

Low-q Tokamak Mode Control in RFX-mod

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Feedback on $m=2$ $n=1$ RWM in $q(a)<2$ experiments

- ☐ Intelligent Shell & Clean Mode Control

Cylindrical MHD model of the RWM feedback

- ☐ sidebands aliasing
- ☐ feedback delays
- ☐ radial/poloidal sensors

RFX-mod $q(a) \leq 2$ tokamak

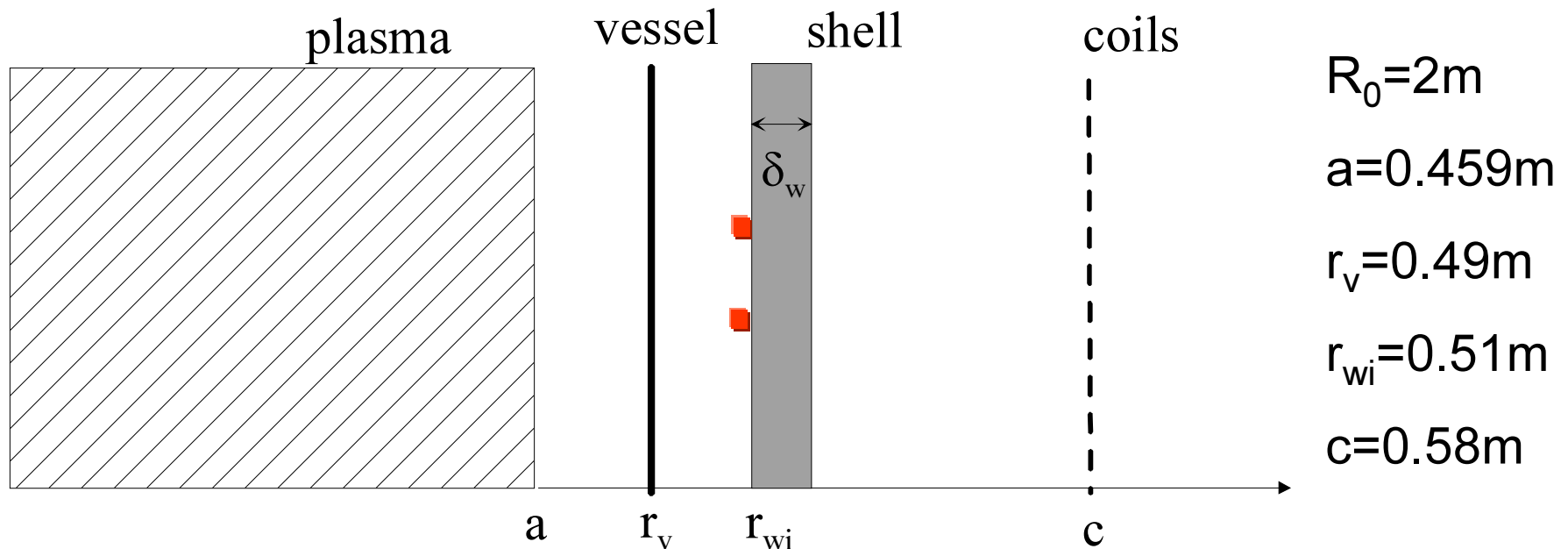
$q(a) \sim 1.8$, $B_\phi(a) = 0.55\text{T}$, $I_p = 150\text{kA}$, pulse length $\sim 1\text{s}$

$n_e R / B_\phi(a) \sim 7$ (density limit for $q(a) \sim 2$)

$T_e \sim 200\text{-}300\text{eV}$, $\beta_N = \beta$ a $B_\phi(a)/I_p \leq 1.5\%$ Tm/MA

Feedback control of $m=2$, $n=1$ RWM

RFX-mod layout



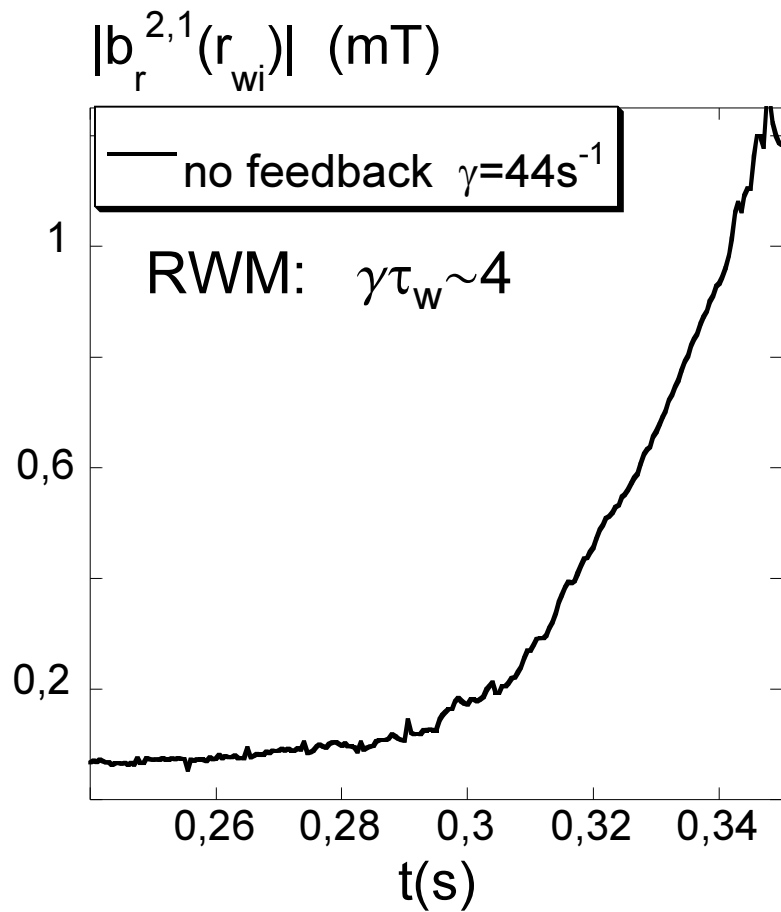
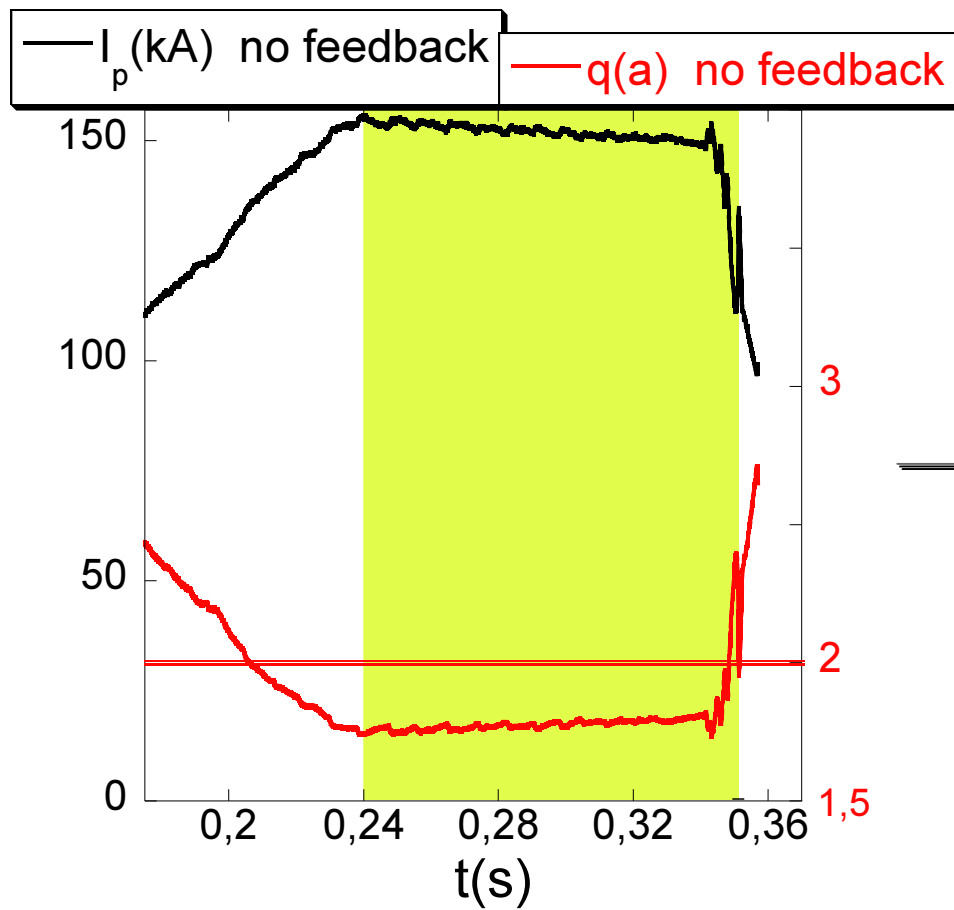
vessel: $\tau_v \approx 2.3\text{ms}$

shell: $\delta_w = 3\text{mm}$, $\tau_w = 100\text{ms}$

coils: $(M_c=4) \times (N_c=48)$, $\Delta\theta_c = 2\pi/M_c$, $\Delta\phi_c = 2\pi/N_c$

sensors: $(M_s=4) \times (N_s=48)$ radial ($\Delta\theta_s = 2\pi/M_s$, $\Delta\phi_s = 2\pi/N_s$) toroidal and poloidal

$m=2, n=1$ RWM



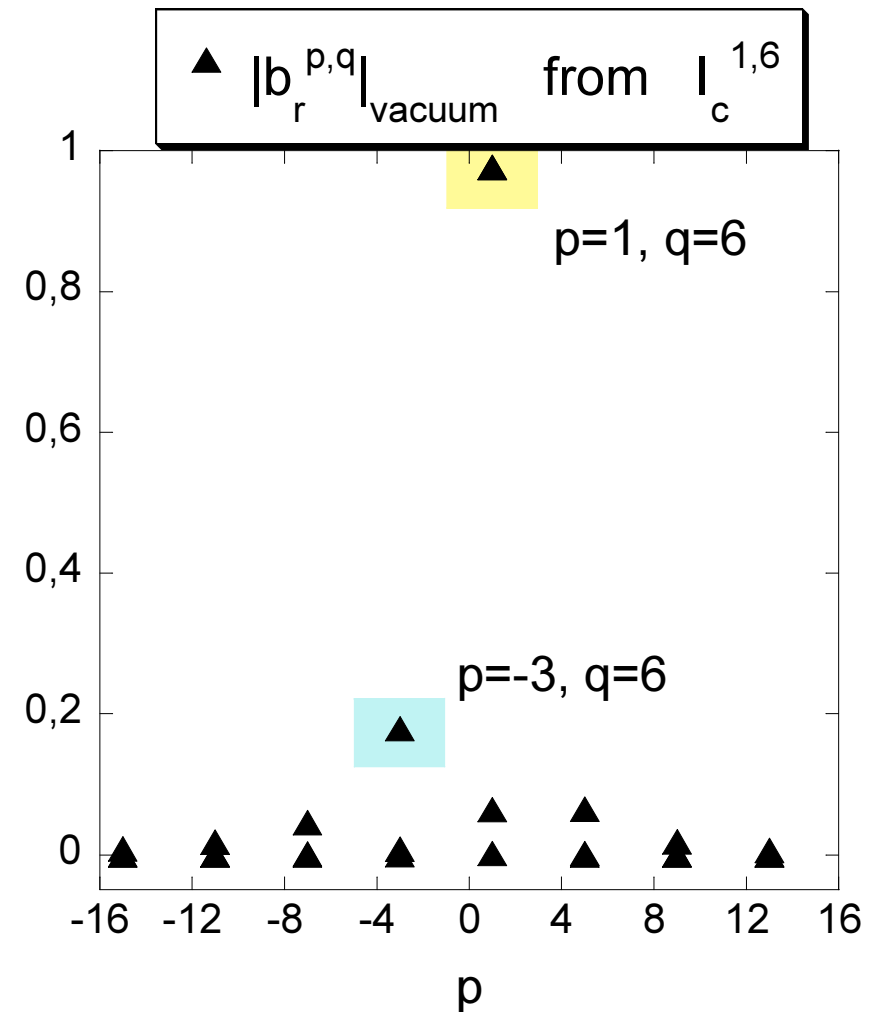
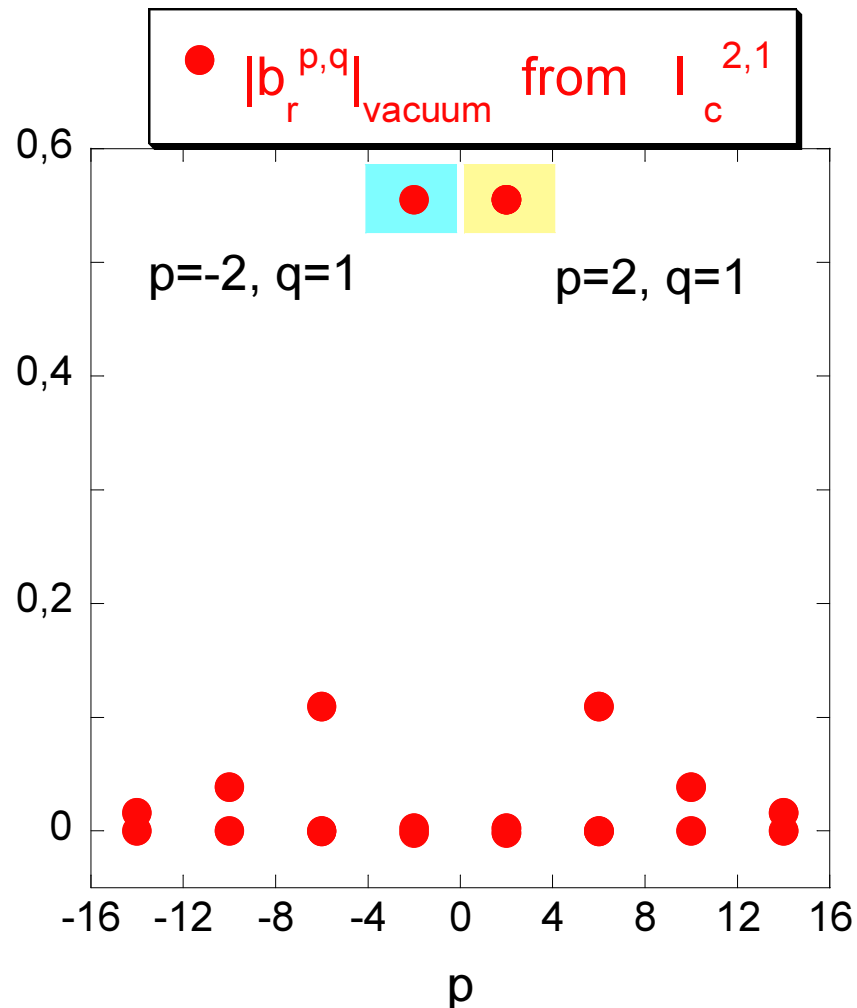
Intelligent Shell (IS) :

$$b_f = b_{r,DFT}^{m,n} = \sum_{p,q} b_r^{p,q}(r_{wi}) f_s(p,q) \quad p = m + l \times M_s, q = n + k \times N_s$$

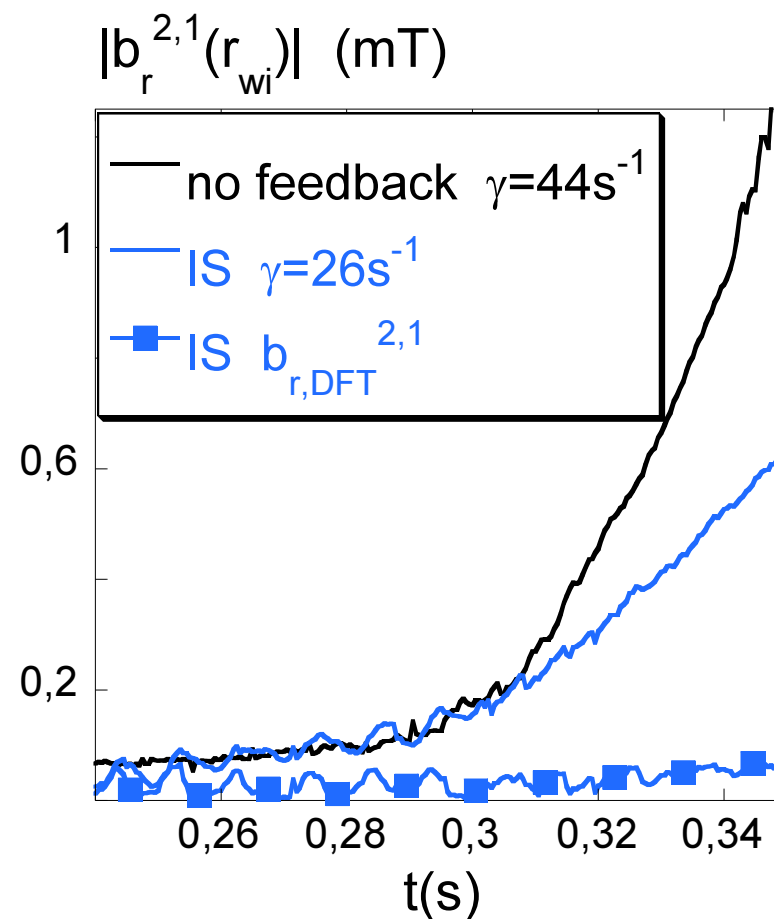
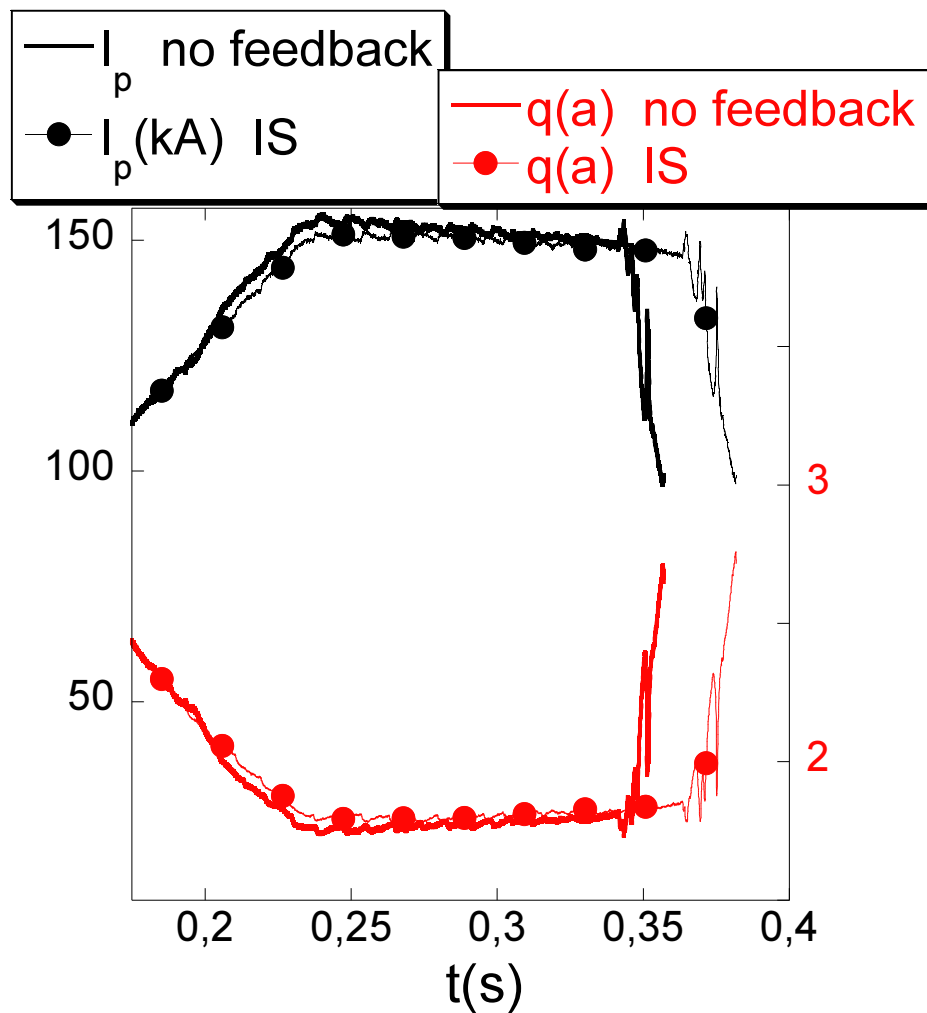
$I_c^{p,q} = I_c^{m,n}$
↑

Intelligent Shell: sidebands pollution

$$M_c = M_s = 4, \quad N_c = N_s = 48$$



Intelligent Shell: $m=2, n=1$ control



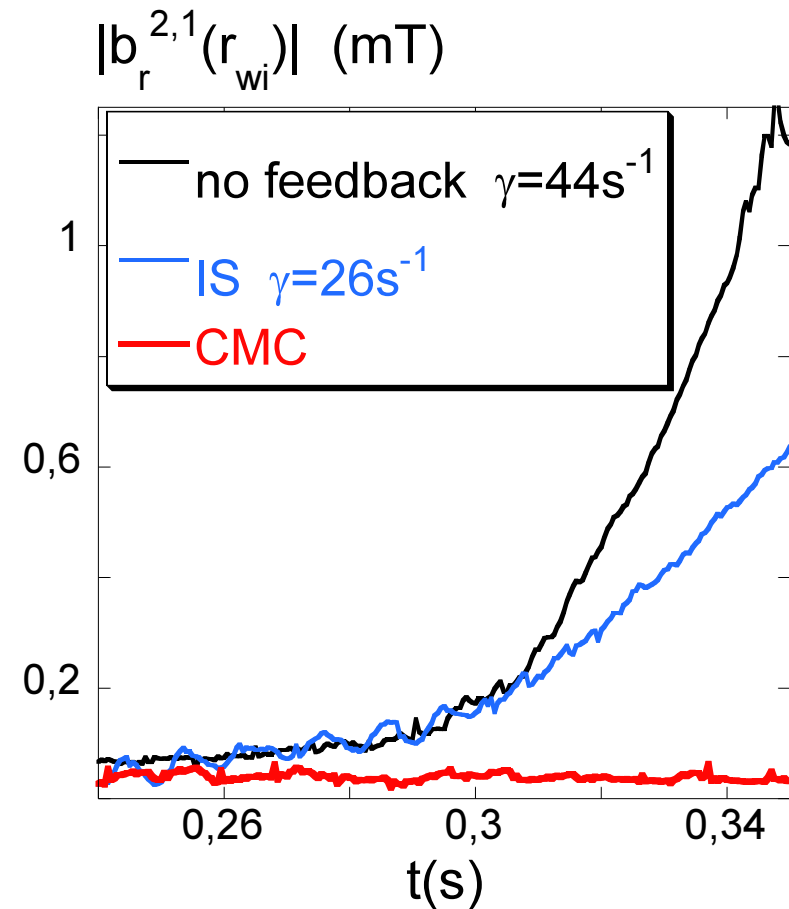
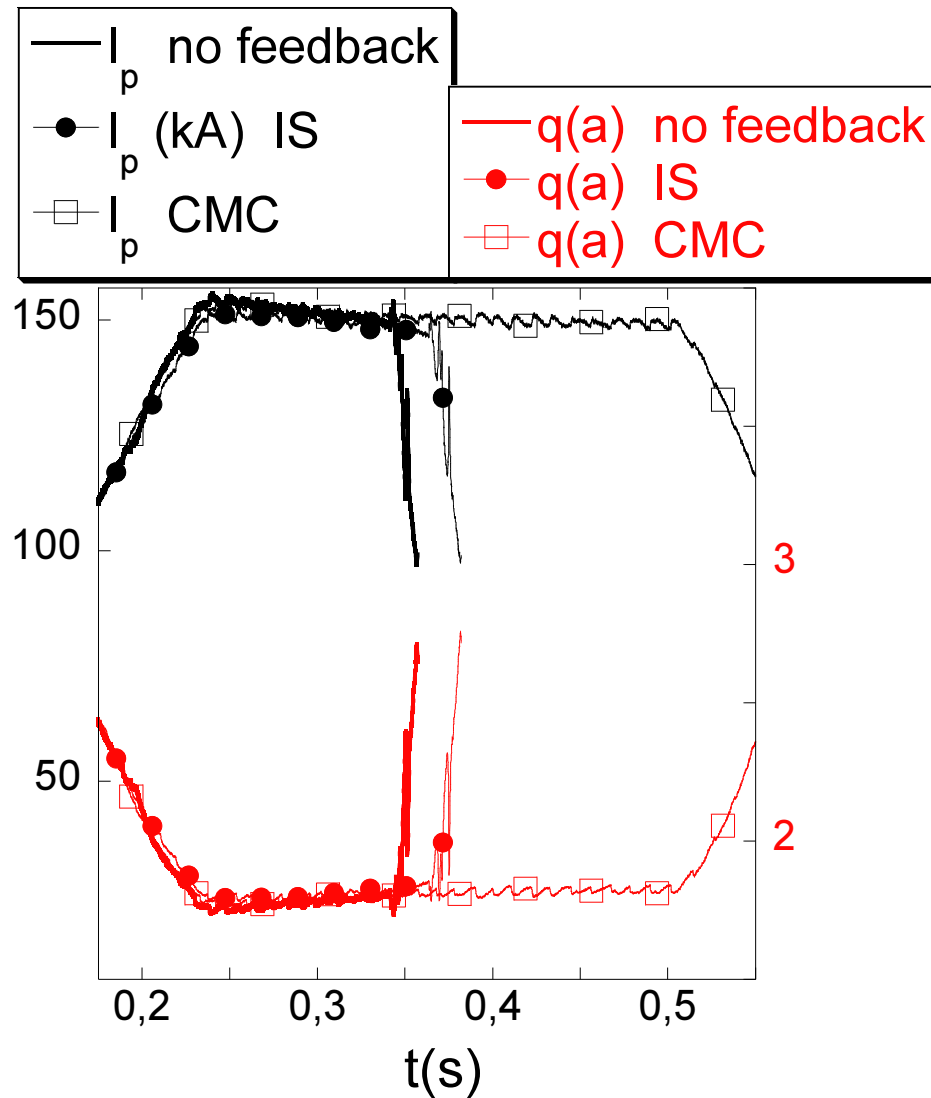
Intelligent Shell (IS):

$$b_f = b_{r,DFT}^{m,n} = \sum_{p,q} b_r^{p,q}(r_{wi}) f_s(p,q) \quad p = m + l \times M_s, q = n + k \times N_s$$
$$\uparrow$$
$$I_c^{p,q} = I_c^{m,n}$$

Clean Mode Control (CMC):

$$b_f = b_{r,DFT}^{m,n} - \int_0^t \left[\sum_{(p,q) \in \mathbb{Z}^2 - (m,n)} \Lambda^{p,q}(t-x) \right] I_c^{m,n}(x) dx \approx b_r^{m,n}(r_{wi})$$

Clean Mode Control on $m=2, n=1$

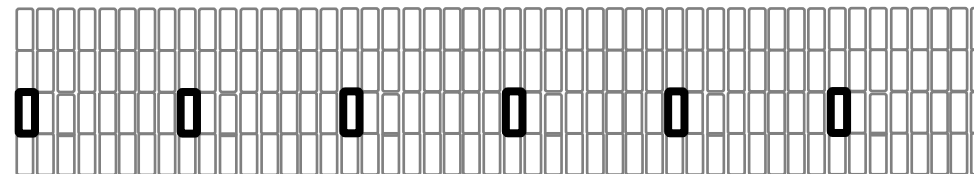
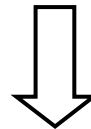
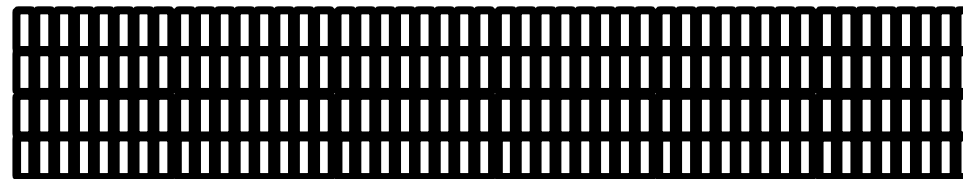


Clean Mode Control on $m=2, n=1$

$I_c^{2,1} < 200A$ ~50-100 times smaller than I_c required by the dynamo modes

CMC works even with $M_c=1$ $N_c=6$

48 (ϕ) x 4 (θ): 100%



6 (ϕ) x 1 (θ): 3.1%

Feedback on $m=2$ $n=1$ RWM in $q(a)<2$ experiments

- ❑ Intelligent Shell & Clean Mode Control (radial sensors)

Cylindrical MHD model of the RWM feedback

- ❑ sidebands aliasing
- ❑ feedback delays
- ❑ radial/poloidal sensors

The MHD model

□ Cylindrical geometry, zero- β equilibrium

□ Newcomb's equation for $\Psi^{p,q} = -i r b_r^{p,q}$, $p=m+l \times M_c$, $q=n+k \times N_c$

□ Vessel
$$\tau_v \left. \frac{\partial \Psi^{p,q}}{\partial t} \right|_{r_v} = r \left. \frac{\partial \Psi^{p,q}}{\partial r} \right|_{r_v^-}^{r_v^+}$$

□ Shell
$$\mu_0 \sigma_w \frac{\partial \Psi^{p,q}}{\partial t} = \frac{\partial^2 \Psi^{p,q}}{\partial r^2}, \quad r \in [r_{wi}, r_{we}]$$

□ Coils
$$r \left. \frac{\partial \Psi^{p,q}}{\partial r} \right|_{c^-}^{c^+} \propto I_c^{p,q}, \quad I_c^{p,q} = I_c^{m,n}$$

$$\tau_c^{m,n} \frac{dI_c^{m,n}}{dt} + I_c^{m,n} \approx \frac{V_c^{m,n}}{R_c}$$

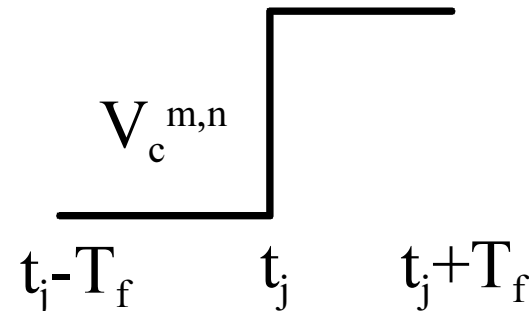
Continuous-time

$$V_c^{m,n}(t) = R_c \left[K_p + K_d \frac{d}{dt} \right] w_f(t)$$

$$\tau_d \frac{d}{dt} w_f(t) + w_f(t) = b_f$$

Delays: 1-pole filter τ_d

Discrete-time



$$\Delta t = \bar{k} T_f$$

$$V_c^{m,n}(t_j \leq t \leq t_j + T_f) = \left[K_p + K_d \frac{d}{dt} \right] w_f(t_{j-\bar{k}})$$

$$T_f \frac{d}{dt} w_f(t) + w_f(t) = b_f$$

Delays:

- ✓ 1-pole filter T_f
- ✓ true delay $\Delta t \div (\Delta t + T_f)$

Intelligent Shell: $m=2$ $n=1$ RWM

$$\begin{aligned} q(0) &= 0.82 \\ q(a) &= 1.78 \end{aligned}$$

$$\longrightarrow \gamma = 47 \text{ s}^{-1}$$

$$b_f = b_{r,\text{DFT}}^{2,1}(r_{wi})$$

$M_c = M_s = 4$, $N_c = N_s = 48$ $\Delta\theta_c = \Delta\theta_s = 2\pi/M_c$, $\Delta\phi_c = \Delta\phi_s = 2\pi/N_c$ **non-stabilizable**

Benchmark: $m=1, n=6$ RWM can be stabilized

$$p = -2, q = 1$$

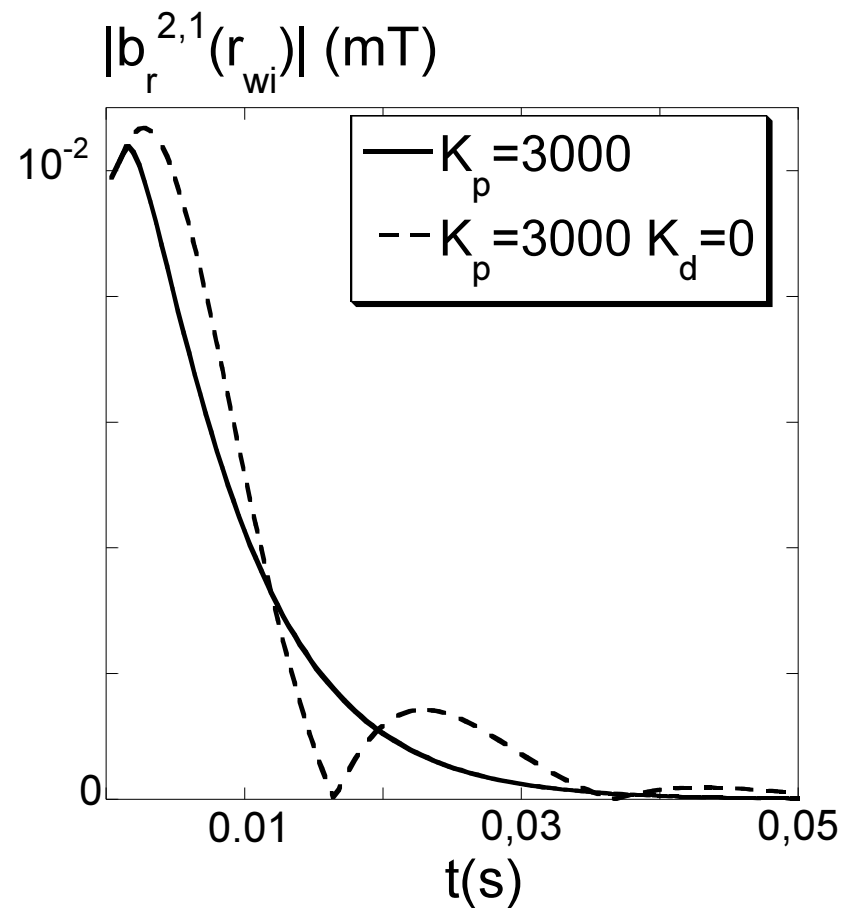
$M_c = M_s = 8$, $N_c = N_s = 6$ $\Delta\theta_c = \Delta\theta_s = 2\pi/M_c$, $\Delta\phi_c = \Delta\phi_s = 2\pi/N_c$ **stabilizable**

$M_c = M_s = 8$, $N_c = N_s = 6$ $\Delta\theta_c = \Delta\theta_s = \pi/M_c$, $\Delta\phi_c = \Delta\phi_s = \pi/N_c$ **non-stabilizable**

CMC simulations: RFX-mod case

$$b_f = b_r^{2,1}(r_{wi})$$

$$T_f=0.4\text{ms}, \quad \Delta t=2T_f, \quad K_d=T_c K_p$$



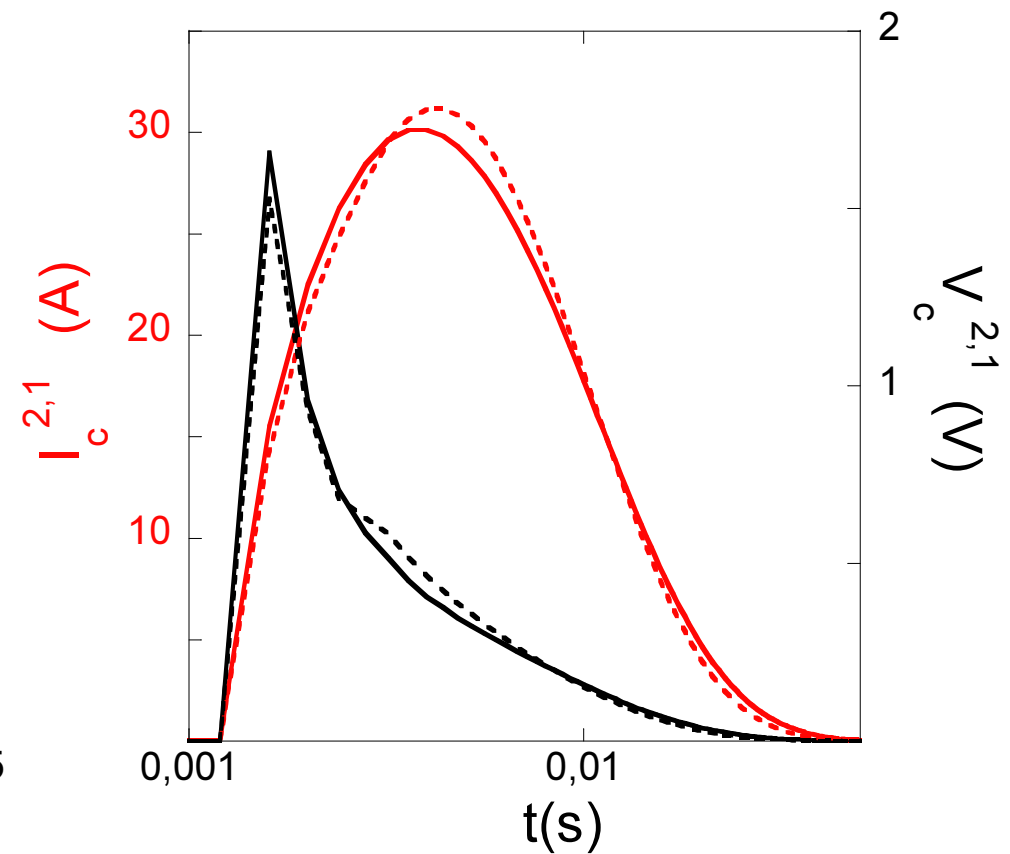
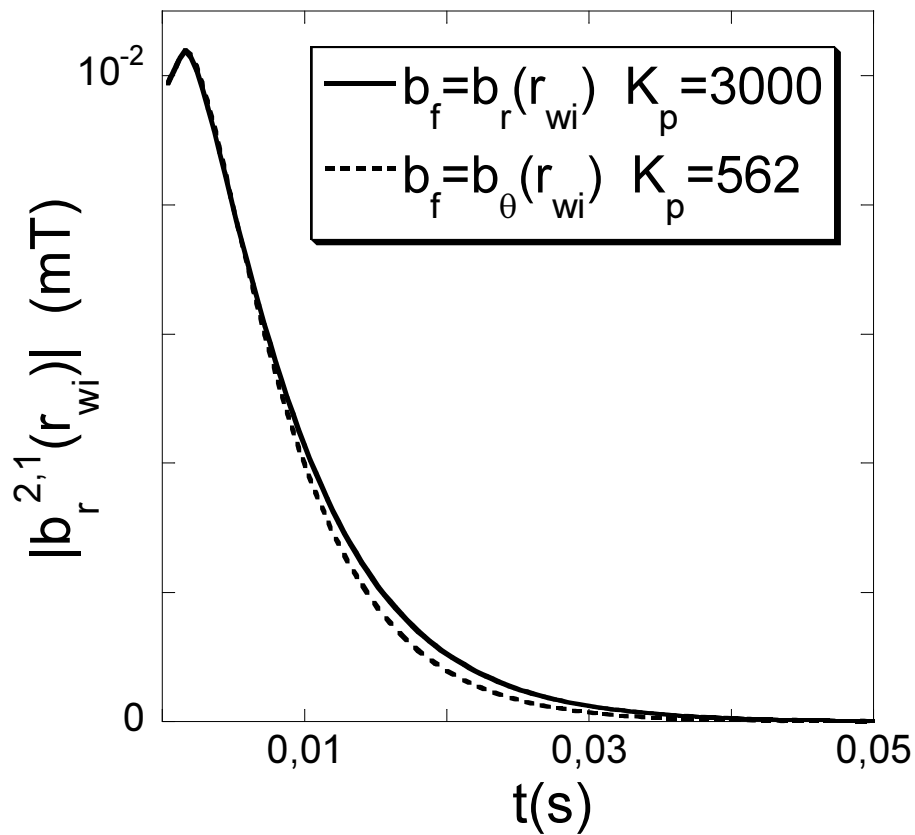
Experimental K_p

Simulations works also with
 $M_c=1, N_c=6$

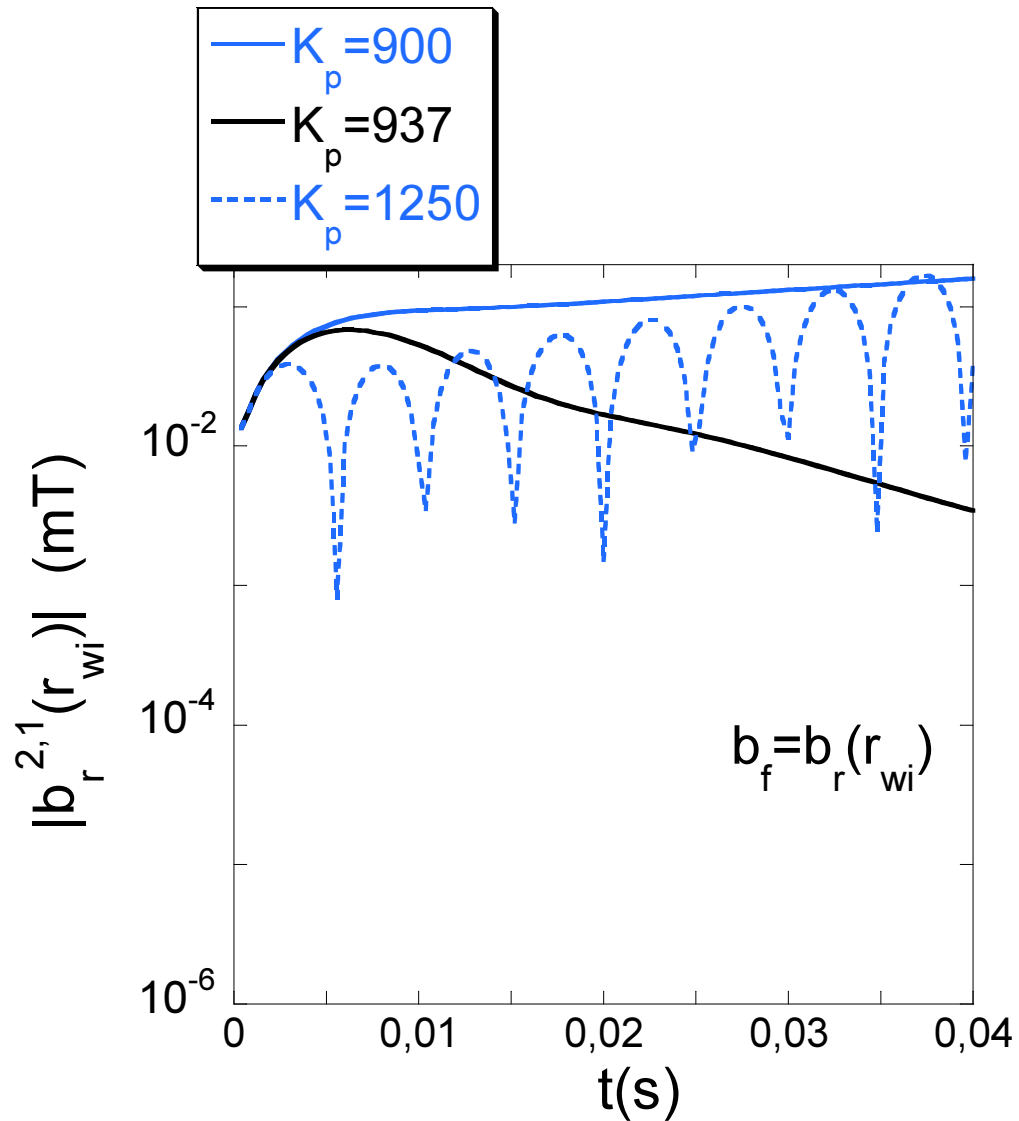
CMC simulations: comparison b_r , b_θ

$$b_f = b_r^{2,1}(r_{wi}), \quad b_f = i b_\theta^{2,1}(r_{wi})$$

$$T_f = 0.4 \text{ ms}, \quad \Delta t = 2T_f, \quad K_d = T_c K_p$$



CMC simulations: $\tau_w=3\text{ms}$



$$\gamma \sim 47 \text{s}^{-1} \rightarrow \gamma \sim 672 \text{s}^{-1}$$

$$T_f = 0.4 \text{ms}, \quad \Delta t = 2T_f$$

$$K_d = \tau_c K_p$$

$$K_{p,\text{max}} / K_{p,\text{min}} < 1.3$$

Continuous-time

$$V_c^{m,n}(t) = R_c \left[K_p + K_d \frac{d}{dt} \right] w_f(t)$$

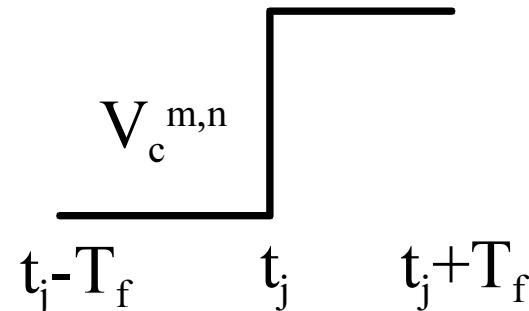
$$\tau_d \frac{d}{dt} w_f(t) + w_f(t) = b_f$$

Delays: 1-pole filter τ_d



$$\tau_d = T_f + \Delta t + T_f$$

Discrete-time



$$\Delta t = \bar{k} T_f$$

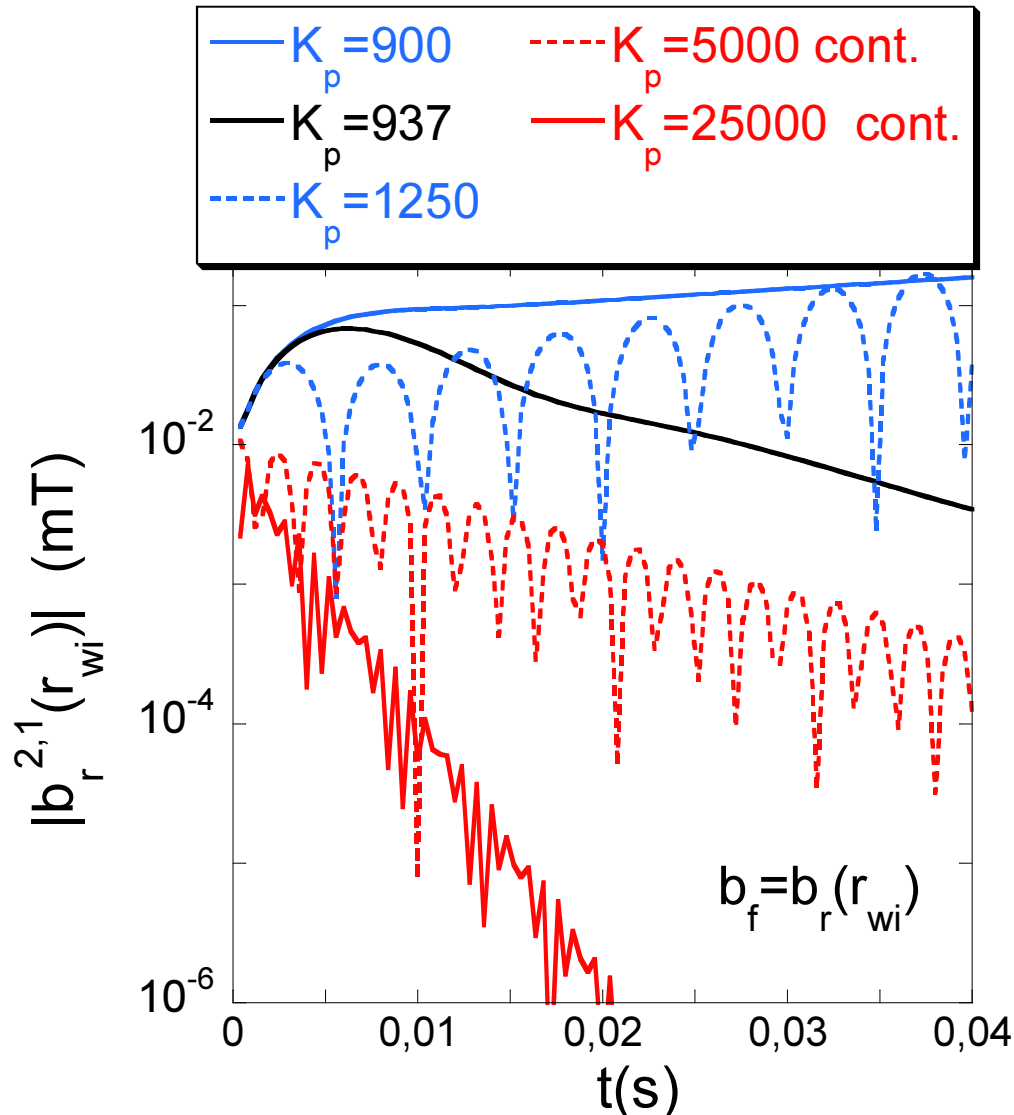
$$V_c^{m,n}(t_j \leq t \leq t_j + T_f) = R_c \left[K_p + K_d \frac{d}{dt} \right] w_f(t_{j-\bar{k}})$$

$$T_f \frac{d}{dt} w_f(t) + w_f(t) = b_f$$

Delays:

- ✓ 1-pole filter T_f
- ✓ true delay $\Delta t \div (\Delta t + T_f)$

CMC simulations: $\tau_w=3\text{ms}$



$$T_f=0.4\text{ms}, \quad \Delta t=2T_f$$

$$K_d=\tau_c K_p$$

Continuous-time model:

$$\tau_d = T_f + T_f + \Delta t = 1.6\text{ms}$$

1-pole filter delays are not equivalent to true delays

CMC br: comparative table

τ_w (ms)	γ (s ⁻¹)	$T_f - \Delta t$	CMC b_r $K_{p,max} / K_{p,min}$
100 ($\tau_w \gg \tau_v$)	47	$T_f = 0.4\text{ms}$ $\Delta t = 2T_f$	17
3 ($\tau_w \approx \tau_v$)	672	$T_f = 0.1\text{ms}$ $\Delta t = T_f$	8
0	1300	$T_f = 0.1\text{ms}$ $\Delta t = T_f$	4.7

$$K_d = T_c K_p$$

Explicit mode control $\tau_w=100\text{ms}$

$$b_f = b_r^{2,1}(r_{wi}) - [b_r^{2,1}(r_{wi})]_{vac}$$

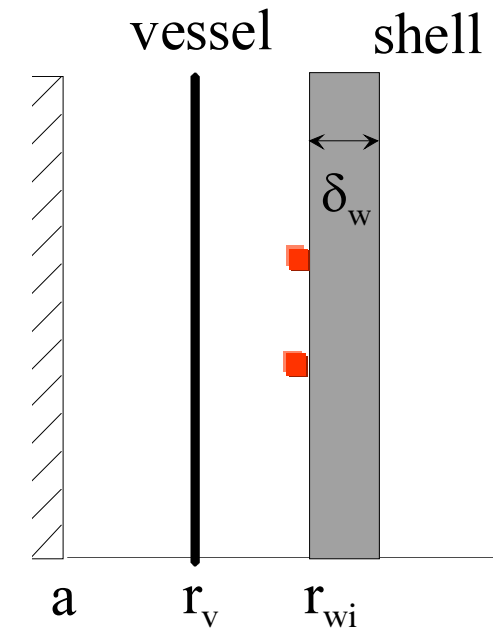
$$T_f=0.1\text{ms}, \quad \Delta t=T_f$$

$$K_d=T_c K_p$$

Not stabilized

CMC b_r, b_θ : comparative table

τ_w (ms)	$T_f - \Delta t$	CMC b_r $K_{p,max} / K_{p,min}$	CMC b_θ $K_{p,max} / K_{p,min}$
100 ($\tau_w \gg \tau_v$)	$T_f = 0.4\text{ms}$ $\Delta t = 2T_f$	17	12
3 ($\tau_w \approx \tau_v$)	$T_f = 0.1\text{ms}$ $\Delta t = T_f$	8	1.6
0	$T_f = 0.1\text{ms}$ $\Delta t = T_f$	4.7	1.07

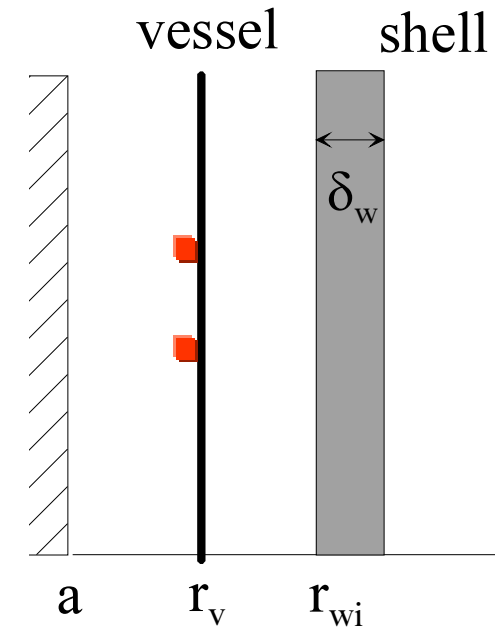


$$K_d = T_c K_p$$

Shielding + discrete-time feedback

CMC b_r , b_θ : comparative table

τ_w (ms)	$T_f - \Delta t$	CMC b_r $K_{p,max} / K_{p,min}$	CMC b_θ $K_{p,max} / K_{p,min}$
100 ($\tau_w \gg \tau_v$)	$T_f = 0.4\text{ms}$ $\Delta t = 2T_f$	17	/
3 ($\tau_w \approx \tau_v$)	$T_f = 0.1\text{ms}$ $\Delta t = T_f$	8	5.3
0	$T_f = 0.1\text{ms}$ $\Delta t = T_f$	4.7	4



$$K_d = T_c K_p$$

Shielding + discrete-time feedback

CMC b_r, b_θ : comparative table

τ_w (ms)	$T_f - \Delta t$	CMC b_r $K_{p,max}/K_{p,min}$	CMC b_θ $K_{p,max}/K_{p,min}$	Explicit MC $b_f = ib_{\theta}^{2,1}(r_{wi}) - [ib_{\theta}^{2,1}(r_{wi})]_{vac}$
100 ($\tau_w \gg \tau_v$)	$T_f=0.4ms$ $\Delta t=2T_f$	17	12	/
3 ($\tau_w \approx \tau_v$)	$T_f=0.1ms$ $\Delta t=T_f$	8	1.6	4
0	$T_f=0.1ms$ $\Delta t=T_f$	4.7	1.07	1.04

$$K_d = T_c K_p$$

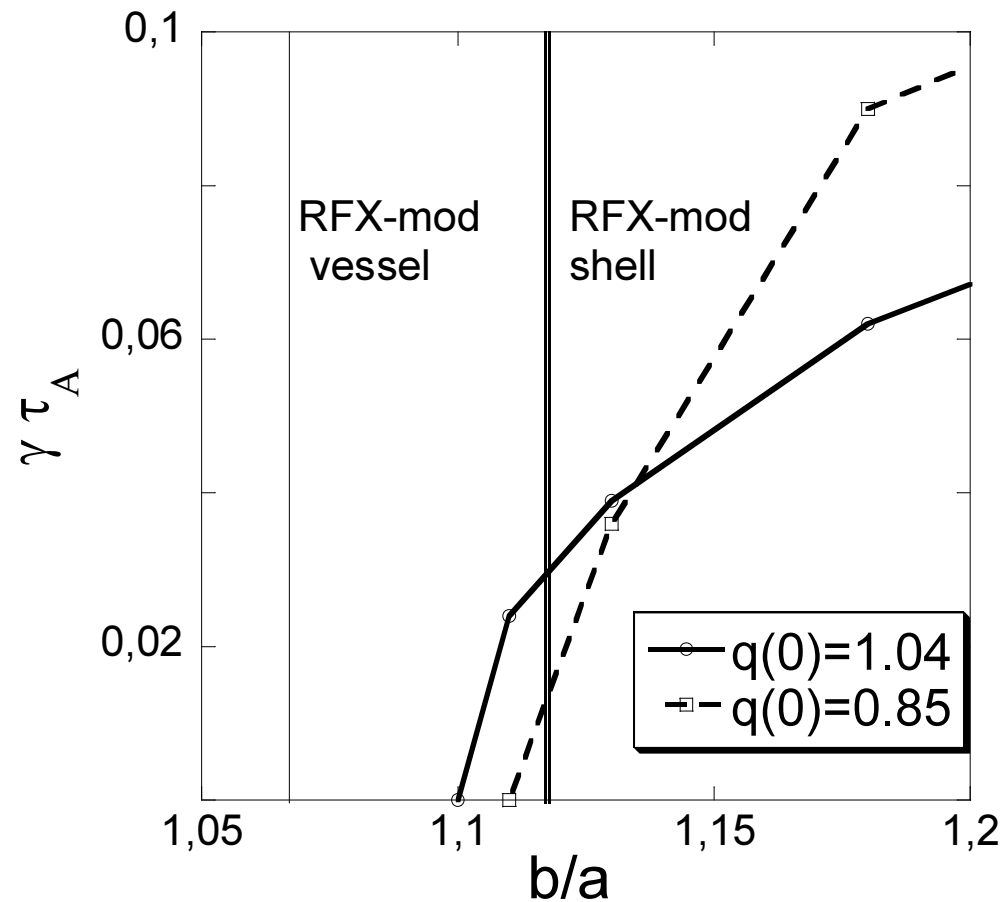
Shielding + discrete-time feedback

- ☐ Stable $q(a) < 2$ operations at RFX-mod allowed by proper feedback
 - ☐ Radial sensors can be used within the CMC → [Y. Liu, J.B. Lister, *Nucl. Fusion* **47** (2007) 648]
 - ☐ Full coils coverage not required ($M_c=1$, $N_c=6$)
 - ☐ Sidebands removal can be considered an electro-technical issue
 - ☐ Future experiments: shaping, sensors downgrade & poloidal
-
- ☐ MHD model reproduces fairly well the experiment
 - ☐ Importance of the discretization of the feedback (not equivalent to 1-pole filter delays)
 - ☐ Poloidal sensors: combined discretization and shielding effects



Spare slides

MARS-F stability analysis for $m=2, n=1$



The ideal mode is stabilized by the vacuum vessel

The MHD model: feedback variable

CMC: $b_f = b_r^{m,n}(r_{wi}) \quad b_{\bar{f}} = i b_{\theta}^{m,n}(r_{wi})$

Raw control: $b_f = b_{r,DFT}^{m,n} = \sum_{p,q} b_r^{p,q}(r_{wi}) f_s(p,q)$
 $b_f = i b_{\theta,DFT}^{m,n} = i \sum_{p,q} b_{\theta}^{p,q}(r_{wi})$

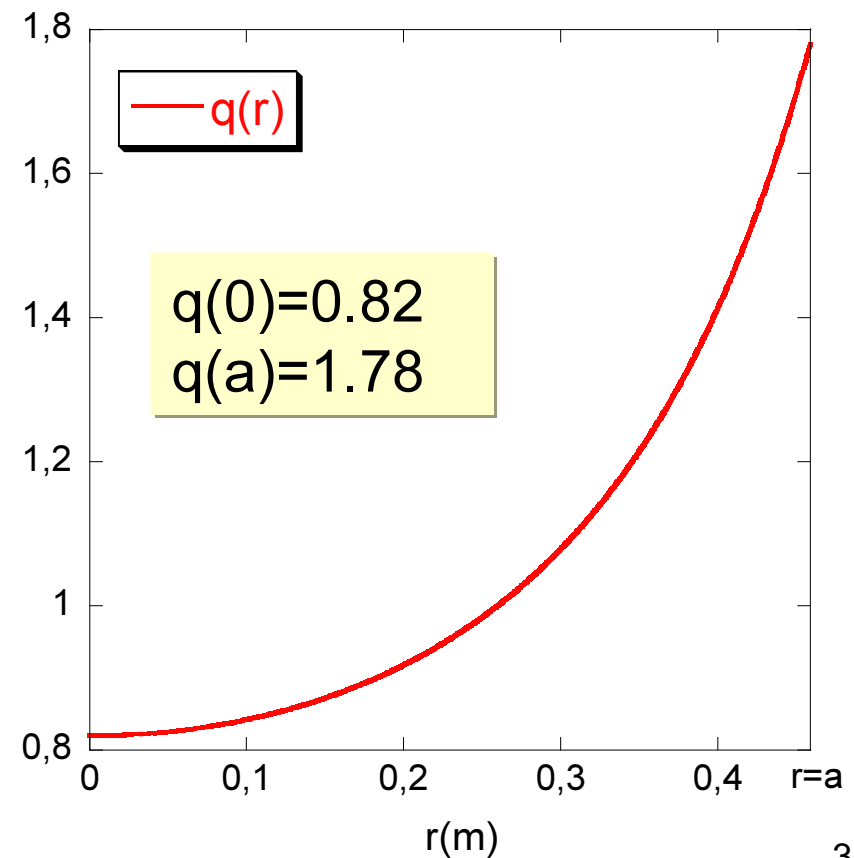
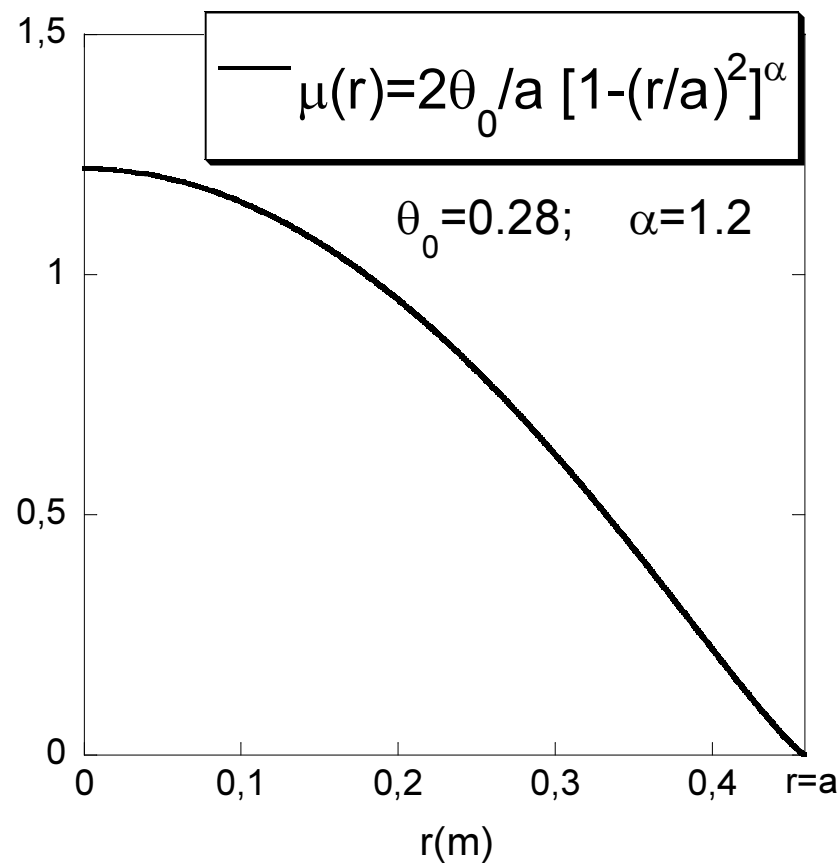
$p = m + l \times M_s, q = n + k \times N_s$

‘Explicit Mode Control’: $b_f = b_r^{m,n}(r_{wi}) - [b_r^{m,n}(r_{wi})]_{\text{vac}}$

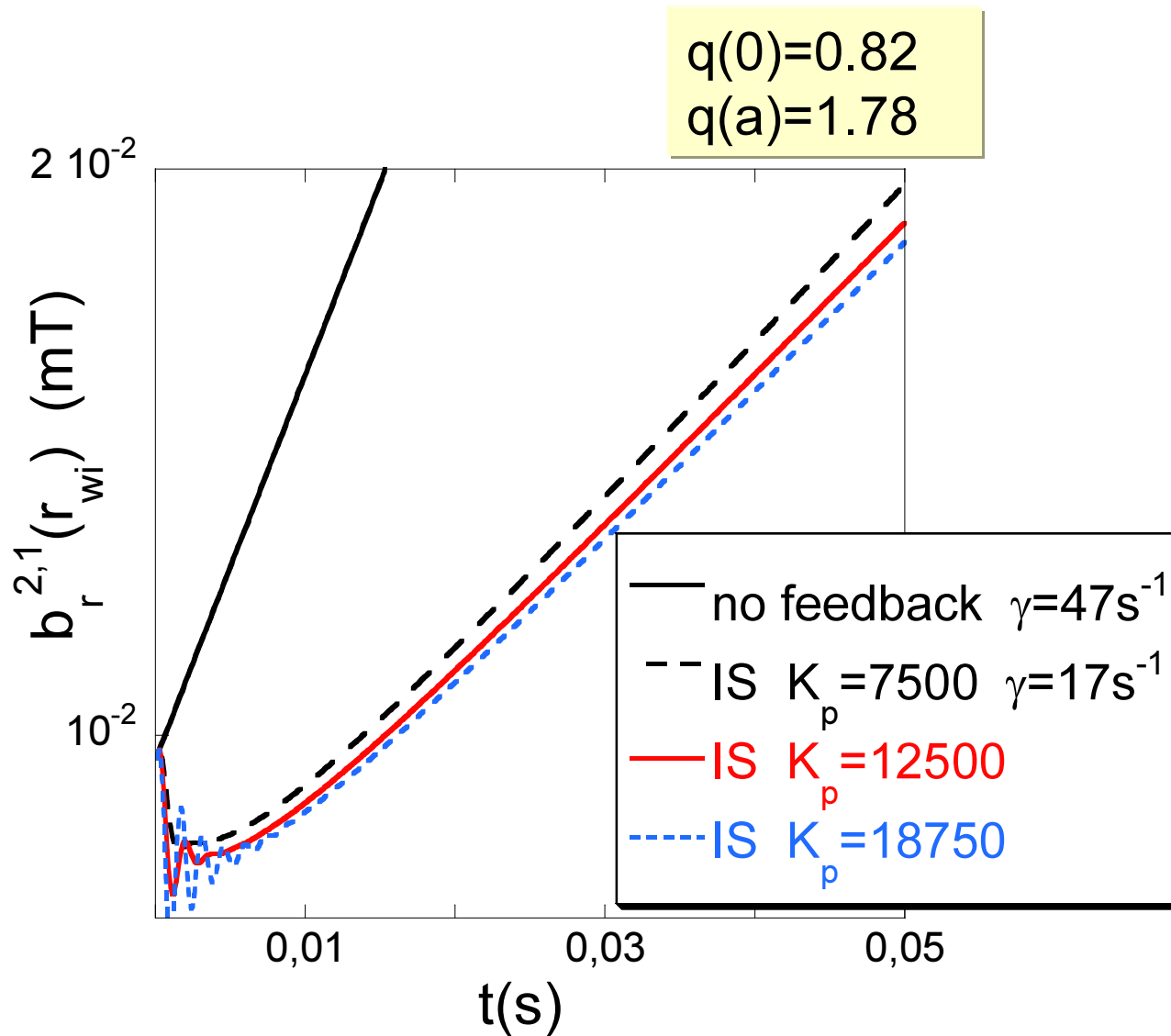
$$b_f = i b_{\theta}^{m,n}(r_{wi}) - i [b_{\theta}^{m,n}(r_{wi})]_{\text{vac}}$$

Equilibrium example

$$\mu_0 \mathbf{J}_0 = \mu(r) \mathbf{B}_0$$



Intelligent Shell: m=2 n=1 RWM



$$b_f = b_{r,DFT}^{2,1}(r_{wi})$$

$$M_c = M_s = 4$$

$$N_c = N_s = 48$$

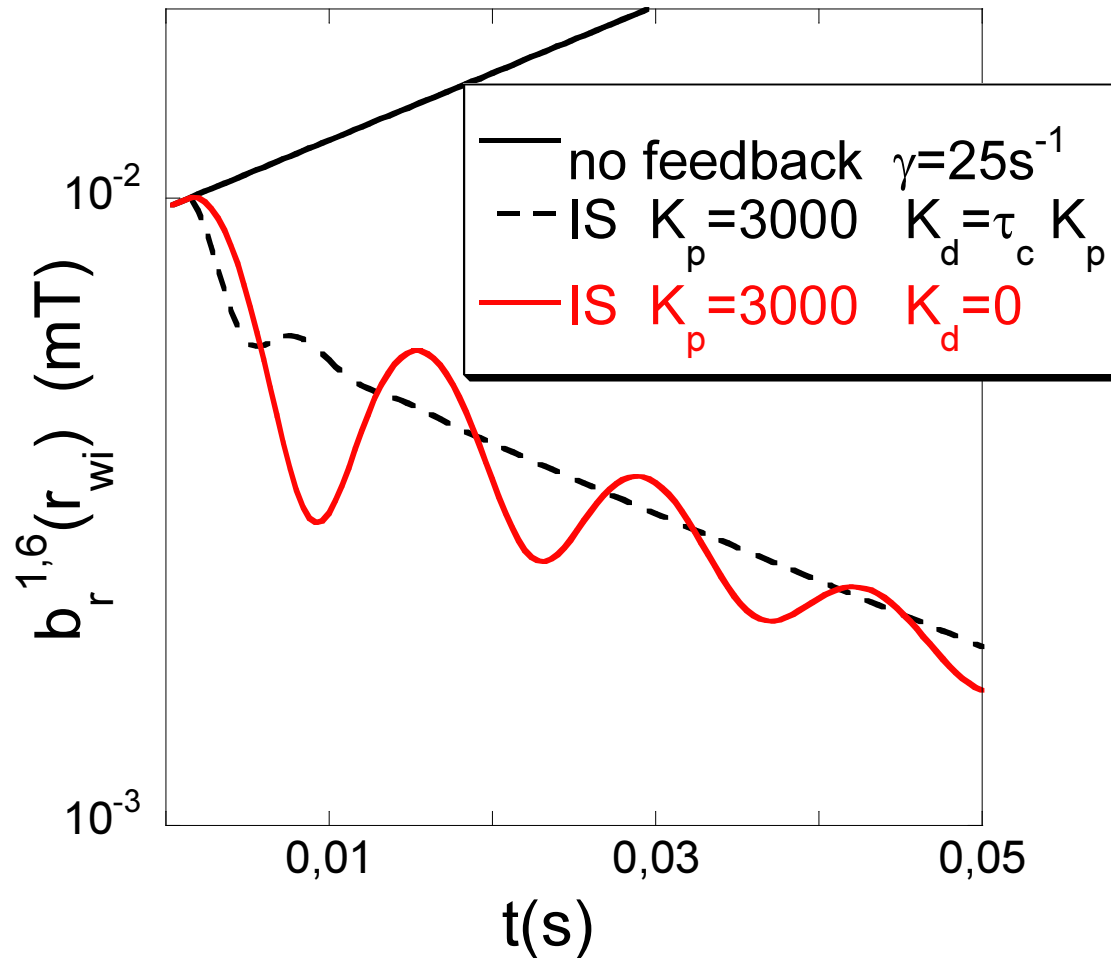
$$\Delta\theta_c = \Delta\theta_s = 2\pi/M_s$$

$$\Delta\phi_c = \Delta\phi_s = 2\pi/N_s$$

$$T_f = 0.1\text{ms}, \Delta t = T_f$$

$$K_d = T_c K_p$$

Intelligent Shell: m=1 n=6 RWM



$$b_f = b_{r,DFT}^{1,6}(r_{wi})$$

$$M_c = M_s = 4$$

$$N_c = N_s = 48$$

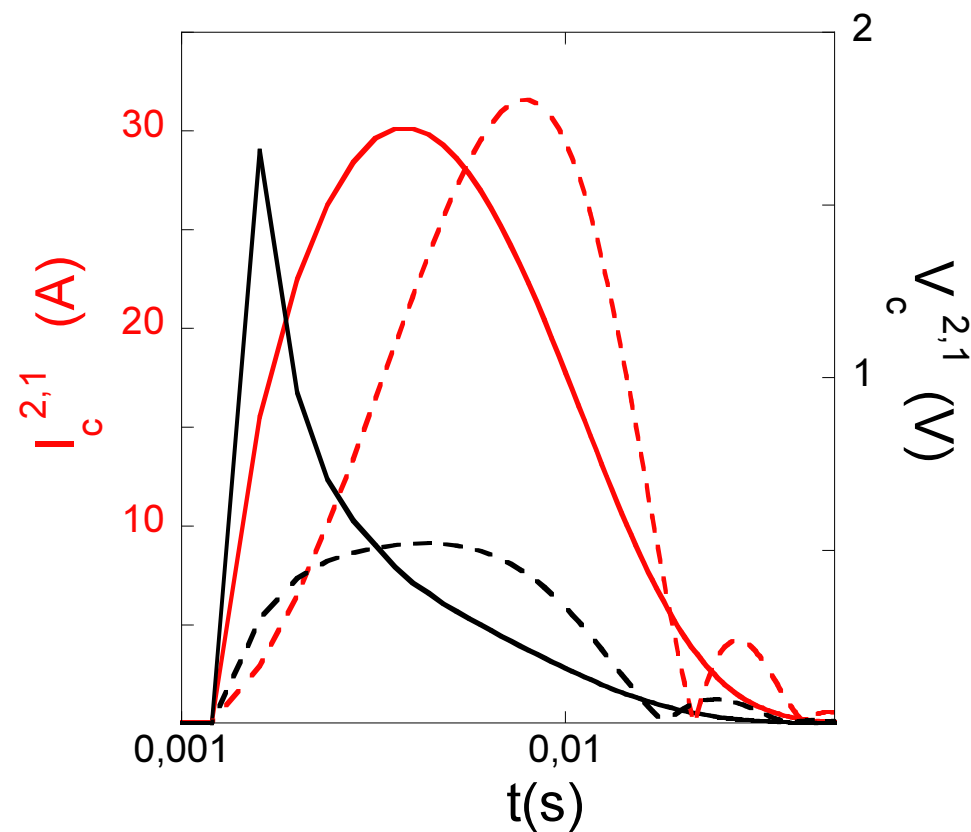
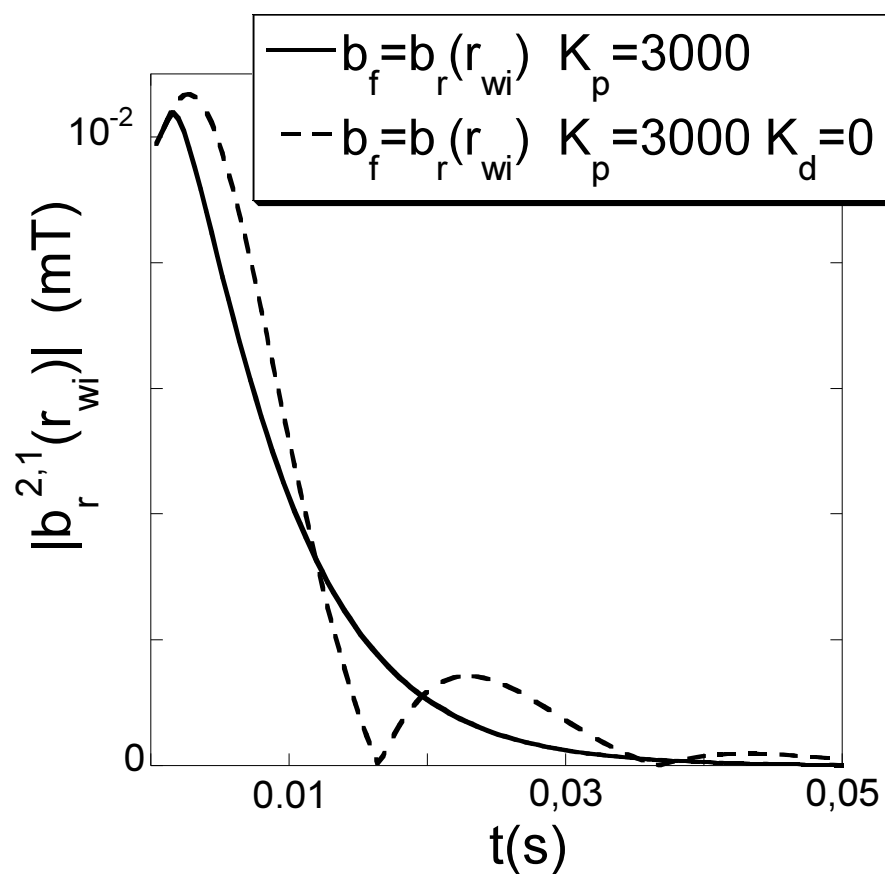
$$\Delta\theta_c = \Delta\theta_s = 2\pi/M_s$$

$$\Delta\phi_c = \Delta\phi_s = 2\pi/N_s$$

$$T_f = 0.4\text{ms}, \quad \Delta t = 2T_f$$

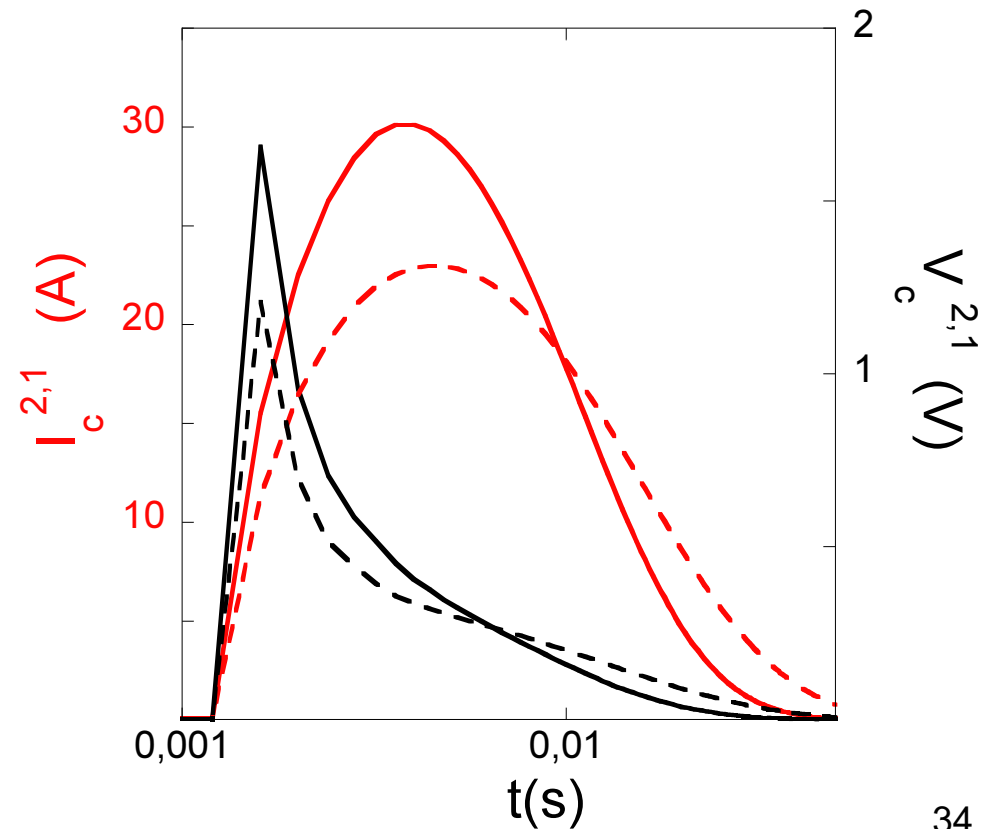
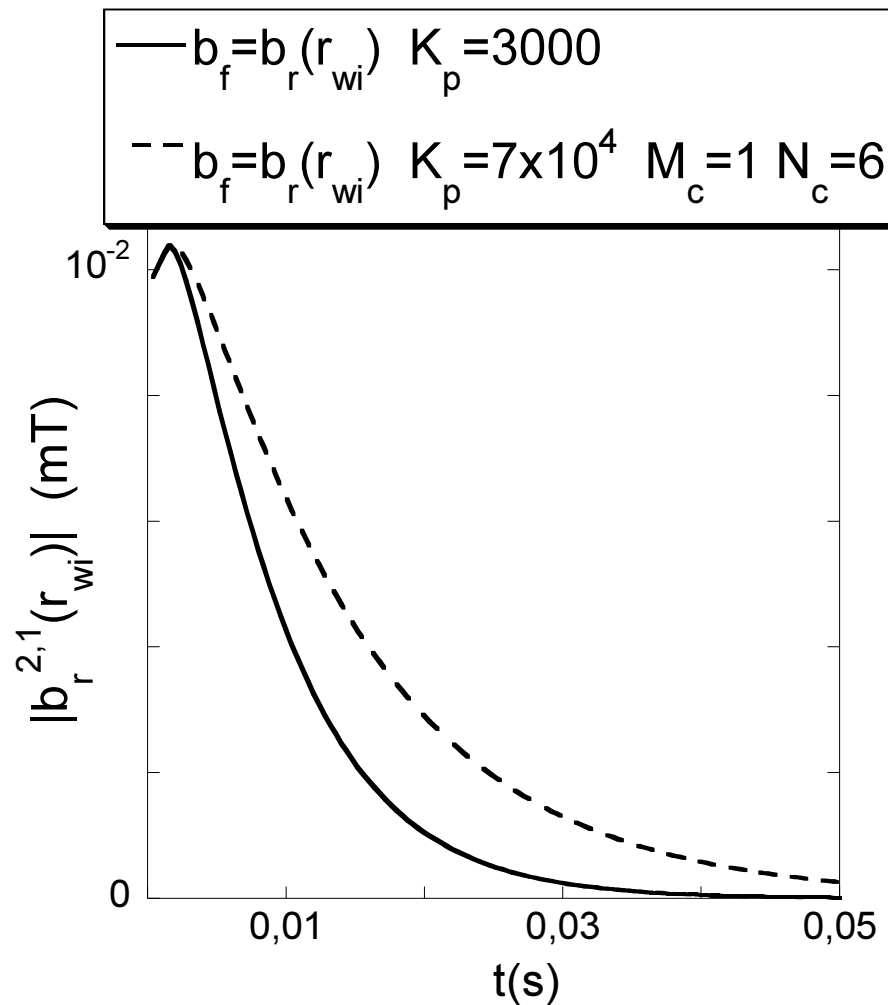
CMC simulations: RFX-mod case

$$T_f = 0.4 \text{ ms}, \quad \Delta t = 2T_f, \quad K_d = T_c K_p$$

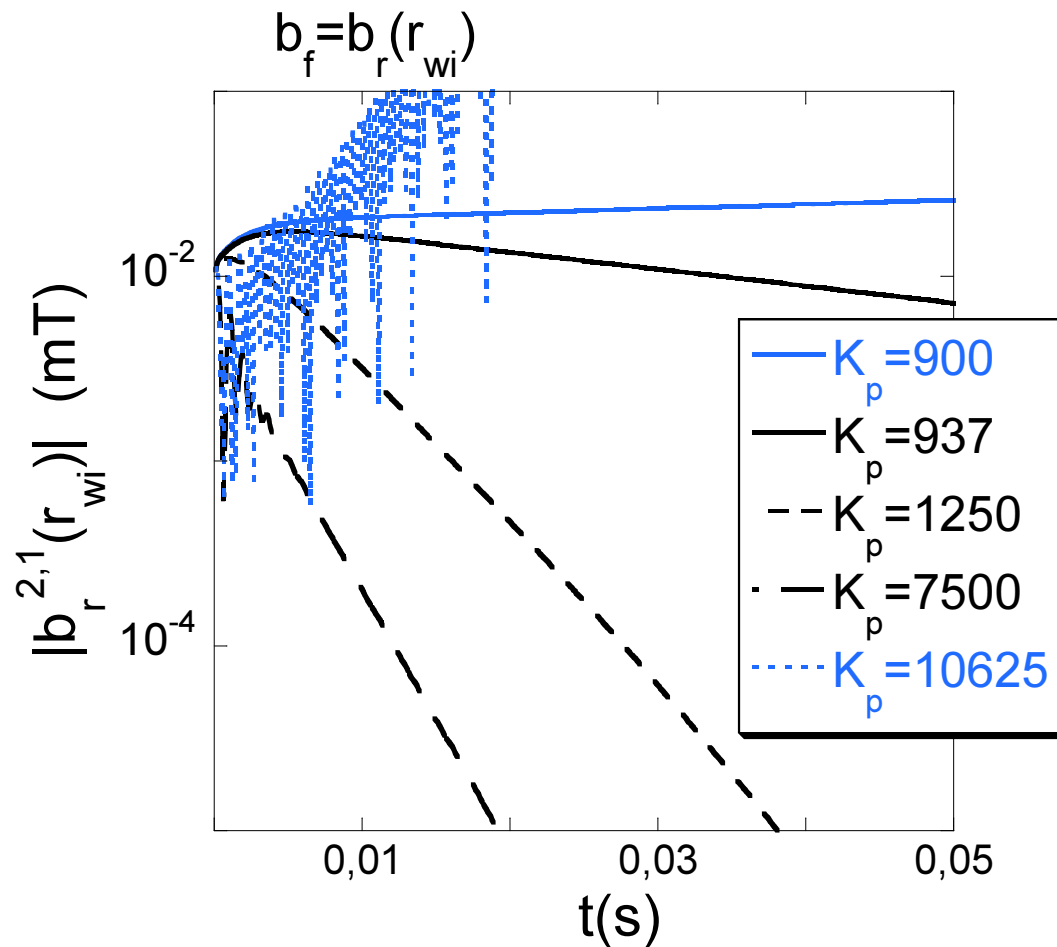


CMC simulations: RFX-mod case

Coils downgrade: $M_c=1$, $N_c=6$



CMC simulations: $\tau_w=3\text{ms}$



RFX-mod upgrade:

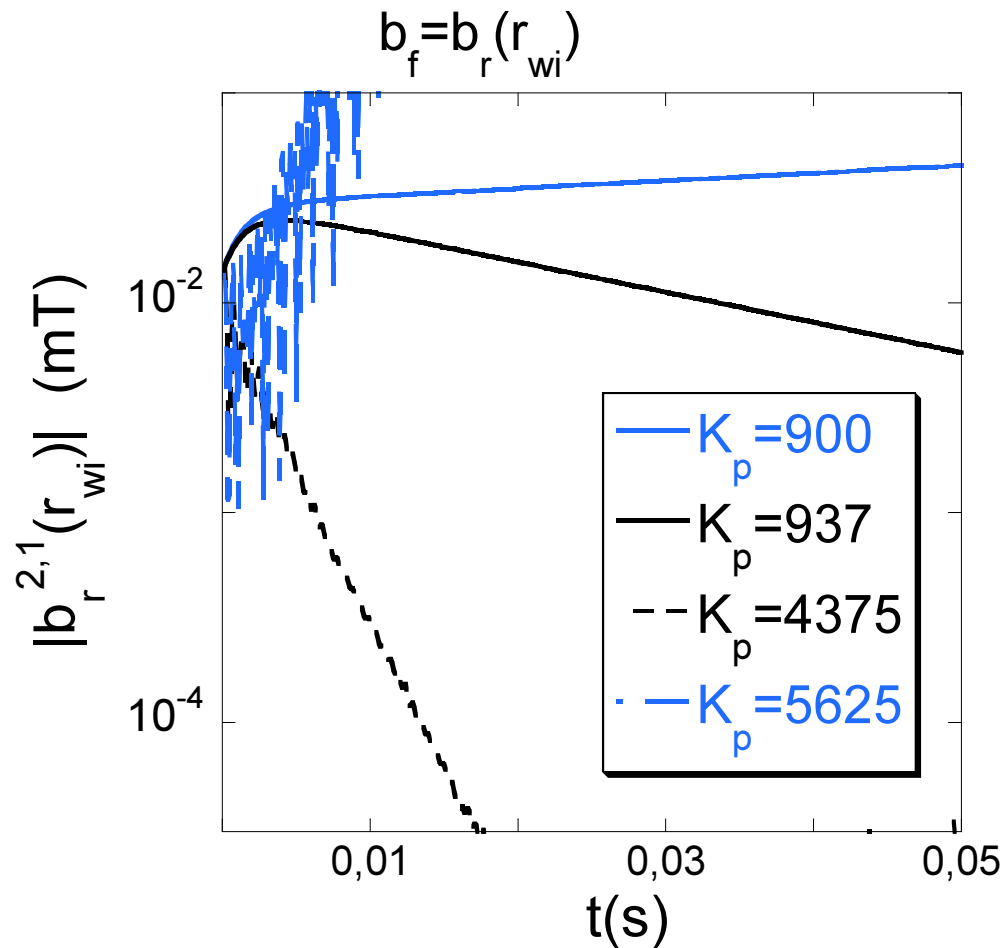
$$T_f = 0.1 \text{ ms}, \quad \Delta t = T_f$$

$$K_d = T_c K_p$$

$$K_{p,\text{max}} / K_{p,\text{min}} \sim 8$$

CMC simulations: $\tau_w=0$

$$\gamma \sim 47 \text{s}^{-1} \rightarrow \gamma \sim 672 \text{s}^{-1} \rightarrow \gamma \sim 1300 \text{s}^{-1}$$



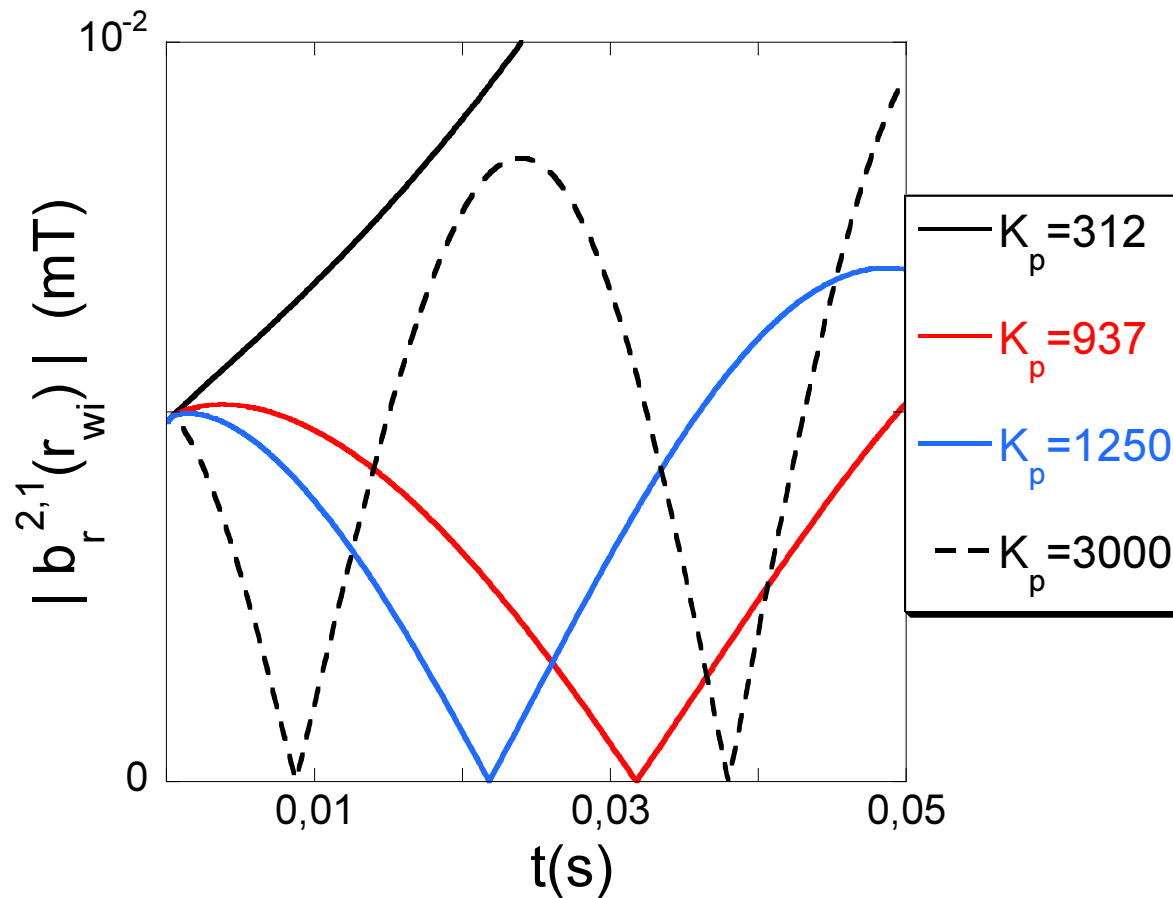
$$T_f = 0.1 \text{ms}, \quad \Delta t = T_f$$

$$K_d = T_c K_p$$

$$K_{p,\max} / K_{p,\min} \sim 4.7$$

Explicit mode control $\tau_w=100\text{ms}$

$$b_f = b_r^{2,1}(r_{wi}) - [b_r^{2,1}(r_{wi})]_{vac}$$

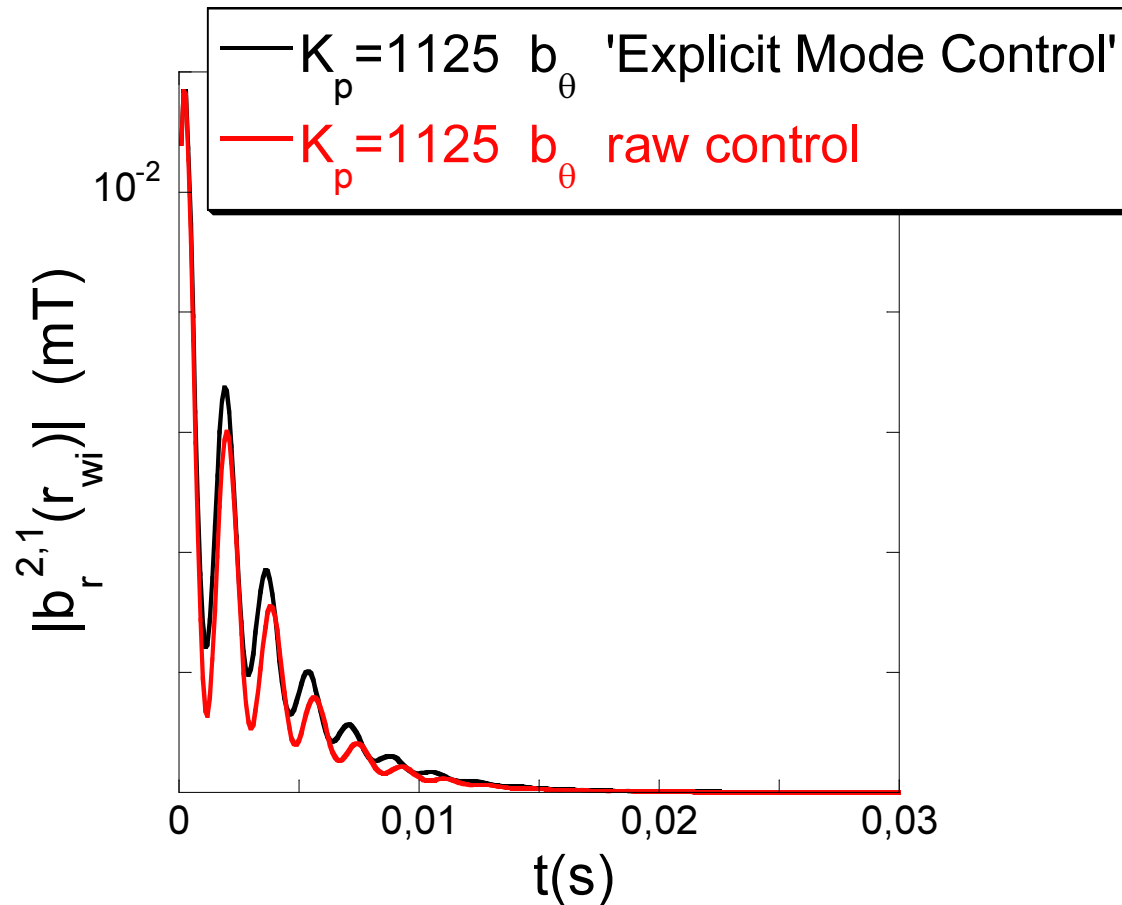


$$T_f = 0.1\text{ms}, \quad \Delta t = T_f$$

$$K_d = T_c K_p$$

Not stabilized

b_θ : 'Explicit Mode Control' vs raw control



$$M_c=M_s=4$$

$$N_c=N_s=48$$

$$\tau_w=3\text{ms}$$

$$T_f=0.1\text{ms}, \quad \Delta t=T_f$$

$$K_d=T_c K_p$$