Integrated modeling of island growth, stabilization and mode locking

Consequences for NTM control on ITER

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Outline

What are the consequences of preventing mode locking for NTM control on ITER?

- Rotation of NTMs
- Generalized Rutherford Equation with ECCD stabilization
- Maximum allowed latency
- Effect of misalignment
NTM control on ITER

- ECCD on ITER
  - Four EC launchers spanning 100° toroidal angle
  - Total power 13.3 MW
  - Destabilizing current drive in X-point likely for locked modes

- Full suppression of islands
- No mode locking (rotation should remain finite)

- Latency and alignment requirements?
- **Rigid body rotation of the island**
  - Speed up by viscous coupling with the bulk plasma
  - Slow down by wall interaction

\[
\frac{d\omega}{dt} = \left( \frac{\omega_0}{\tau_{M0}} - \frac{\omega}{\tau_M} \right) - \frac{1}{m \tau_{A0}^2 C_W \omega} \left( \frac{W}{a} \right)^3
\]

\[
\tau_M = \frac{\tau_{M0}}{1 + C_M \frac{W}{a}}
\]

- **Islands equal to or larger than 5 cm lock**

Rotation model by La Haye et al NF pp. 451 vol 46 2006
Locking of $q=2/1$ modes

Island rotation frequency for constant island widths ($q=2/1$)

Rotation frequency $\omega$ [rad/s] vs. Time $t$ [s]

- $w=3$ cm (solid line)
- $w=4$ cm (dashed line)
- $w=5$ cm (dotted line)
- $w=6$ cm (dashed-dotted line)
Locking time comparable to growth time

![Graph showing the relationship between locking time and island width. The graph plots locking time and growth time at locking width as a function of island width. The x-axis represents island width (w) in meters, ranging from 0 to 0.3. The y-axis represents the logarithm of locking time (t) in seconds, ranging from $10^{-2}$ to $10^2$. The graph indicates that as the island width increases, both the locking time and growth time decrease.]
Island growth

- Generalized Rutherford Equation
- Classical growth rate

\[ \frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta'_0 \]
Island growth

- Generalized Rutherford Equation
- Classical growth rate
- Neoclassical bootstrap growth rate
  - Flattening of pressure profile leading to a decreased bootstrap current

\[
\frac{\tau_r}{r_s} \frac{d w}{d t} = r_s \Delta' + \frac{16 \mu_0 L_q r_s j_{BS}}{B_p \pi} \frac{4}{3w} f \left( \frac{w}{w_{\text{marg}}} \right)
\]
Island growth

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\[
\frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta'_0 + \frac{16 \mu_0 L_q r_s j_{BS}}{B_p \pi} \frac{4}{3w} f \left( \frac{w}{w_{\text{marg}}} \right)
\]

- Classical growth rate found by relating it to the bootstrap term

\[
\Delta'_0 = -\Delta'_{BS} \left( w = w_{\text{sat}} \right)
\]
Bootstrap term models

NTM growth rate from the Generalized Rutherford Equation for $q=2\cdot1$

Polarization ($w_{marg}=2 \text{ cm}$)

- Island width $w$ [m]
- NTM growth rate $\frac{dw}{dt}$ [m/s]
Bootstrap term models

NTM growth rate from the Generalized Rutherford Equation for $q=2/1$

- Polarization ($w_{marg}=2$ cm)
- Polarization ($w_{marg}=6$ cm)
Bootstrap term models

NTM growth rate from the Generalized Rutherford Equation for $q=2/1$

Graph showing:
- Blue line: Polarization ($w_{\text{marg}}=2 \text{ cm}$)
- Dotted blue line: Polarization ($w_{\text{marg}}=6 \text{ cm}$)
- Red line: Transport ($w_{\text{marg}}=2 \text{ cm}$)

Graph axes:
- Vertical axis: NTM growth rate $dw/dt$ [m/s]
- Horizontal axis: Island width $w$ [m]
Bootstrap term models

NTM growth rate from the Generalized Rutherford Equation for $q=2/1$ for different polarization and transport conditions with marginal island widths of 2 cm and 6 cm.
Bootstrap term models

NTM growth rate from the Generalized Rutherford Equation for $q=2/1$

- Polarization ($w_{\text{marg}}=2 \text{ cm}$)
- Polarization ($w_{\text{marg}}=6 \text{ cm}$)
- Transport ($w_{\text{marg}}=2 \text{ cm}$)
- Transport ($w_{\text{marg}}=6 \text{ cm}$)

$w_{\text{lock}}=5 \text{ cm}$
Electron Cyclotron Current Drive

\[
\frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta'_0 + r_s \Delta'_BS + r_s \Delta'_CD
\]

\[
r_s \Delta'_CD = -K_{CD} P_{EC} F_{CD} \left( w^* = \frac{w}{w_{dep}}, x_{norm} = \frac{x_{dep}}{\max(w, w_{dep})} \right)
\]

ECCD model by De Lazzari et al NF pp. 075002 vol 49 2009
Simulation example

$q = 2/1 \quad w_{\text{seed}} = 4 \text{ cm} \quad \text{(polarization model} \quad w_{\text{marg}} = 2 \text{ cm})$
Simulation example

$q = 2/1, w_{\text{seed}} = 4 \text{ cm} \quad \text{(polarization model} \ w_{\text{marg}} = 2 \text{ cm)}$

- Island width $w$ [m]

- Island rotation $\phi$ [rad/s]

- Time $t$ [s]
Simulation example

$q=2/1 \quad w_{seq}=4 \text{ cm (polarization model } w_{marg}=2 \text{ cm)}$

$P=13.3 \text{ MW}$

$\theta=0.95 \text{ s}$
Simulation example

$q = 2/1 \quad w_{\text{seq}} = 4 \text{ cm} (\text{polarization model } w_{\text{marg}} = 2 \text{ cm})$

$P = 13.3 \text{ MW}$

Island width $w$ [m]

Island rotation $\varphi$ [rad/s]

$\varphi = \begin{cases} 
0 & \text{if } t_d = 0.90 \text{ s} \\
0 & \text{if } t_d = 0.95 \text{ s}
\end{cases}$
Maximum allowed latency

Polarization ($w_{marg} = 2$ cm)
Maximum allowed latency

- Polarization ($w_{marg} = 2$ cm)
- Transport ($w_{marg} = 6$ cm)
Effect of misalignment on island growth

![Graph showing the effect of misalignment on island growth. The x-axis represents the island width (w [m]), and the y-axis represents the NTM growth rate (dw/dt [m/s]). The graph includes two curves: one for No ECCD and another for x_{dep}=0.0 cm.](image)
Effect of misalignment on island growth

No ECCD

$x_{dep} = 0.0 \, \text{cm}$

$x_{dep} = 0.7 \, \text{cm}$
Effect of misalignment on island growth

![Graph showing the effect of misalignment on island growth. The graph plots island width (w) against NTM growth rate (dw/dt) for different values of x_{dep}. The legend indicates three scenarios: No ECCD, x_{dep} = 0.0 cm, x_{dep} = 0.7 cm, and x_{dep} = 0.8 cm. The graph demonstrates how misalignment affects the growth rate of islands.]
Effect of misalignment on island growth

- Maximum allowed deviation 0.7 cm to 1.0 cm
Effect of misalignment on island growth

- Maximum allowed deviation 0.7 cm to 1.0 cm
- No effect on latency
Conclusions

Due to limited actuator span, mode locking should be avoided

- Dependence of rotation on width requires combined model
- Maximum allowed latency in order of 1 second
- Maximum allowed deviation 0.7 cm to 1.0 cm from GRE
  - Misalignment does not affect latency
- Similar results for 3/2 NTM
- Results relevant for design of ITER NTM controller
  - Reported mirror settling time of 2.5 s is longer

Settling time from Collazos et al IEEE TPS NF pp. 441 vol 38 2010
Terminology

- **Maximum allowed latency** = the longest time between island seeding and start of ECCD deposition that results in full suppression of the island before mode-locking.

- **Mode-locking** = stop of the island rotation, this means that the island locks to the error fields.

- **$x_{\text{dep}}$** = radial misalignment related to the resonant surface position.

- **Island width** = Average of island width on LFS and HFS in the equatorial plane.

- **Locking time** = the time it takes for an island of constant width to stop rotating.
Bootstrap models

\[
\begin{align*}
  f_{\text{tra}}(w, w_{\text{marg}}) &= \left( \frac{w^2}{w^2 + w_{\text{marg}}^2} \right) \\
  f_{\text{pol}}(w, w_{\text{marg}}) &= \left( 1 - \frac{w_{\text{marg}}^2}{3w^2} \right)
\end{align*}
\]
## Simulation parameters

<table>
<thead>
<tr>
<th>Constants and ITER parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>$4\pi \cdot 10^{-7} \frac{N}{A^2}$</td>
</tr>
<tr>
<td>$R_b$</td>
<td>6.2 m</td>
</tr>
<tr>
<td>$a$</td>
<td>2.0 m</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.7</td>
</tr>
<tr>
<td>$Z_{eff}$</td>
<td>1.7</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>NTM parameters</th>
<th>$q = 2/1$</th>
<th>$q = 3/2$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s$</td>
<td>1.55 m</td>
<td>1.3 m</td>
<td>Position of the resonant surface(^{22})</td>
</tr>
<tr>
<td>$j_{BS}(r_s)$</td>
<td>73 kA m(^{-2})</td>
<td>94 kA m(^{-2})</td>
<td>Bootstrap current\cite Bertelli forthcoming(^{68})</td>
</tr>
<tr>
<td>$T_e(r_s)$</td>
<td>5.6 keV</td>
<td>7.6 keV</td>
<td>Electron temperature(^{22})</td>
</tr>
<tr>
<td>$B_p(r_s)$</td>
<td>0.97 T</td>
<td>1.07 T</td>
<td>Poloidal magnetic field(^{22})</td>
</tr>
<tr>
<td>$L_n(r_s)$</td>
<td>0.87 m</td>
<td>0.88 m</td>
<td>Gradient length of the q-profile(^{22})</td>
</tr>
<tr>
<td>$w_{marg}$</td>
<td>2-6 cm</td>
<td>2-6 cm</td>
<td>Marginal island width\cite Bertelli forthcoming(^{68})</td>
</tr>
<tr>
<td>$w_{sat}$</td>
<td>32 cm</td>
<td>25 cm</td>
<td>Saturated island width(^{6})</td>
</tr>
<tr>
<td>$w_{LSM , dep}$</td>
<td>2.4 cm</td>
<td>3.7 cm</td>
<td>Deposition width at flux surface with LSM TORBEAM simulation by Bertelli et. al.</td>
</tr>
<tr>
<td>$j_{CD,W , LSM}$</td>
<td>$1.32 \cdot 10^{-2}$ A m(^{-2}) W</td>
<td>$1.22 \cdot 10^{-2}$ A m(^{-2}) W</td>
<td>Current drive density per watt with LSM TORBEAM simulation by Bertelli et. al.</td>
</tr>
<tr>
<td>$w_{LSM , dep}$</td>
<td>3.9 cm</td>
<td>5.5 cm</td>
<td>Deposition width at flux surface with USM TORBEAM simulation by Bertelli et. al.</td>
</tr>
<tr>
<td>$j_{CD,W , USM}$</td>
<td>$9.62 \cdot 10^{-3}$ A m(^{-2}) W</td>
<td>$1.07 \cdot 10^{-2}$ A m(^{-2}) W</td>
<td>Current drive density per watt with USM TORBEAM simulation by Bertelli et. al.</td>
</tr>
<tr>
<td>$\epsilon = \frac{r_s}{R_0}$</td>
<td>0.25</td>
<td>0.21</td>
<td>Inverse aspect ratio</td>
</tr>
<tr>
<td>$\eta_{NC}$</td>
<td>$1.44 \cdot 10^{-8}$ $\Omega$m</td>
<td>$7.73 \cdot 10^{-9}$ $\Omega$m</td>
<td>Neoclassical conductivity (equation 2.4)</td>
</tr>
<tr>
<td>$\tau_i$</td>
<td>293 s</td>
<td>383 s</td>
<td>Resistive time scale (equation 2.3)</td>
</tr>
</tbody>
</table>

<table>
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<th>$q = 2/1$</th>
<th>$q = 3/2$</th>
<th></th>
</tr>
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<tr>
<td>$\omega_0$</td>
<td>$420 \cdot 2\pi$ rad/s</td>
<td>$578 \cdot 2\pi$ rad/s</td>
<td>Result of ITER scenario 2 simulations</td>
</tr>
<tr>
<td>$\tau_{A0}$</td>
<td>3.0 $\mu$s</td>
<td></td>
<td>Assumes toroidal rotation of the island</td>
</tr>
<tr>
<td>$\tau_{M0}$</td>
<td>$\tau_E = 3.7$ s</td>
<td></td>
<td>Equal to ITER energy confinement time</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>0.188 s</td>
<td>0.125 s</td>
<td>Resitive wall simulations with VALEN</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$\frac{1}{14}$</td>
<td>$\frac{1}{11}$</td>
<td>Obtained through fits on DIII-D shots</td>
</tr>
<tr>
<td>$C_2$</td>
<td>20</td>
<td>12</td>
<td>Obtained through fits on DIII-D shots</td>
</tr>
</tbody>
</table>
Simulation details

- Seed island size varied
- Delay between seeding and start of ECCD varied
- Largest delay that still results in full suppression before locking is the maximum allowed latency

Simulation scenarios

- Polarization 2 cm
- Transport 6 cm

Resolution: dt=50 ms and dw_seed=0.5 cm

Implemented in MathWorks Simulink

- Runga-Kutta 5th order with 4th order error estimate (var. step)
- Rel error 1e-6
q=3/2 latency

![Graph showing the relationship between maximum allowed latency (t) and seed island width (w_{seed}) for different transport and polarization cases.](image)

- **Polarization** (w_{marg} = 2 cm)
- **Transport** (w_{marg} = 6 cm)
- τ_{SM} = 2.5 s
Effect of misalignment on latency

- **No effect**
Extra simulations

Effect of rotation on the allowable latency for $q=2/1$

- Polarization ($w_{\text{marg}} = 2 \text{ cm}$)
- Double rotation
- Transport ($w_{\text{marg}} = 6 \text{ cm}$)
- Double rotation

Effect of ECCD power on the allowable latency for $q=2/1$

- Polarization ($w_{\text{marg}} = 2 \text{ cm}$)
- 10 MW
- 20 MW
- Transport ($w_{\text{marg}} = 6 \text{ cm}$)
- 10 MW
- 20 MW

Effect of modulation on the allowable latency for $q=2/1$

- Polarization ($w_{\text{marg}} = 2 \text{ cm}$)
- Modulation ($D=0.5$)
- Transport ($w_{\text{marg}} = 6 \text{ cm}$)
- Modulation ($D=0.5$)
Stabilization maps $q=2/1$

Detection limits for $q=2/1$ polarization $w_{\text{marg}}=2 \text{ cm}$

Detection limits for $q=2/1$ transport $w_{\text{marg}}=6 \text{ cm}$
Tearing modes

- Flux surfaces characterized by safety factor
  \[ q = \frac{m}{n} \]

- Reconnection of magnetic field lines

- Rational values of q
  - q=2/1 (and q=3/2)

- Affected by non-inductive current