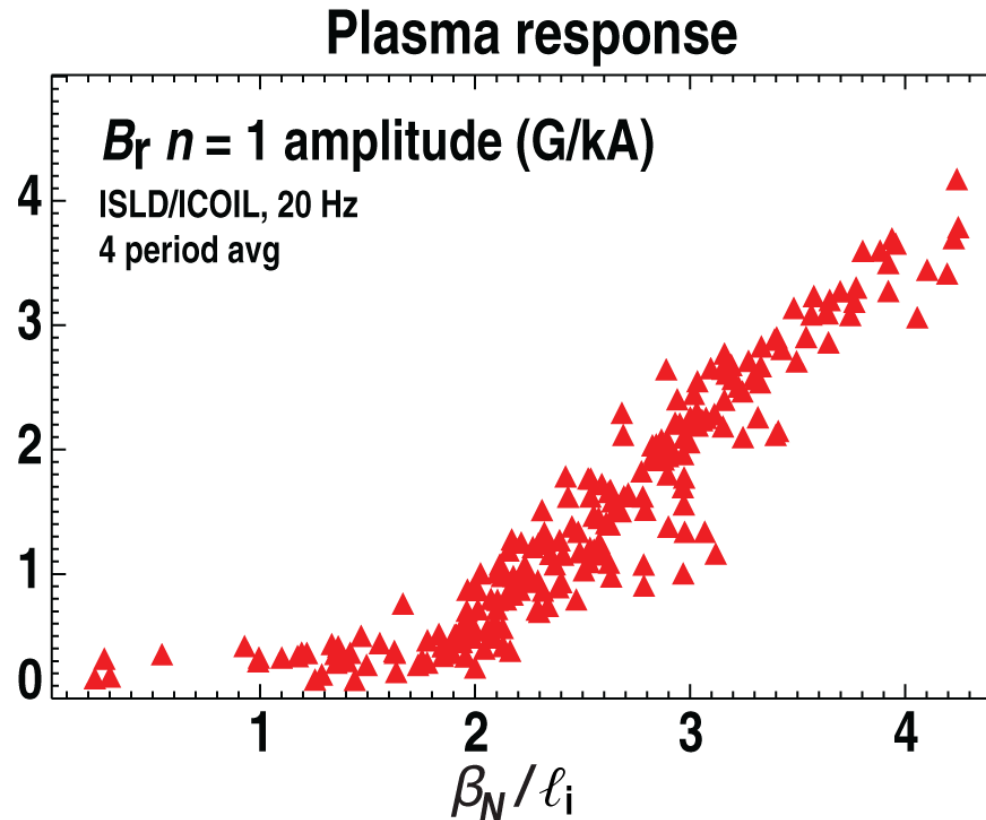
- Data from 14 shots, obtained during AT scenario development day
- Increase of δB^{plas} with β_N widely observed (DIII-D, NSTX, JET)
 - Some devices/scenarios do exhibit deviations from monotonicity

Plasma response dependencies consistent with ideal MHD expectations for RWM stability, below no-wall limit



- See inverse dependence on internal inductance (current profile broadness)
 - Scatter due to wide variations in β_N

Plasma response dependencies consistent with ideal MHD expectations for RWM stability, below no-wall limit

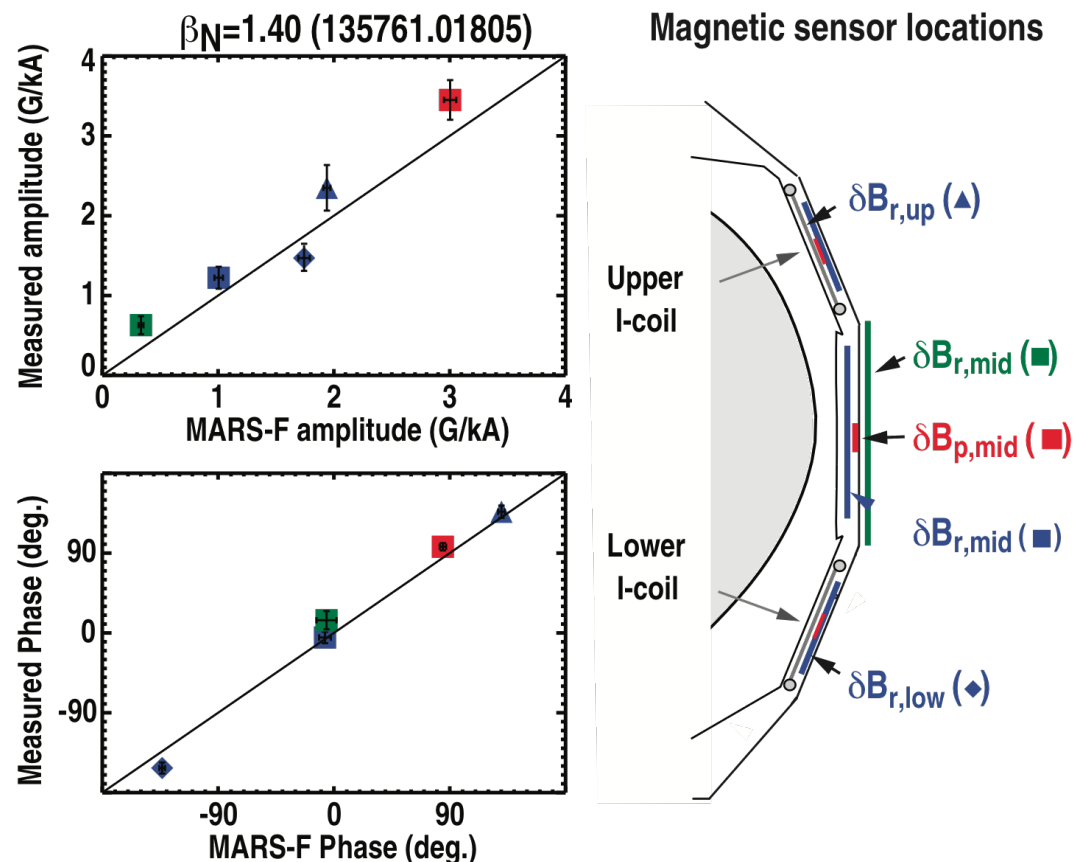


- Clear dependence of plasma response on β_N / ℓ_i normalization

Measured plasma response consistent with linear, ideal MHD below no-wall limit

- Linear ideal MHD calculations (MARS-F) in good agreement with magnetic plasma response measurements

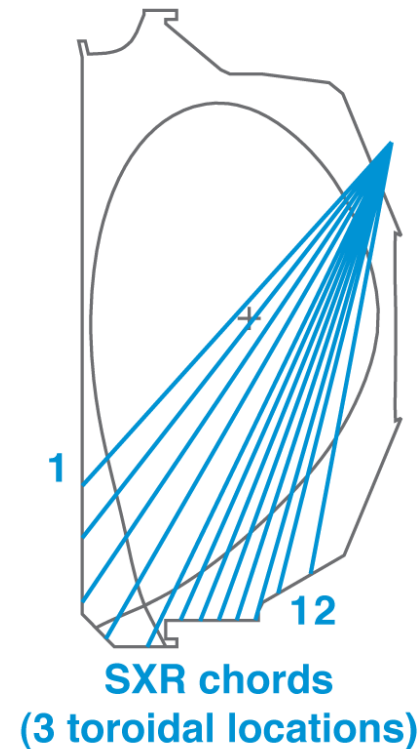
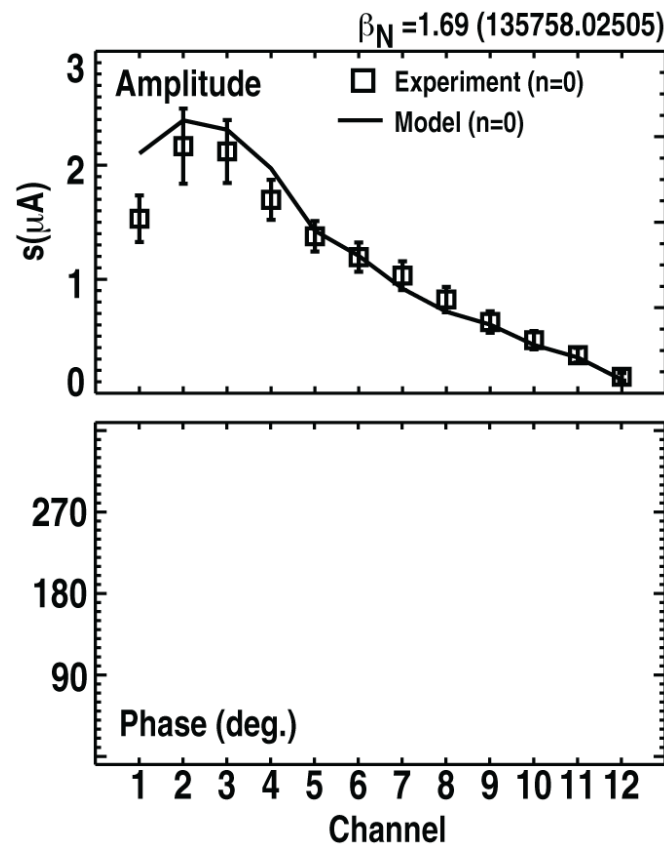
[M.J. Lanctot et al., *Phys. Plasmas* 2010]



Measured plasma response consistent with linear, ideal MHD below no-wall limit

- Linear ideal MHD calculations (MARS-F) in good agreement with SXR profile measurements of plasma response

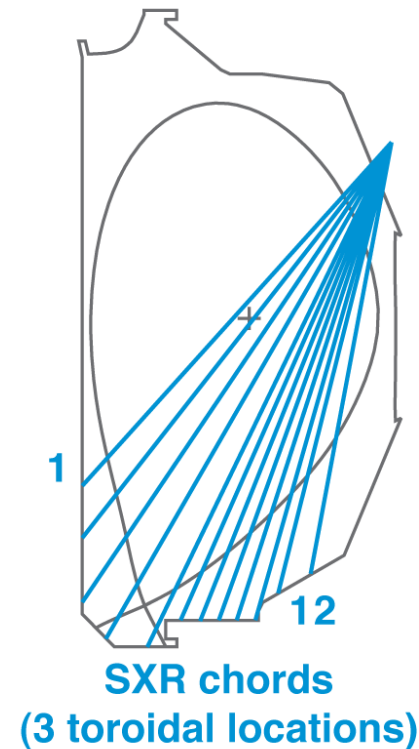
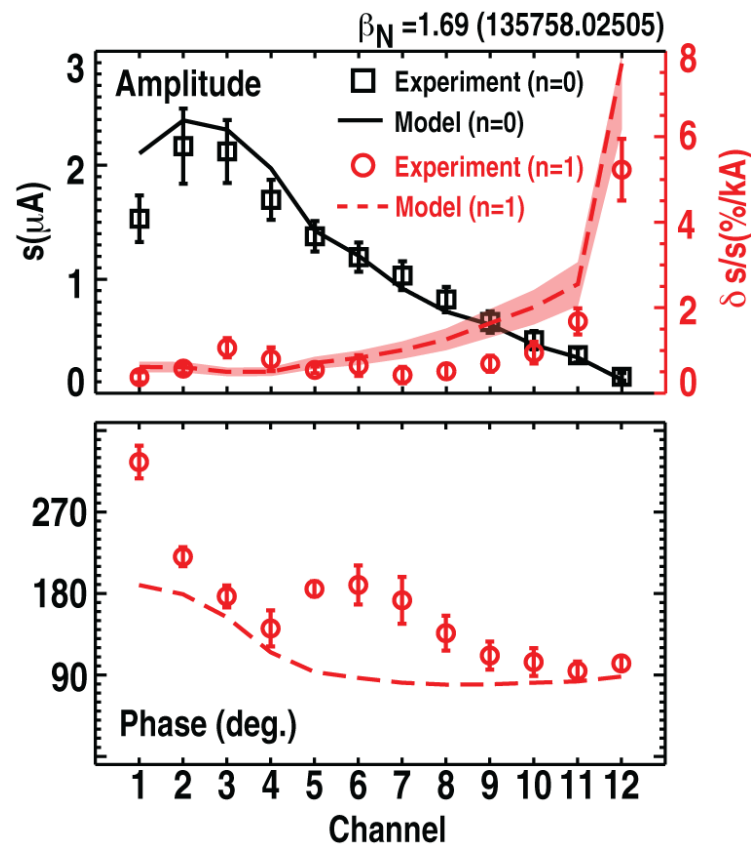
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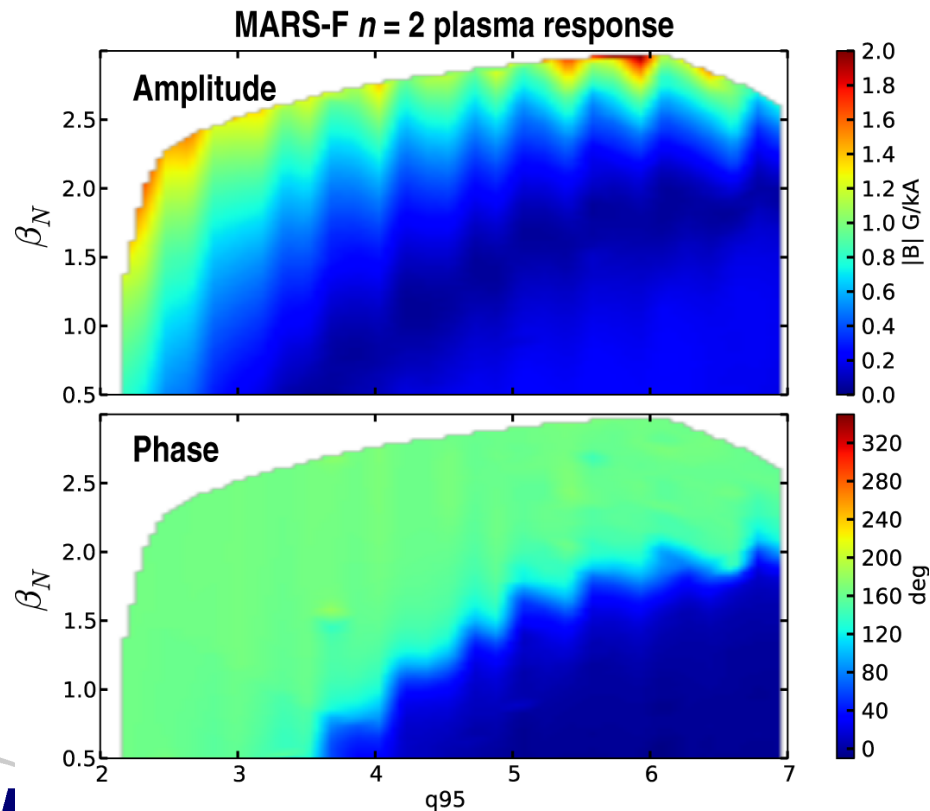
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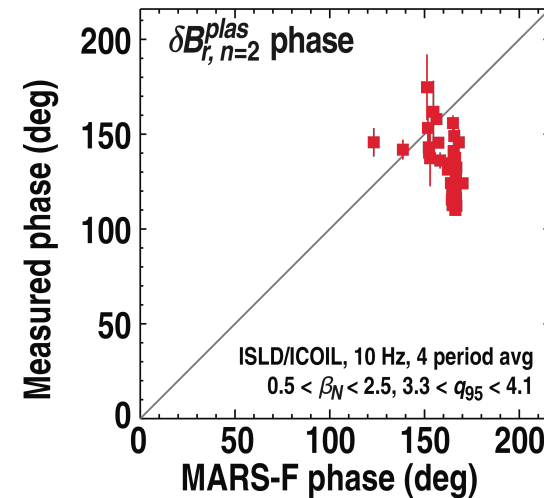
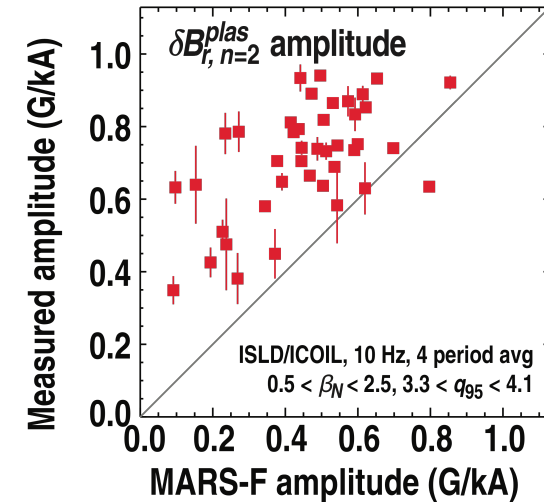
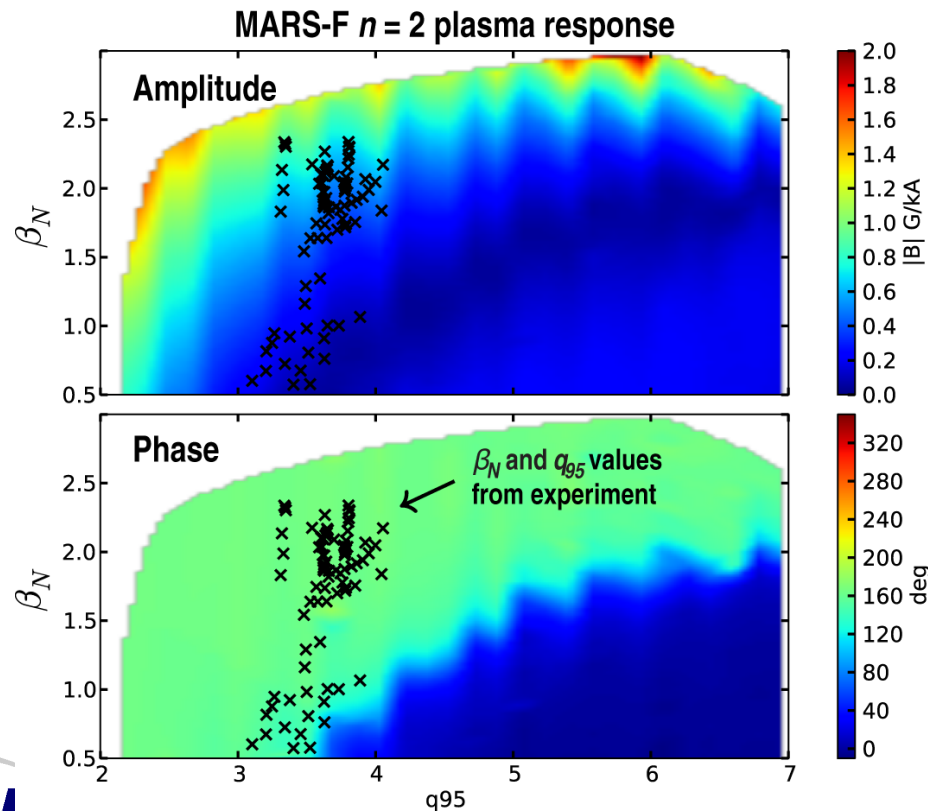
Measurements of $n = 2, 3$ plasma response compared with linear, ideal MHD

- Plasma response to ac $n = 2$ perturbations predicted for a range of β_N , q_{95} values using MARS-F



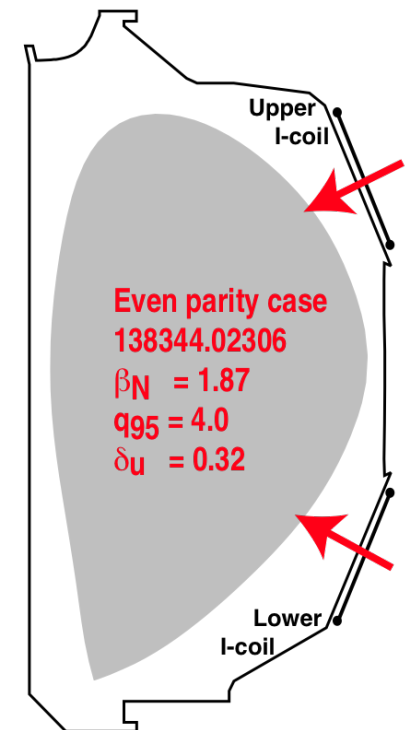
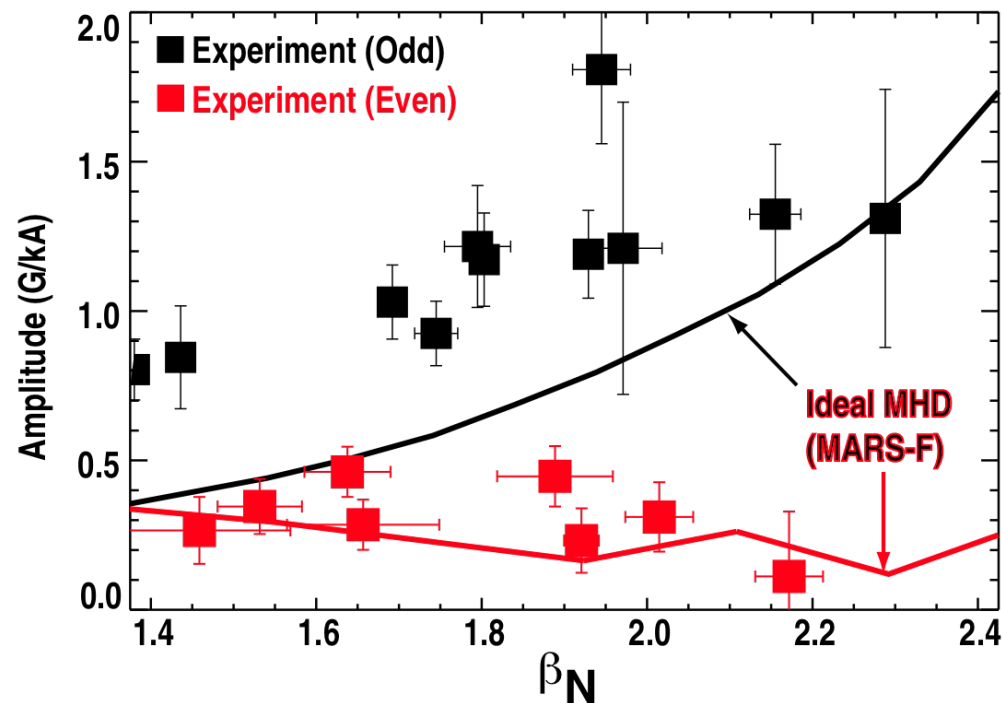
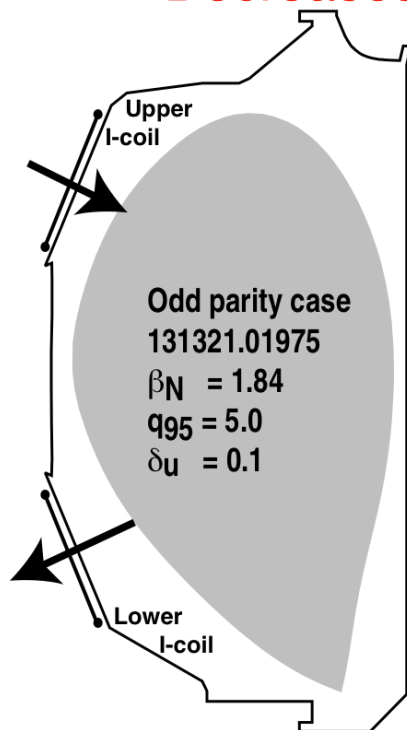
Measurements of $n = 2, 3$ plasma response compared with linear, ideal MHD

- Plasma response to ac $n = 2$ perturbations predicted for a range of β_N , q_{95} values using MARS-F
- Preliminary comparison with experiment yields qualitative agreement



Measurements of $n = 2, 3$ plasma response compared with linear, ideal MHD

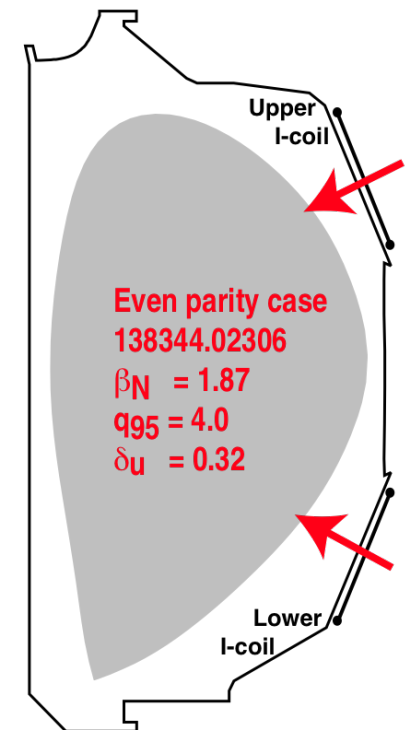
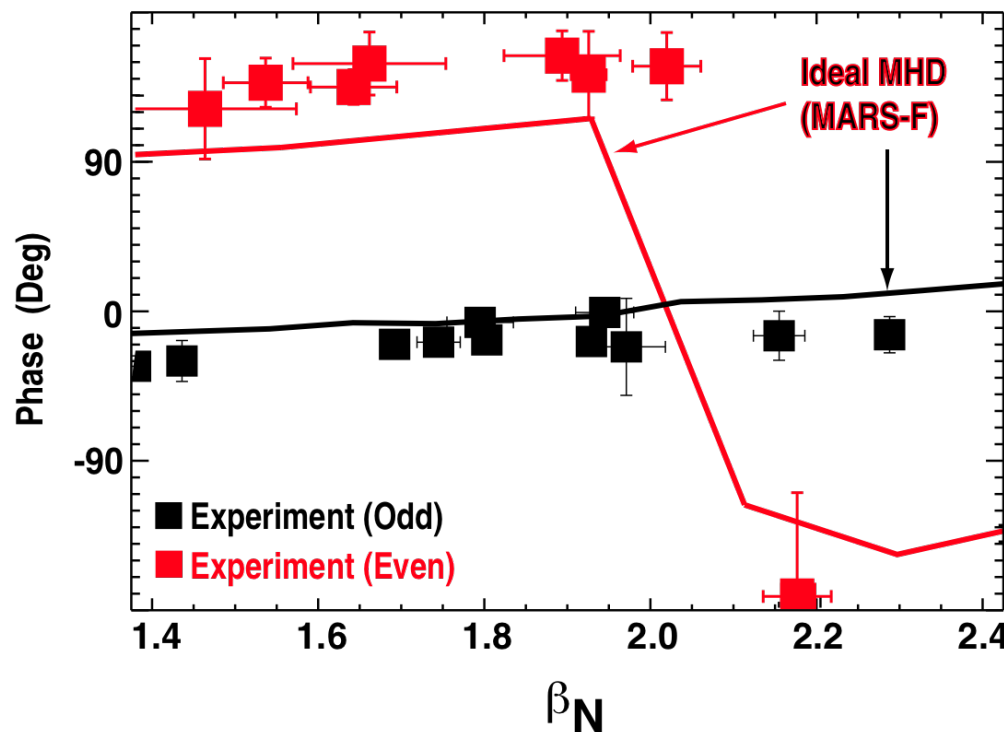
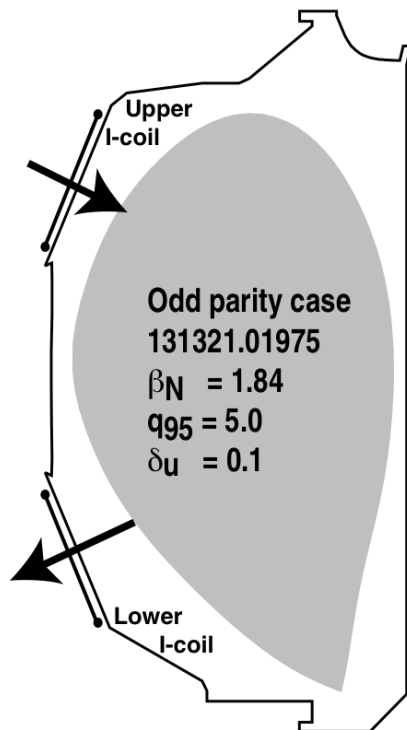
- Probe plasma with static $n=3$ fields using odd and **even** parity
- Measurements and modeling (MARS-F) show plasma δB at midplane
 - Increases with β_N for odd parity field
 - **Decreases with β_N for even parity field**



[Lanctot, et al., Phys. Plasmas 2011]

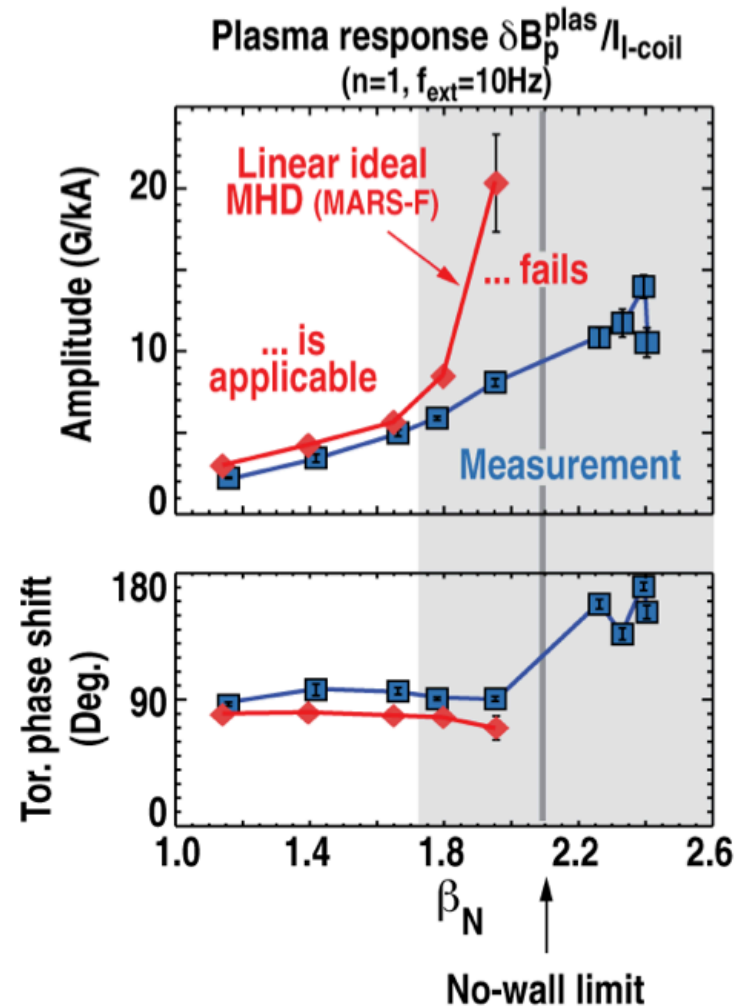
Measurements of $n = 2, 3$ plasma response compared with linear, ideal MHD

- Measured phase of **even parity** response field drifts by 60° with β_N in agreement with linear ideal MHD model
 - Odd parity phase is relatively constant



Linear, ideal MHD sufficient to predict plasma response below the no-wall limit

- Scan of β_N dependence of $n = 1$ plasma response reveals limitation of linear ideal MHD
 - Ideal MHD works for $\beta < 0.8 \beta^{nw}$
 - Diverges near no-wall limit
 - Predicts instability above no-wall limit
- Progress in describing observed stability above no-wall limit with kinetic modifications to ideal MHD energy principle



[Lanctot, *et al.*, *Phys. Plasmas* 2010]

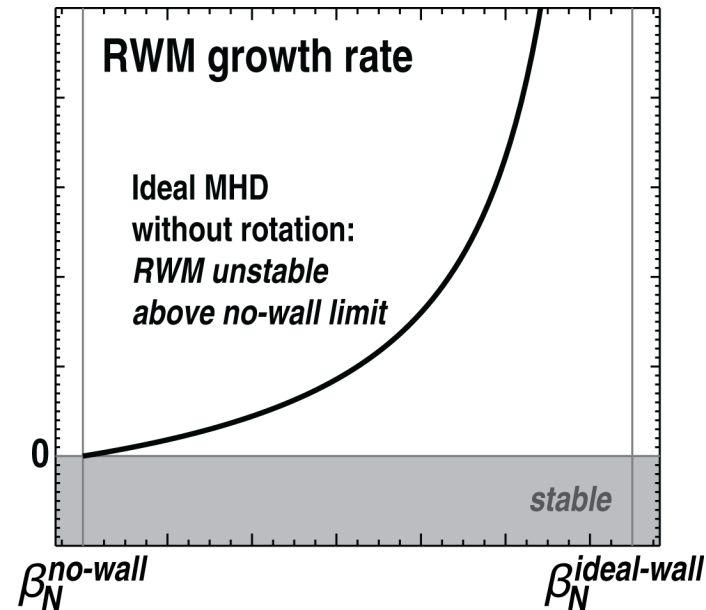
Uncovering kinetic modifications to ideal MHD

Kinetic wave-particle damping leads to enhanced RWM stability above no-wall limit

- **Ideal MHD energy principle modified to include kinetic damping physics**

[Hu and Betti, PRL, 2004].

$$\gamma\tau_w = -\frac{\delta W_{nw}}{\delta W_{iw}}$$



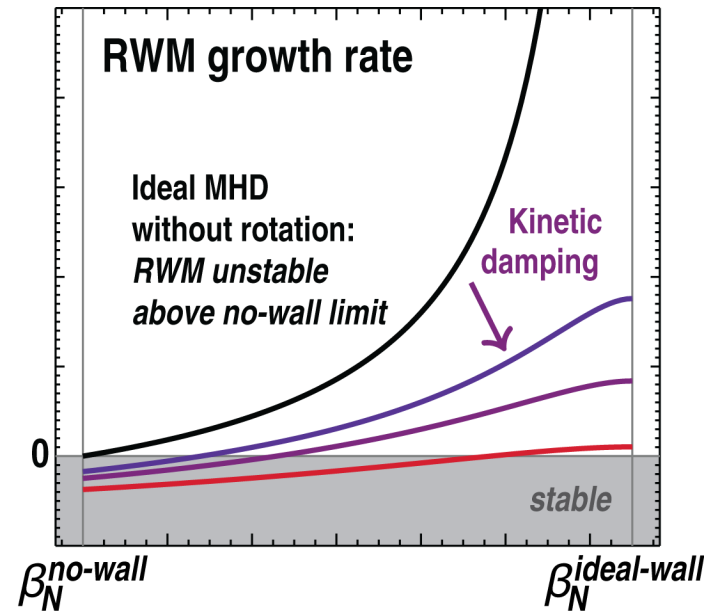
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$$\gamma\tau_w = -\frac{\delta W_{nw}}{\delta W_{iw}} \rightarrow -\frac{\delta W_{nw} + \delta W_K}{\delta W_{iw} + \delta W_K}$$

$$\delta W_K = \frac{1}{2} \int \vec{\xi}_\perp \cdot \vec{\nabla} \cdot \tilde{P}_K d\vec{v}$$



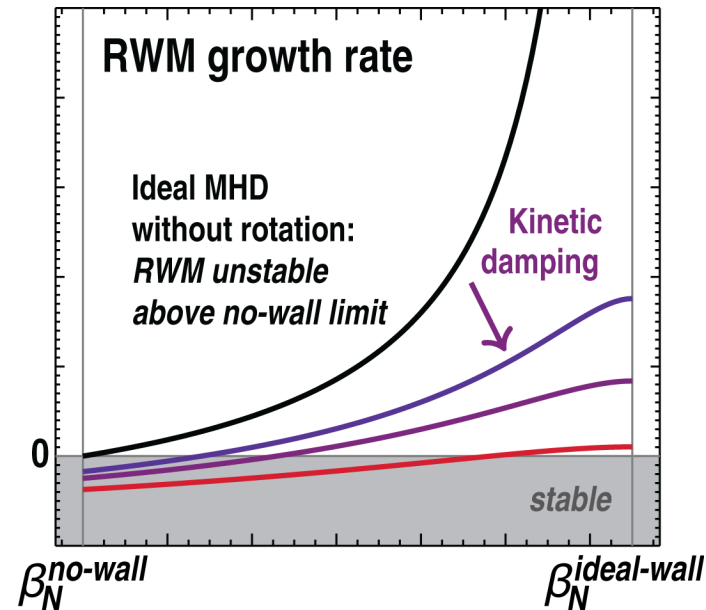
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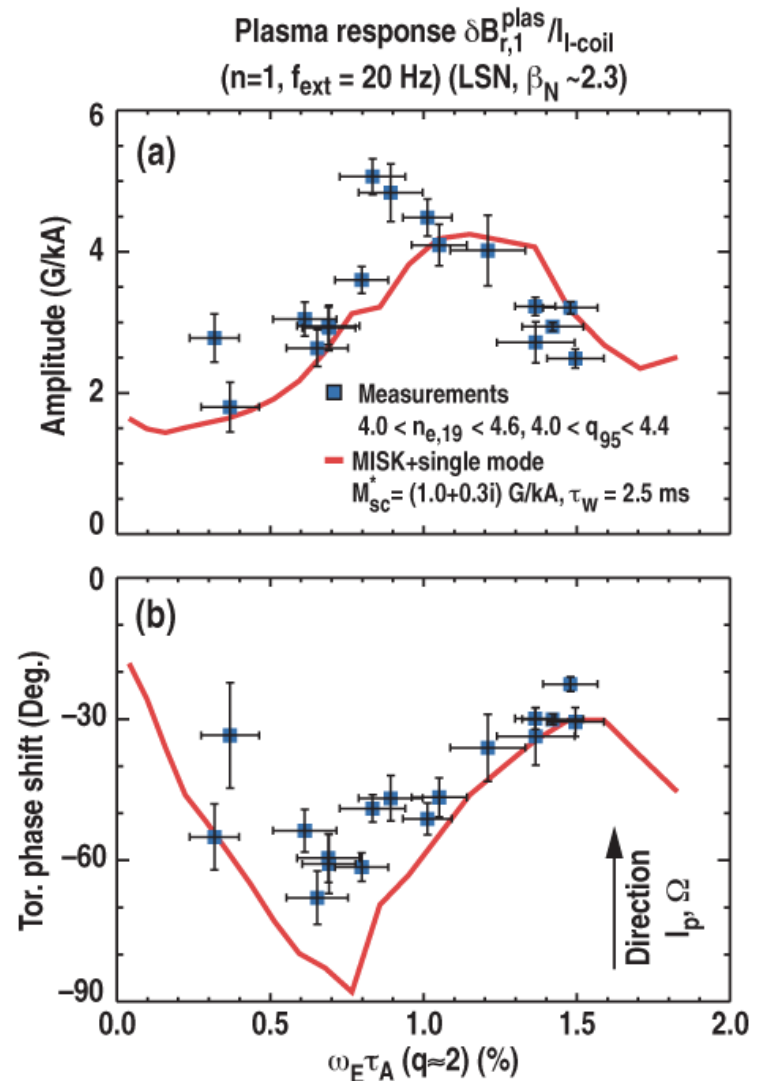


- Kinetic energy principle δW_K allows for energy exchange between RWM and kinetic particle populations:
 - Resonances between motion of *trapped particles* and *plasma rotation*
 - **Non-resonant** effects that depend on alignment of distribution function gradients and the RWM eigenfunction
 - Several codes incorporate this physics: MISK, HAGIS, MARS-K

Kinetic RWM stability effects investigated in stable plasmas above the no-wall limit

- Measurements of **plasma response** to slowly rotating $n = 1$ perturbations used to compare theory and experiment
- **Rotation scan** revealed evidence of **trapped particle resonances** in DIII-D; complemented NSTX work on the RWM stability threshold

[Berkery, *et al*, PRL, 2010]



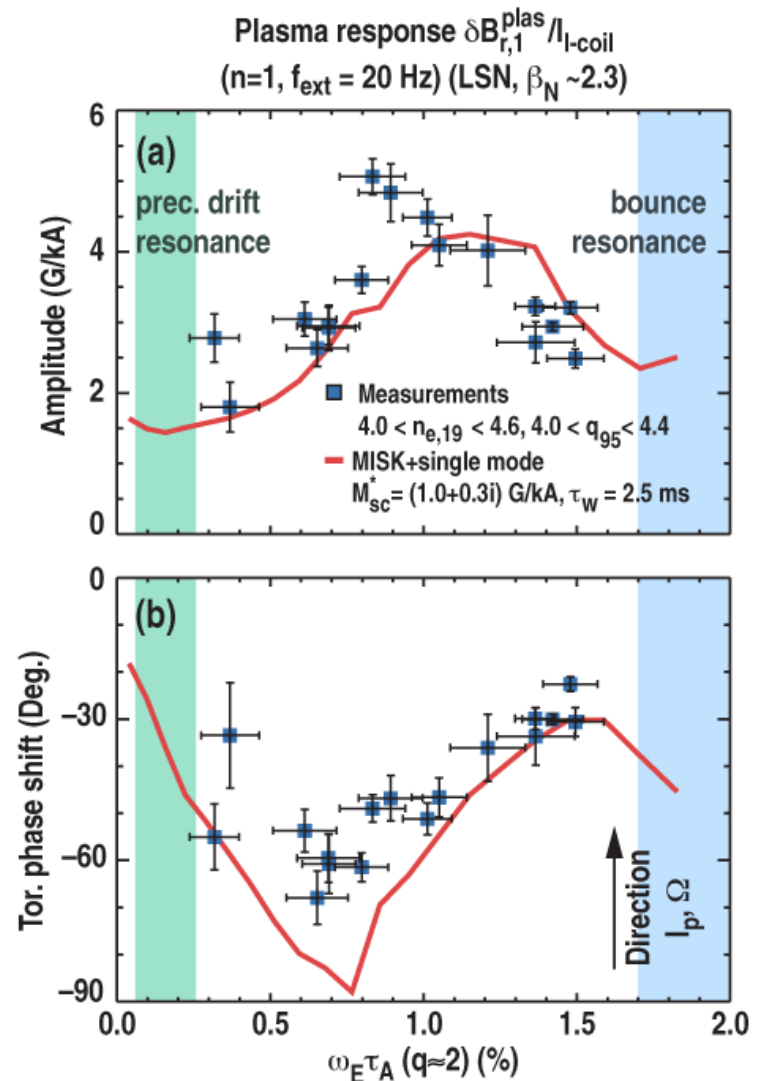
[Reimerdes, *et al*, PRL, 2011]



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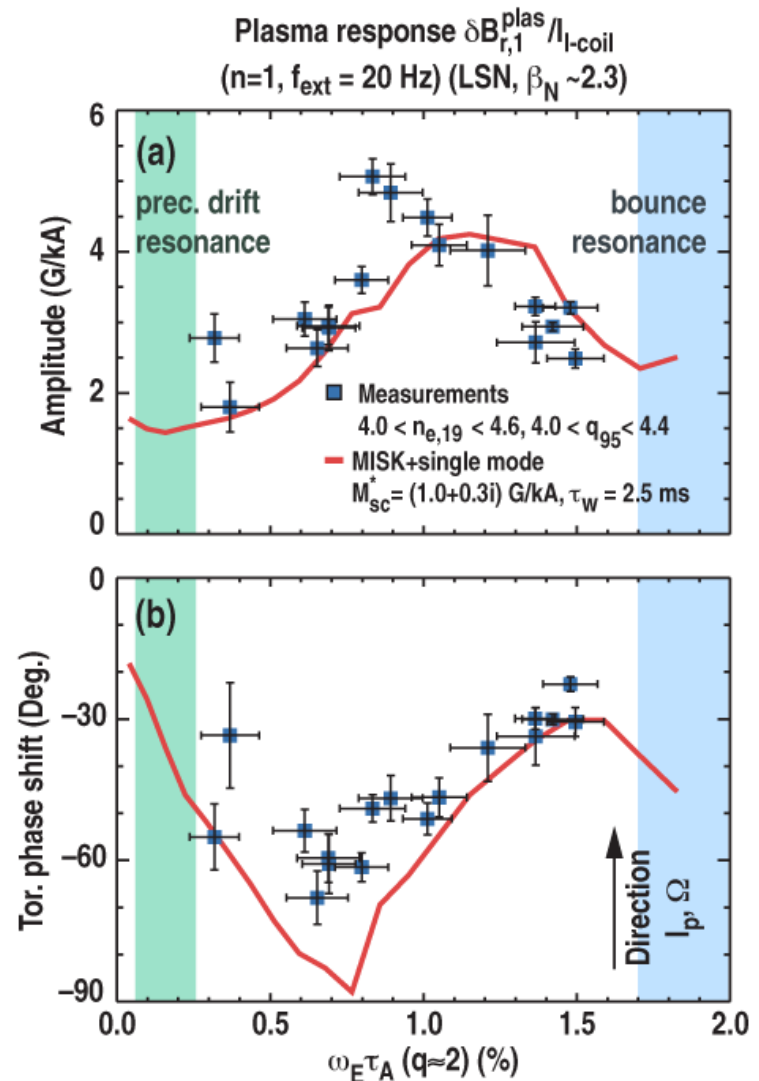


Kinetic RWM stability effects investigated in stable plasmas above the no-wall limit

- Measurements of **plasma response** to slowly rotating $n = 1$ perturbations used to compare theory and experiment
- Rotation scan** revealed evidence of **trapped particle resonances** in DIII-D; complemented NSTX work on the RWM stability threshold
- Ratio of $\beta_{fast} / \beta_{thermal}$ investigated in MAST using **density scans**
- Recent DIII-D experiment: use **off-axis NBI** to impact **trapped ion fraction**

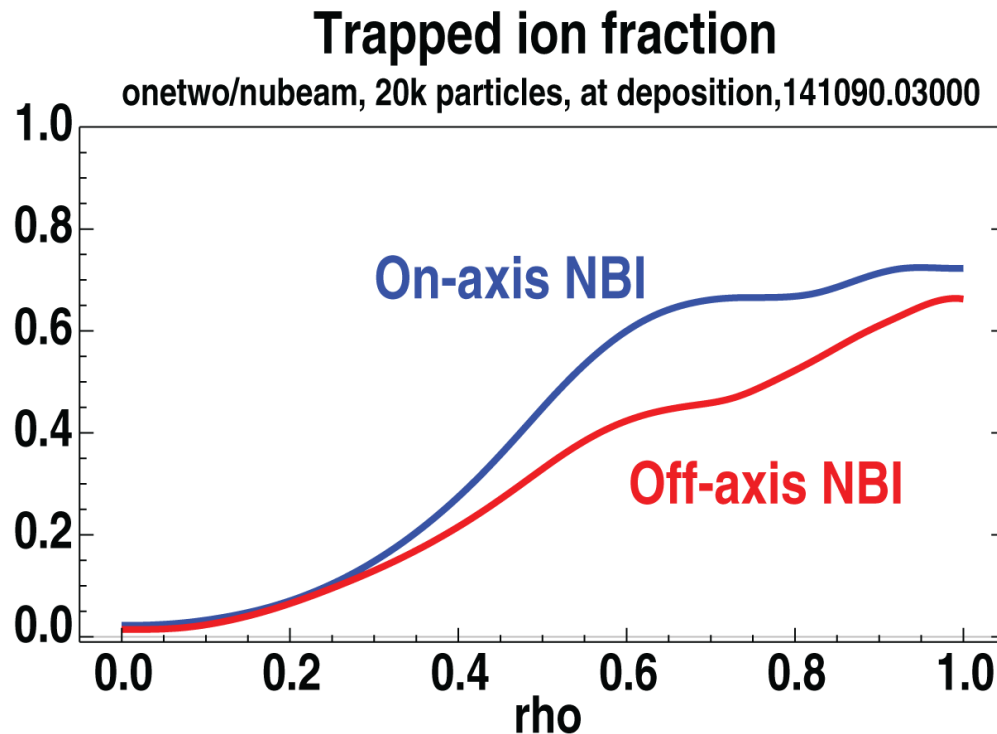
[Berkery, *et al*, PRL, 2010]

[Chapman, *et al*, PPCF, 2011]



[Reimerdes, *et al*, PRL, 2011]

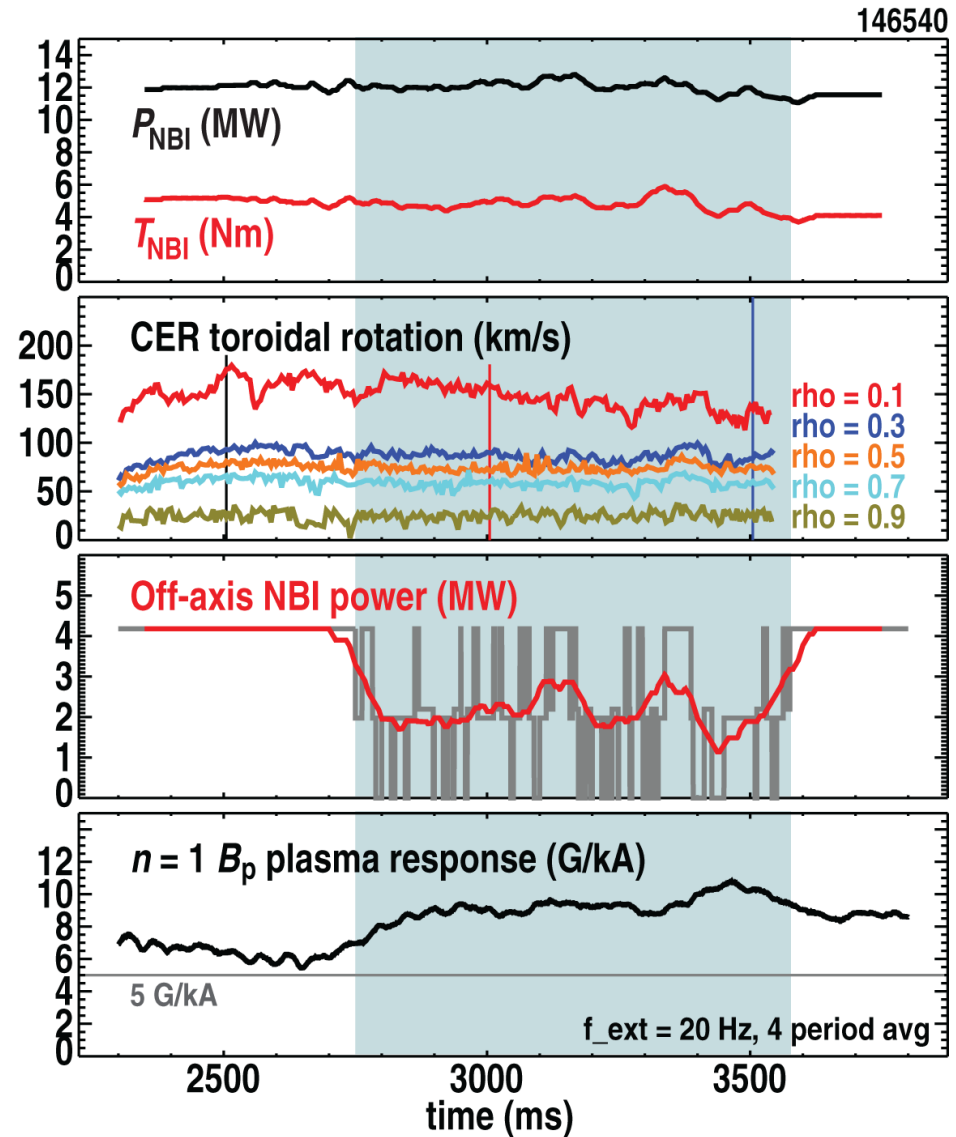
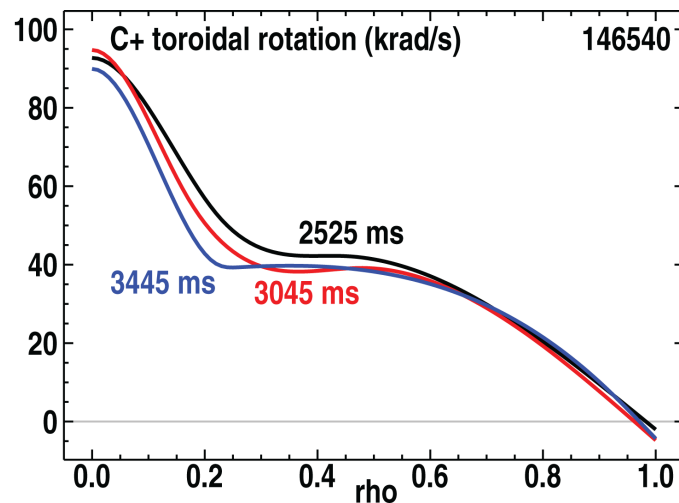
Off-axis NBI expected to decrease RWM stability



- **Transport modeling: reduced trapped ion fraction with off-axis NBI**, due to more favorable alignment of injection angle with field line pitch.
- **Reduced RWM stability expected with off-axis NBI**; stabilizing effect of passing particles expected to be localized near resonant surfaces, small.

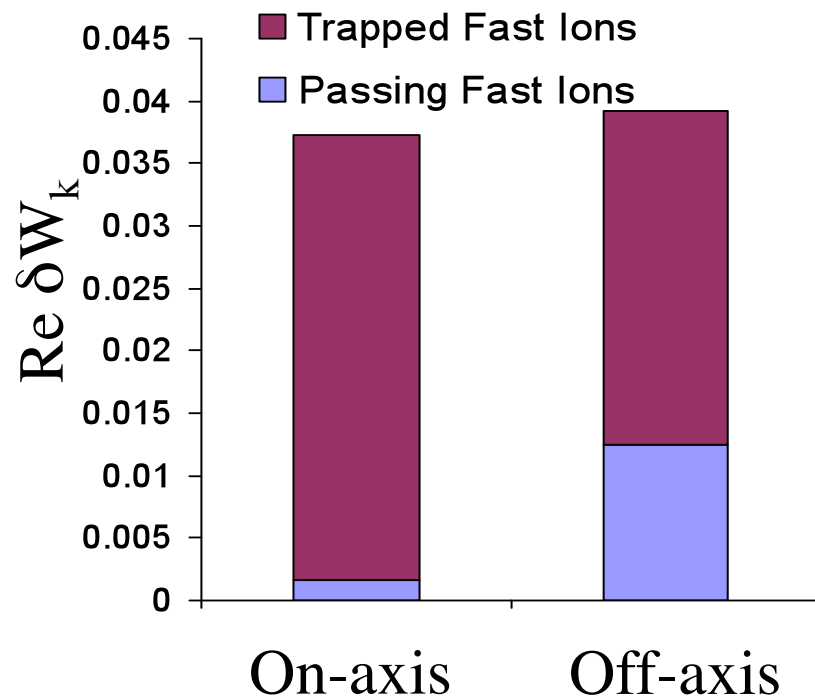
Off-axis NBI leads to increased RWM stability

- Off-axis NBI power modulated at constant β_N , ℓ_i , density
 - Minor variations in rotation profile
- Plasma response increases with decreased off-axis power
 - Opposite of expectation from considerations of trapped ion fraction



Including finite orbit width effects results in enhanced damping of RWM with off-axis NBI

- TRANSP predicts fast ion distribution function
- HAGIS evolves interaction between fast ions and RWM

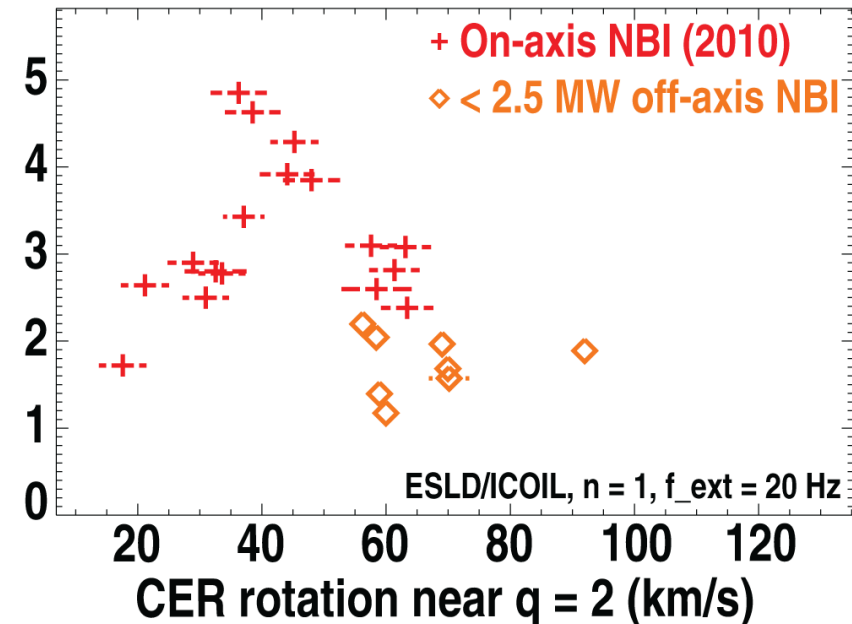


- Scanned radial peak of distribution function using simplified model for F_h
- Passing fast ion damping sensitive to location of peak of F_h with respect to rational surfaces

Stabilizing effect of off-axis NBI observed over a range of rotation

- Existing 2010 dataset extended to higher rotation

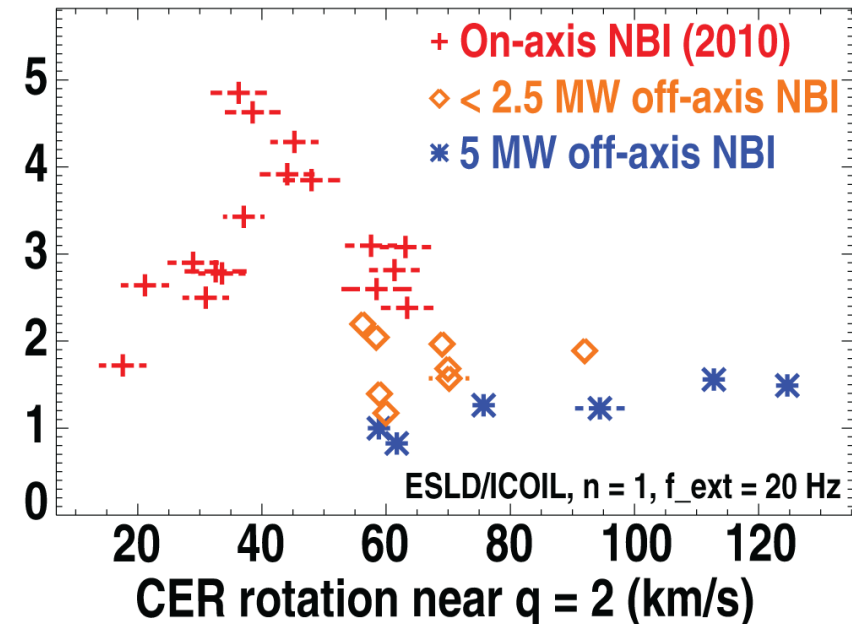
Radial field plasma response amplitude (G/kA)



Stabilizing effect of off-axis NBI observed over a range of rotation

- Existing 2010 dataset extended to higher rotation
- See ~50% *reduction* in plasma response amplitude with 5 MW off-axis NBI, at intermediate rotation.

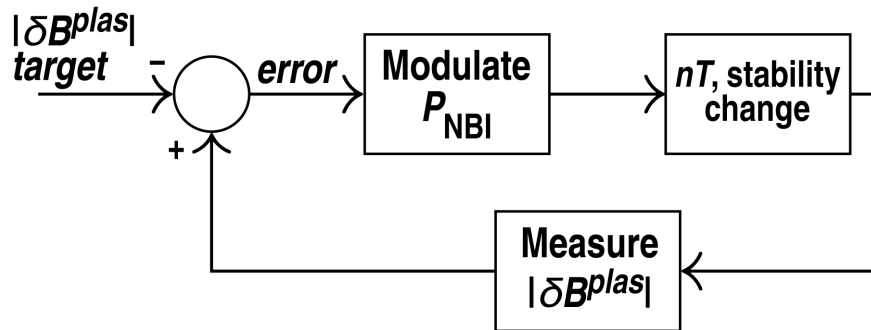
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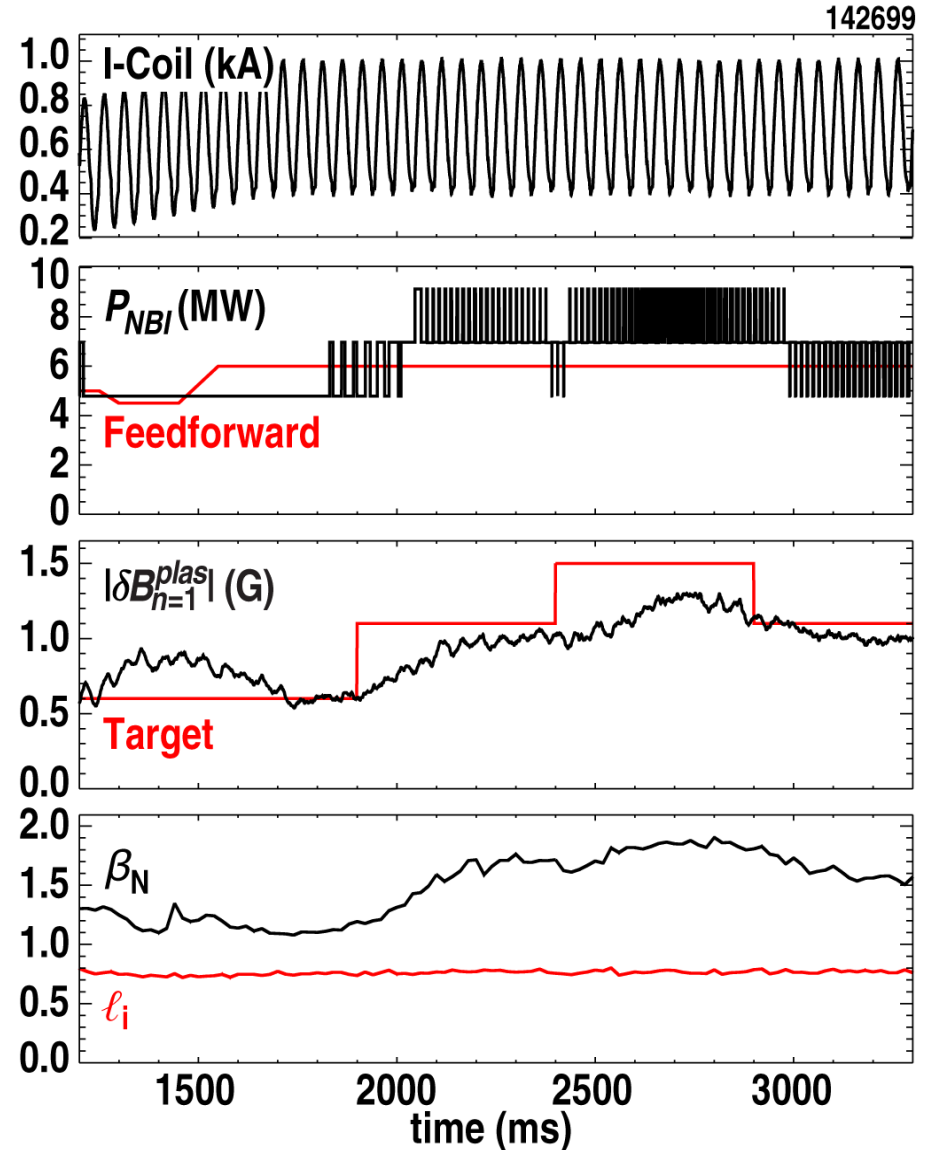
- Continued damping at increased rotation qualitatively consistent with theoretical expectations.
- Resonances with additional bounce frequency harmonics encountered as rotation increases.

Controlling proximity to the RWM stability limit

RWM stability directly controlled with NBI for first time



- Plasma response measurement input to NBI control algorithm
- Plasma response settles to a value near the target on a timescale close to $\tau_E \sim 100$ ms.
- β_N changes linearly with plasma response amplitude – expected below no-wall limit.



Observed δB^{plas} dependencies suggest 2 parameter dynamical model

- Consider a model of the form

$$\frac{d}{dt} |\delta B_{n=1}^{\text{plas}}| = -\frac{1}{\tau} |\delta B_{n=1}^{\text{plas}}| + c P_{\text{NBI}}.$$

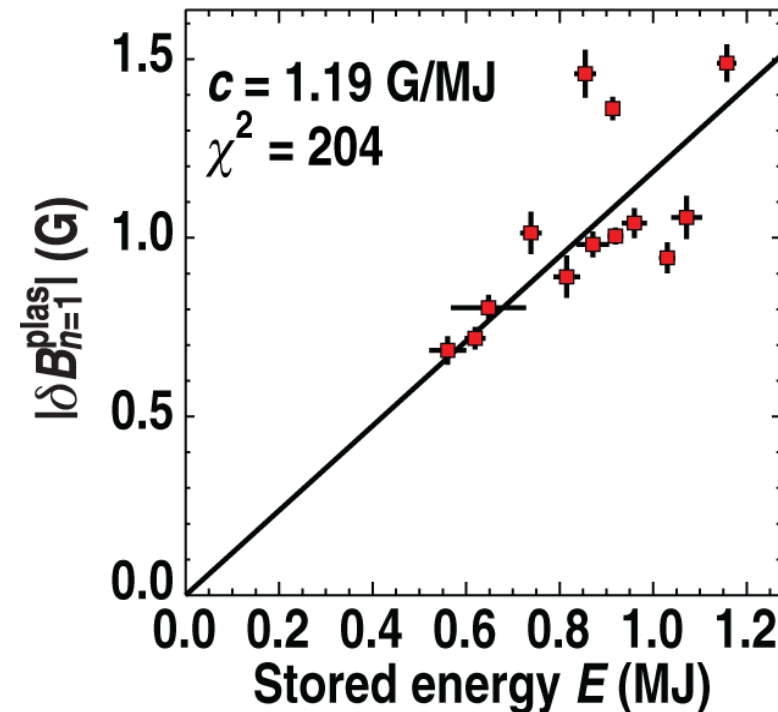
- 2 parameters can be estimated by fitting experimental data
 - Time-constant τ
 - Beams coupling coefficient c

- Equivalent to a model* for the plasma stored energy E , if

$$\tau = \tau_E \quad \text{and} \quad |\delta B_{n=1}^{\text{plas}}| = cE = c \frac{I_p B_t}{2\mu_0 a} \beta_N$$

- Expect this scaling to work for constant I_p , B_t , a .

Least squares fit to $|\delta B_{n=1}^{\text{plas}}| = cE$



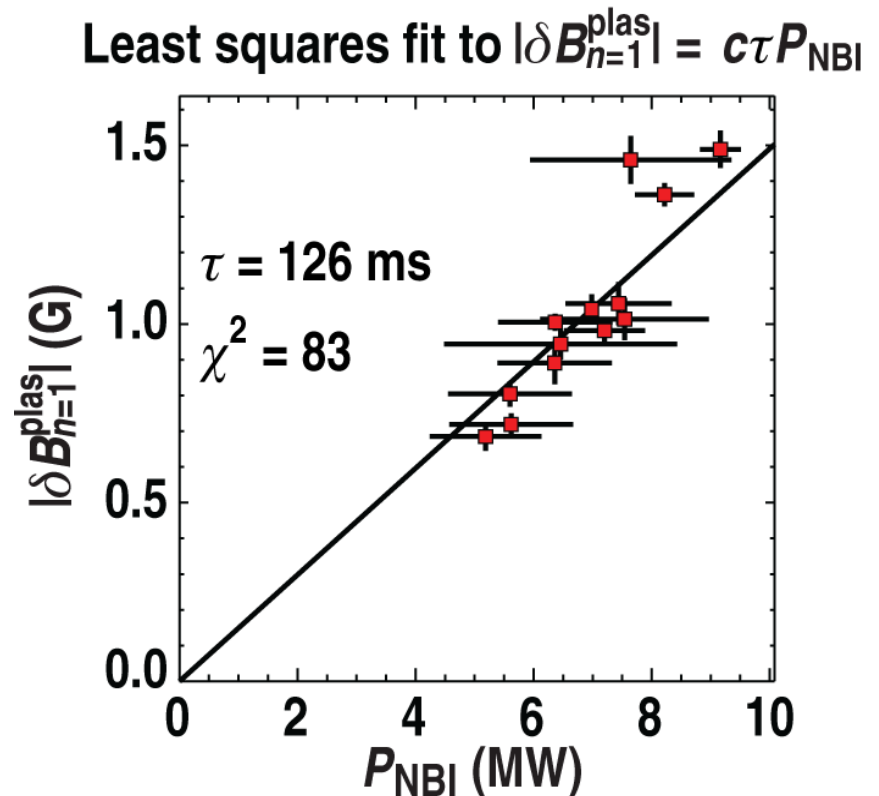
*J. T. Scoville, et al., *Fusion Eng. and Design* **45**, A367 (2003)

Time-constant obtained from fit to steady-state limit

- In the steady state limit, obtain

$$|\delta B_{n=1}^{\text{plas}}| = c\tau_E P_{\text{NBI}}.$$

- So time constant can be obtained from least squares fit if c is known.
- These linear relationships will not necessarily hold at high β_N above the no-wall limit.
- However, super-linear dependence of $|\delta B^{\text{plas}}|$ with β_N suggests that a linear controller would naturally avoid the RWM's true marginal stability point.



Conclusions

- **Ideal MHD describes perturbed equilibria below no-wall beta limit**
 - Large body of measurements consistent with ideal MHD at low $\beta < \beta^{nw}$
 - Magnetic ($n = 1, 2, 3$) and SXR profile measurements ($n = 1$) compared with theory
- **Kinetic modifications to ideal MHD needed above no-wall limit**
 - Experimental evidence for wave – particle interactions uncovered, qualitatively consistent with kinetic theory.
 - Off-axis NBI used to probe kinetic damping in recent experiment, lead to increased damping of RWM
 - New data obtained for comparisons with theory.
 - Preliminary calculations indicate finite ion orbit width effects may be important
- **Direct RWM stability control demonstrated using NBI feedback**
 - Linear feedback dynamics obtained below no-wall limit
 - Possible solution for maximizing β while avoiding unstable RWM
 - Challenges expected above no-wall limit: rotation, kinetic effects impact feedback linearity