

Scaling of Error-Field Penetration Threshold in Tokamak Plasmas

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Introduction

- Locked mode formation in low density target plasmas seriously limits available experimental operating space.
- Locked modes thought to be due to *error-field driven magnetic reconnection* triggered when resonant component of error-field (at rational surface) exceeds some *threshold value*.
- Observed threshold in present experiments is small (but manageable): *i.e.*,
$$(b_r/B_T)_{crit} > 10^{-4}.$$
- Threshold generally *decreases* with increasing machine size. What is expected penetration threshold for ITER?

Empirical Scaling Laws

- Scaling studies of penetration threshold with “engineering parameters”, n_e , B_T , R_0 , performed on many tokamaks.^a
- Writing

$$(b_r/B_T)_{\text{crit}} \sim n_e^{\alpha_n} B_T^{\alpha_B} R_0^{\alpha_R},$$

all studies agree that $\alpha_n \simeq 1$: *i.e.*, density scaling is *linear*.

- Measured values of α_B lie in range -2.9 to -1.0 .
- Value of α_R cannot be directly measured, but can be inferred from dimensionless scaling arguments:

$$\alpha_R = 2 \alpha_n + 1.25 \alpha_B.$$

^aCompass-C/D, Tector, Alcator C-mod, DIII-D, JET

Extrapolation to ITER

- Extrapolation from JET ^a to ITER ^b yields

$$1.3 \times 10^{-5} < (b_r/B_T)_{crit} < 2.7 \times 10^{-4}.$$

- Proposed ITER error-field correction system designed to reduce resonant error-fields to level

$$b_r/B_T \simeq 5 \times 10^{-5}.$$

- Large uncertainty in extrapolation of penetration threshold to ITER (and, hence, in adequacy of error-field correction system). Could reduce uncertainty by developing error-field penetration theory consistent with experimental data.

^a $n_e = 1.6 \times 10^{19} \text{ m}^{-3}$, $B_T = 3.5 \text{ T}$, $R_0 = 2.95 \text{ m}$, $(b_r/B_T)_{crit} = 1.1 \times 10^{-4}$.

^b $n_e = 2 \times 10^{19} \text{ m}^{-3}$, $B_T = 5.3 \text{ T}$, $R_0 = 6.2 \text{ m}$.

Fitzpatrick (1993) Theory ^a

- Response of plasma governed by *linear resistive-viscous MHD*.
- Scaling of penetration threshold with standard dimensionless parameters:

$$(b_r/B_T)_{\text{crit}} \sim \beta^{-1/6} \nu_*^{1/6} \rho_*^{4/3}.$$

- Scaling with engineering parameters:^b

$$(b_r/B_T)_{\text{crit}} \sim n_e^0 B_T^{-13/15} R_0^{-13/12}.$$

- Predicted scaling highly inconsistent with experimental data, since no density dependence.

^aNucl. Fusion 33, 1049 (1993).

^bAssuming ohmic power balance, $\tau_M \sim \tau_E$, and (dimensionally consistent) neo-Alcator energy transport: $B_T \tau_E \sim n_e B_T R_0^{13/4} \sim \beta^{5/4} \nu_*^{-1/4} \rho_*^{-7/2}$.

Cole & Fitzpatrick (2006) Theory^a

- Response of plasma governed by linear resistive-viscous *drift*-MHD.
- Scaling of penetration threshold with dimensionless parameters:

$$(b_r/B_T)_{\text{crit}} \sim \nu_*^{1/4} \rho_*^{5/4}.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{\text{crit}} \sim n_e^{1/4} B_T^{-23/20} R_0^{-15/16}.$$

- Slight improvement in density scaling, but dependence still much too weak.

^aPlasma Phys. **13**, 032503 (2006).

Cole, Hegna, Callen (2008) Theory ^a

- Like Cole & Fitzpatrick theory, except also takes *neoclassical flow damping* into account.
- Scaling of penetration threshold with dimensionless parameters: ^b

$$(b_r/B_T)_{\text{crit}} \sim \beta \nu_*^{-1/2} \rho_*^{3/2}.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{\text{crit}} \sim n_e^{1/2} B_T^{-13/10} R_0^{-5/8}.$$

- Further improvement in density scaling, but dependence still too weak.

^aPlasma Phys. **15**, 056102 (2008).

^bAssuming flow damping in $1/\nu$ regime.

Fitzpatrick (2011) Theory ^a

- Like Cole, Hegna, Callen theory, except response of plasma in vicinity of rational surface governed by *nonlinear island physics*.
- Scaling of penetration threshold with dimensionless parameters: ^b

$$(b_r/B_T)_{\text{crit}} \sim \beta \rho_*.$$

- Scaling with engineering parameters:

$$(b_r/B_T)_{\text{crit}} \sim n_e B_T^{-9/5} R_0^{-1/4}.$$

- Scaling fairly consistent with experimental data. Leads to predicted ITER penetration threshold of $(b_r/B_T)_{\text{crit}} \sim 5 \times 10^{-5}$.

^a<http://farside.ph.utexas.edu/papers/nonlinear.pdf>

^bAssuming plasma response in so-called “polarization regime.”

Island Width Evolution

- New expression for penetration threshold obtained from recently developed magnetic island model.^a
- Time evolution of island width, w , governed by^b

$$4I_1 \tau_R \frac{d}{dt} \left(\frac{w}{r_s} \right) = \Delta' + 2m_\theta \left(\frac{w_v}{w} \right)^2 \cos \phi - I_p \beta_0 \left(\frac{w_0}{r_s} \right)^2 \frac{r_s^3}{w^3 + \rho_s^3}.$$

- Second term on r.h.s. represents error-field drive. Third term describes stabilizing influence of *ion polarization current*.

^aR. Fitzpatrick, F.L. Waelbroeck, Phys. Plasmas **17**, 062503 (2010).

^bSee paper for complete list of definitions.

Ion Polarization Stabilization

- Large island limit ($w \gg \rho_s$): Acceleration of ion fluid flowing around curved island separatrix produces perpendicular ion polarization current. Parallel return current has strong *stabilizing* effect on island that scales as w^{-3} .
- Small island limit ($w \simeq \rho_s$): Ion fluid decouples from magnetic flux-surfaces and flows straight through island separatrix. Much smaller ion polarization current generated. Parallel return current has weak *stabilizing* effect on island that scales as w^0 .
- Polarization term used in model is interpolation between large and small island limits.

Island Phase Evolution

- Time evolution of island phase, ϕ , governed by ^a

$$-2m_\theta \left(\frac{w_v}{w_0}\right)^2 \left(\frac{w}{r_s}\right)^2 \sin \phi = 4\beta_0 \left(\frac{v_{\phi i}}{\tau_M \omega_{*i}^2}\right)^{1/2} \left(\frac{1}{\omega_{*i}} \frac{d\phi}{dt} - v_{nc}\right).$$

- Left hand side is electromagnetic *locking torque* due to error-field. Right-hand side is viscous torque due to combination of *anomalous ion perpendicular viscosity* and *neoclassical ion toroidal viscosity*.

^aSee paper for complete list of definitions.

Penetration Regimes

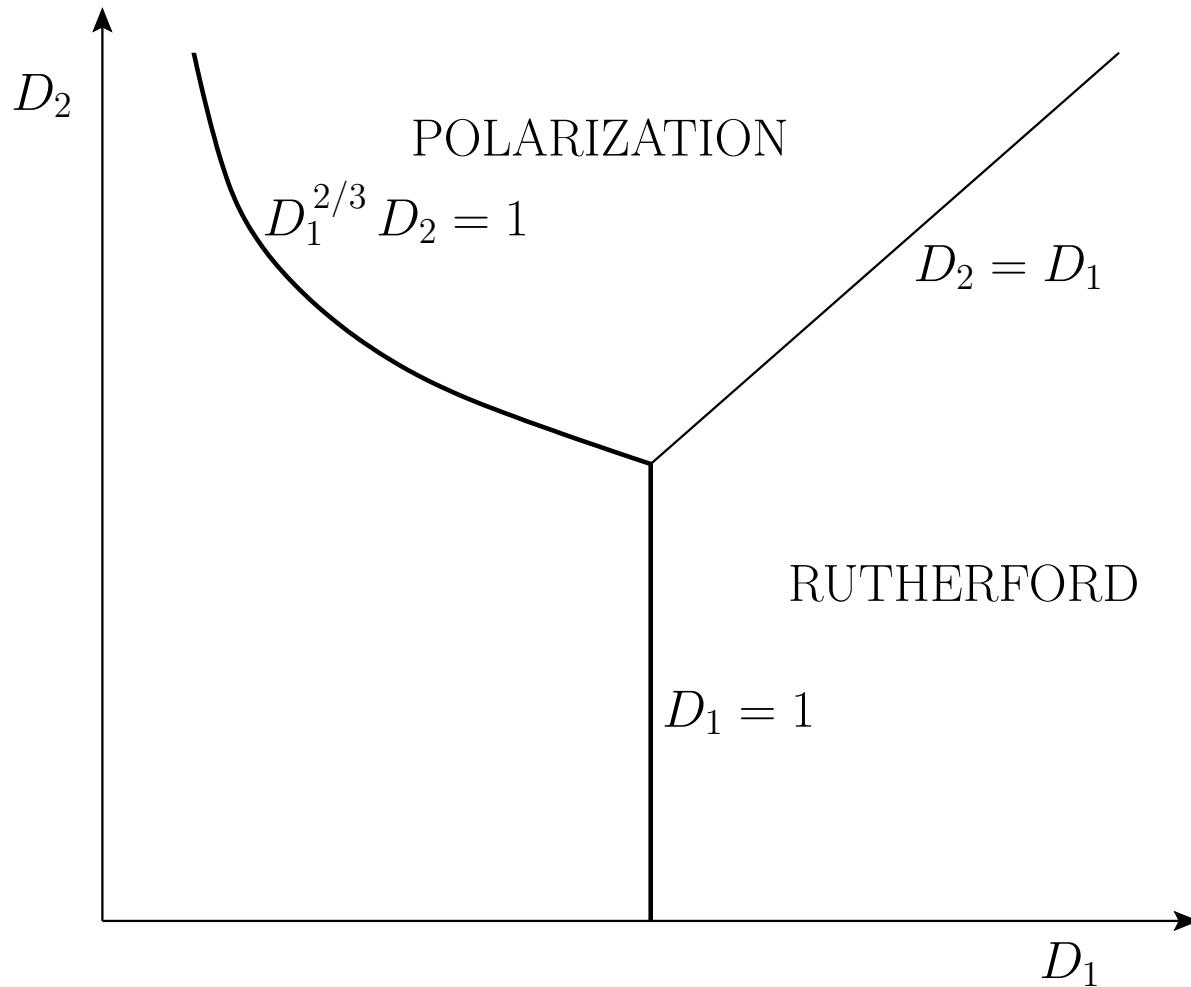
- Island model predicts two distinct regimes for error-field penetration depending on values of

$$D_1 \sim \frac{\tau_R \omega_{*i}}{(\tau_D \tau_M \omega_{*i}^2)^{1/4}} \beta^{1/2},$$

$$D_2 \sim (\tau_D \tau_M \omega_{*i})^{1/2} \beta \rho_*.$$

- Here, τ_R is resistive t.s., τ_M is momentum confinement t.s., and τ_D is neoclassical toroidal flow damping t.s.
- D_1 and D_2 parameterize relative importance of *neoclassical rotation* and the *ion polarization current*, respectively, in suppressing error-field driven magnetic reconnection.

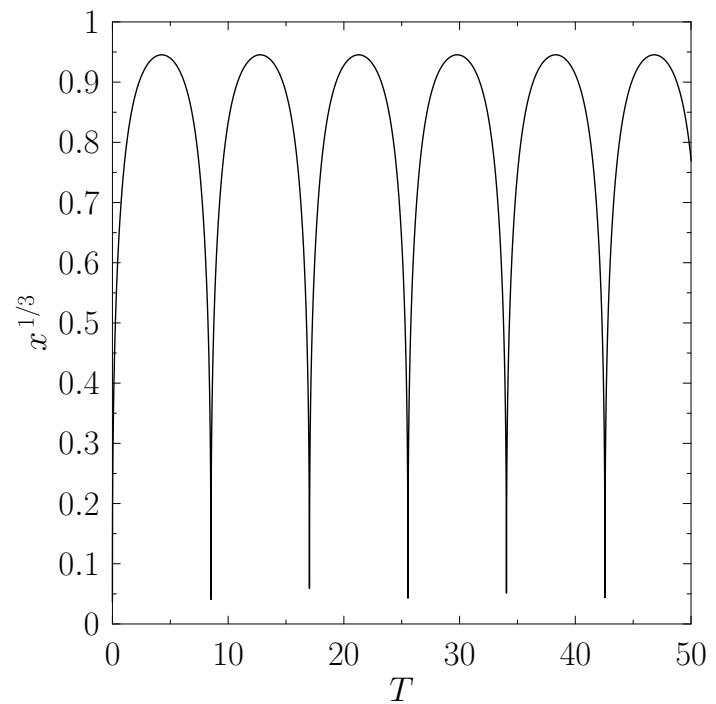
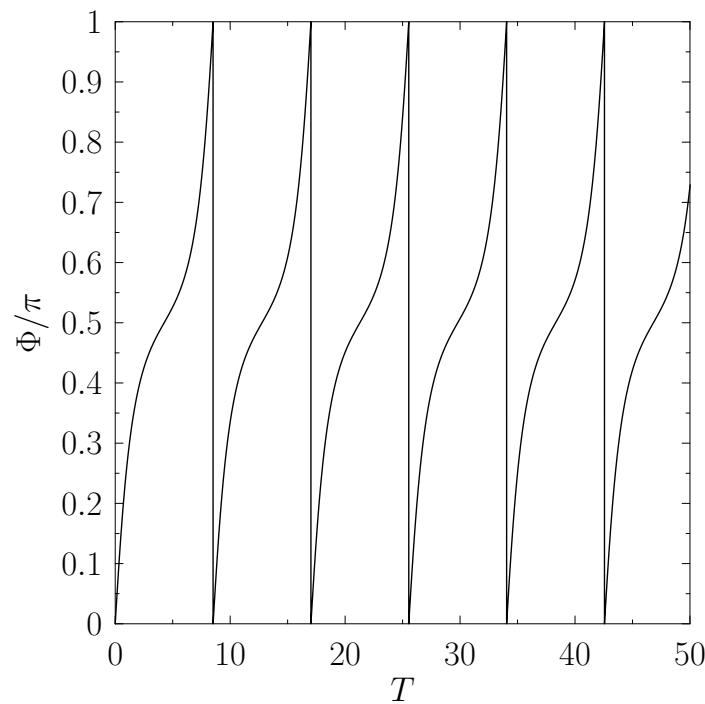
Penetration Regimes



Rutherford Regime

- Suppression of driven magnetic reconnection due to *neoclassical plasma rotation*, which prevents island from locking in phase with resonant error-field. Stabilizing effect of ion polarization current negligible.
- Penetration occurs when electromagnetic locking torque becomes large enough to overwhelm viscous torque, and allows island to lock to resonant error-field.
- Prior to penetration, island width “pulsates”, since island spends as much time in stabilizing phase of error-field, as in destabilizing phase.

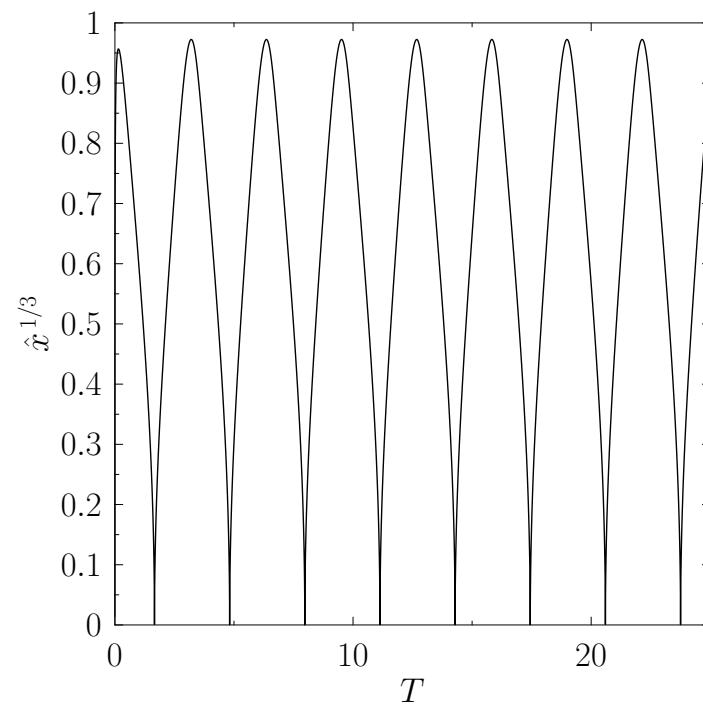
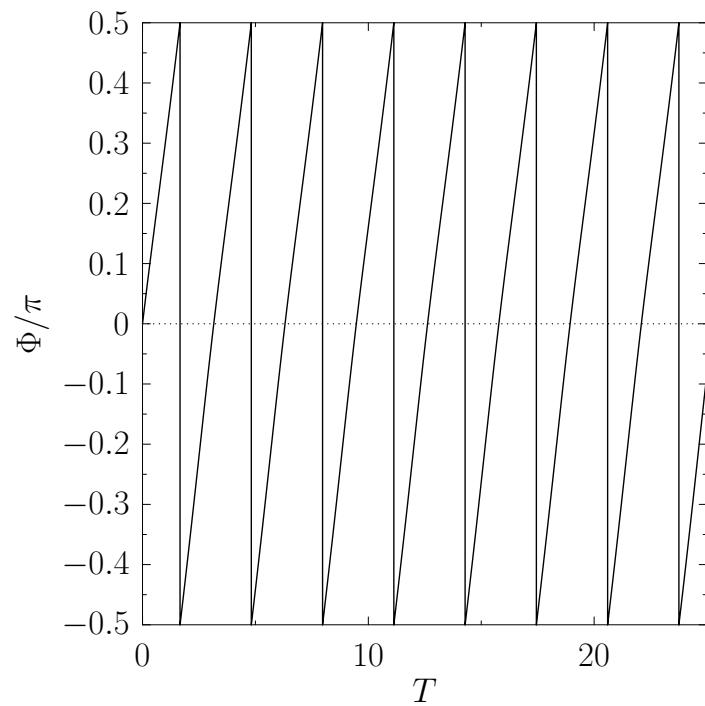
Rutherford Regime



Polarization Regime

- Suppression of driven magnetic reconnection due to *ion polarization current*, which essentially prevents island width from exceeding ρ_s , and, hence, prevents locking to resonant error-field.
- Penetration occurs when error-field drive overwhelms polarization current stabilization, allowing island to grow to large amplitude, and triggering locking of island to resonant error-field.
- Prior to penetration, island width “pulsates” due to balance between error-field destabilization (which varies with island phase) and polarization current stabilization.

Polarization Regime



Scaling of Penetration Threshold

- Rutherford regime:

$$(b_r/B_T)_{crit} \sim \beta \nu_*^{-1} \rho_*^2 \sim n_e^0 B_T^{-4/5} R_0^{-1}.$$

- Polarization regime:

$$(b_r/B_T)_{crit} \sim \beta \rho_* \sim n_e B_T^{-9/5} R_0^{-1/4}.$$

- Rutherford regime scaling is almost indistinguishable from original Fitzpatrick (1993) scaling, and cannot explain experimental data.
- Polarization regime scaling is consistent with experimental data.

Conclusions

- Observed scaling of error-field penetration threshold in tokamaks with engineering parameters—in particular, *linear* scaling with plasma density—indicates that threshold largely determined by *ion polarization current*.
- Since stabilizing effect of ion polarization current only manifests itself in *nonlinear* island physics, this necessitates nonlinear response model for plasma in immediate vicinity of rational surface. However, in absence of ion polarization current, such a model fails as badly as a linear response model.
- Given that error-field penetration threshold is governed by ion polarization current, seems highly likely that threshold for *neoclassical tearing modes* is also determined by this effect.