

# Uncovering RWM stability limits in tokamaks

By

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In collaboration with

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Presented at

**16<sup>th</sup> Workshop on MHD Stability Control**

**21 November 2011**



**COLUMBIA UNIVERSITY**  
IN THE CITY OF NEW YORK

# Measurements of plasma response to applied perturbations used to understand, control RWM stability

- **Motivation: need to validate theories of RWM stability for ITER, beyond**
  - Resistive wall modes arise from the interaction between an external kink mode and wall eddy currents. RWMs can be *global, beta-limiting instabilities*.
  - Recent experiments on several devices have shown complex dependence of stability on plasma rotation, *lack of a critical rotation threshold*.
- **Compare driven response of stable plasma with theory**
  - $\delta B/B < 10^{-3}$  – refer to this state as a *perturbed equilibrium*
- **Ideal MHD describes perturbed equilibria below no-wall beta limit**
  - Large body of measurements consistent with ideal MHD when  $\beta < \beta^{no-wall}$
- **Kinetic modifications to ideal MHD needed above no-wall limit**
  - Evidence for wave–particle interactions uncovered, qualitatively consistent with theory.
  - Off-axis NBI used to probe kinetic damping in recent experiment
- **Direct stability control demonstrated using NBI feedback**
  - Feedback dynamics are linear below no-wall limit.



# Outline

## 1. Making perturbative measurements of RWM stability

- Measure plasma response  $\delta B^{\text{plas}}$  to applied fields in stable plasmas

## 2. Linking plasma response and ideal MHD theory

- Ideal MHD describes experiments for  $\beta < \beta^{\text{no-wall}}$ , plasma rotation sufficiently high

## 3. Uncovering kinetic modifications to ideal MHD

- Kinetic modifications important when  $\beta > \beta^{\text{no-wall}}$

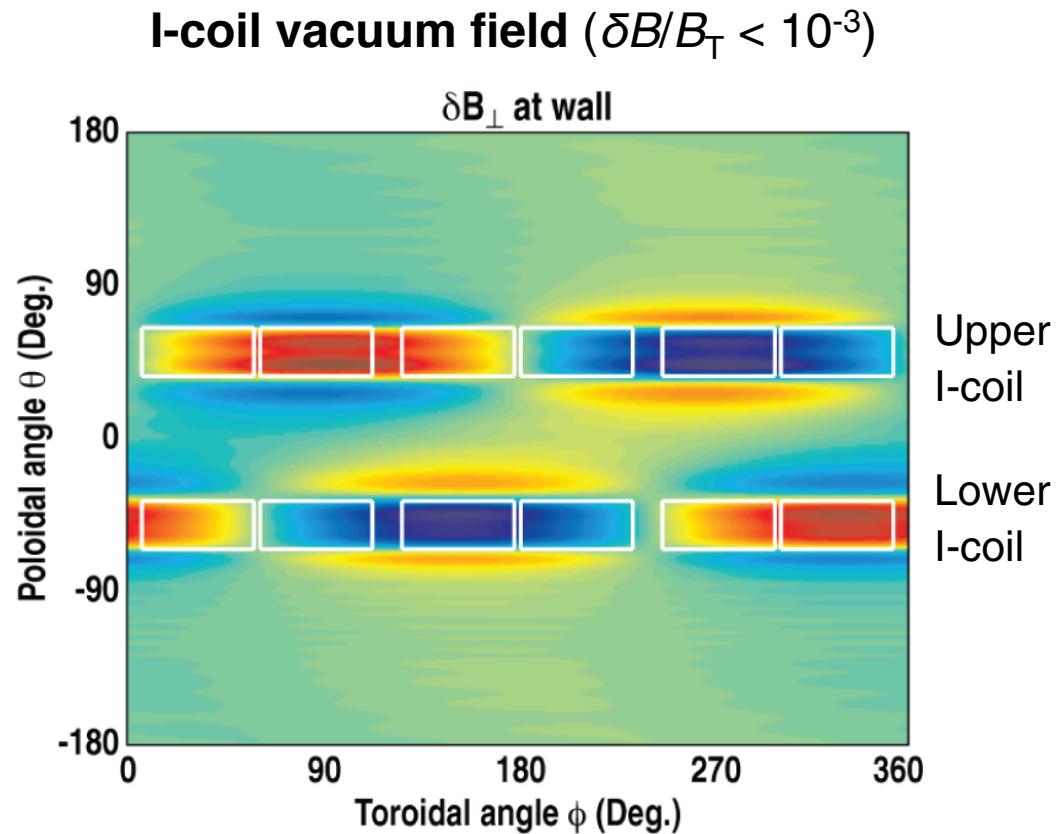
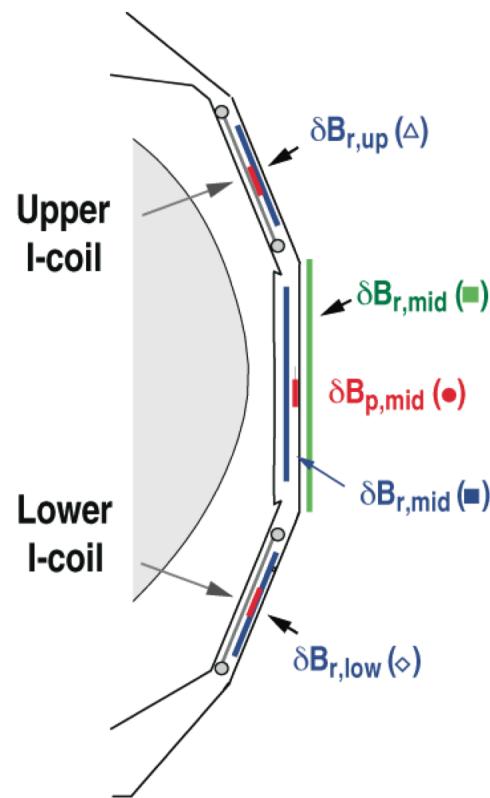
## 4. Controlling the proximity to the RWM stability limit

- Direct control of RWM stability margin demonstrated using NBI feedback

# Perturbative measurements of RWM stability

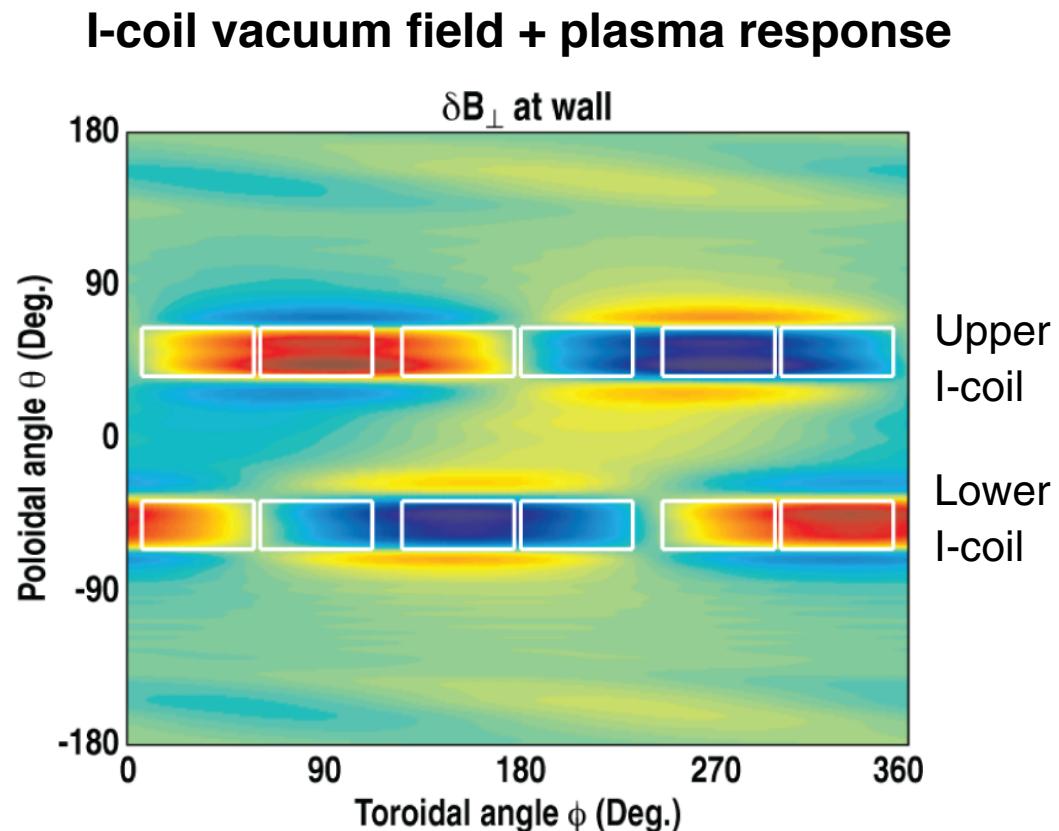
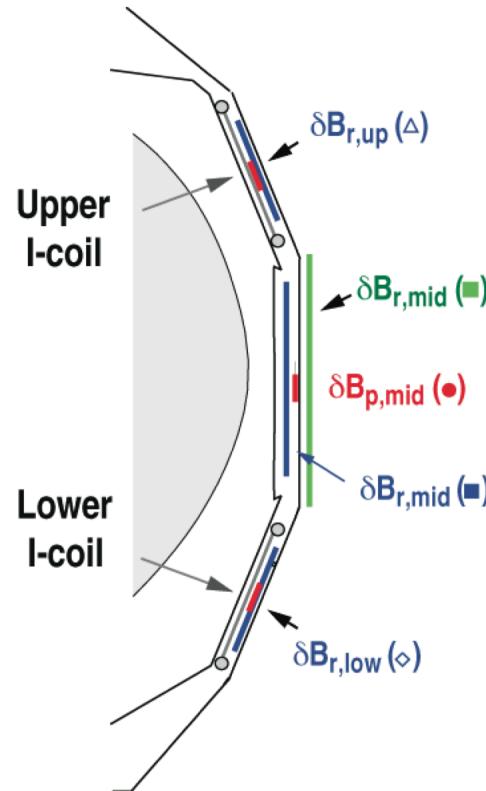


# DIII-D tokamak is well equipped to create and measure perturbed equilibria



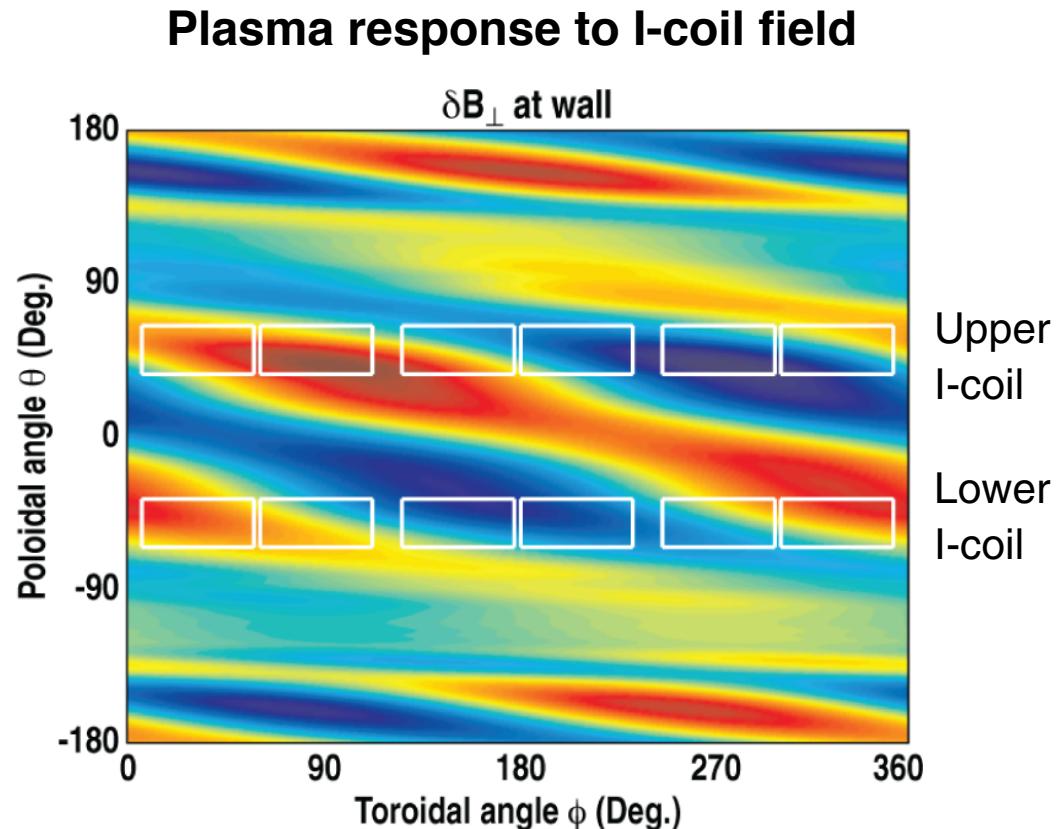
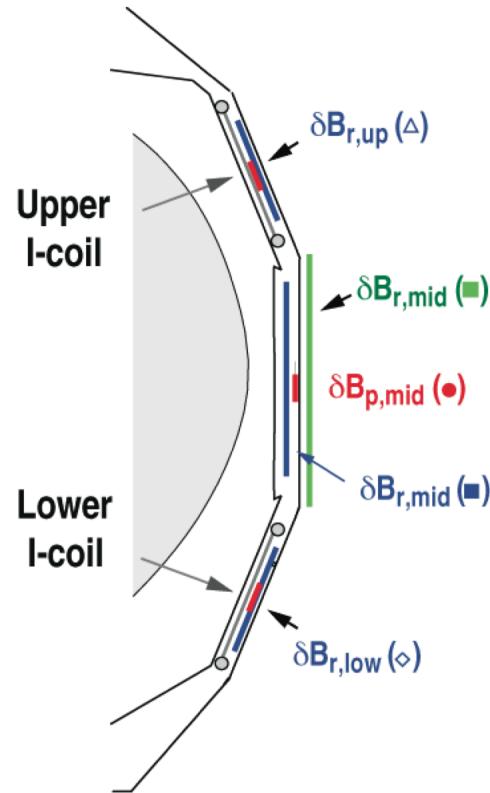
**Spectrum of applied I-coil field chosen to resonate with plasma kink mode**

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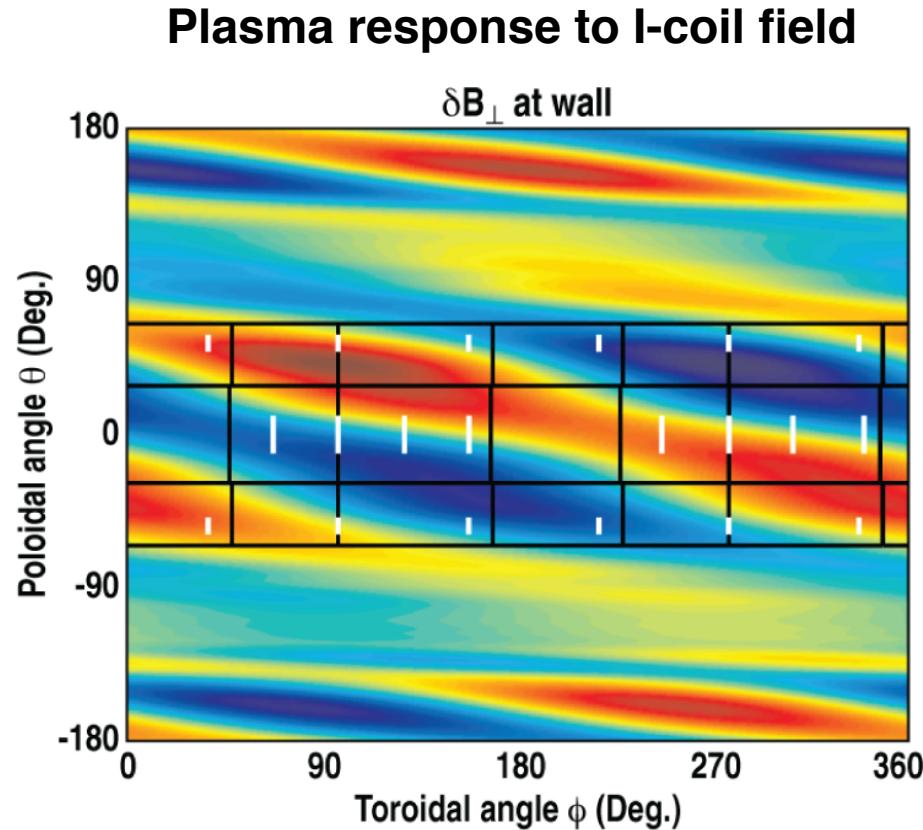
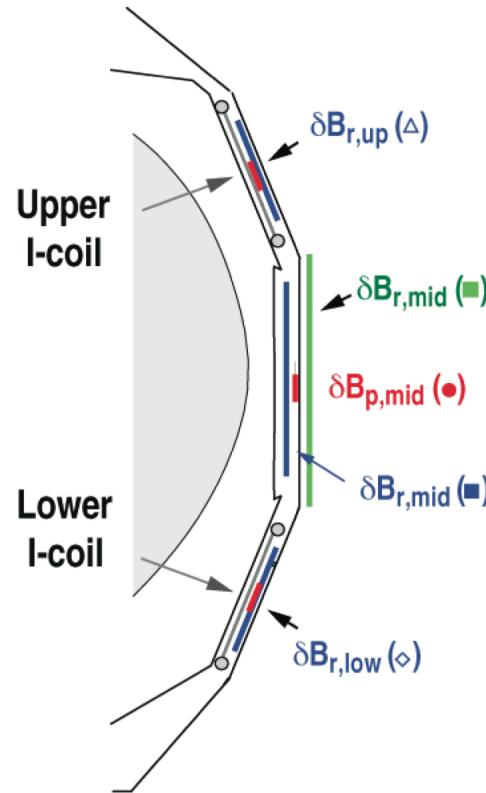
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**Spectrum of applied I-coil field chosen to resonate with plasma kink mode**

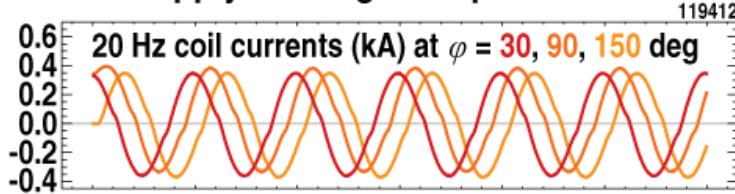
# DIII-D tokamak is well equipped to create and measure perturbed equilibria



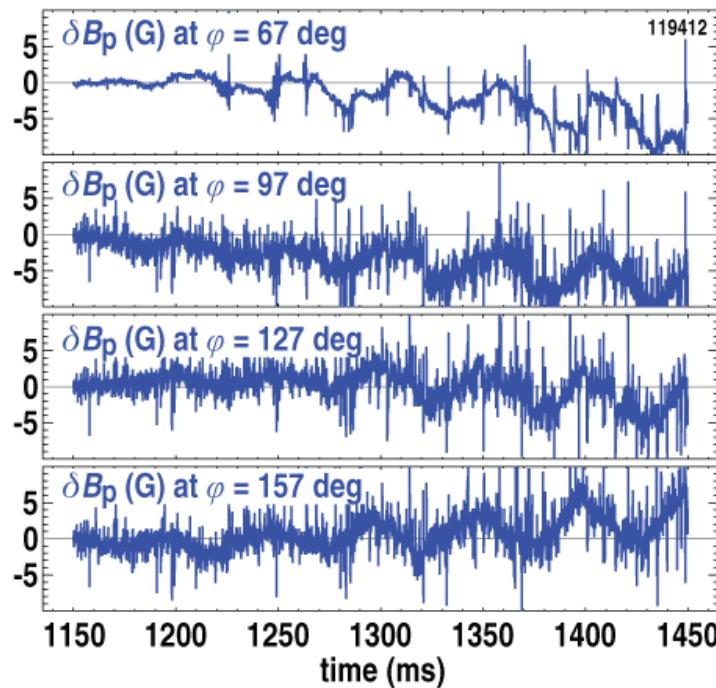
Fits to magnetic measurements yield amplitude and toroidal phase of plasma response

# Single-frequency analysis yields plasma response to applied, rotating perturbation

## 1. Apply rotating $n = 1$ perturbation

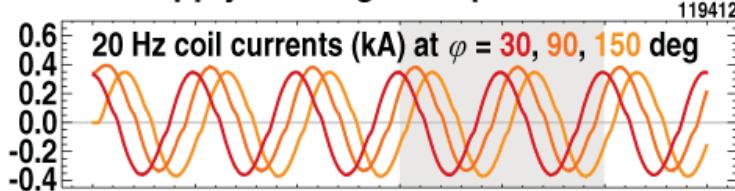


- **Apply rotating perturbation** near natural rotation frequency of RWM, ~20 Hz in DIII-D.

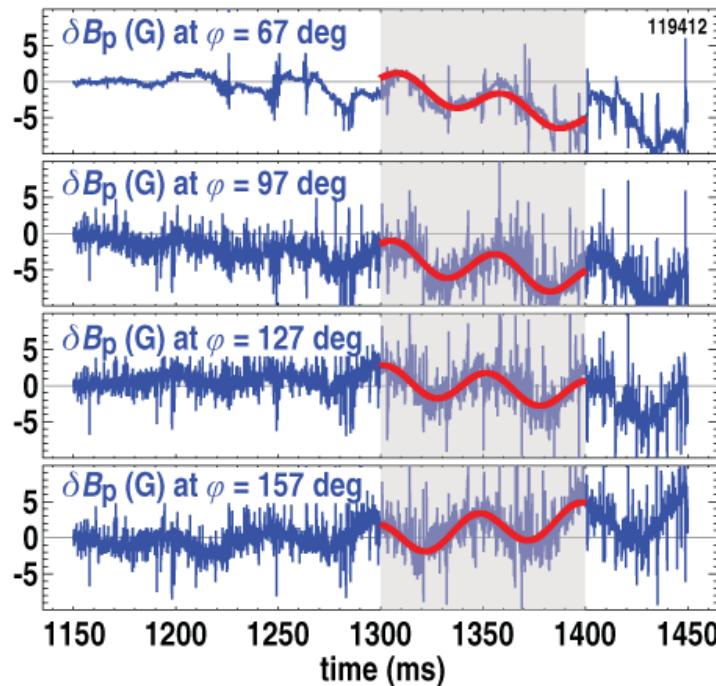


# Single-frequency analysis yields plasma response to rotating, 3D perturbation

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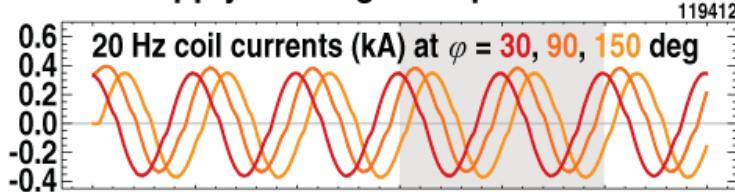
## 2. Fourier analysis gives resonant response



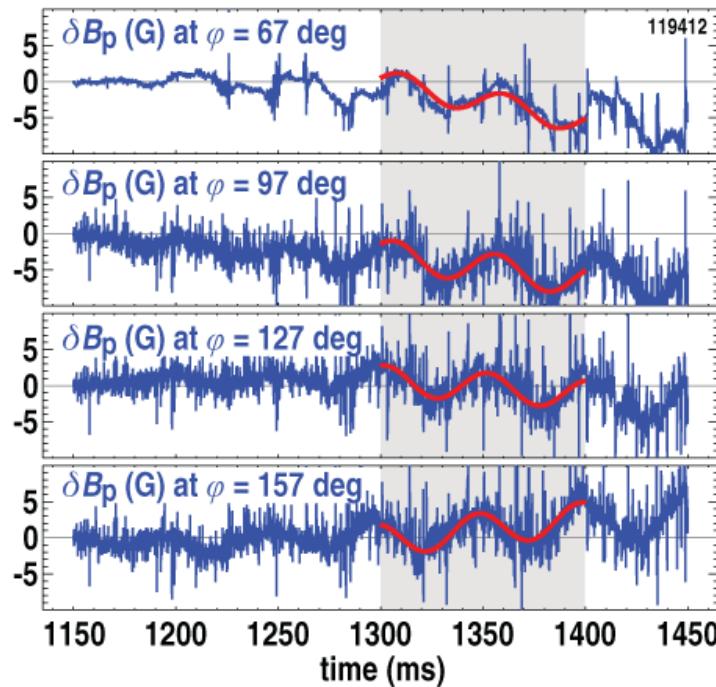
- **Apply rotating perturbation** near natural rotation frequency of RWM, ~20 Hz in DIII-D.
- **Measure response using synchronous detection.** Fourier analyze over a sliding window containing several oscillation periods
- In real-time analysis and feedback, sliding averaging window leads to delay of half the window size,  $\tau_{\text{lag}} \sim 100$  ms.

# Single-frequency analysis yields plasma response to rotating, 3D perturbation

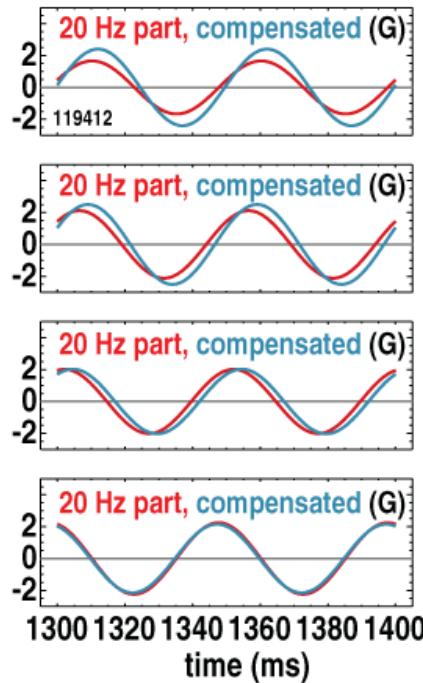
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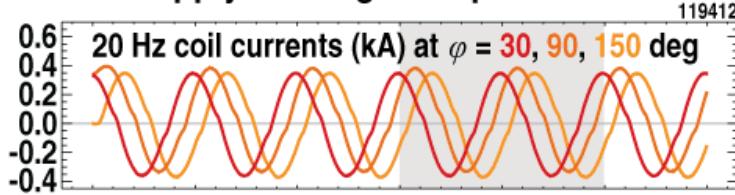


## 3. Subtract direct ac pickup from coils

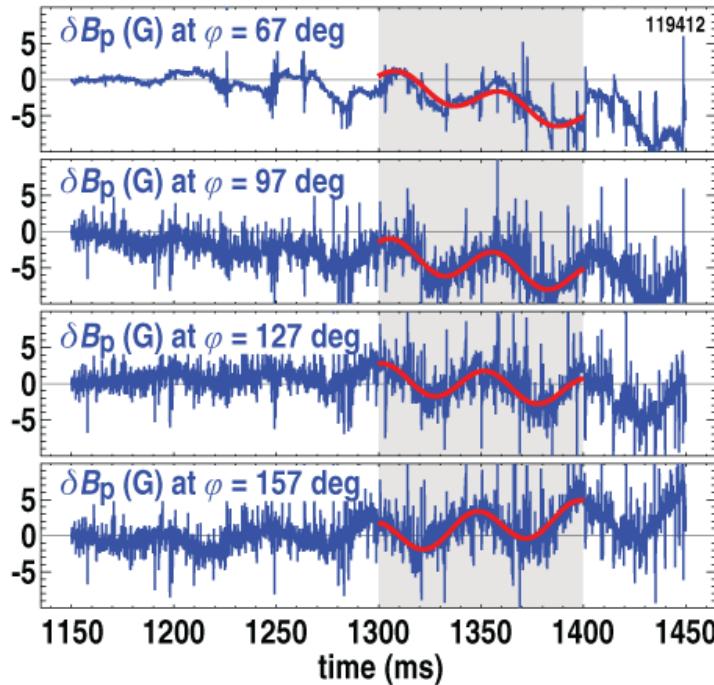


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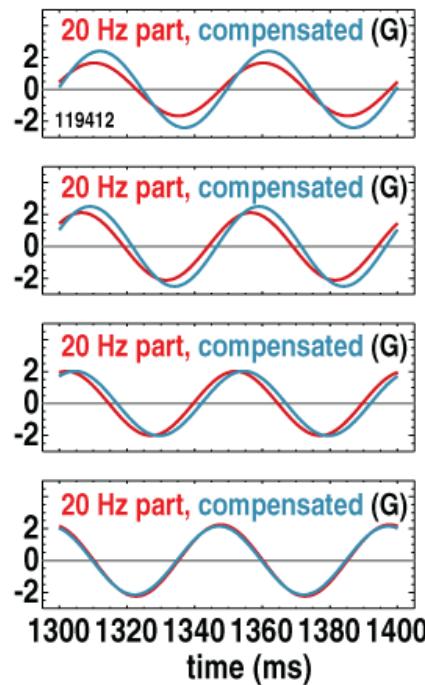
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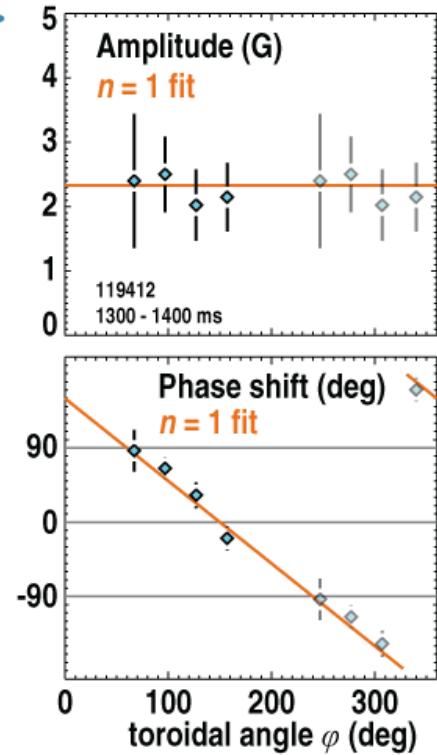
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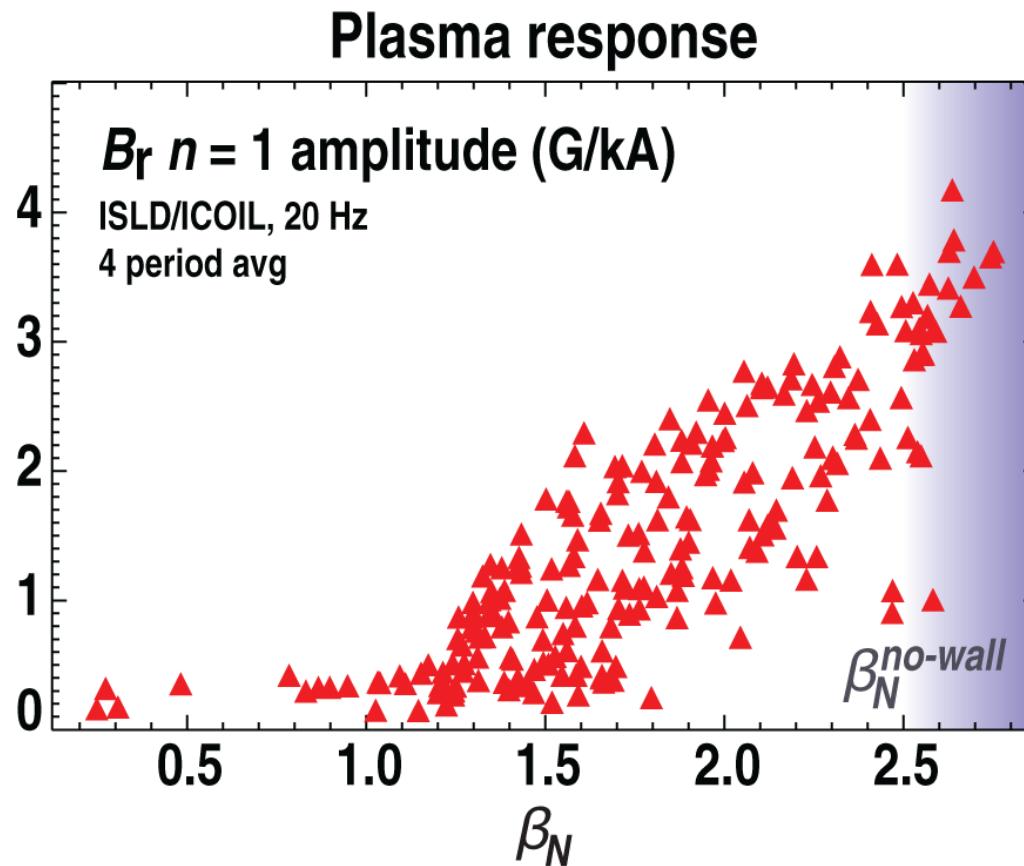


## 4. Toroidal mode number fit



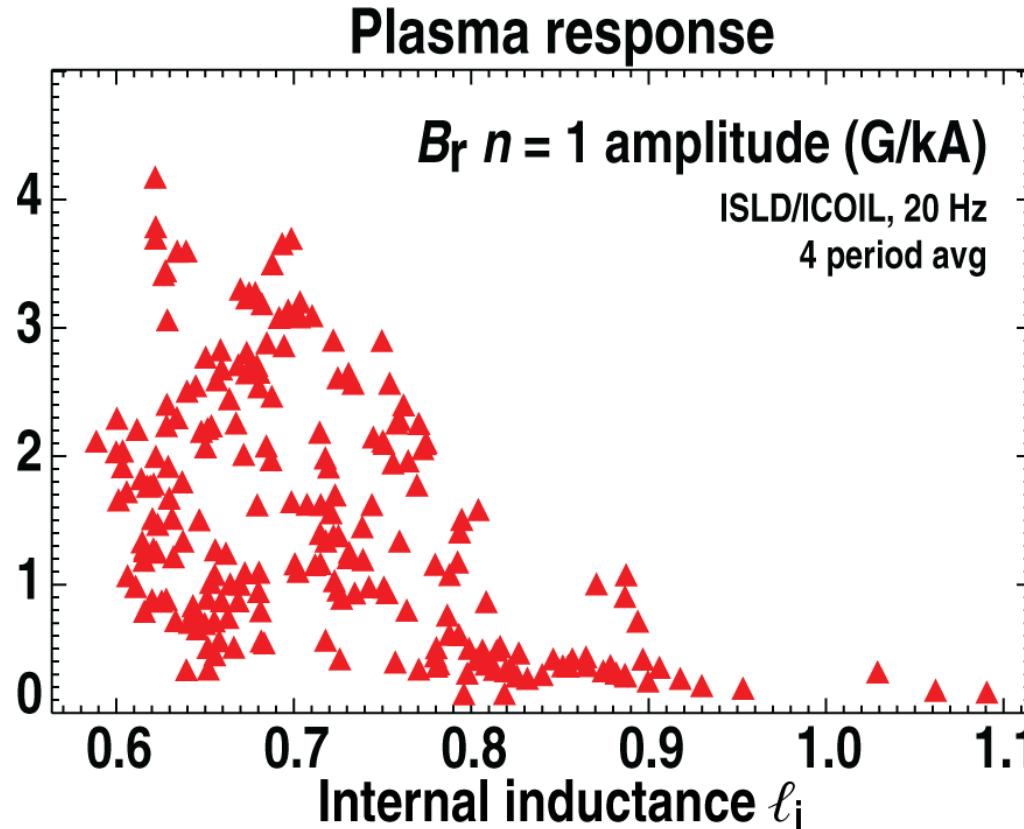
# **Link between plasma response and ideal MHD theory below no-wall limit**

# Plasma response dependencies consistent with ideal MHD expectations for RWM stability, below no-wall limit



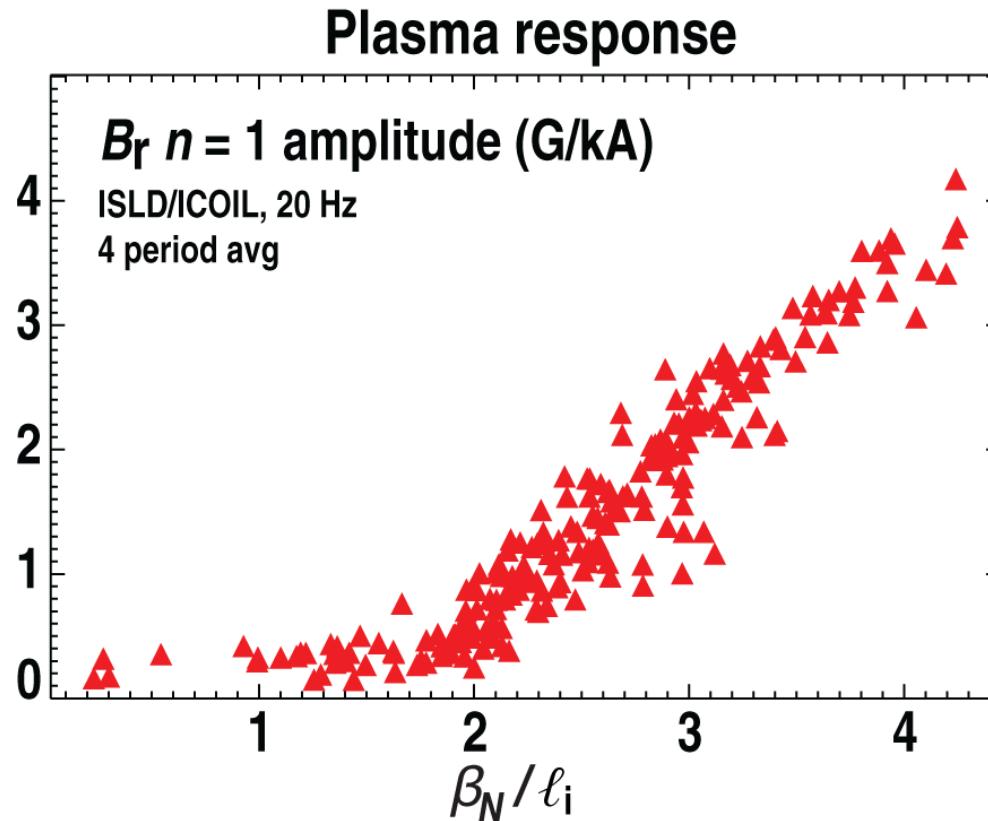
- Data from 14 shots, obtained during AT scenario development day
- Increase of  $\delta B^{\text{plas}}$  with  $\beta_N$  widely observed (DIII-D, NSTX, JET)
  - Some devices/scenarios do exhibit deviations from monotonicity

# Plasma response dependencies consistent with ideal MHD expectations for RWM stability, below no-wall limit



- See inverse dependence on internal inductance (current profile broadness)
  - Scatter due to wide variations in  $\beta_N$

# Plasma response dependencies consistent with ideal MHD expectations for RWM stability, below no-wall limit

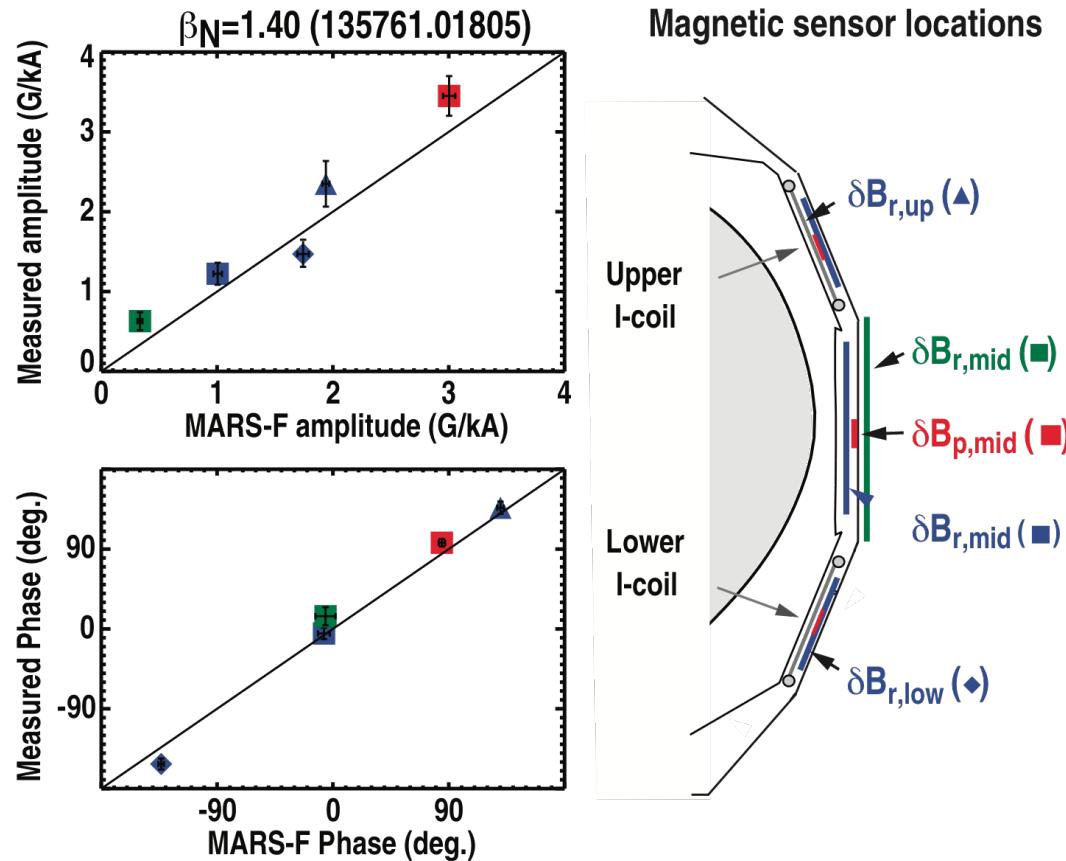


- Clear dependence of plasma response on  $\beta_N/\ell_i$  normalization

# Measured plasma response consistent with linear, ideal MHD below no-wall limit

- Linear ideal MHD calculations (MARS-F) in good agreement with magnetic plasma response measurements

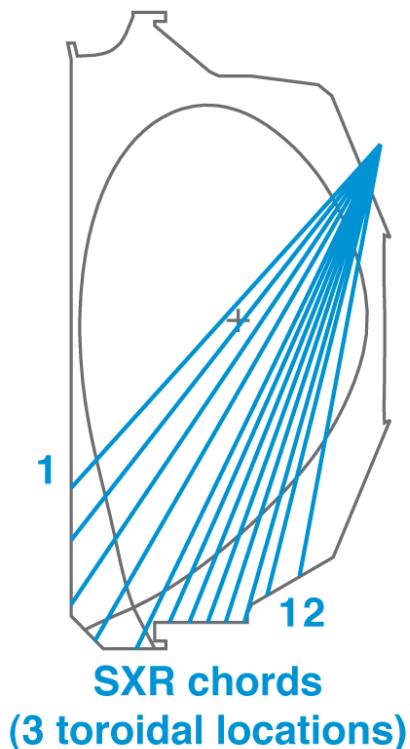
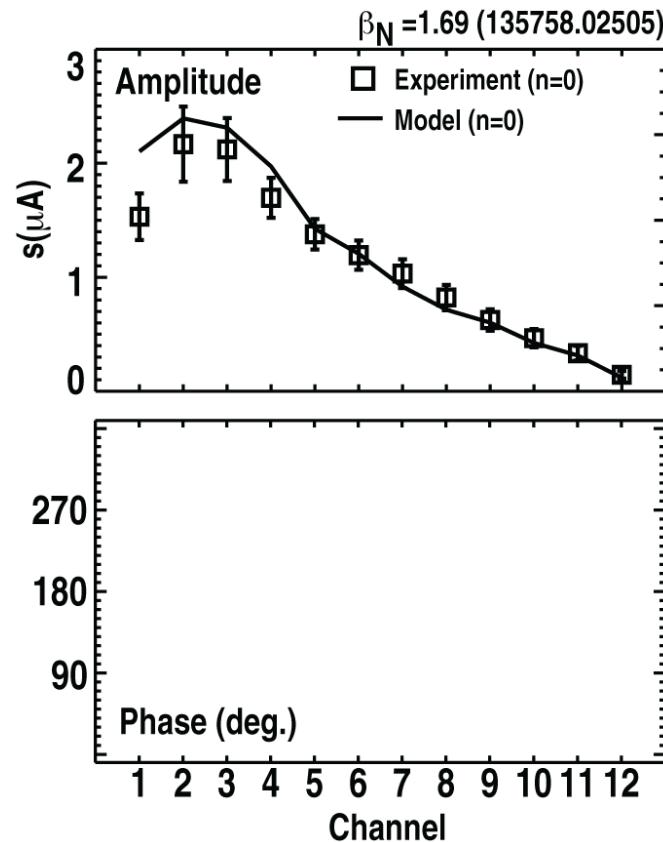
[M.J. Lanctot et al., *Phys. Plasmas* 2010]



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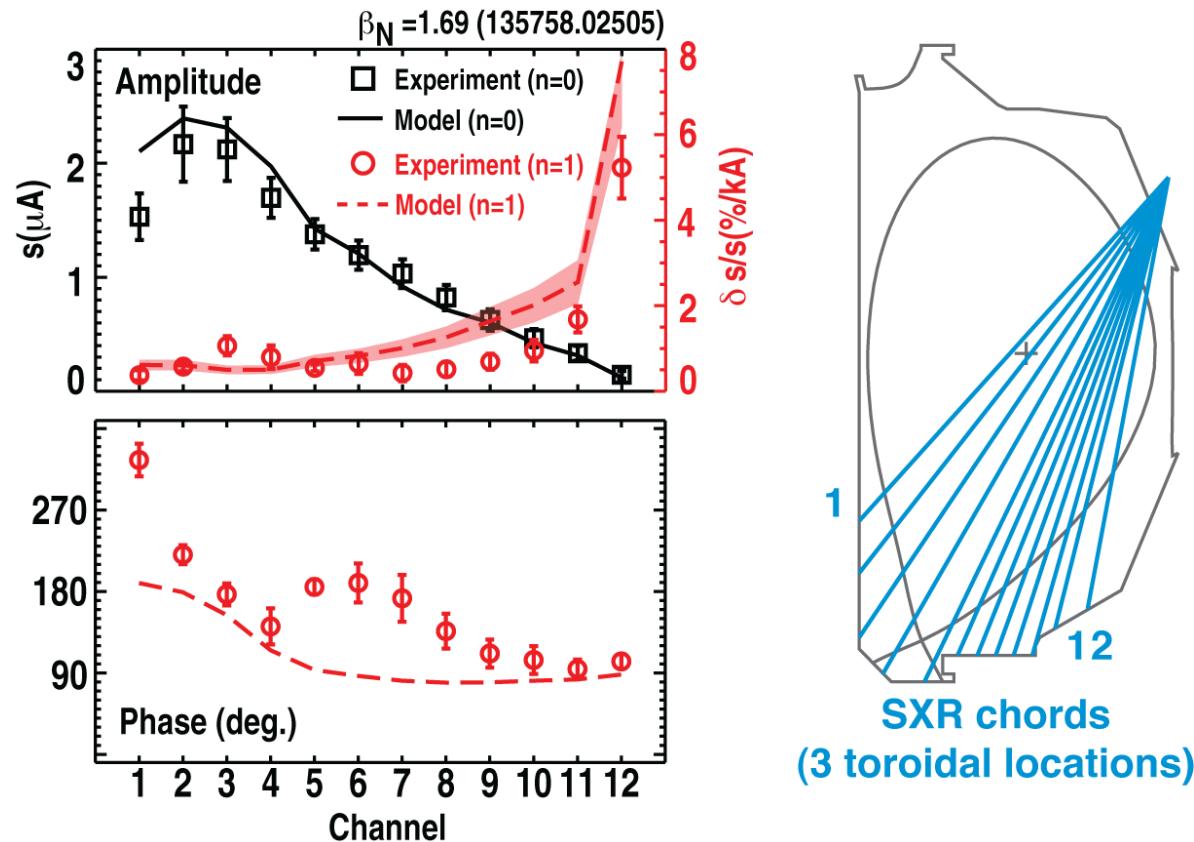
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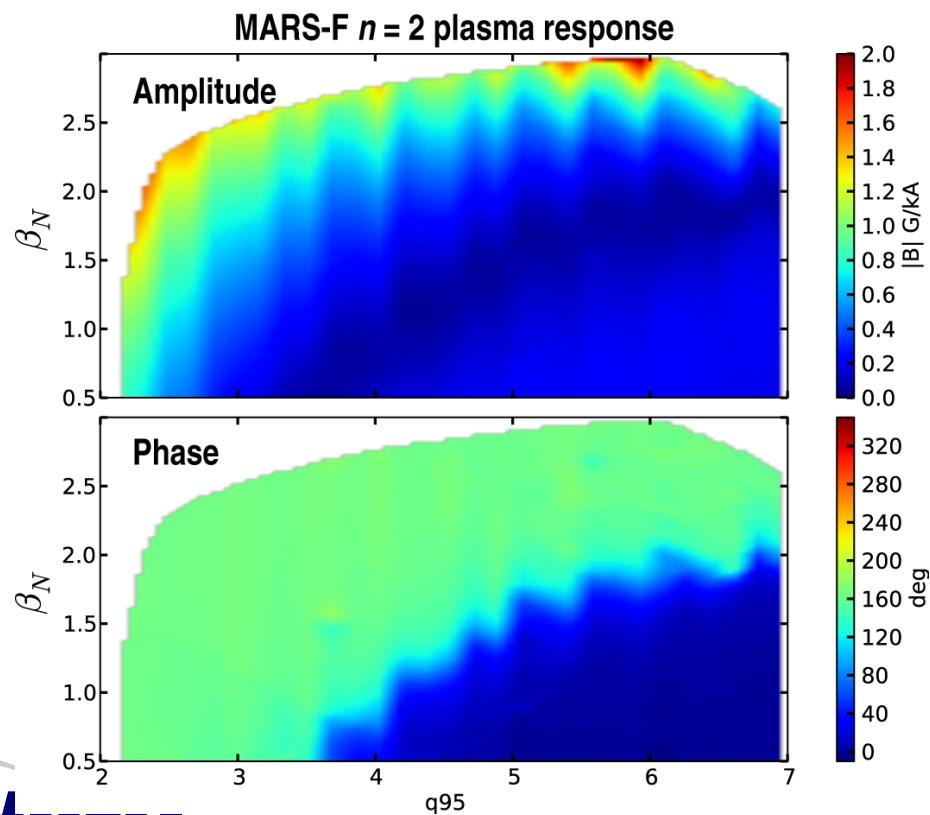
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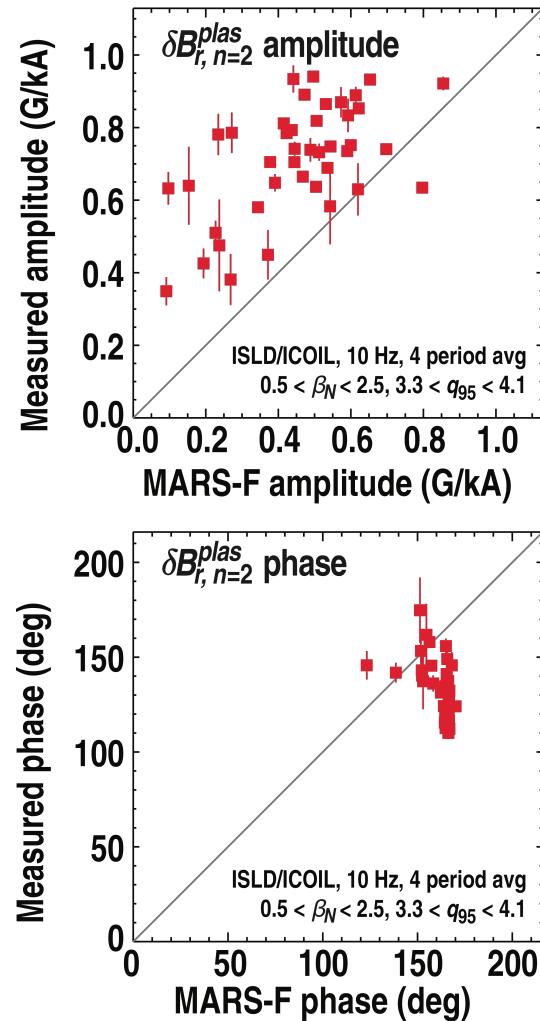
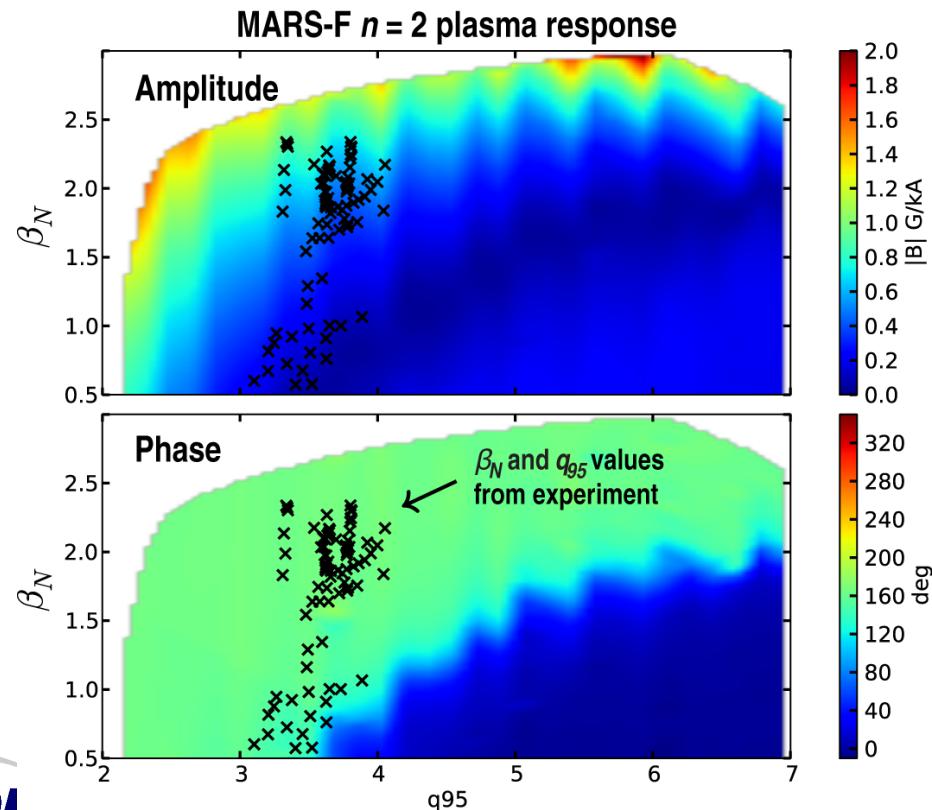
# Measurements of $n = 2, 3$ plasma response compared with linear, ideal MHD

- Plasma response to ac  $n = 2$  perturbations predicted for a range of  $\beta_N$ ,  $q_{95}$  values using MARS-F



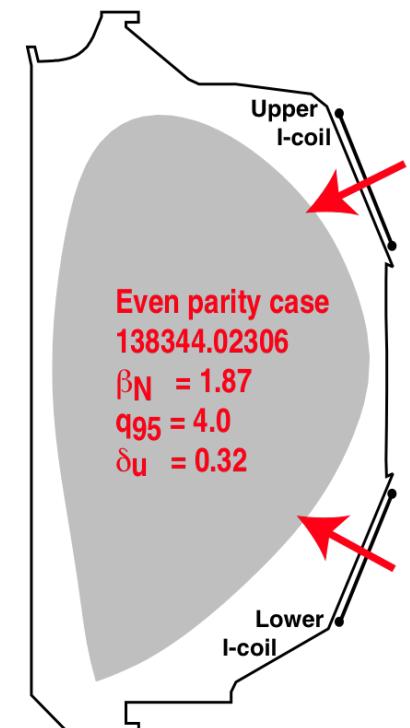
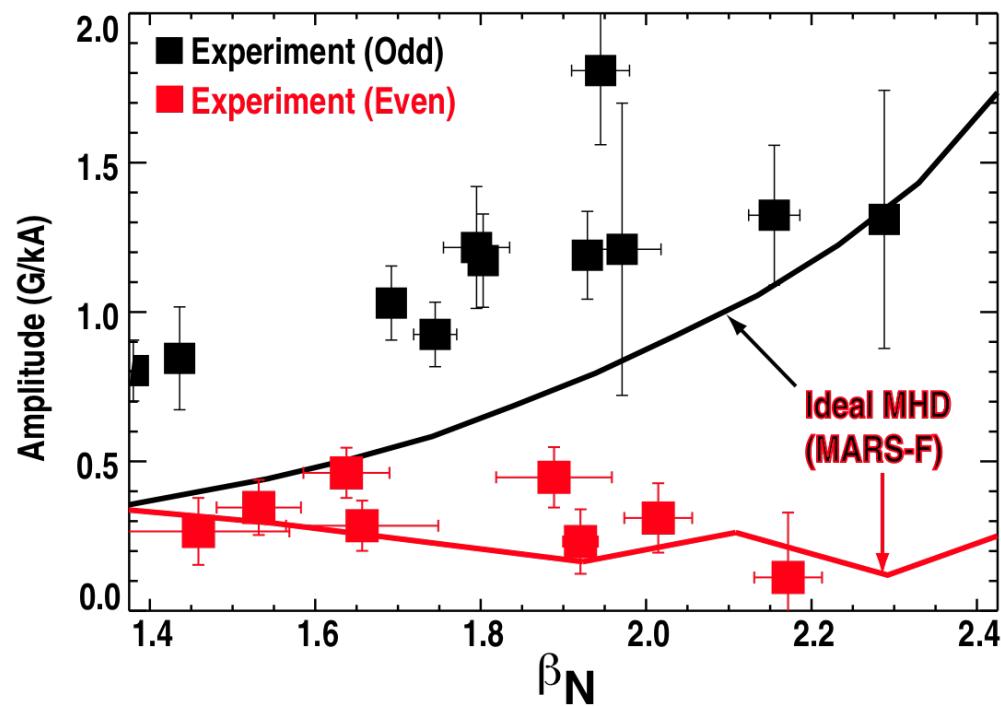
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- Plasma response to ac  $n = 2$  perturbations predicted for a range of  $\beta_N$ ,  $q_{95}$  values using MARS-F
- Preliminary comparison with experiment yields qualitative agreement



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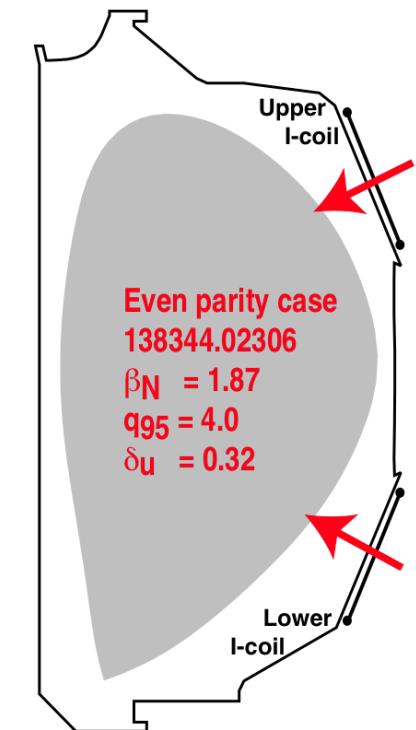
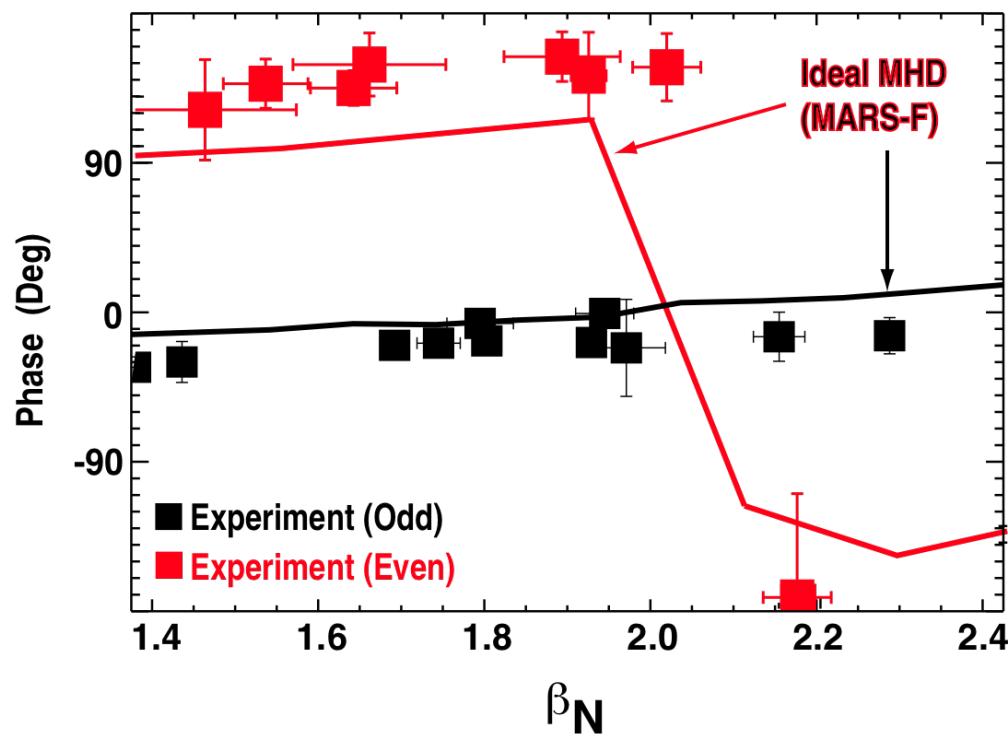
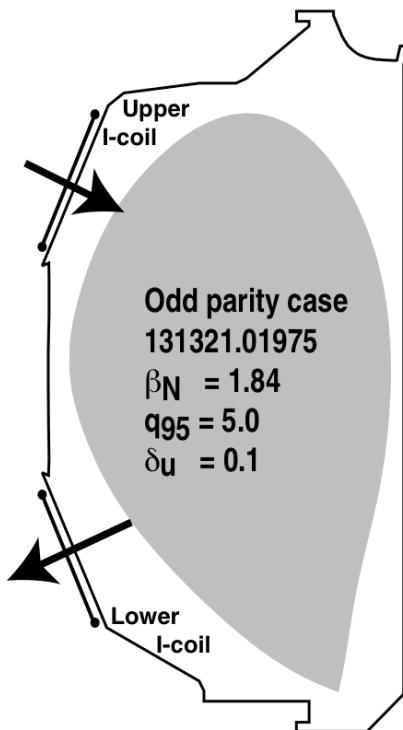
- Probe plasma with static  $n=3$  fields using odd and even parity
- Measurements and modeling (MARS-F) show plasma  $\delta B$  at midplane
  - Increases with  $\beta_N$  for odd parity field
  - Decreases with  $\beta_N$  for even parity field



[Lanctot, et al., Phys. Plasmas 2011]

# Measurements of $n = 2, 3$ plasma response compared with linear, ideal MHD

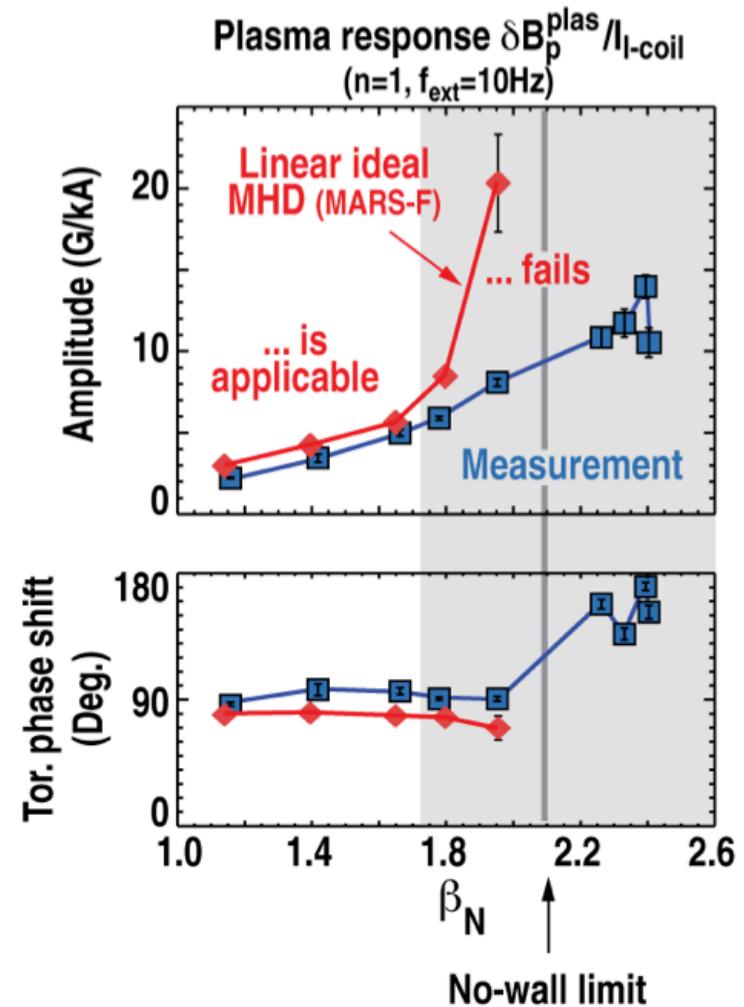
- Measured phase of **even parity** response field drifts by  $60^\circ$  with  $\beta_N$  in agreement with linear ideal MHD model
  - Odd parity phase is relatively constant



[Lanctot, et al., *Phys. Plasmas* 2011]

# Linear, ideal MHD sufficient to predict plasma response below the no-wall limit

- Scan of  $\beta_N$  dependence of  $n = 1$  plasma response reveals limitation of linear ideal MHD
  - Ideal MHD works for  $\beta < 0.8 \beta^{nw}$
  - Diverges near no-wall limit
  - Predicts instability above no-wall limit
- Progress in describing observed stability above no-wall limit with kinetic modifications to ideal MHD energy principle



[Lanctot, *et al.*, *Phys. Plasmas* 2010]

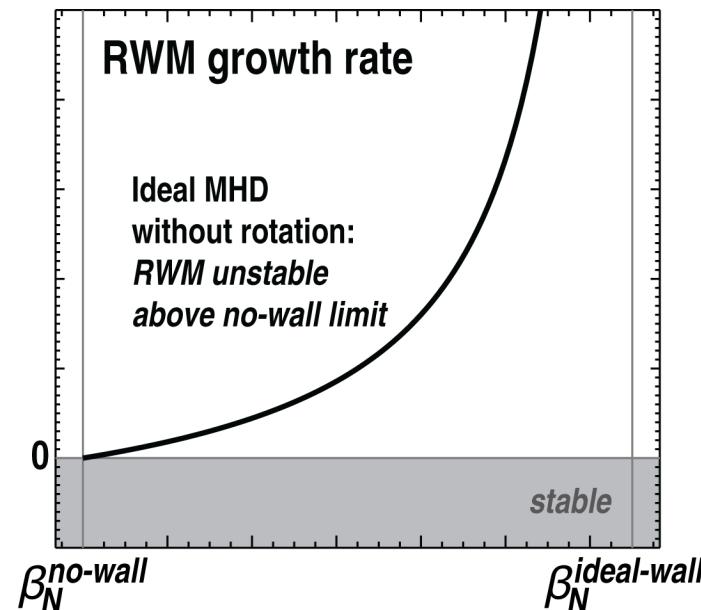
# Uncovering kinetic modifications to ideal MHD



# Kinetic wave-particle damping leads to enhanced RWM stability above no-wall limit

- Ideal MHD energy principle modified to include kinetic damping physics  
[Hu and Betti, PRL, 2004].

$$\gamma\tau_w = -\frac{\delta W_{nw}}{\delta W_{iw}}$$



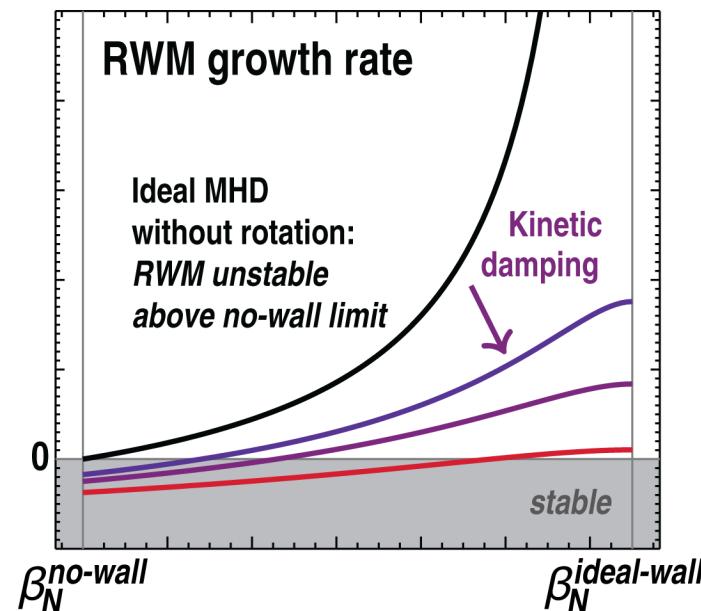
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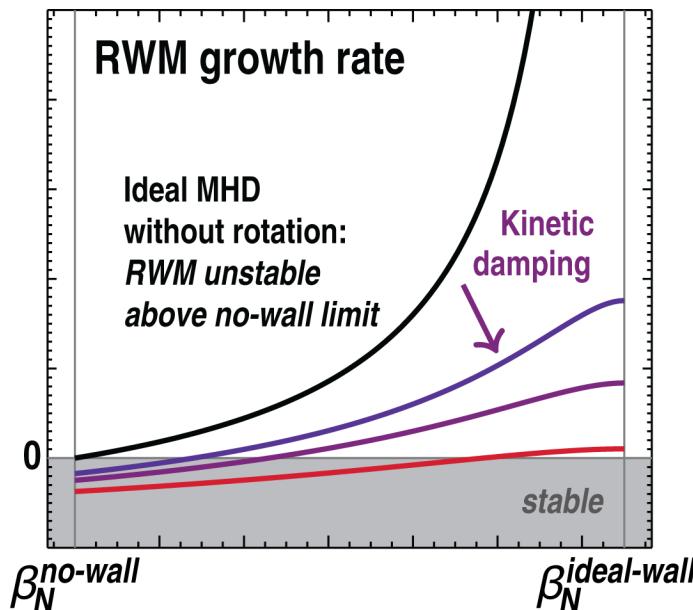


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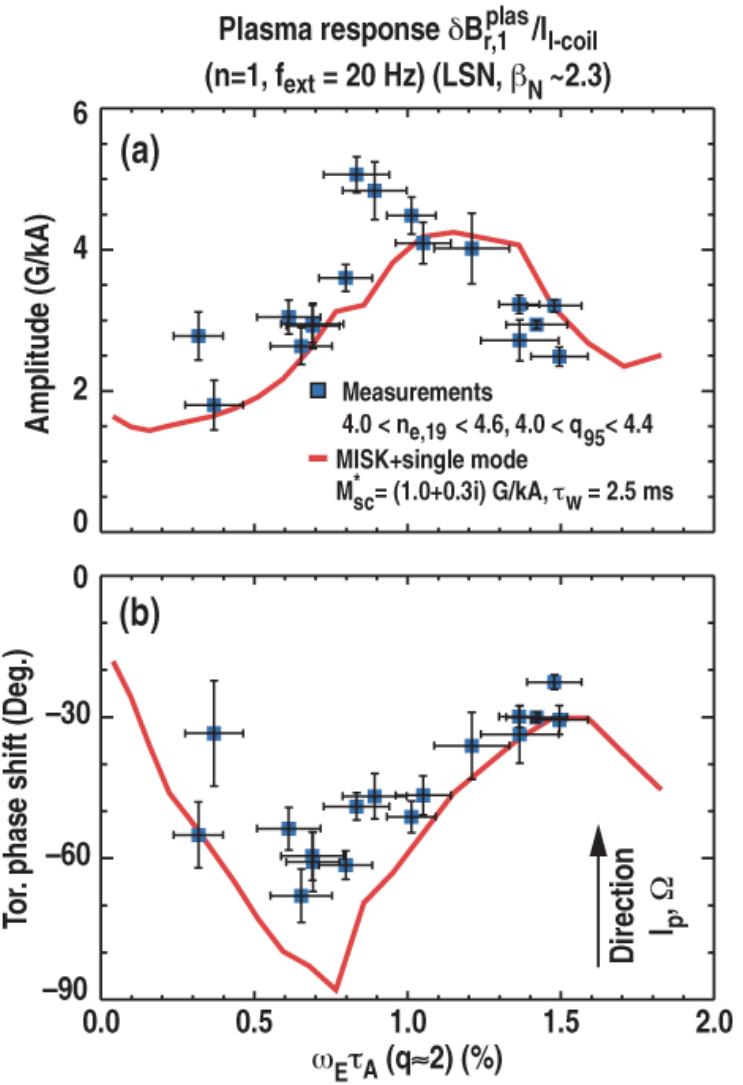


- Kinetic energy principle  $\delta W_K$  allows for energy exchange between RWM and kinetic particle populations:
  - Resonances between motion of *trapped particles* and *plasma rotation*
  - *Non-resonant* effects that depend on alignment of distribution function gradients and the RWM eigenfunction
  - Several codes incorporate this physics: MISK, HAGIS, MARS-K

# Kinetic RWM stability effects investigated in stable plasmas above the no-wall limit

- Measurements of **plasma response** to slowly rotating  $n = 1$  perturbations used to compare theory and experiment
- **Rotation scan** revealed evidence of **trapped particle resonances** in DIII-D; complemented NSTX work on the RWM stability threshold

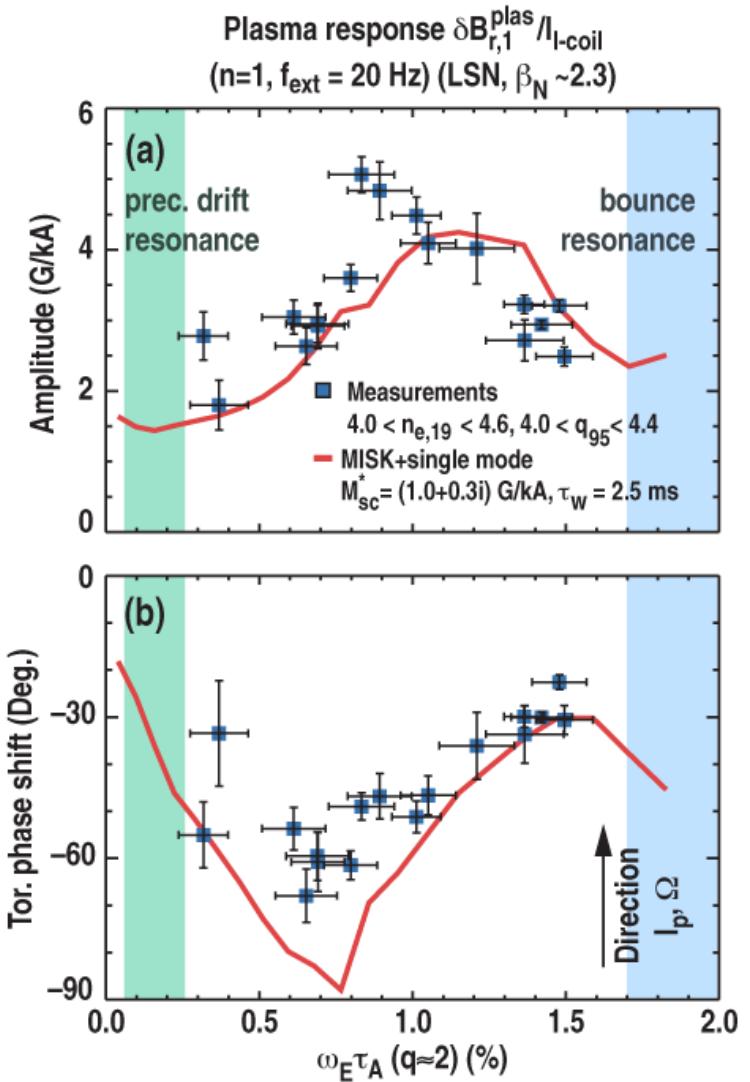
[Berkery, et al, PRL, 2010]



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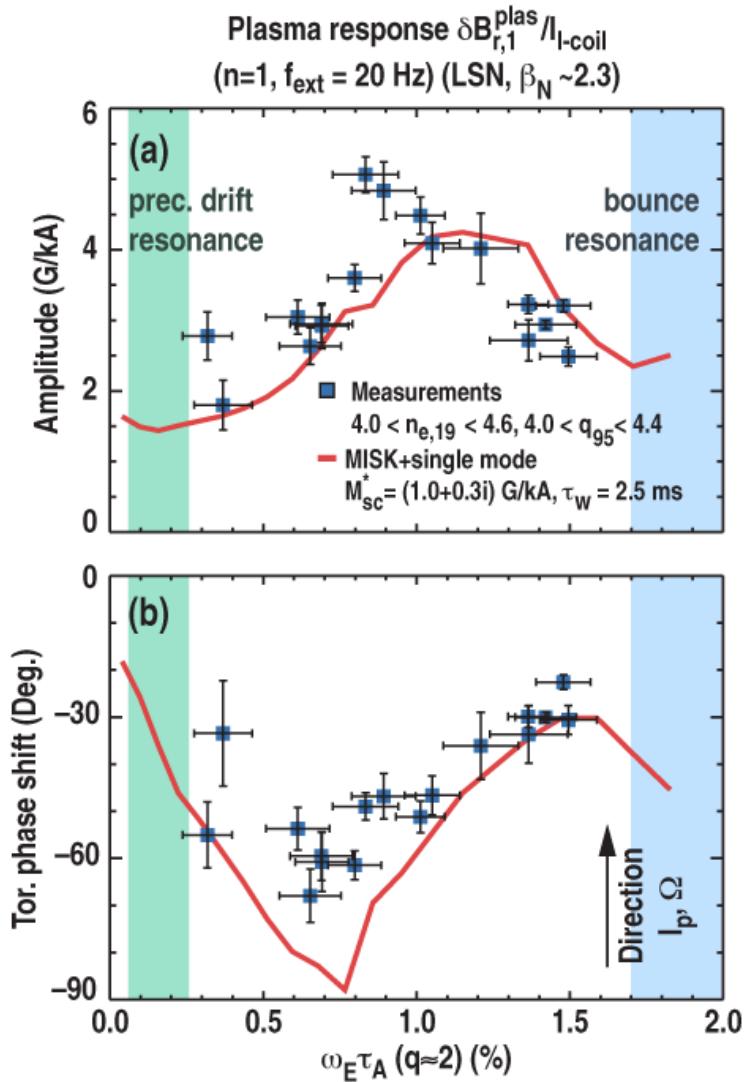
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- Ratio of  $\beta_{fast} / \beta_{thermal}$  investigated in MAST using **density scans**
- Recent DIII-D experiment: use **off-axis NBI** to impact **trapped ion fraction**

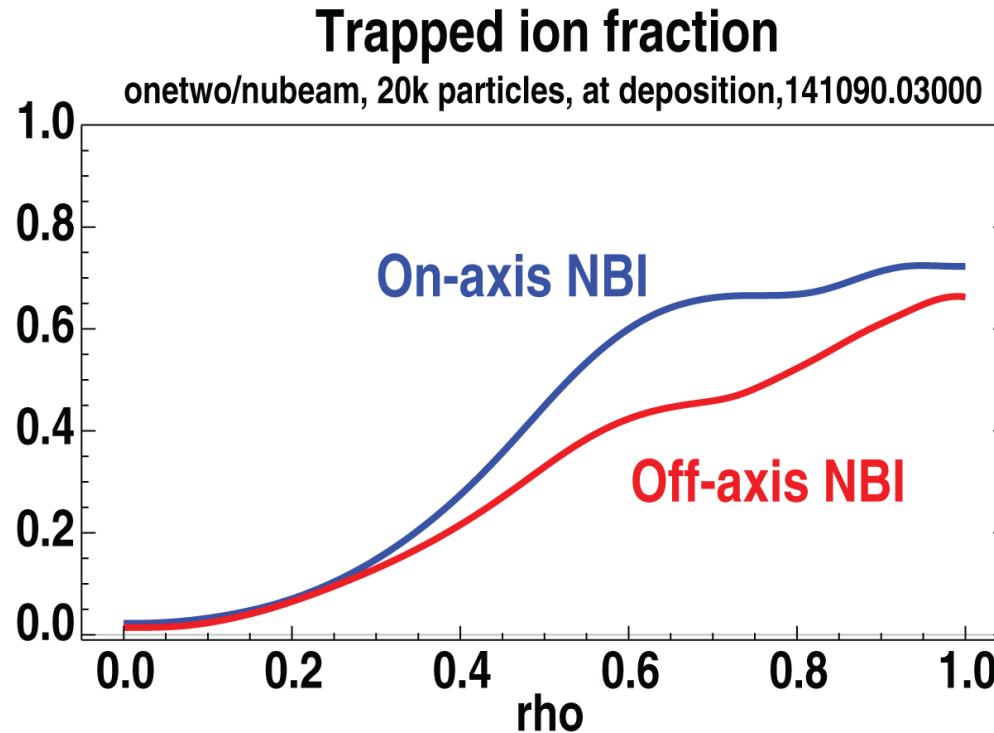


[Berkery, et al, PRL, 2010]

[Chapman, et al, PPCF, 2011]



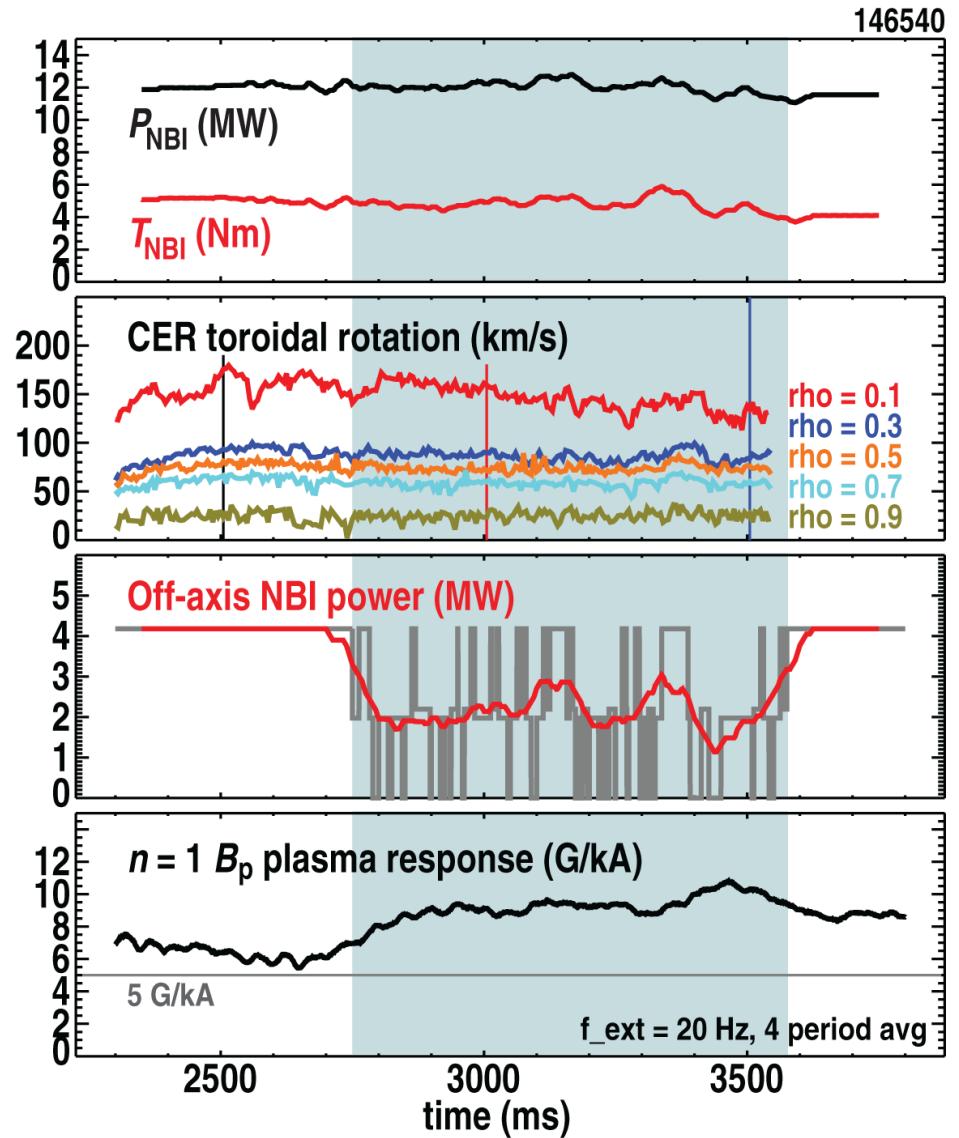
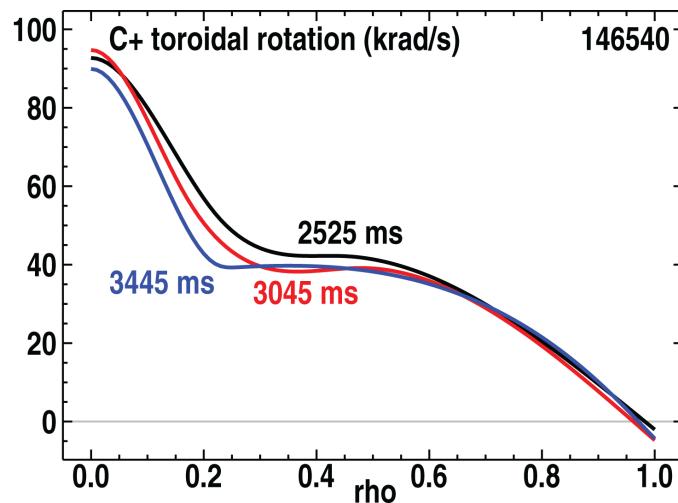
# Off-axis NBI expected to decrease RWM stability



- **Transport modeling: reduced trapped ion fraction with off-axis NBI**, due to more favorable alignment of injection angle with field line pitch.
- **Reduced RWM stability expected with off-axis NBI**; stabilizing effect of passing particles expected to be localized near resonant surfaces, small.

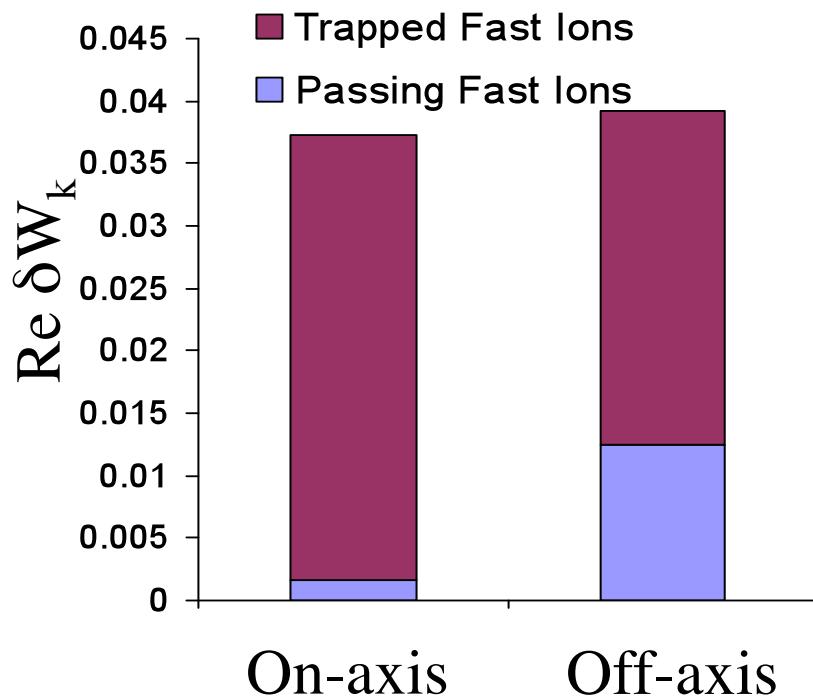
# Off-axis NBI leads to *increased* RWM stability

- Off-axis NBI power modulated at constant  $\beta_N$ ,  $\ell_i$ , density
  - Minor variations in rotation profile
- Plasma response increases with decreased off-axis power
  - Opposite of expectation from considerations of trapped ion fraction



# Including finite orbit width effects results in enhanced damping of RWM with off-axis NBI

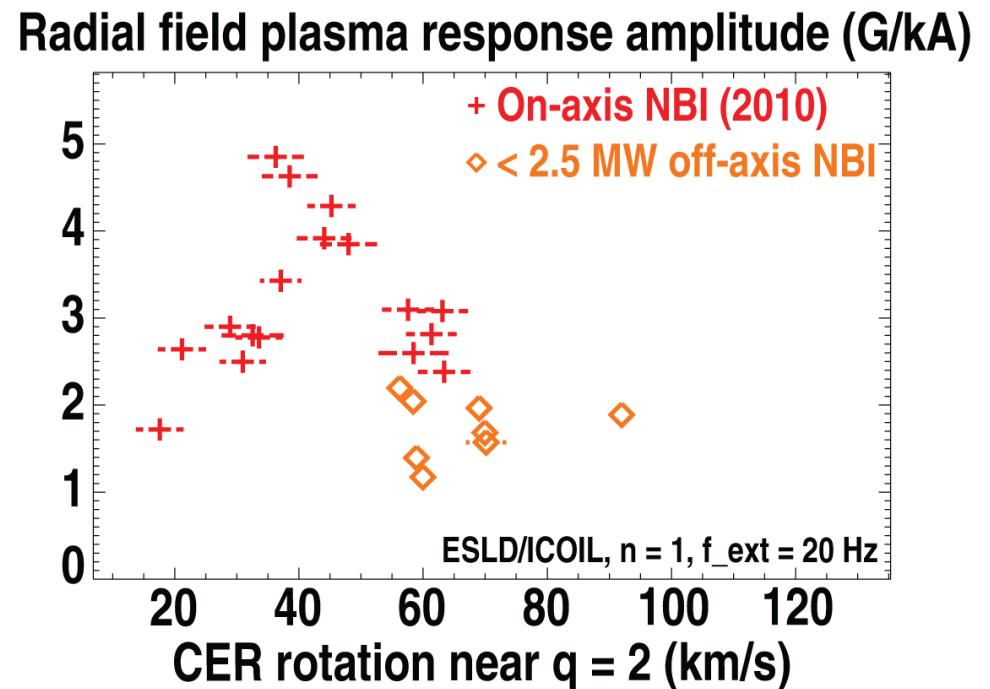
- TRANSP predicts fast ion distribution function
- HAGIS evolves interaction between fast ions and RWM



- Scanned radial peak of distribution function using simplified model for  $F_h$
- Passing fast ion damping sensitive to location of peak of  $F_h$  with respect to rational surfaces

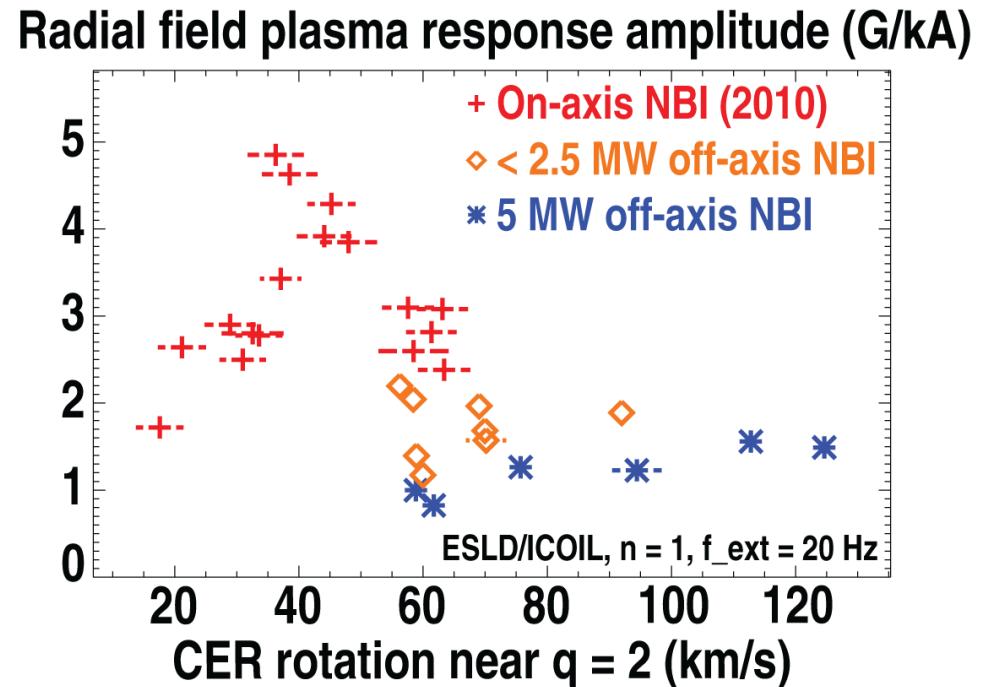
# Stabilizing effect of off-axis NBI observed over a range of rotation

- Existing 2010 dataset extended to higher rotation



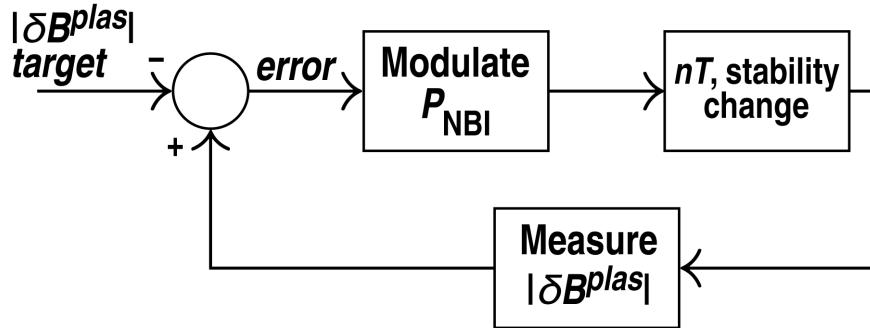
# Stabilizing effect of off-axis NBI observed over a range of rotation

- Existing 2010 dataset extended to higher rotation
- See ~50% *reduction* in plasma response amplitude with 5 MW off-axis NBI, at intermediate rotation.
- Continued damping at increased rotation qualitatively consistent with theoretical expectations.
- Resonances with additional bounce frequency harmonics encountered as rotation increases.

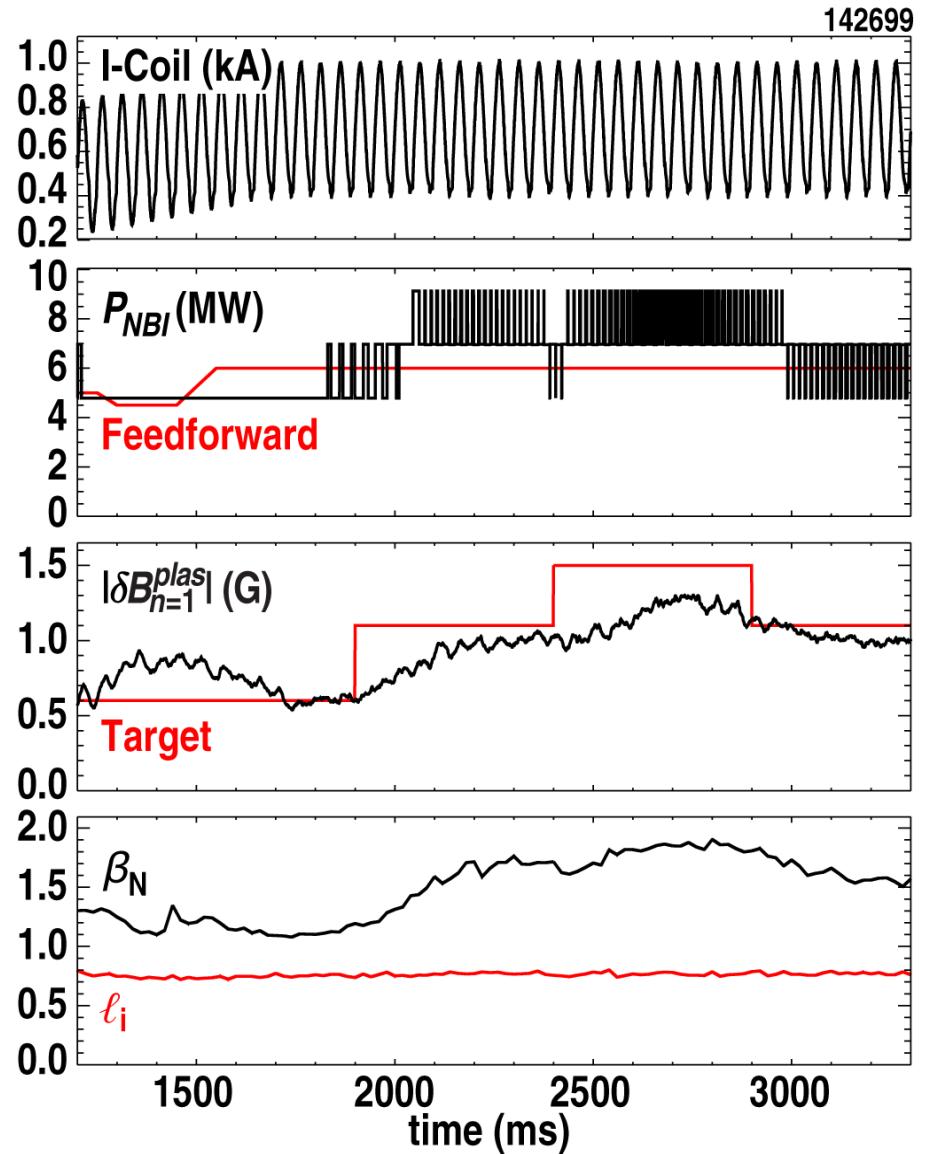


# Controlling proximity to the RWM stability limit

# RWM stability directly controlled with NBI for first time

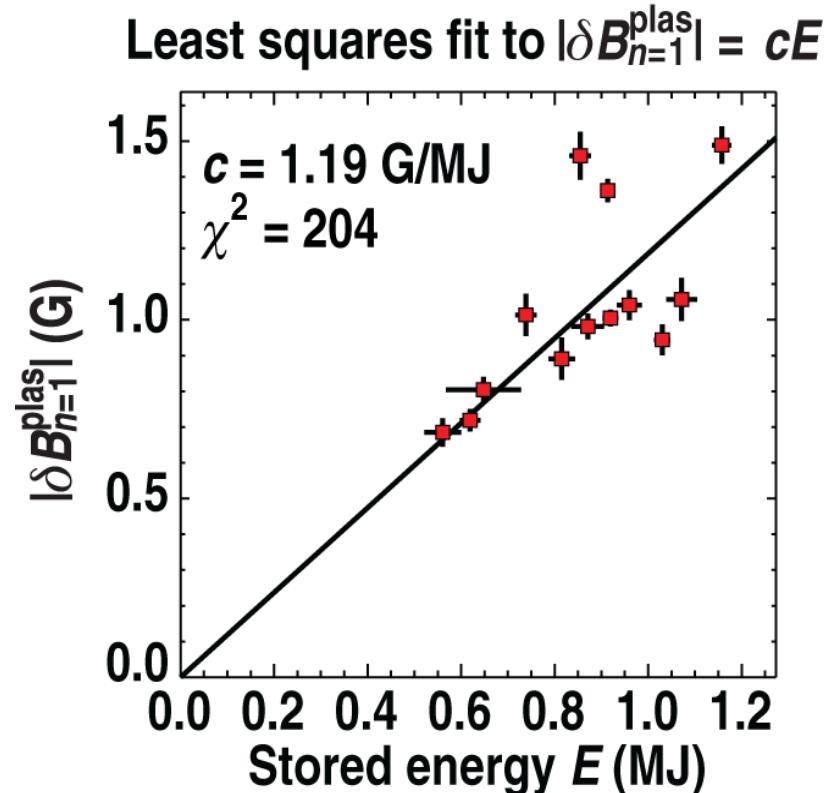


- Plasma response measurement input to NBI control algorithm
- Plasma response settles to a value near the target on a timescale close to  $\tau_E \sim 100$  ms.
- $\beta_N$  changes linearly with plasma response amplitude – expected below no-wall limit.



# Observed $\delta B_{n=1}^{\text{plas}}$ dependencies suggest 2 parameter dynamical model

- Consider a model of the form
$$\frac{d}{dt} |\delta B_{n=1}^{\text{plas}}| = -\frac{1}{\tau} |\delta B_{n=1}^{\text{plas}}| + c P_{\text{NBI}}.$$
- 2 parameters can be estimated by fitting experimental data
  - Time-constant  $\tau$
  - Beams coupling coefficient  $c$
- Equivalent to a model\* for the plasma stored energy  $E$ , if
$$\tau = \tau_E \quad \text{and} \quad |\delta B_{n=1}^{\text{plas}}| = cE = c \frac{I_p B_t}{2\mu_0 a} \beta_N$$
- Expect this scaling to work for constant  $I_p$ ,  $B_t$ ,  $a$ .



\*J. T. Scoville, et al., *Fusion Eng. and Design* **45**, A367 (2003)

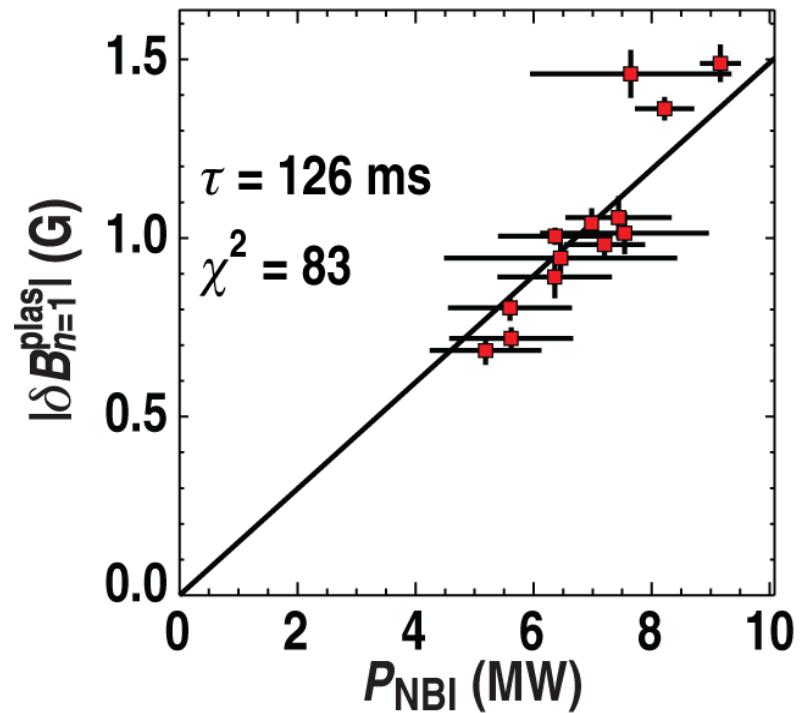
# Time-constant obtained from fit to steady-state limit

- In the steady state limit, obtain

$$|\delta B_{n=1}^{\text{plas}}| = c\tau_E P_{\text{NBI}}.$$

- So time constant can be obtained from least squares fit if  $c$  is known.
- These linear relationships will not necessarily hold at high  $\beta_N$  above the no-wall limit.
- However, super-linear dependence of  $|\delta B_{n=1}^{\text{plas}}|$  with  $\beta_N$  suggests that a linear controller would naturally avoid the RWM's true marginal stability point.

Least squares fit to  $|\delta B_{n=1}^{\text{plas}}| = c\tau P_{\text{NBI}}$



# Conclusions

- **Ideal MHD describes perturbed equilibria below no-wall beta limit**
  - Large body of measurements consistent with ideal MHD at low  $\beta < \beta^{nw}$
  - Magnetic ( $n = 1, 2, 3$ ) and SXR profile measurements ( $n = 1$ ) compared with theory
- **Kinetic modifications to ideal MHD needed above no-wall limit**
  - Experimental evidence for wave – particle interactions uncovered, qualitatively consistent with kinetic theory.
  - Off-axis NBI used to probe kinetic damping in recent experiment, lead to increased damping of RWM
  - New data obtained for comparisons with theory.
  - Preliminary calculations indicate finite ion orbit width effects may be important
- **Direct RWM stability control demonstrated using NBI feedback**
  - Linear feedback dynamics obtained below no-wall limit
  - Possible solution for maximizing  $\beta$  while avoiding unstable RWM
  - Challenges expected above no-wall limit: rotation, kinetic effects impact feedback linearity