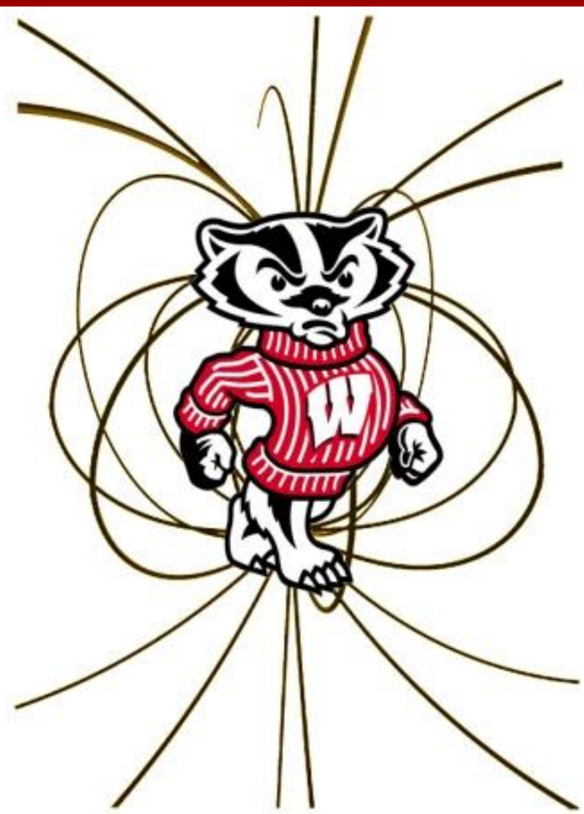
Vertical Field Penetration and Mode-Locking of Global Modes with a Rotating Conducting Wall



Carlos Paz-Soldan

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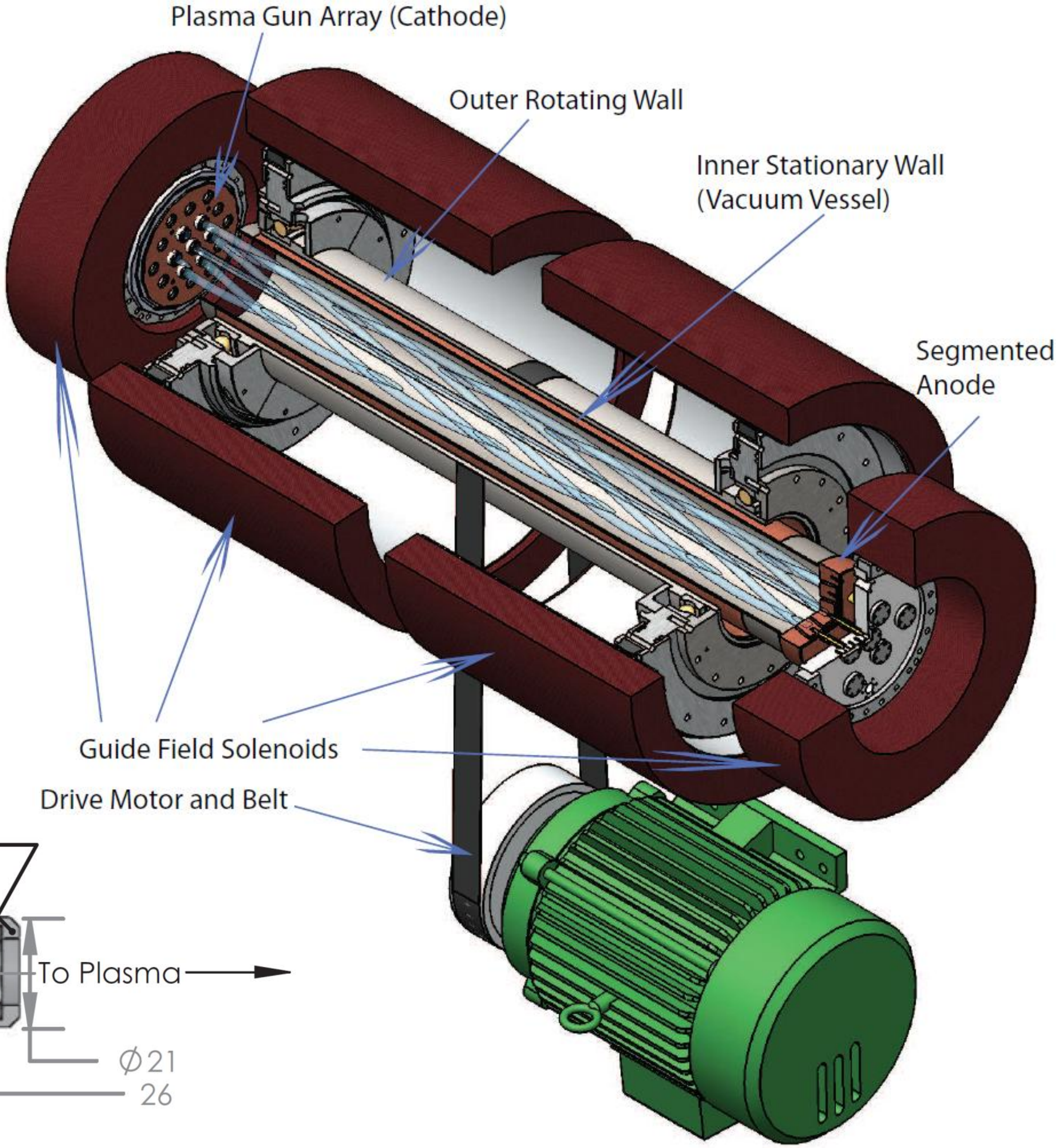
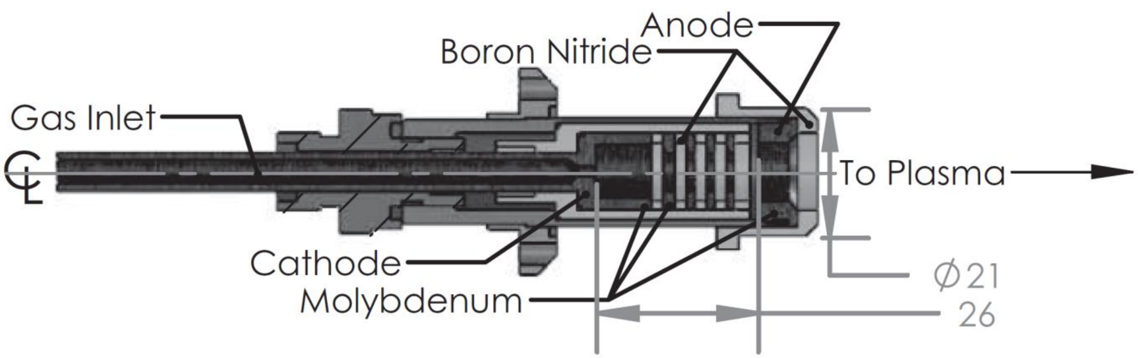
University of Wisconsin-Madison

MHD Workshop Talk, Nov 21st 2011

Experiment overview

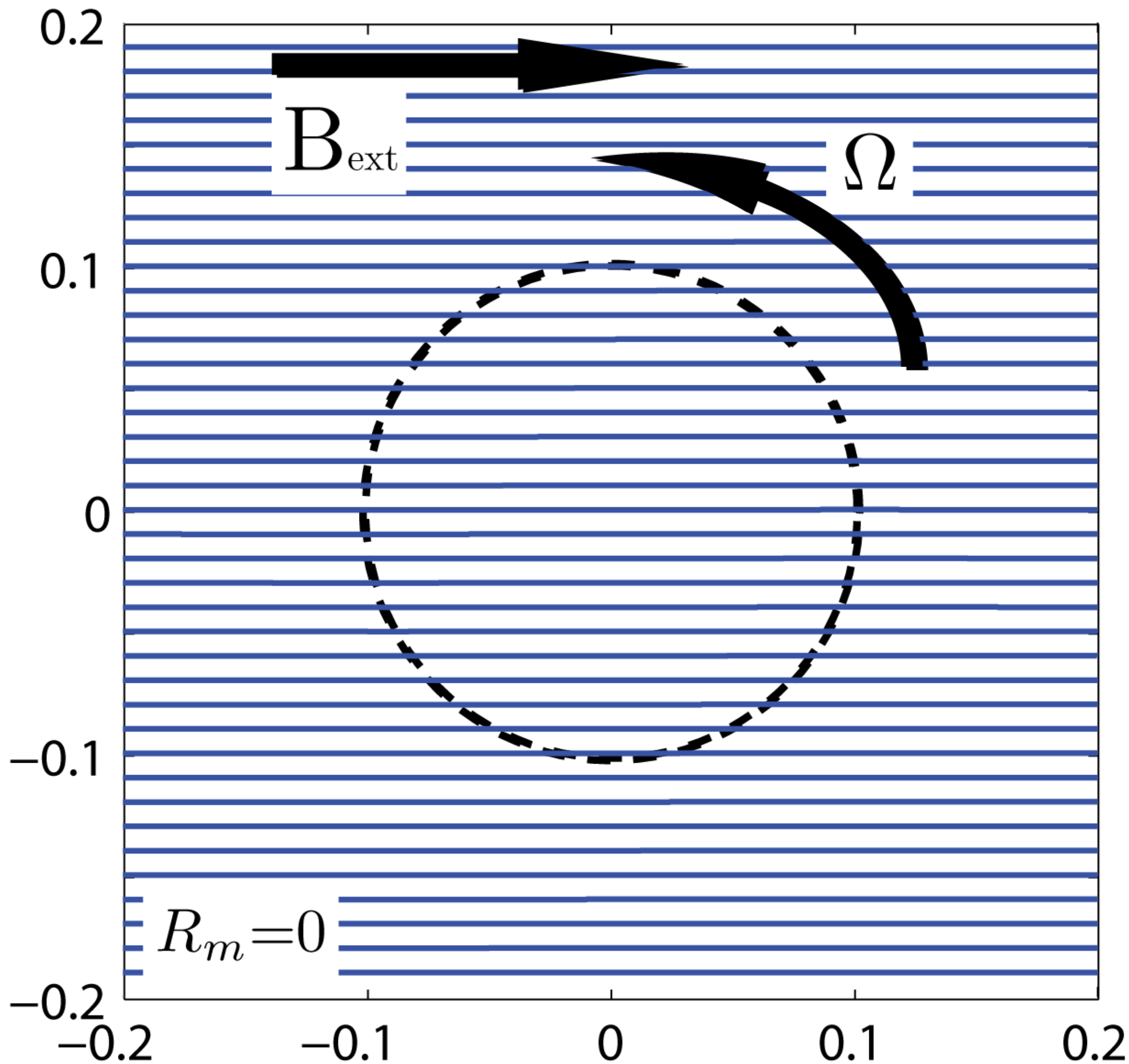
$$B_z \approx 0.1 \text{ T}$$
$$I_p \approx 7 \text{ kA}$$
$$L = 1.2 \text{ m}$$
$$R = 8 \text{ cm}$$
$$t \approx 20 \text{ ms}$$
$$\tau_w = 7 \text{ ms}$$
$$\tau_A \approx 2\ \mu\text{s}$$
$$T_e \approx 3.5 \text{ eV}$$
$$n_e \approx 10^{20} \text{ m}^{-3}$$



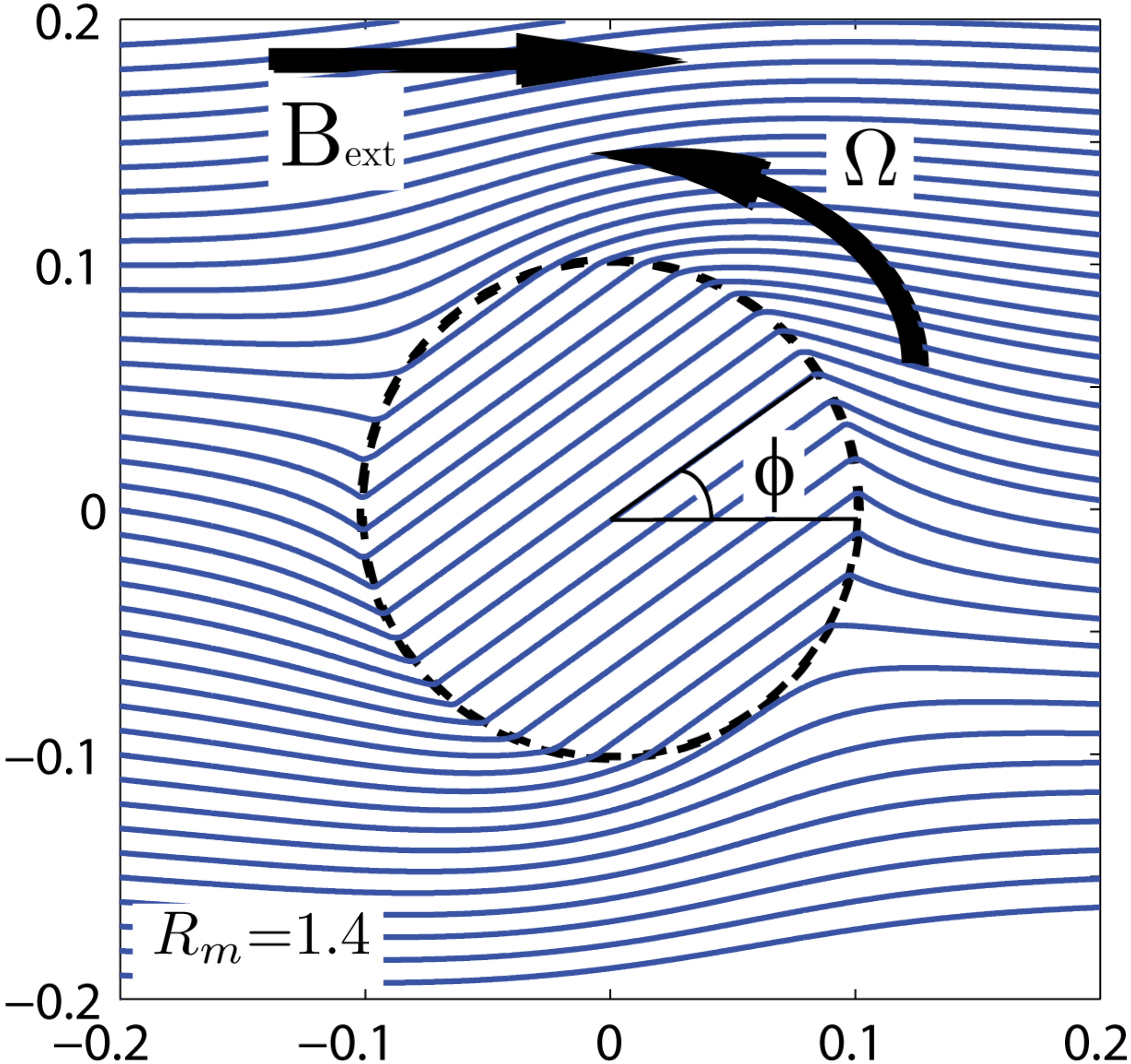


Paz-Soldan et al, Rev. Sci. Instrum. (2010)

Consider vertical field penetration



Consider vertical field penetration



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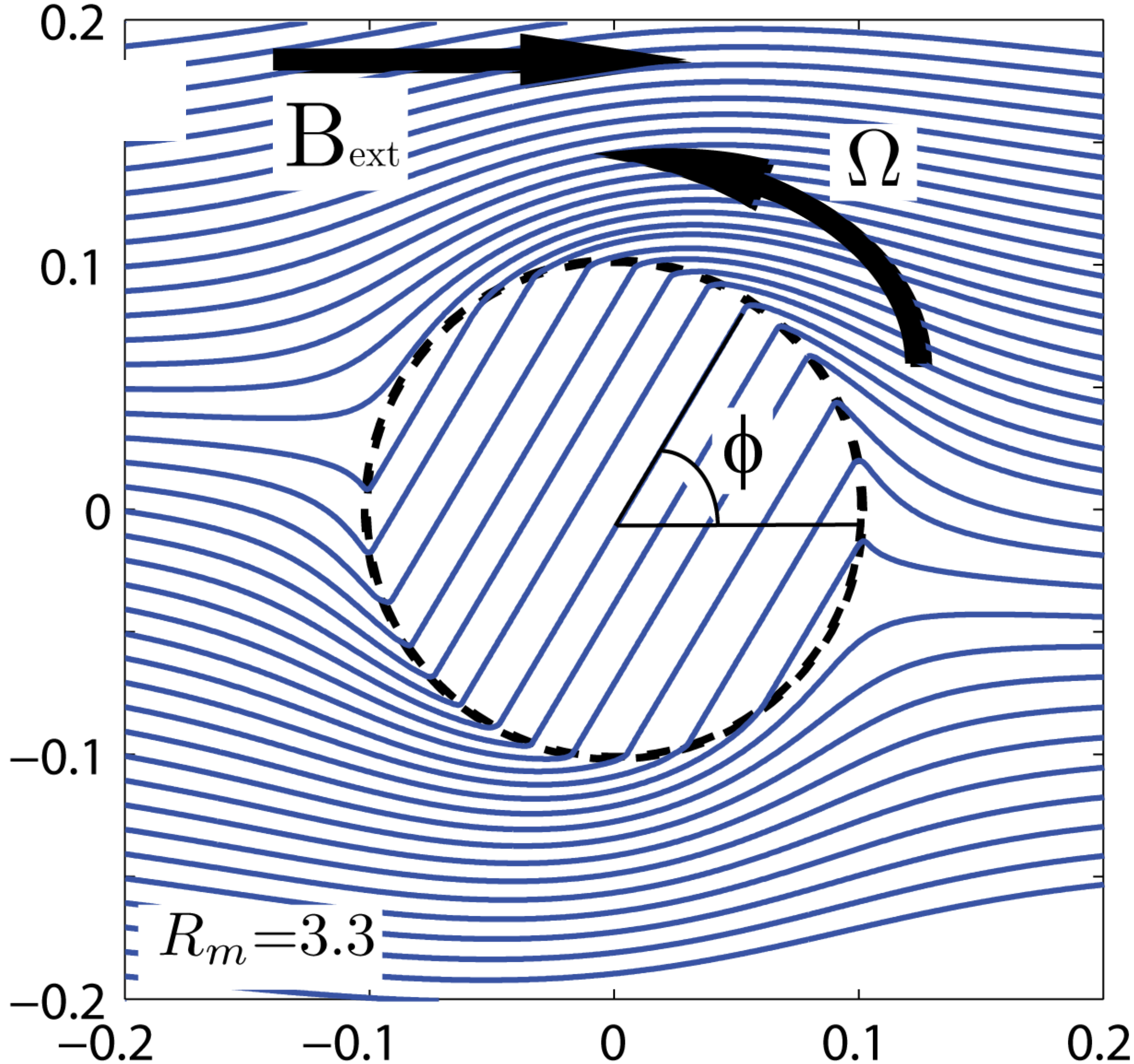
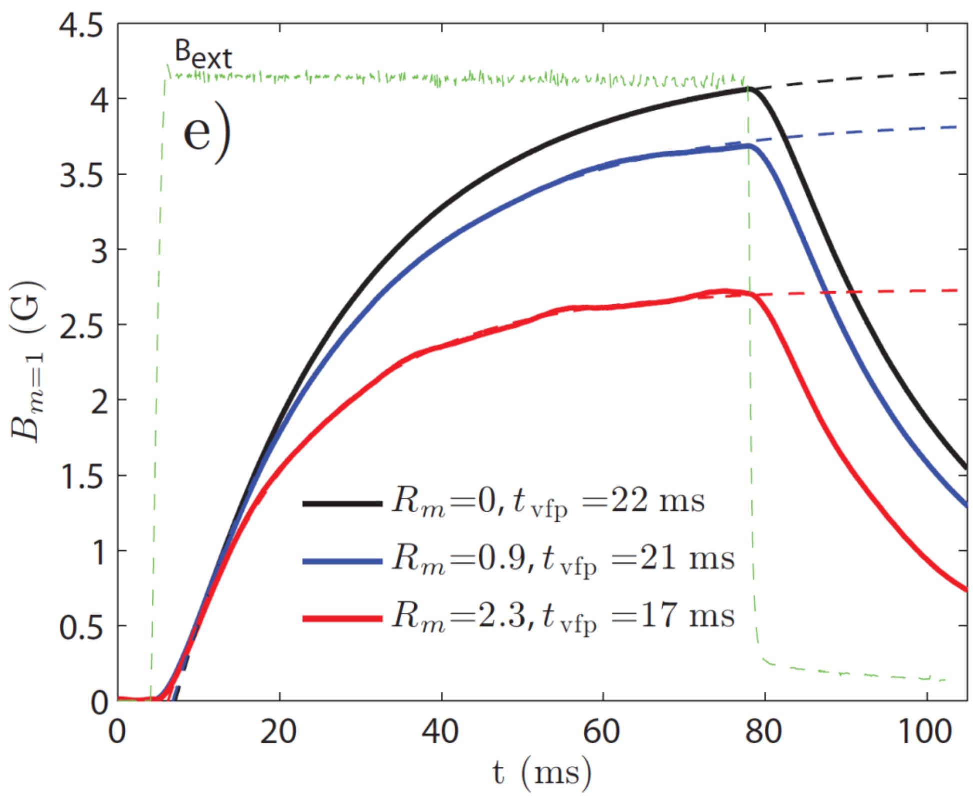
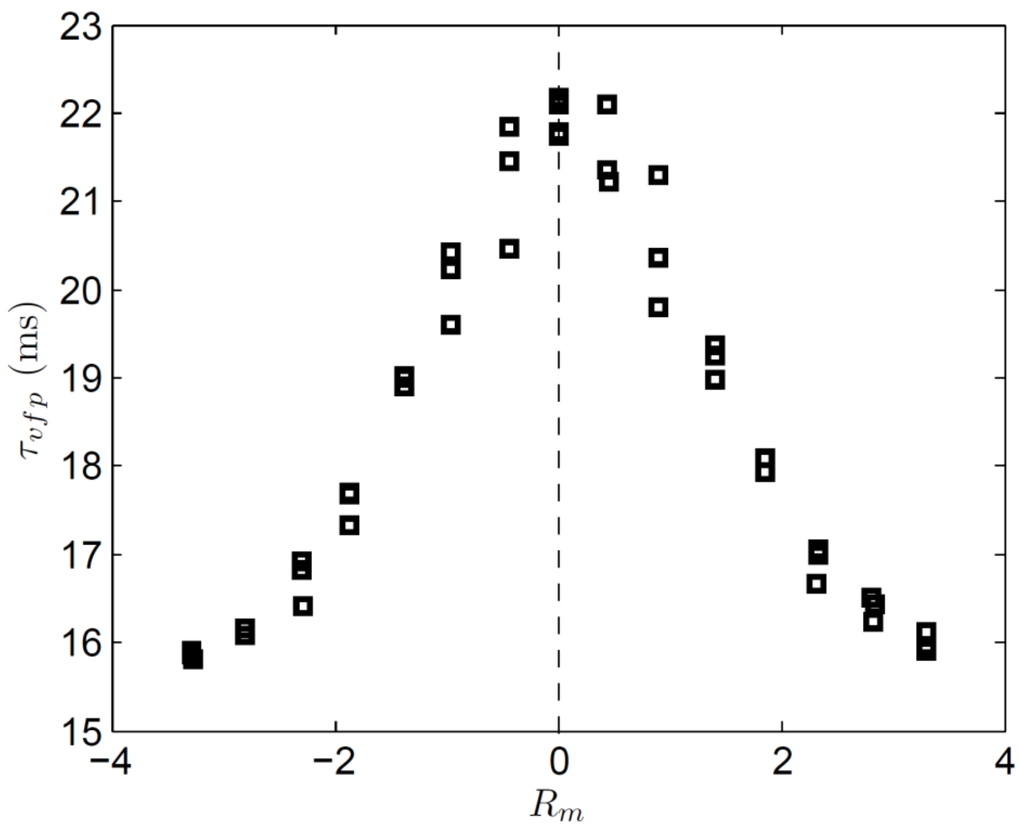


Penetration time *decreases* as wall rotates

- Error field reaches a *smaller* value *faster*
- This is a consequence of *differential* rotation





Model for thin-wall calculations:

- Vacuum field structure is (long cylinder approximation)

$$B_r(r,\theta) = \Re\left[(A_0 - A_1 r^{-2})e^{-i\theta}\right]$$

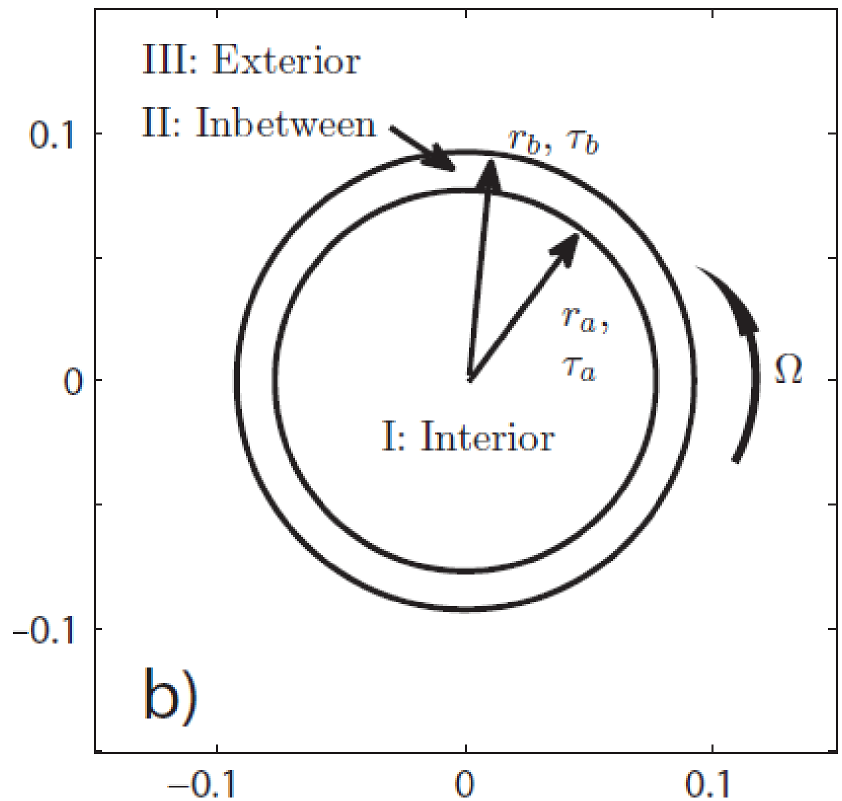
$$B_\theta(r,\theta) = \Re\left[(-i(A_0 + A_1 r^{-2}))e^{-i\theta}\right]$$

- Thin-wall matching:

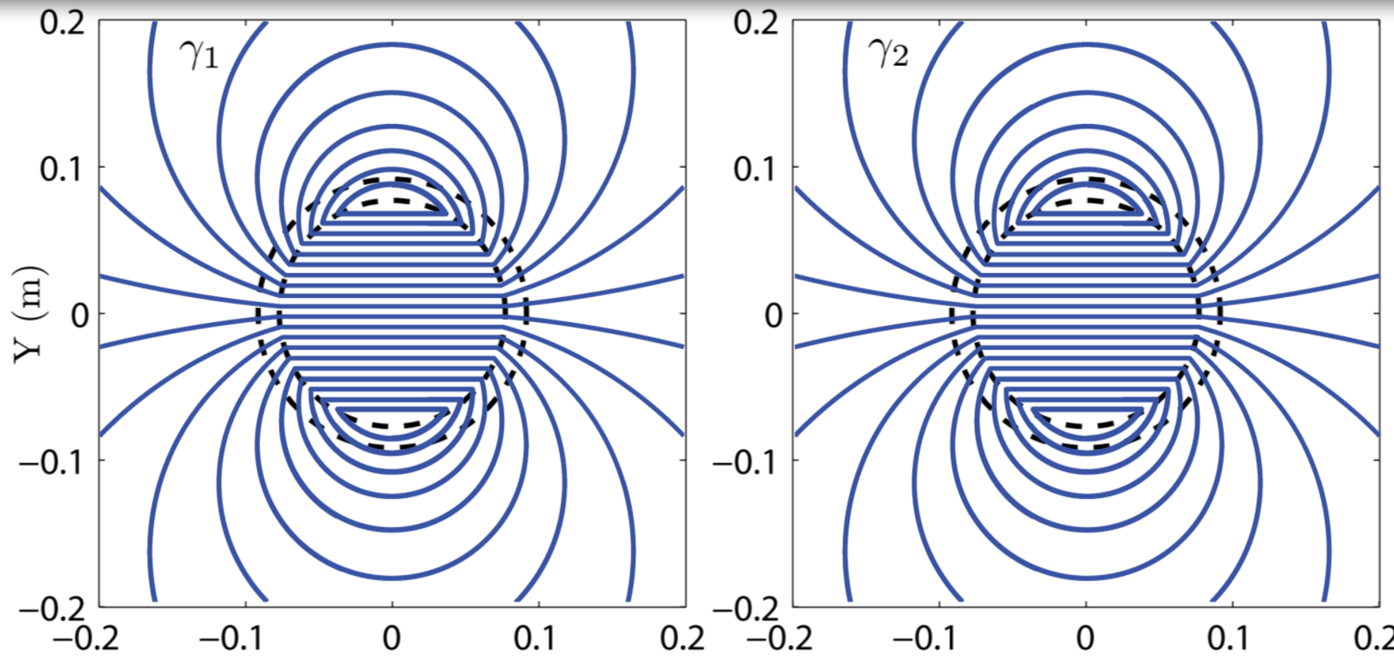
$$B_r\Big|_{r=r_w^-}^{r=r_w^+} = 0$$

$$B_\theta\Big|_{r=r_w^-}^{r=r_w^+} = i(\gamma + i\Omega)\tau_w B_r$$

- Matrix for unknown coefficients generated
  - Non-trivial solutions are the Eigenmodes

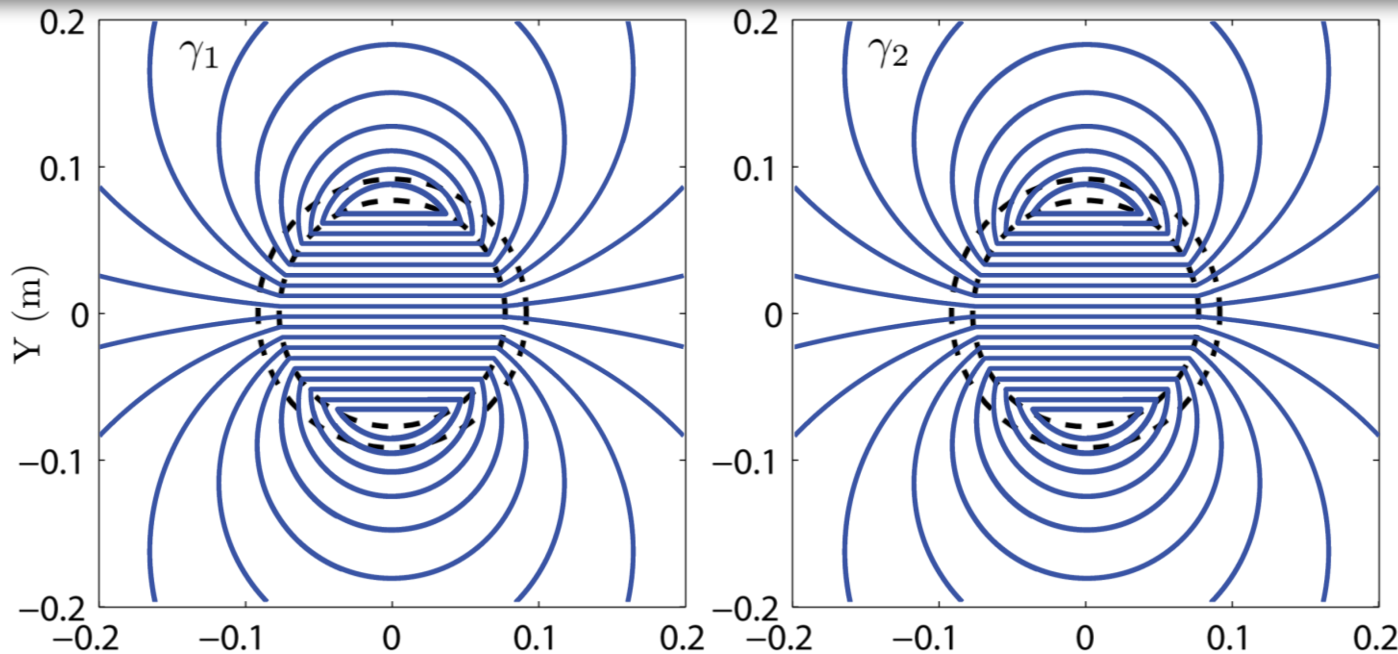


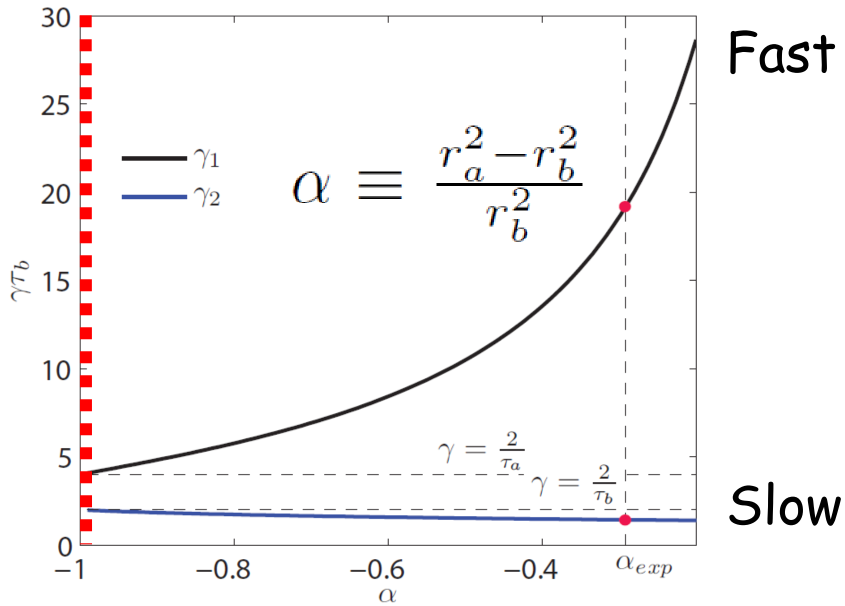
Consider single wall m=1 Eigenmodes



$$\gamma = \frac{2}{\tau_a} - i\Omega$$

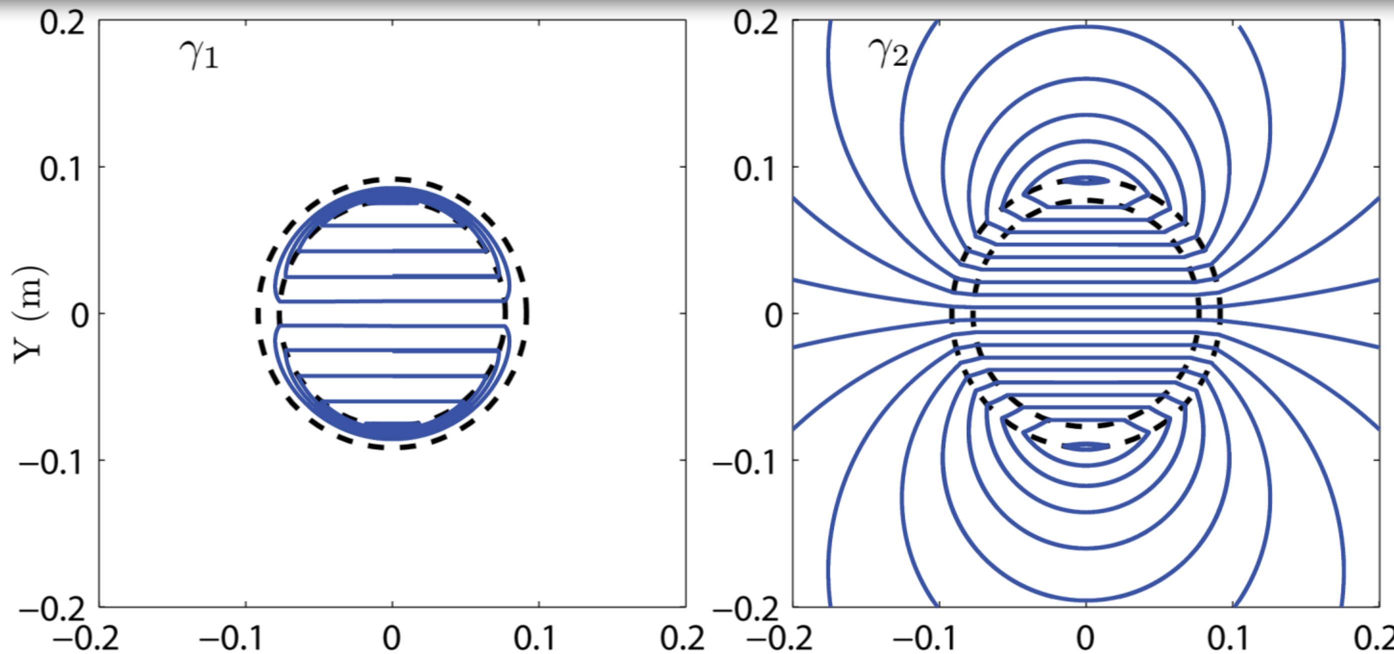
Introduce coupling from second wall

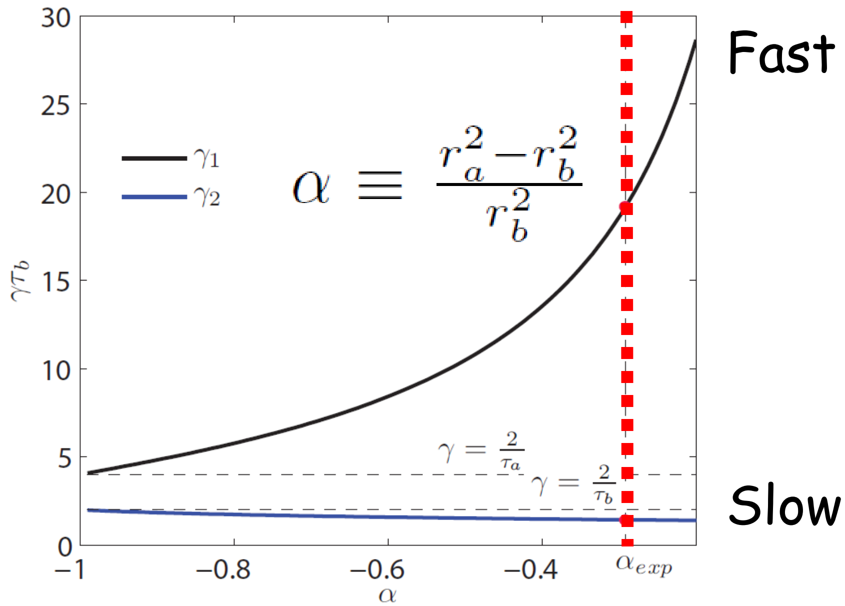




$$\gamma = -\frac{1}{\tau_a \tau_b \alpha}\left[\tau_a + \tau_b \pm \left[(\tau_a + \tau_b)^2 + 4\alpha\tau_a\tau_b\right]^{\frac{1}{2}}\right]$$

Introduce coupling from second wall





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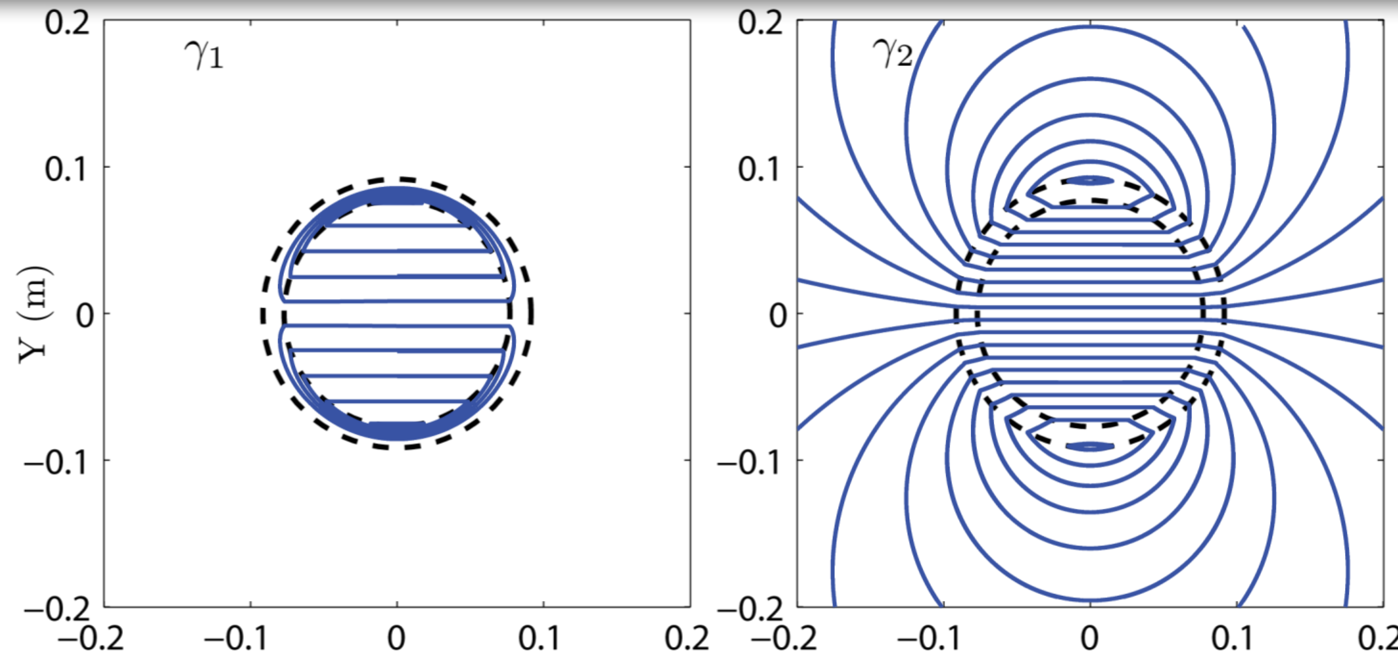
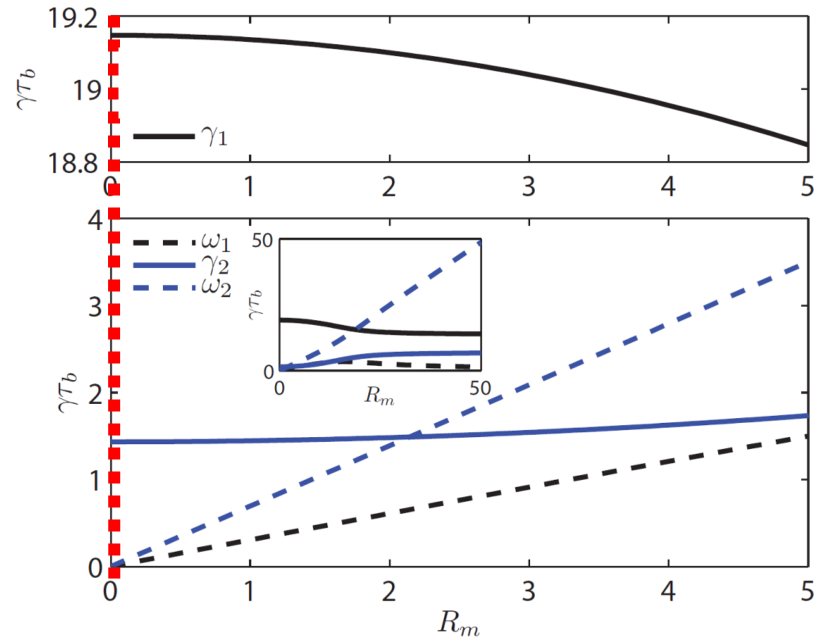
Introduce differential wall rotation





Fast

Slow

$$\gamma = -\frac{\tau_a + \tau_b}{\tau_a \tau_b \alpha} - i\frac{\Omega}{2} \pm \frac{1}{\tau_a \tau_b \alpha}\left[\left[(\tau_a+\tau_b)^2 + 4\alpha\tau_a\tau_b - \frac{\Omega^2}{4}(\tau_a\tau_b\alpha)^2\right] + i\Omega\tau_a\tau_b\alpha(\tau_a - \tau_b)\right]^{\frac{1}{2}}$$

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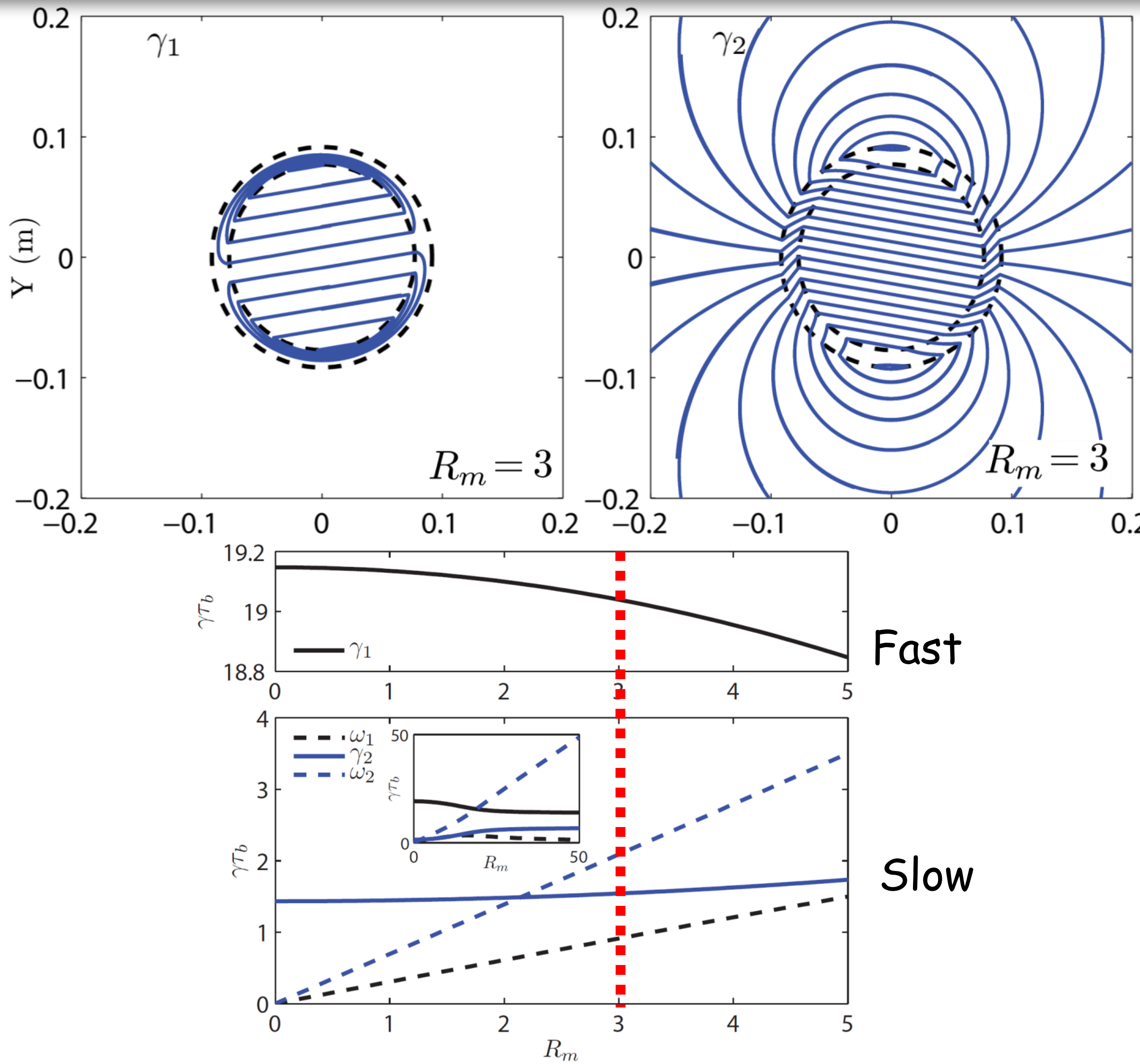

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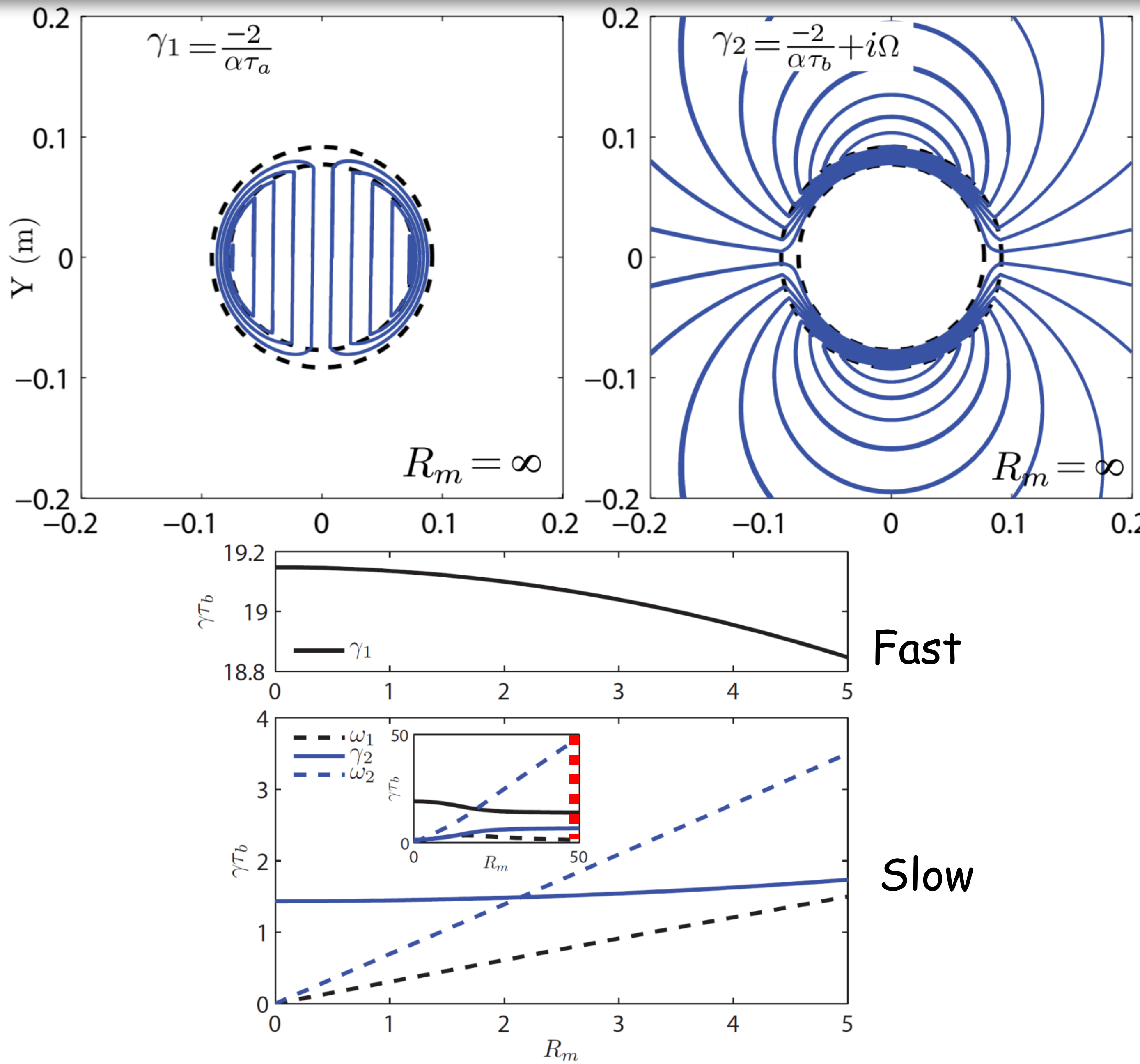

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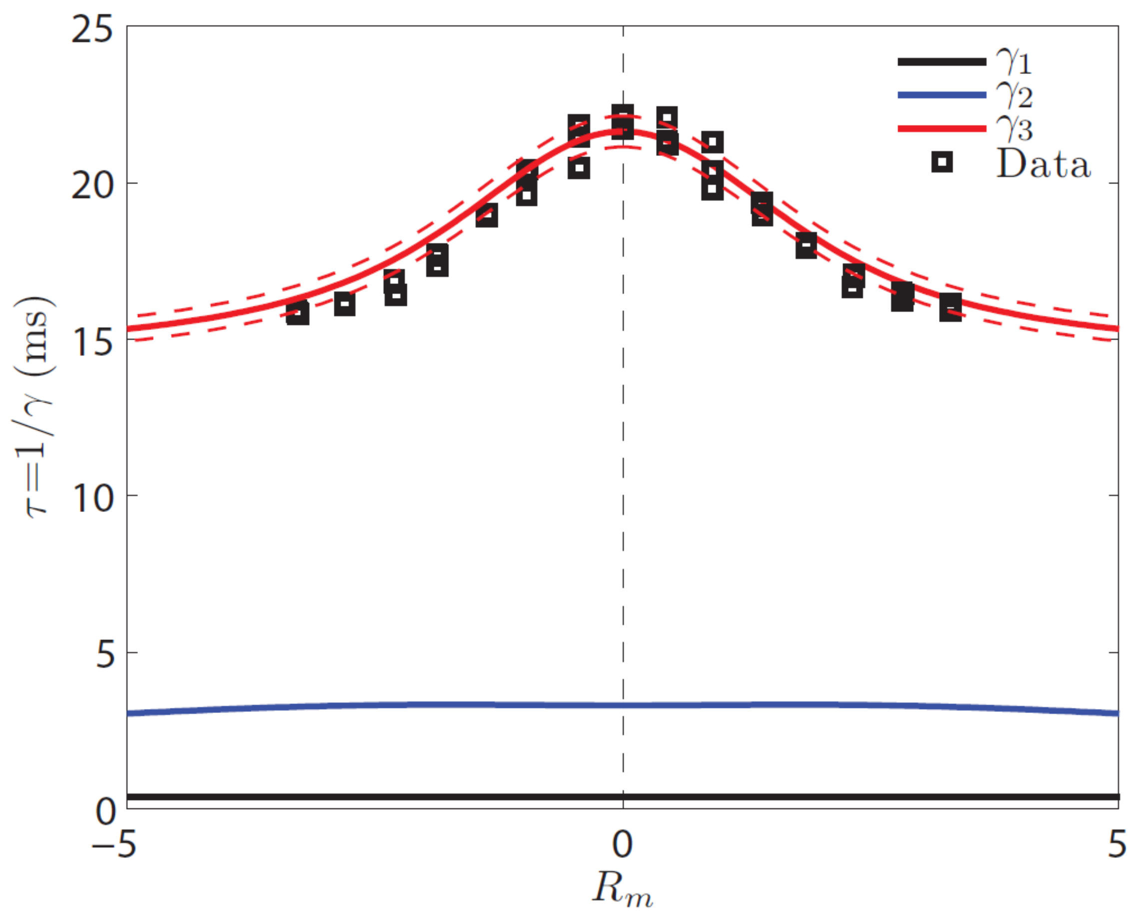
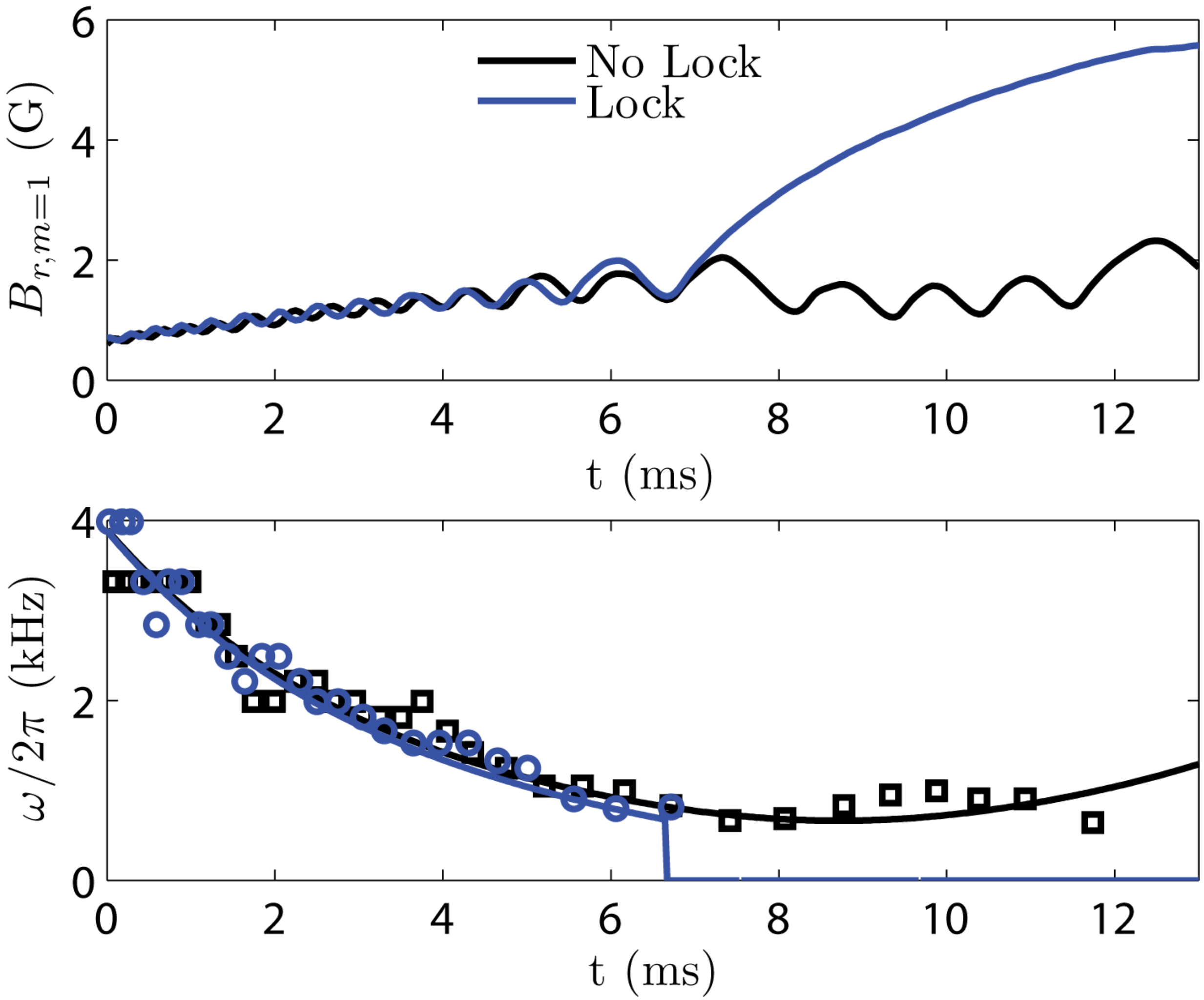
Speeding up of slow root is the observation

- Results of calculation consistent with data
  - Dotted lines are +/- 2% on wall time
  - No free parameters
- This would not be observed on single-wall system



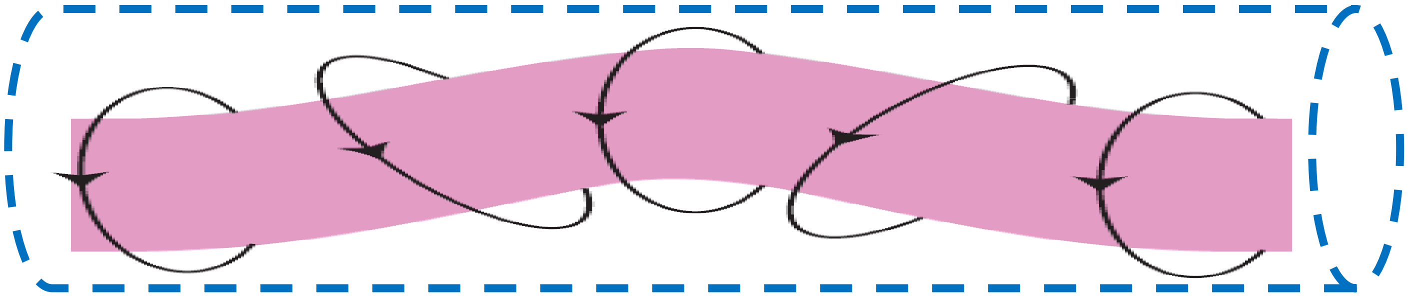
Mode locking observed in device

- We explore this phenomenon in greater detail



Simple torque balance model

$$\int_{\mathcal{V}} \vec{r} \times \left[\rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}\right)\right] d\mathcal{V} = \int_{\mathcal{V}} \vec{r} \times \left[-\nabla P - \nabla\cdot\bar{\bar{\Pi}} + \nabla\cdot\bar{\bar{T}}\right] d\mathcal{V}$$



Volume $\mathcal{V}$
Surface $\mathcal{S}$

Simple torque balance model

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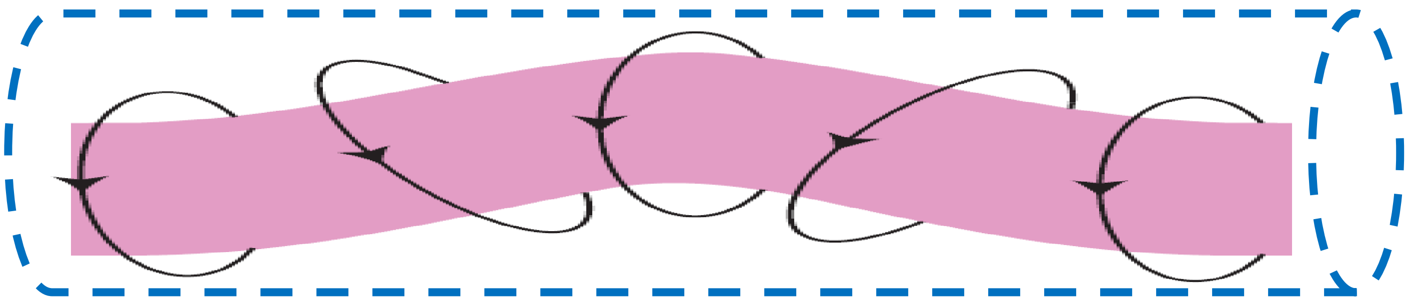

Volume $\mathcal{V}$
Surface $\mathcal{S}$

- Treat plasma as rigid rotor, such that $V_\theta = r\omega$
  - Viscosity is infinite, dissipation present, like a motor

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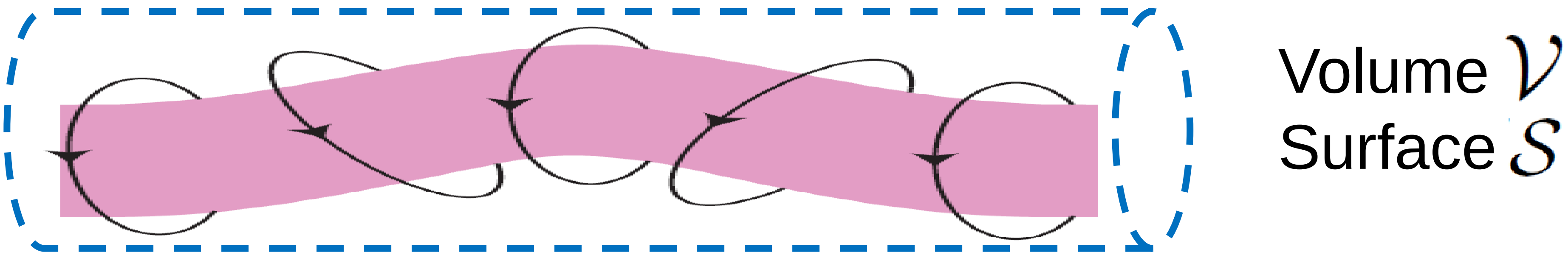

- Treat plasma as rigid rotor, such that $V_\theta = r\omega$
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- Torques act on whole plasma

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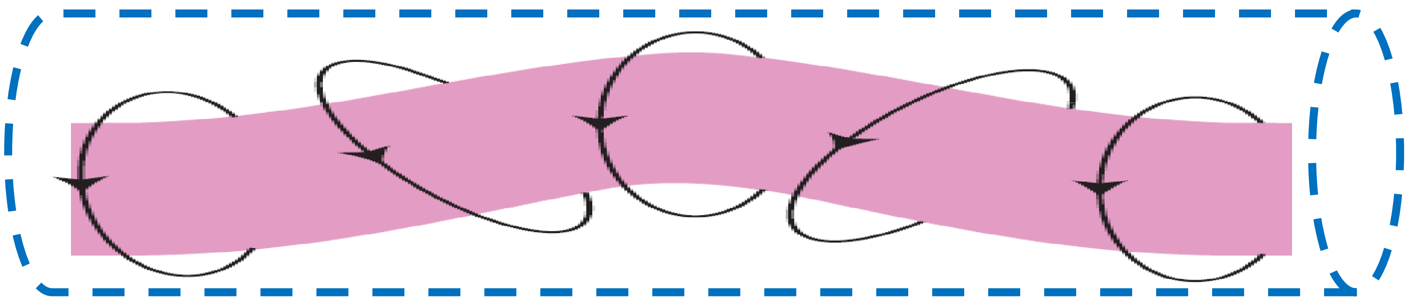

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Surface $\mathcal{S}$

- Treat plasma as rigid rotor, such that $V_\theta = r\omega$
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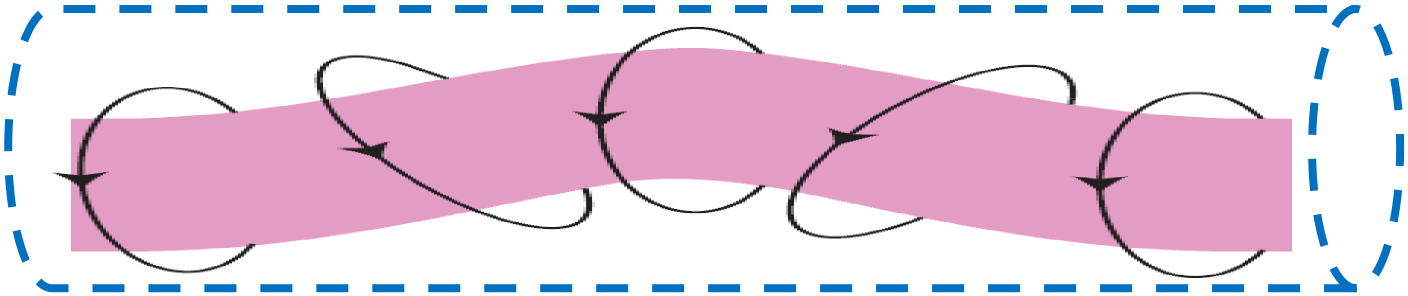
- Torques act on whole plasma
- Inertia is negligible

Simple torque balance model

$$0 = r_a \int_{\mathcal{S}} \hat{\theta} \cdot \frac{1}{\mu_0} \left[\vec{B}\vec{B} - \frac{1}{2} B^2 \bar{\bar{I}} \right] \cdot d\mathcal{S}$$



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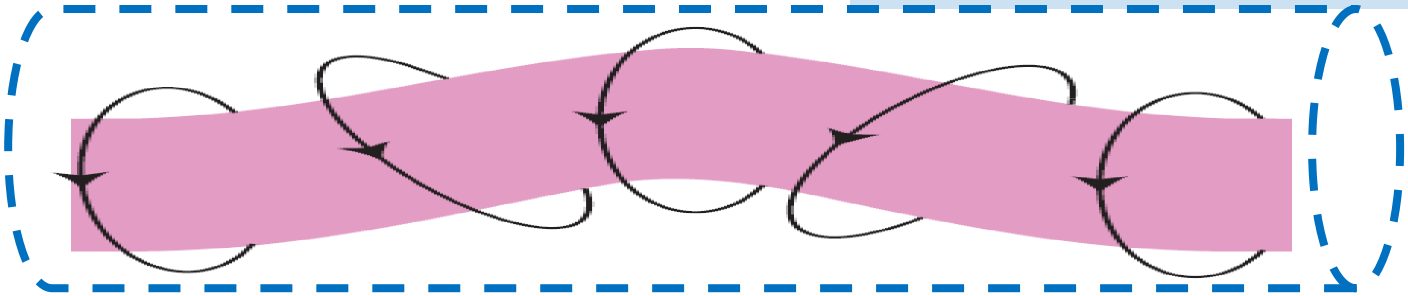
Surface $\mathcal{S}$

- Treat plasma as rigid rotor, such that $V_\theta = r\omega$
  - Viscosity is infinite, dissipation present, like a motor
- Torques act on whole plasma
- Inertia is negligible
- Only electromagnetic torques remain
  - Field is decomposed: $\vec{B} = B_{m=0} + B_{\rm ext} + \tilde{B} e^{-i\omega t}$

Simple torque balance model

$$I\dot{\omega} = 0 = \underbrace{A_{\mathrm{res}}(\Omega_0 - \omega)}_{\Gamma_{res}} - \underbrace{A_{EM}\frac{\omega}{1+(\omega\tau_w)^2}}_{\Gamma_{EM}} + \underbrace{A_{\mathrm{ext}}\sin(\theta)}_{\Gamma_{\mathrm{ext}}}$$



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EM Torque extended for wall rotation

- Perform a cycle-average, torque takes this form

$$\Gamma_{EM} = \frac{L r_a^2}{\mu_0} \int_0^{2\pi} \left\langle \tilde{B}_\theta \tilde{B}_r \right\rangle d\theta$$

- We know the field structure, experimental geometry

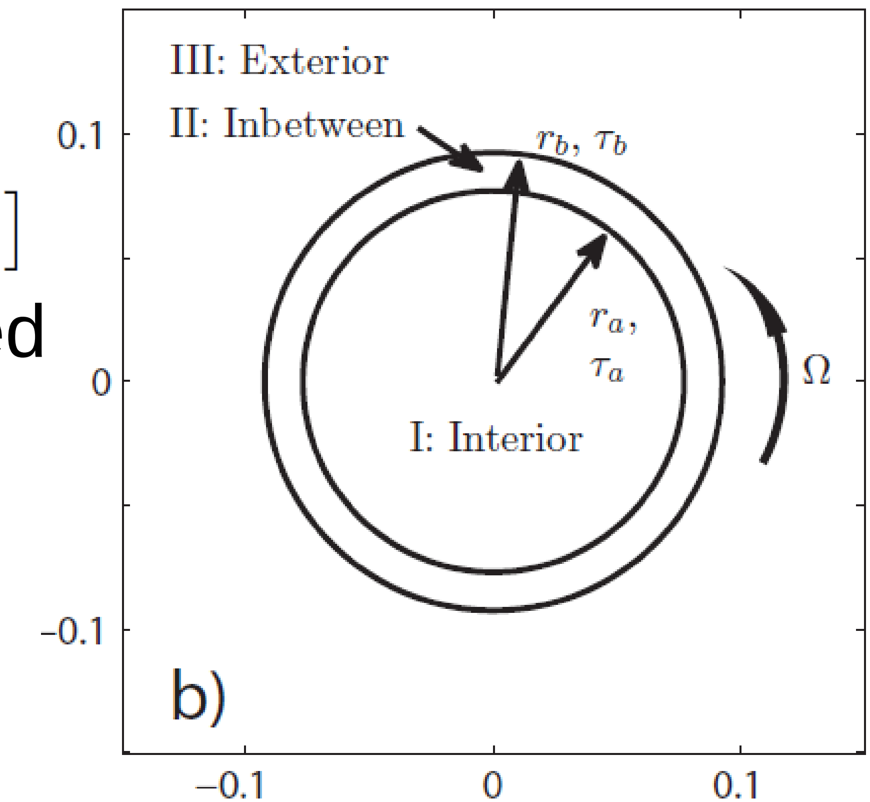
$$B_r(r,\theta) = \Re\left[(A_0 - A_1 r^{-2})e^{-i\theta}\right]$$

$$B_\theta(r,\theta) = \Re\left[(-i(A_0 + A_1 r^{-2}))e^{-i\theta}\right]$$

- Thin-wall matching again used

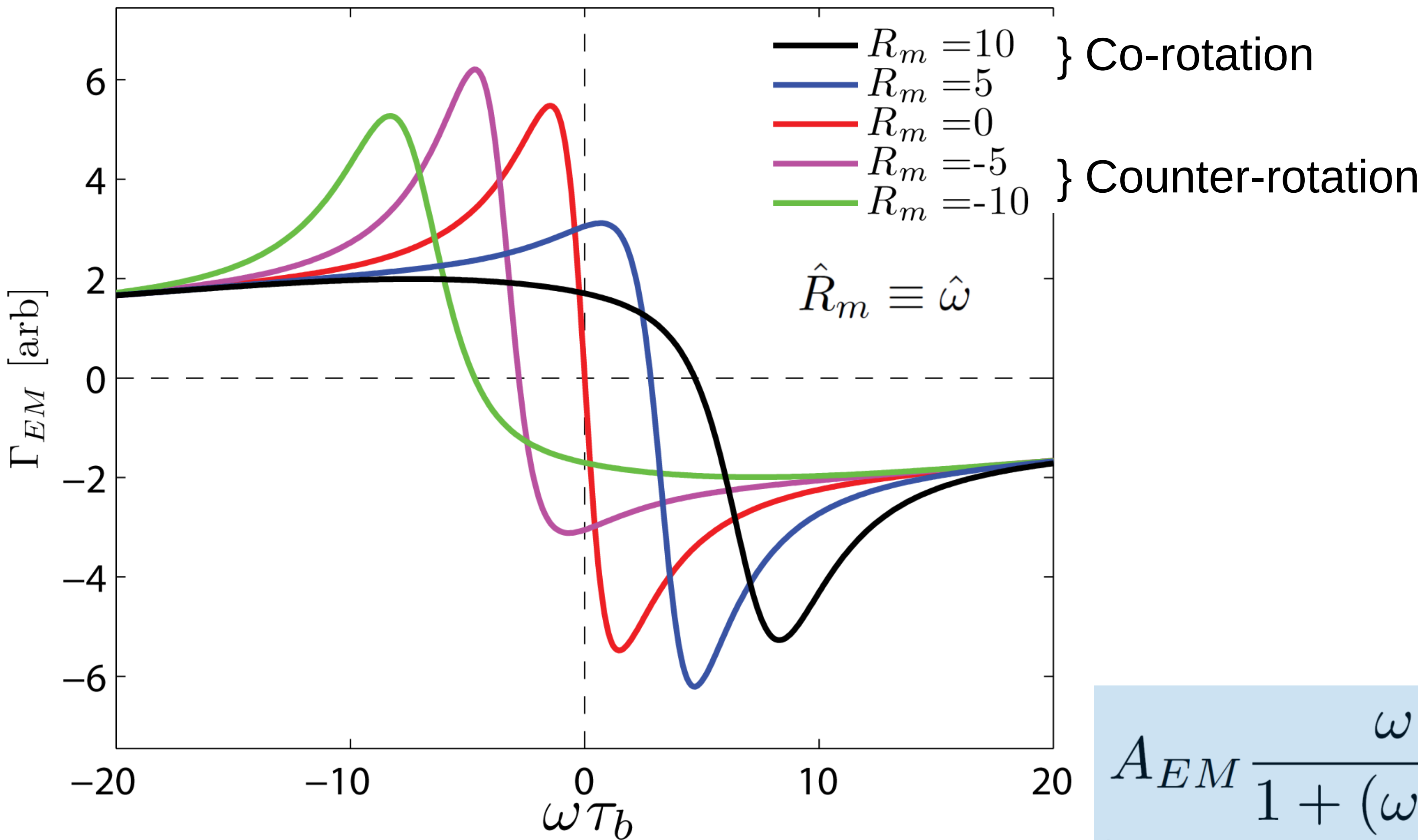
$$B_r\Big|_{r=r_w^-}^{r=r_w^+} = 0$$

$$B_\theta\Big|_{r=r_w^-}^{r=r_w^+} = i(\gamma + i\Omega)\tau_w B_r$$



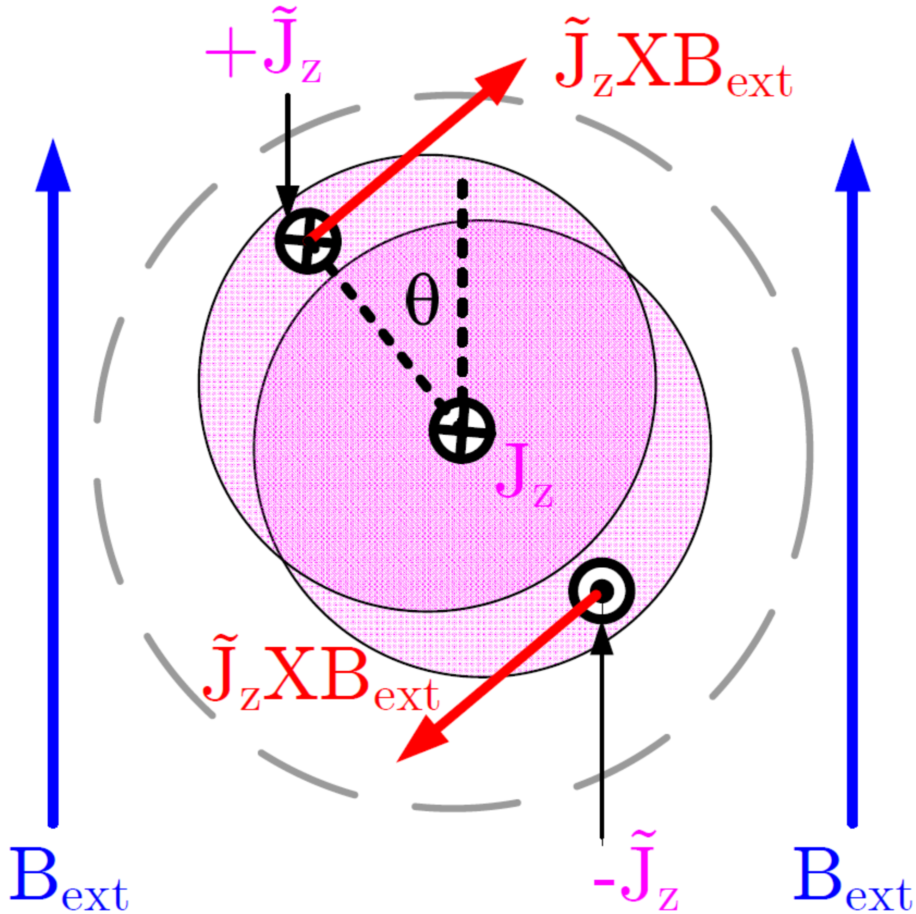
EM torque extended to treat differential rotation

- Curves are Doppler shifted and skewed by wall rotation



$$\underbrace{A_{EM} \frac{\omega}{1 + (\omega \tau_w)^2}}_{\Gamma_{EM}}$$

Error field torque depends on alignment

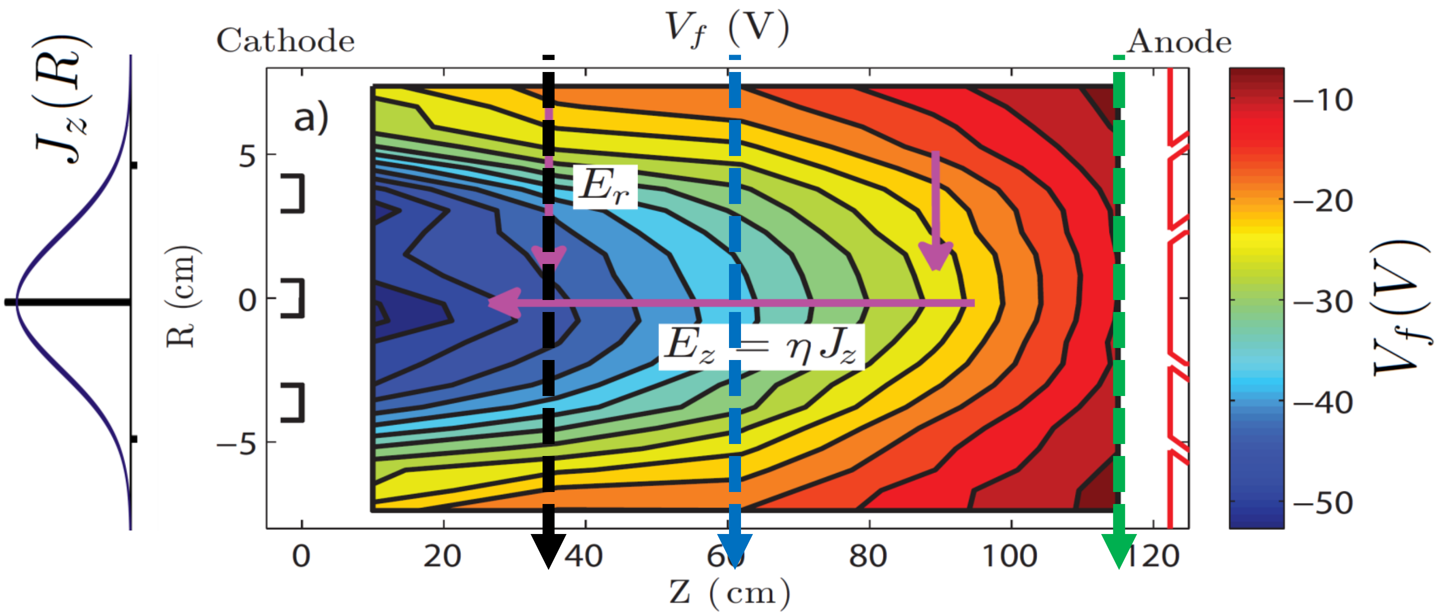


$$\Gamma_{ext} = 2\left(\frac{L\pi}{\mu_0}\right) B_{ext}\tilde{b}\sin\theta$$

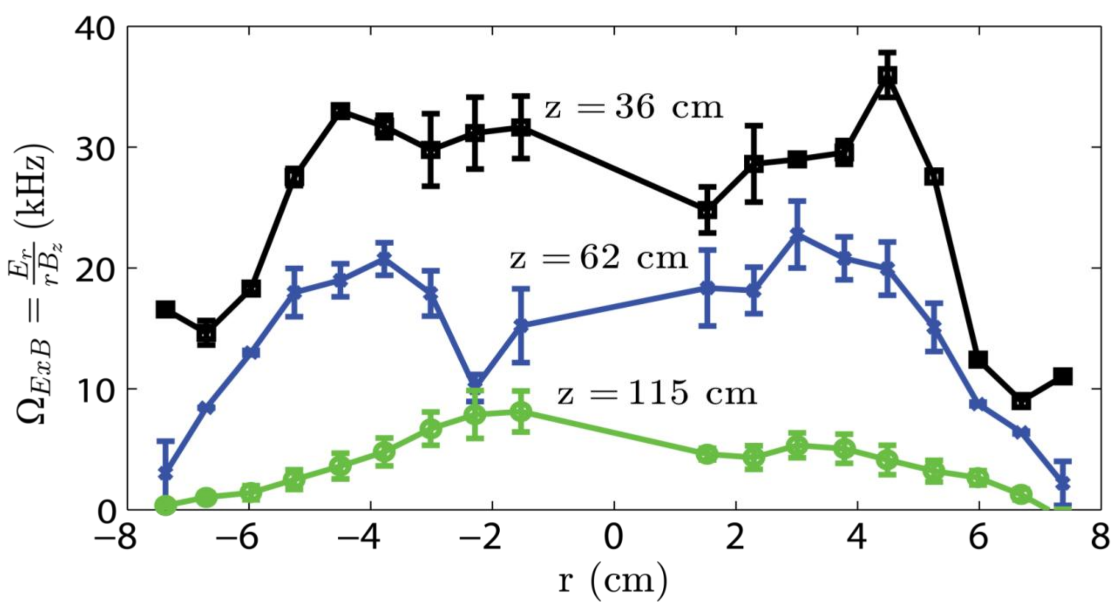
$$\Gamma_{ext} = \underbrace{A_{ext}\sin(\theta)}_{\Gamma_{ext}}$$

- We leave general the relationship between $\tilde{b}$ and B_{ext}
  - Both are measured

Offset torque from ExB rotation



• We measure a radial electric field in the plasma



Phenomenological restoring torque

- Consider consequence of: $\omega = \Omega_{\mathrm{ExB}} + \delta\omega$

$$E_r + V_\theta B_z = \eta_\perp J_r \qquad V_\theta = r\omega$$

$$J_r = \frac{rB_z}{\eta_\perp}\delta\omega$$

$$\Gamma_{res} = \hat{\theta} \cdot r_a \int_{\mathcal{V}} \vec{J}_0 \times \vec{B}_0 d\mathcal{V}$$

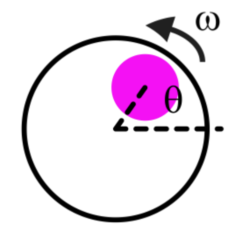
$$= L \int_0^{2\pi} d\theta \int_0^{r_p} r^2 J_r B_z dr$$

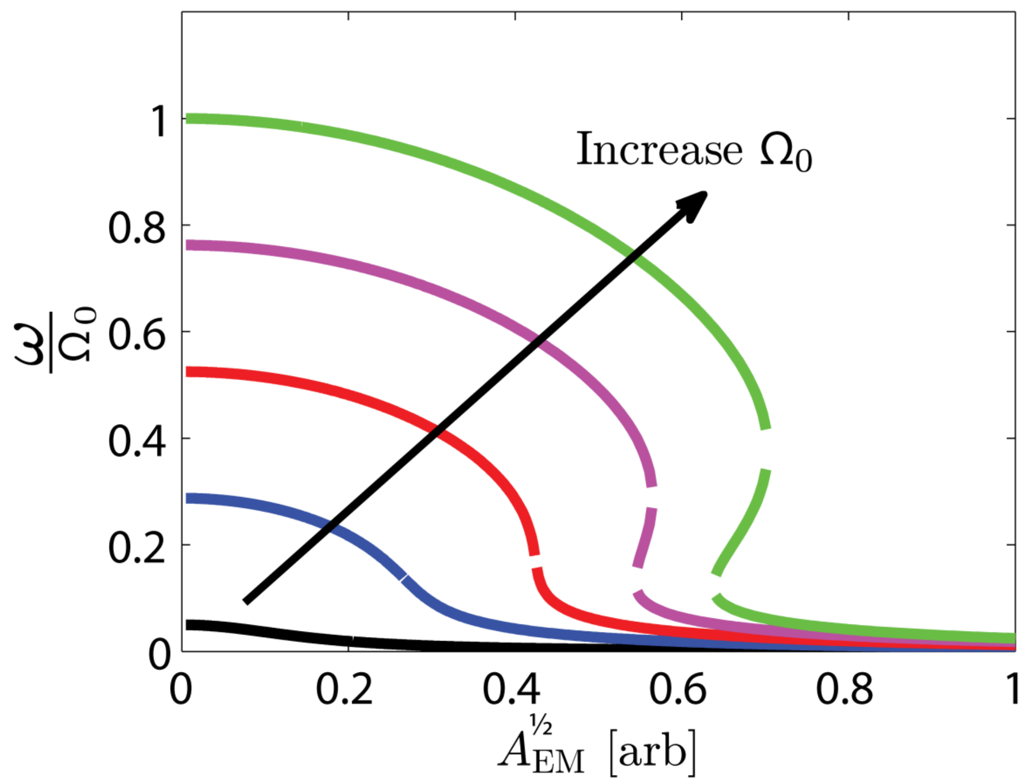
$$= \frac{\pi}{4} \frac{L B_z^2 r_p^4}{\eta_\perp} (\Omega_{\mathrm{ExB}} - \omega) = \underbrace{A_{\mathrm{res}}(\Omega_0 - \omega)}_{\Gamma_{res}}$$

- Would be infinite in ideal MHD
- Comparable to other torques in our cold plasma

Canonical torque balance

$$I\dot{\omega} = 0 = \underbrace{A_{\rm res}(\Omega_0 - \omega)}_{\Gamma_{res}} - \underbrace{A_{EM}\frac{\omega}{1+(\omega\tau_w)^2}}_{\Gamma_{EM}}$$





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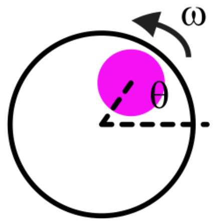
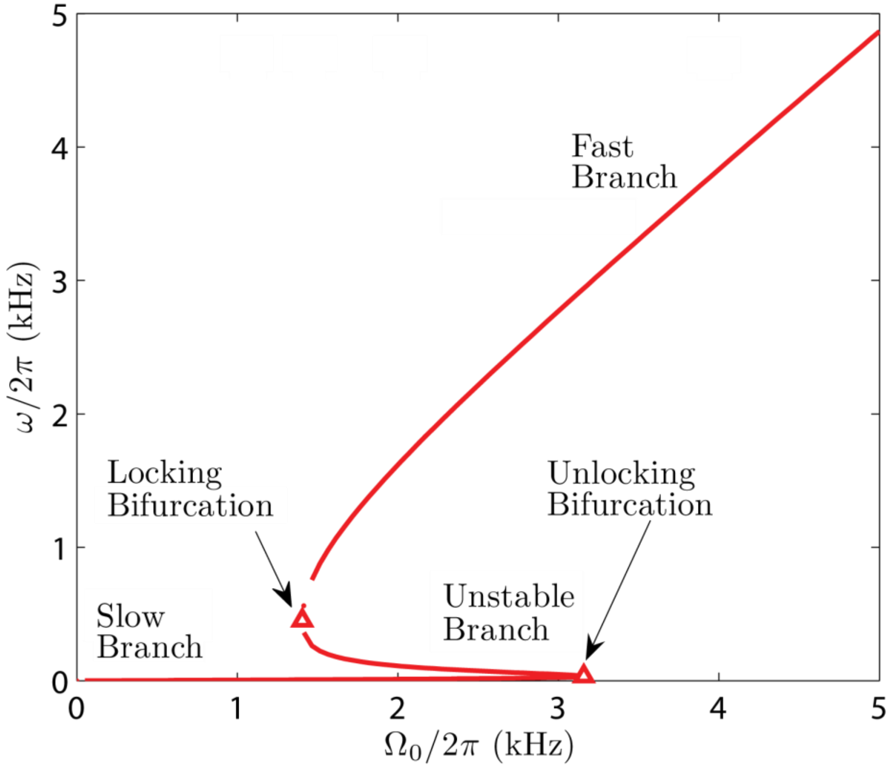
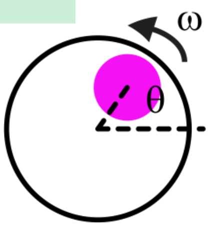
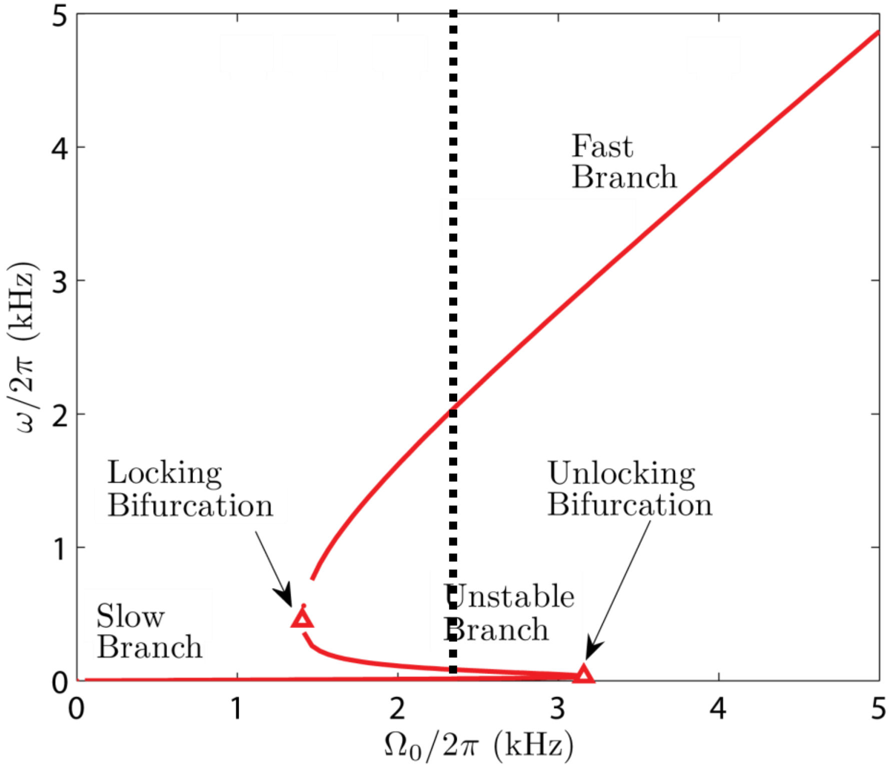




Error field broadens torque curve

$$I\dot{\omega} = 0 = \underbrace{A_{\text{res}}(\Omega_0 - \omega)}_{\Gamma_{res}} - \underbrace{A_{EM}\frac{\omega}{1+(\omega\tau_w)^2}}_{\Gamma_{EM}} + \underbrace{A_{\text{ext}}\sin(\theta)}_{\Gamma_{\text{ext}}}$$





Error field broadens torque curve

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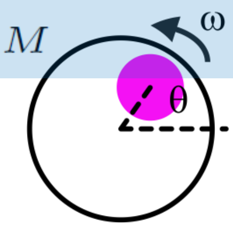
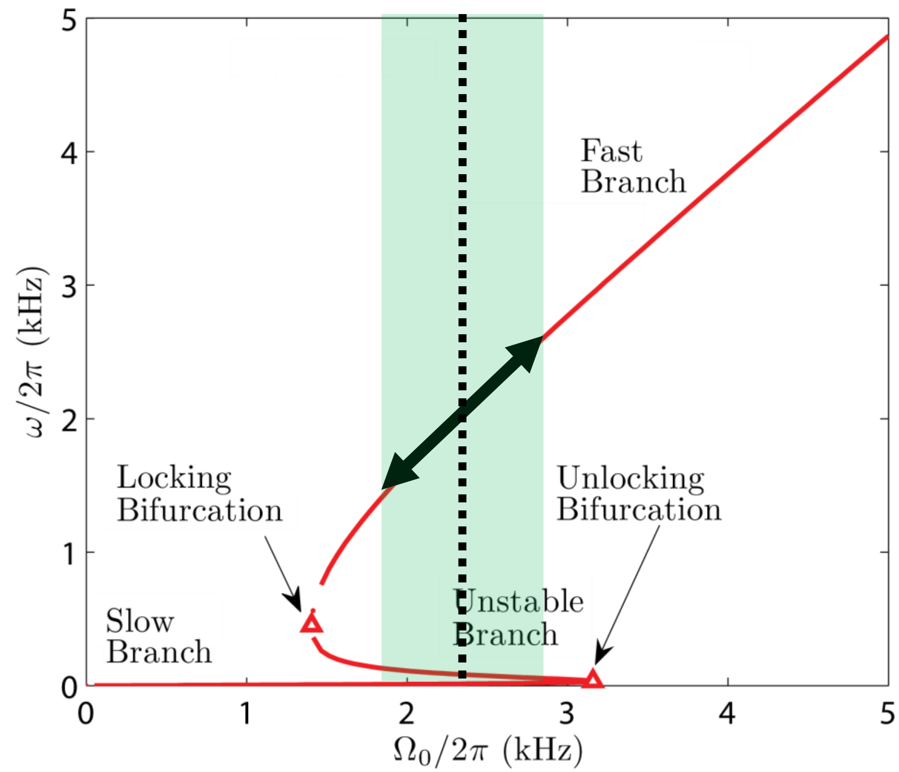
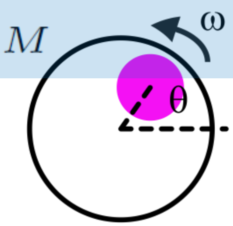


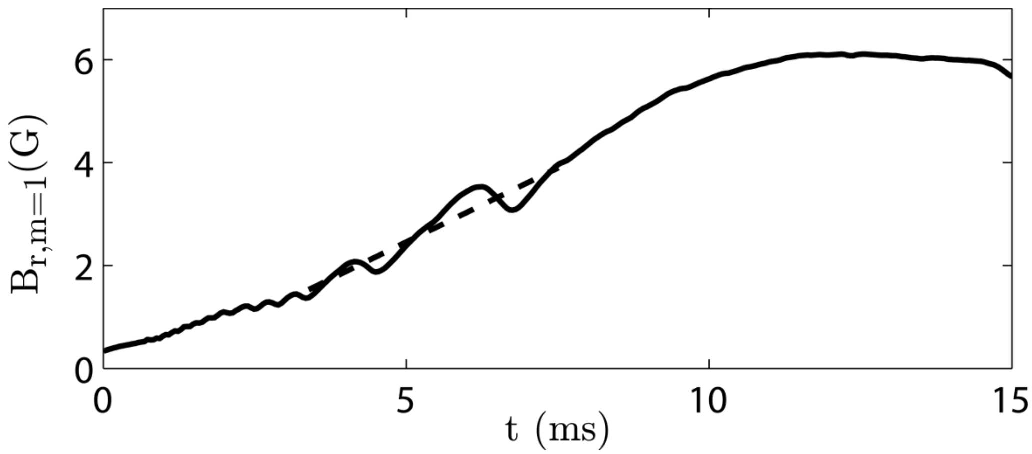


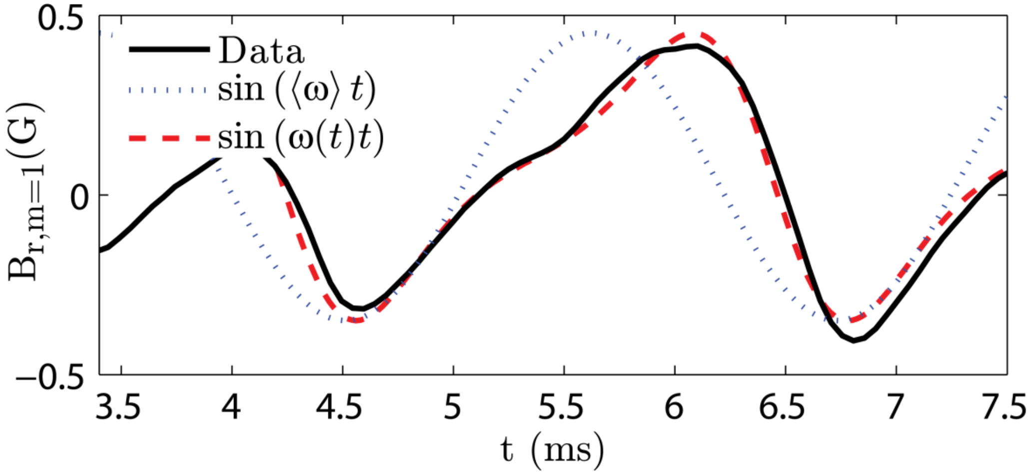
$$\Omega_{0,\text{eff}} = \Omega_0 + \frac{A_{\text{ext}}}{A_{\text{res}}}\sin\theta$$

ω is not constant through a cycle

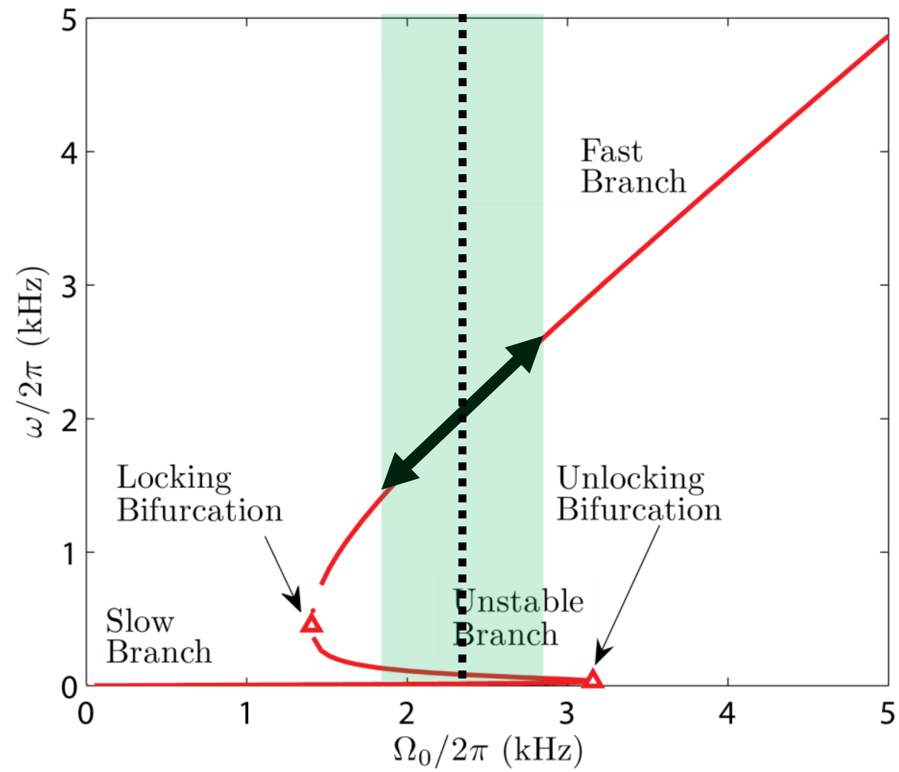
$$I\dot{\omega} = 0 = \underbrace{A_{\text{res}}\left(\left(\Omega_0 + \frac{A_{\text{ext}}}{A_{\text{res}}}\sin(\theta)\right) - \omega\right)}_{\Gamma_{res}} - \underbrace{A_{EM}\frac{\omega}{1+(\omega\tau_w)^2}}_{\Gamma_{EM}}$$







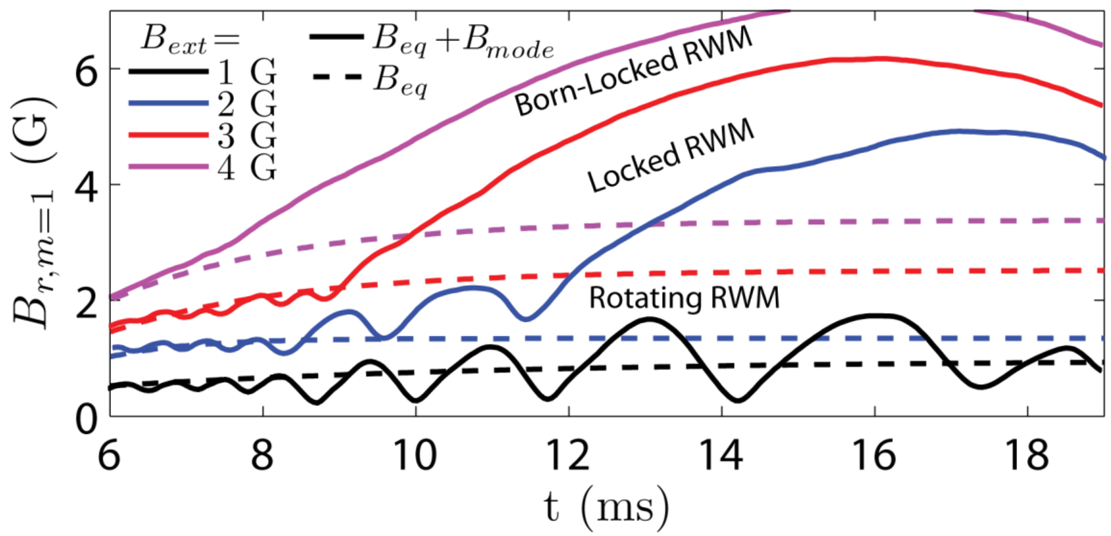
$$\omega(t) = \langle\omega\rangle + \tilde{\omega}\sin(\langle\omega\rangle t)$$

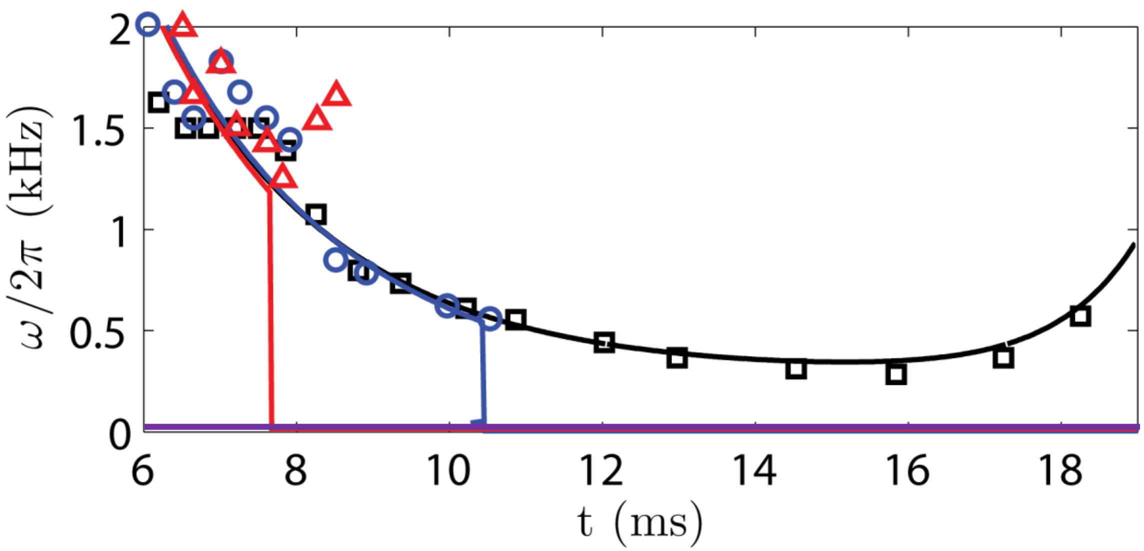


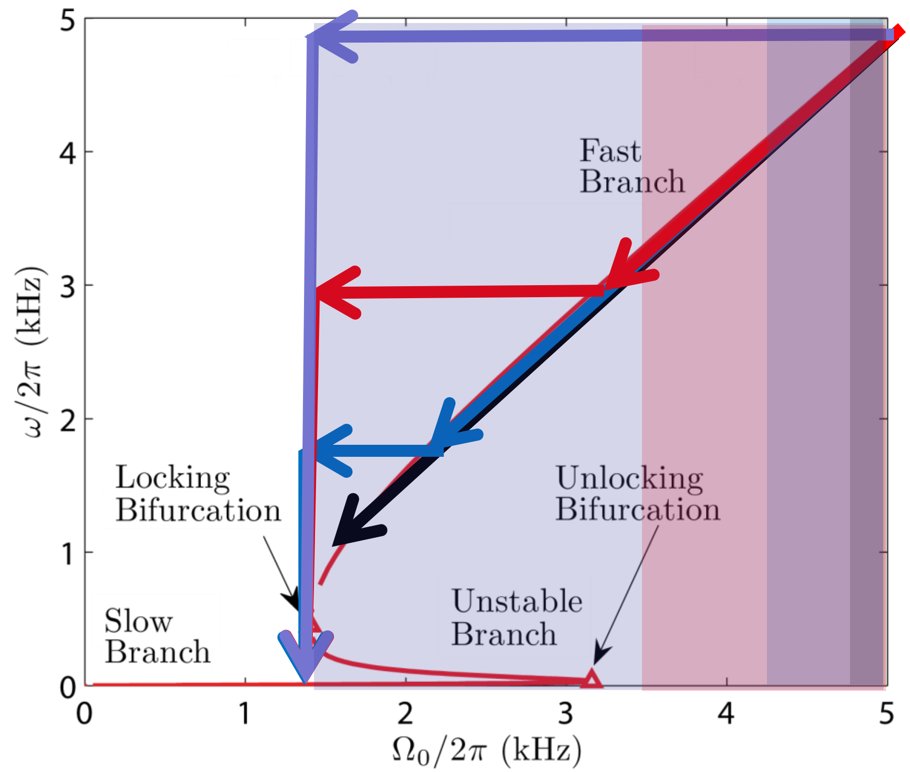
$$\Omega_{0,\text{eff}} = \Omega_0 + \frac{A_{\text{ext}}}{A_{\text{res}}}\sin\theta$$

m = 1 error field aids mode-locking

- Error field defines locking regime

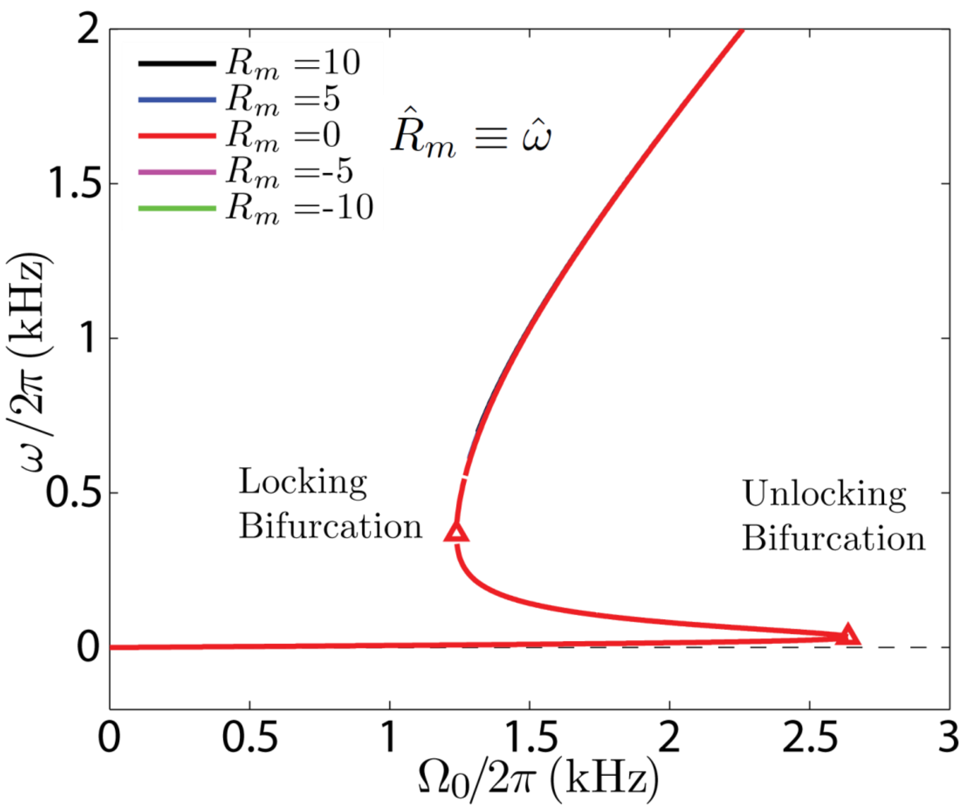


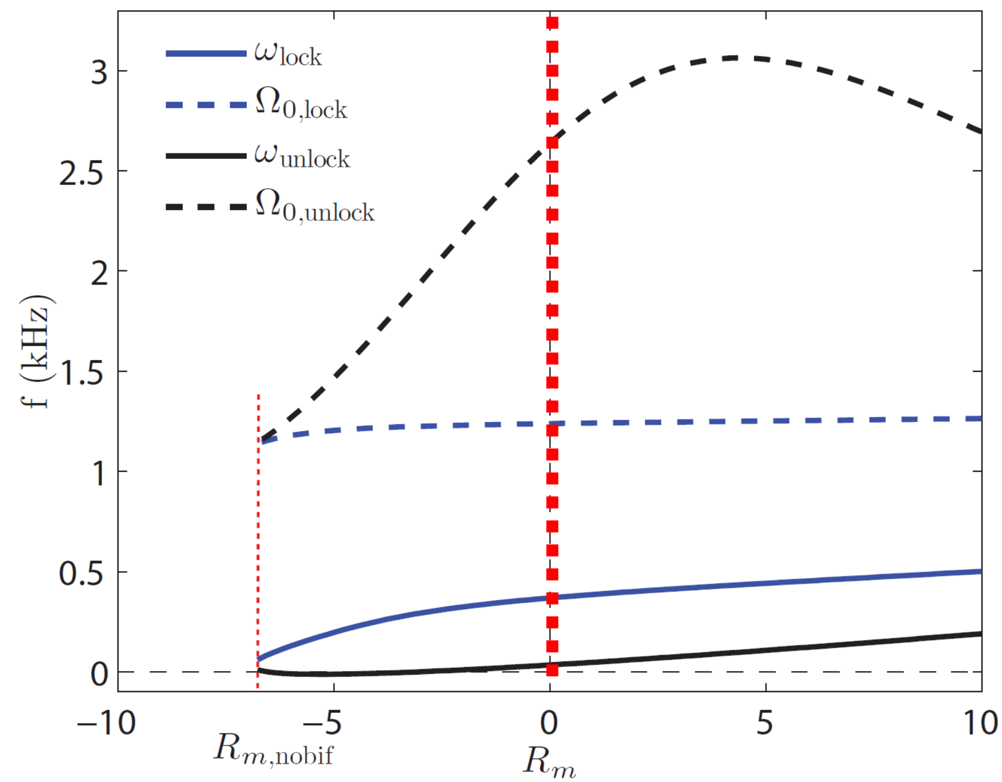




$$\Omega_{0,\mathrm{eff}} = \Omega_0 + \frac{A_{\mathrm{ext}}}{A_{\mathrm{res}}} \sin\theta$$

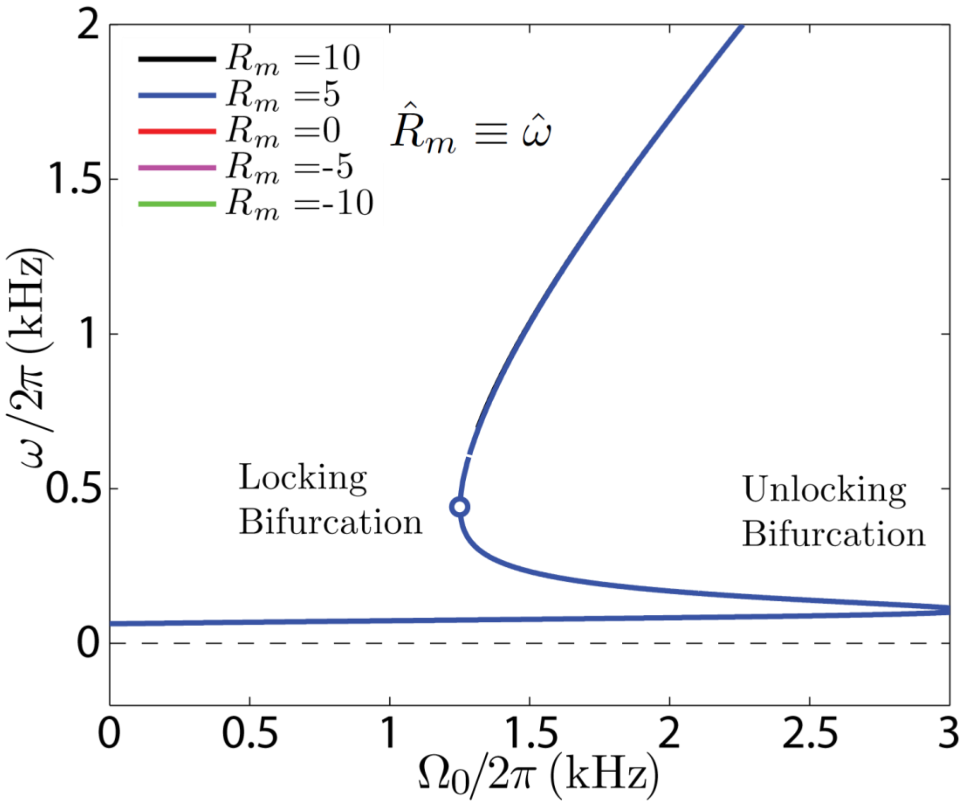
Bifurcations altered by wall rotation

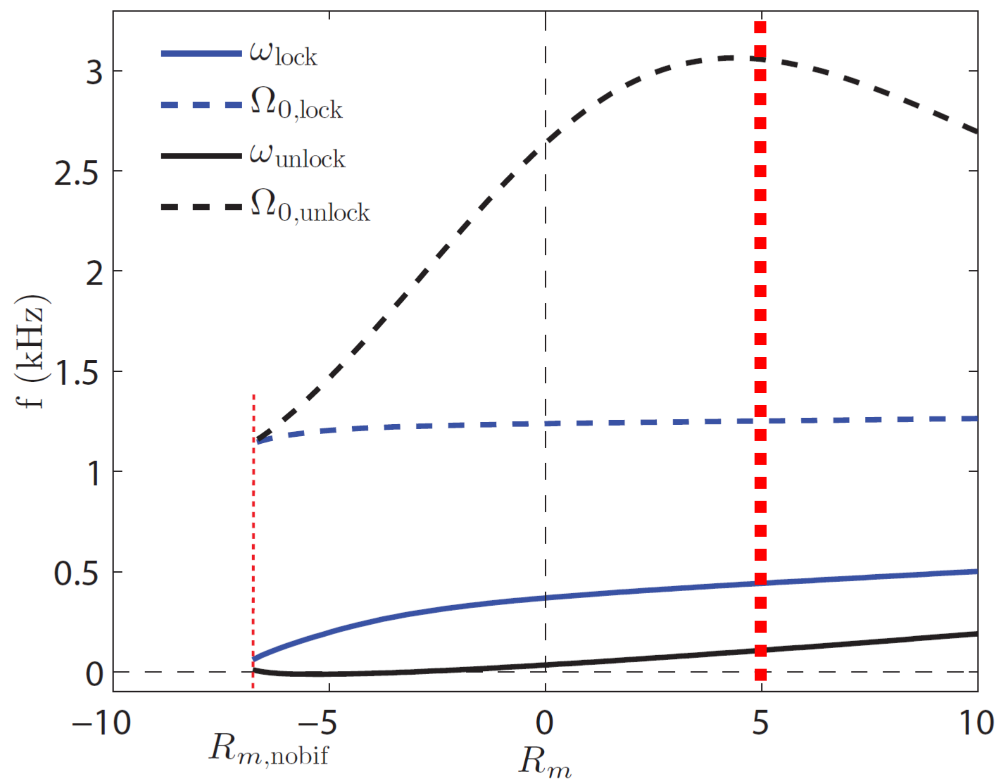




$$\hat{R}_m \equiv \hat{\omega}$$

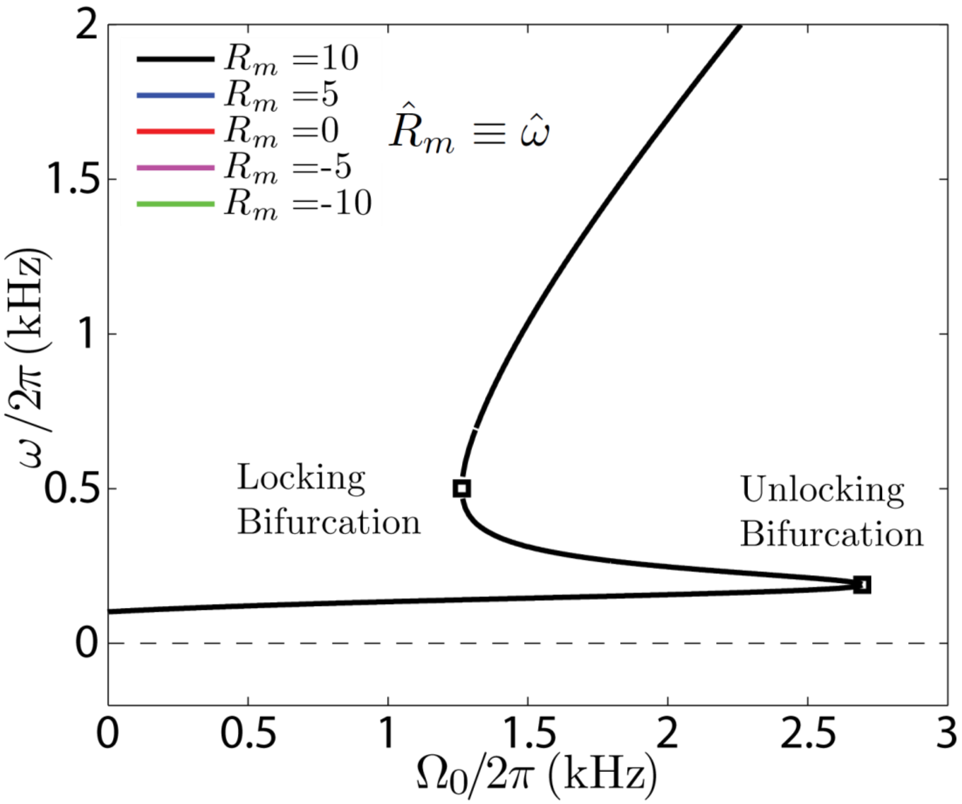
Bifurcations altered by wall rotation

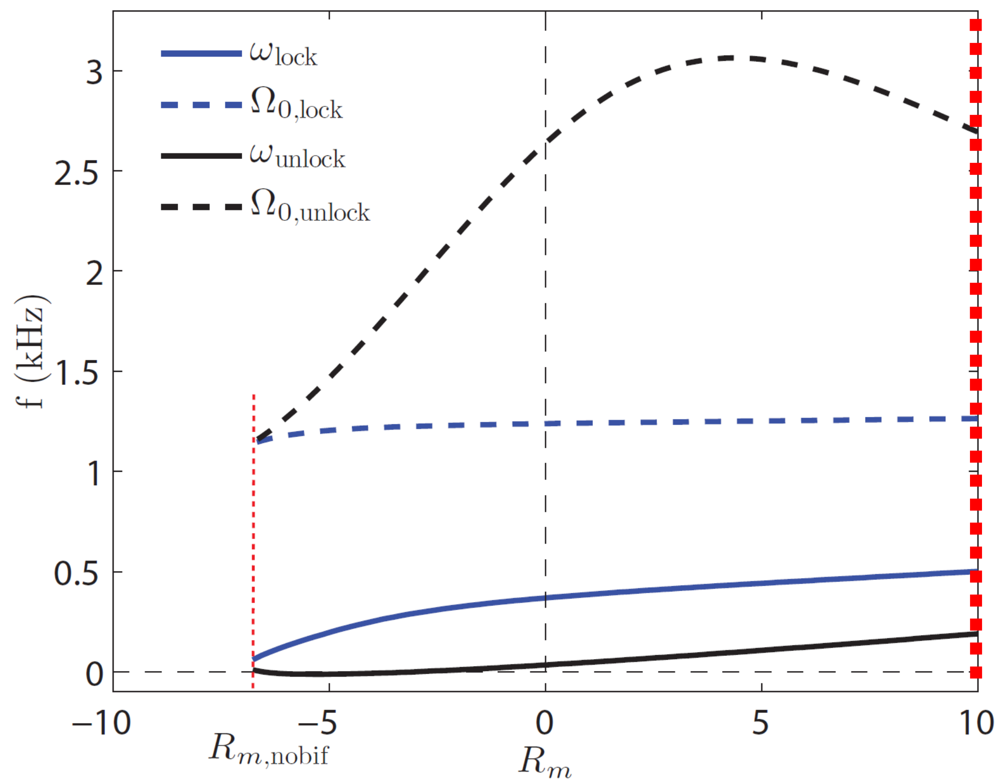




$$\hat{R}_m \equiv \hat{\omega}$$

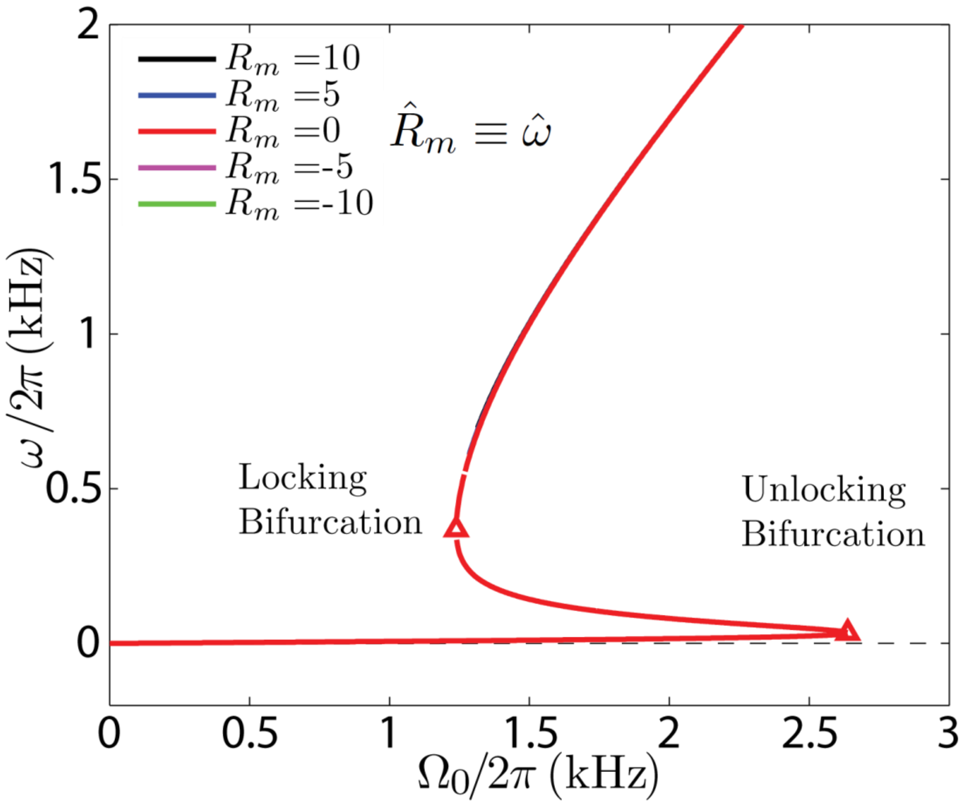
Bifurcations altered by wall rotation

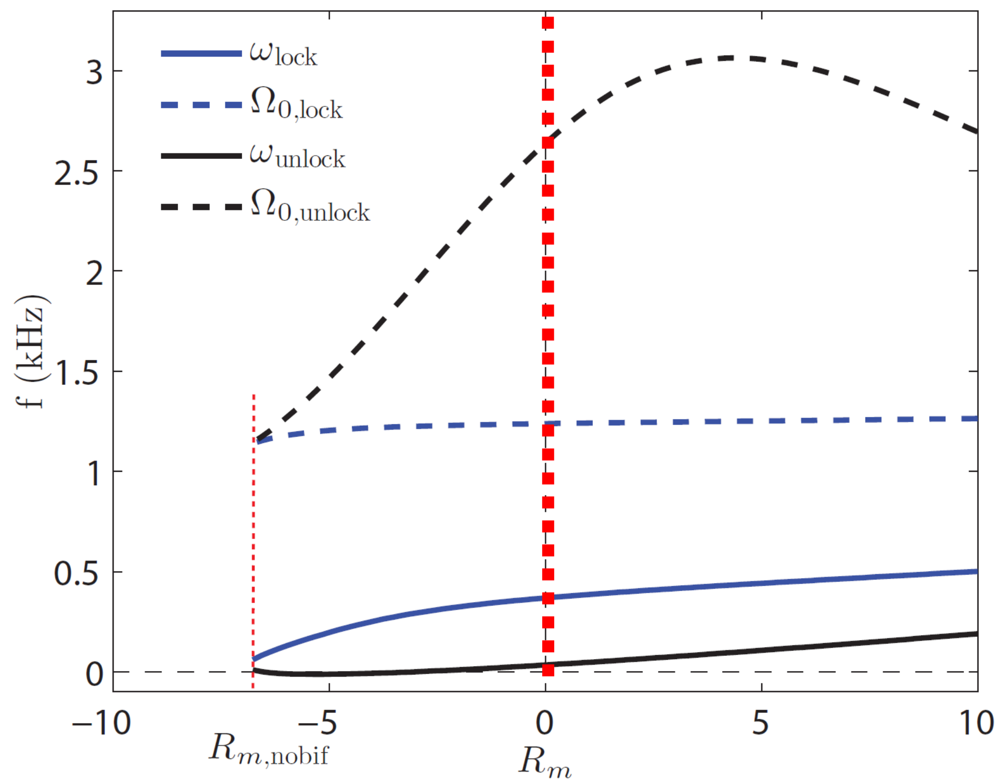




$$\hat{R}_m \equiv \hat{\omega}$$

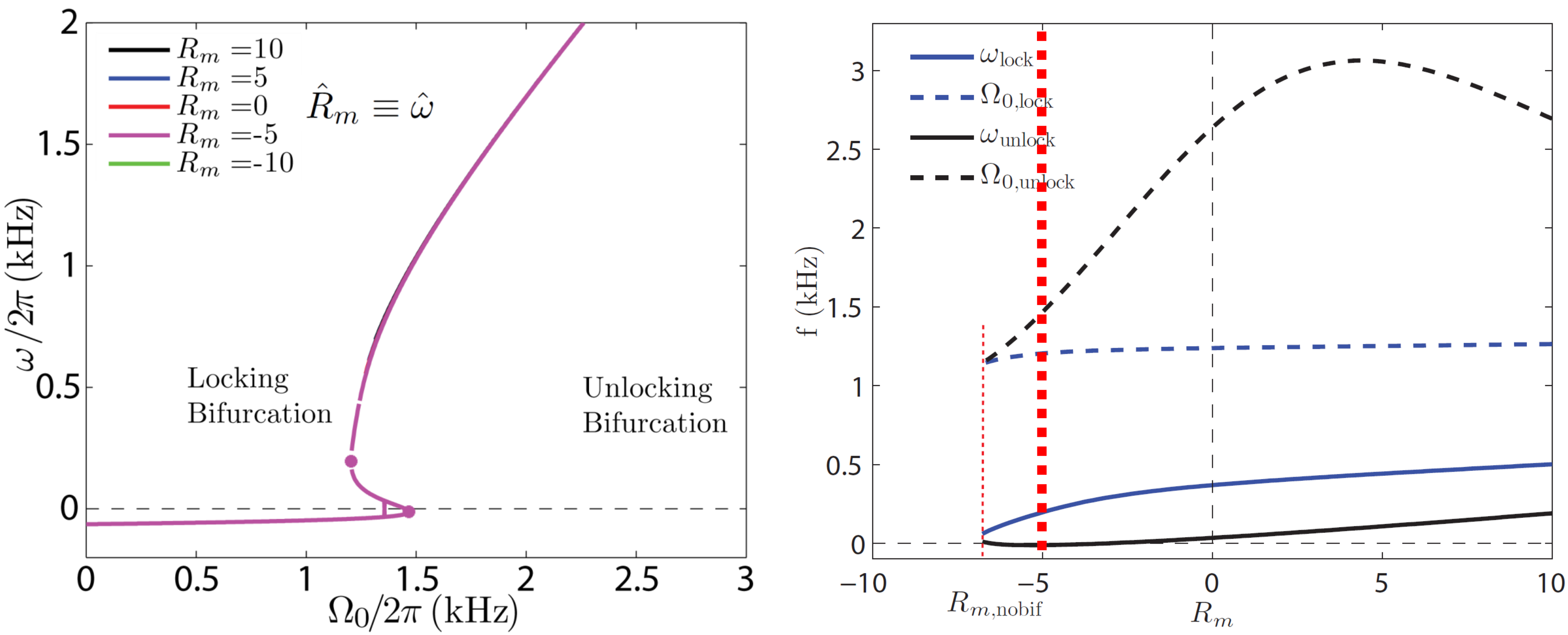
Bifurcations altered by wall rotation





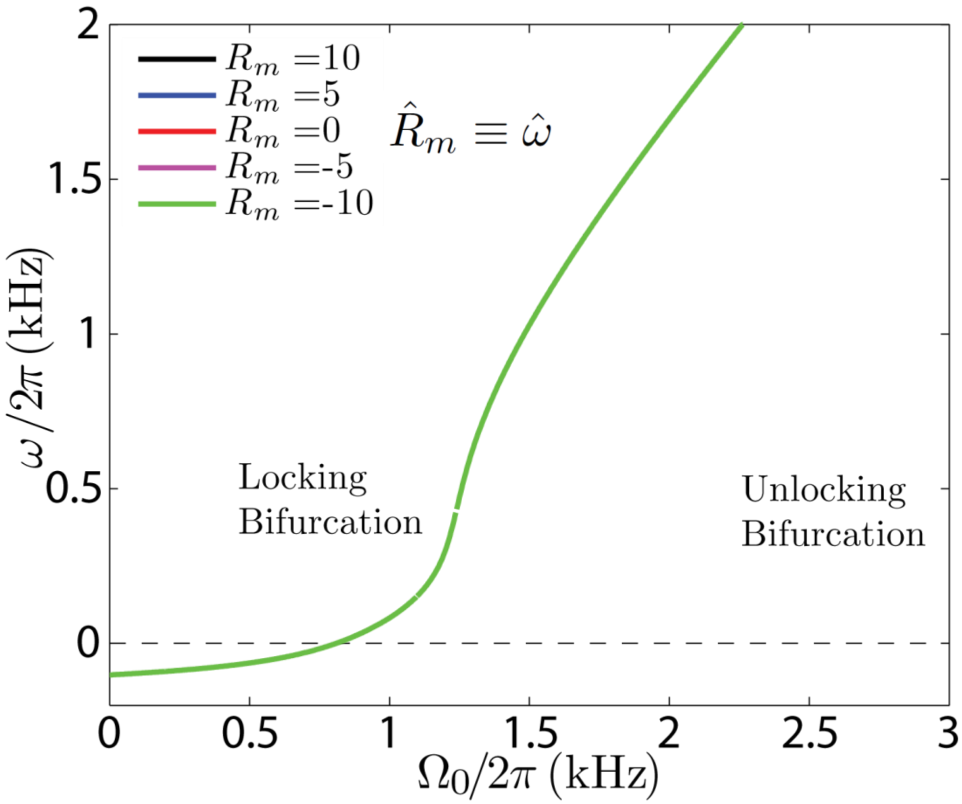
$$\hat{R}_m \equiv \hat{\omega}$$

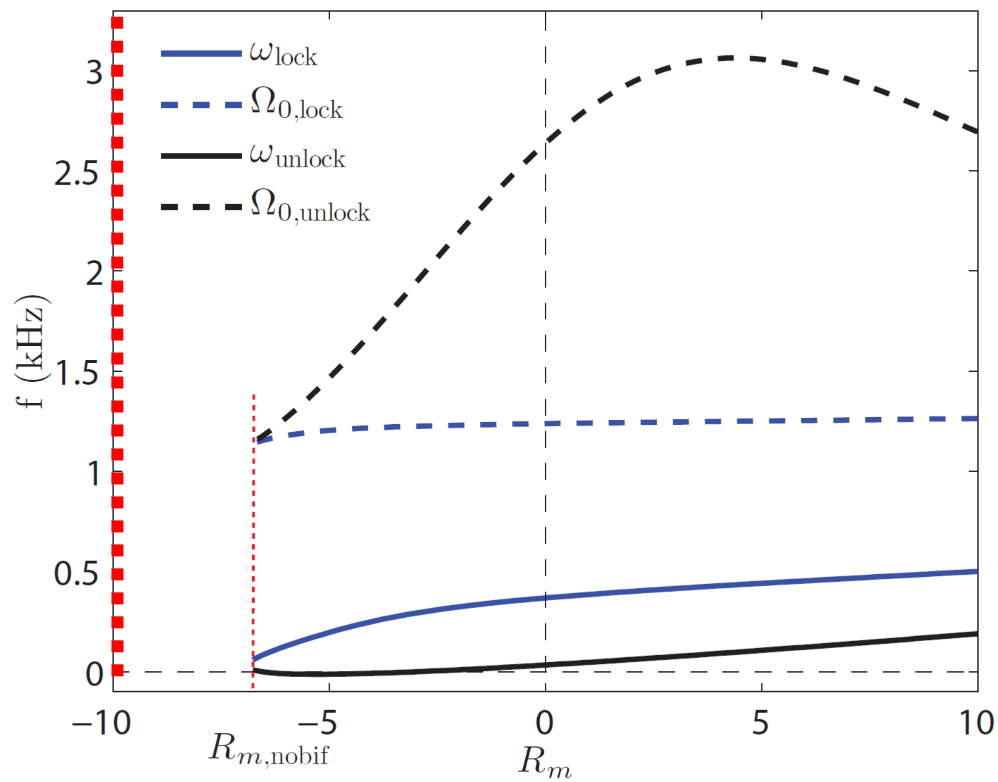
Bifurcations altered by wall rotation



$$\hat{R}_m \equiv \hat{\omega}$$

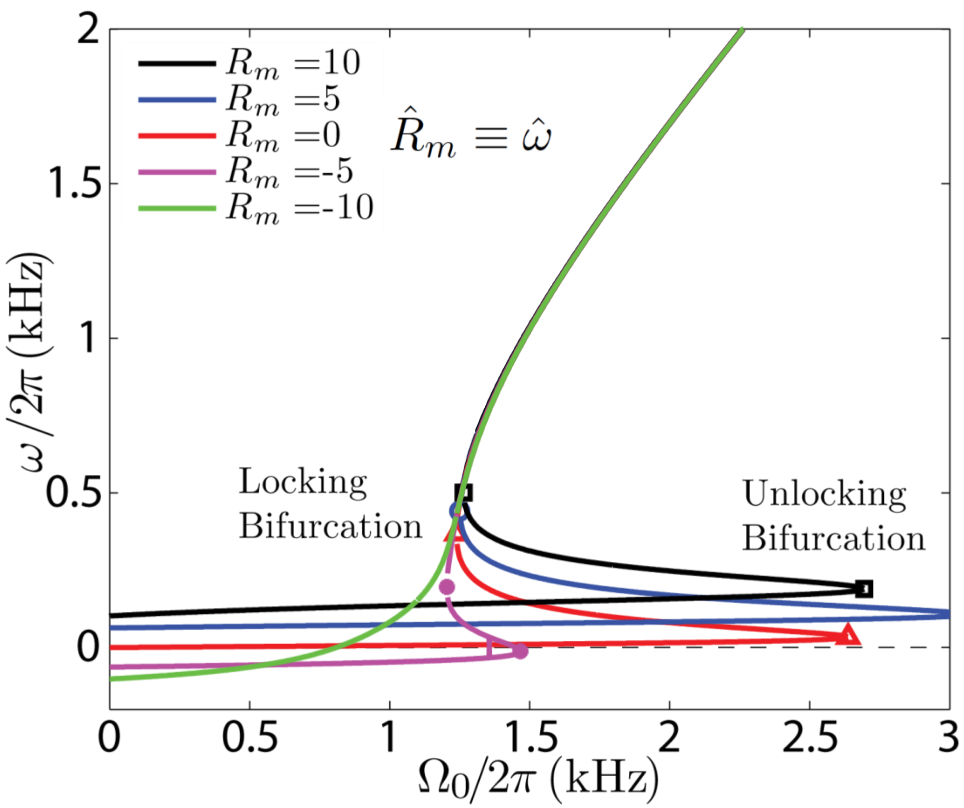
Bifurcations altered by wall rotation

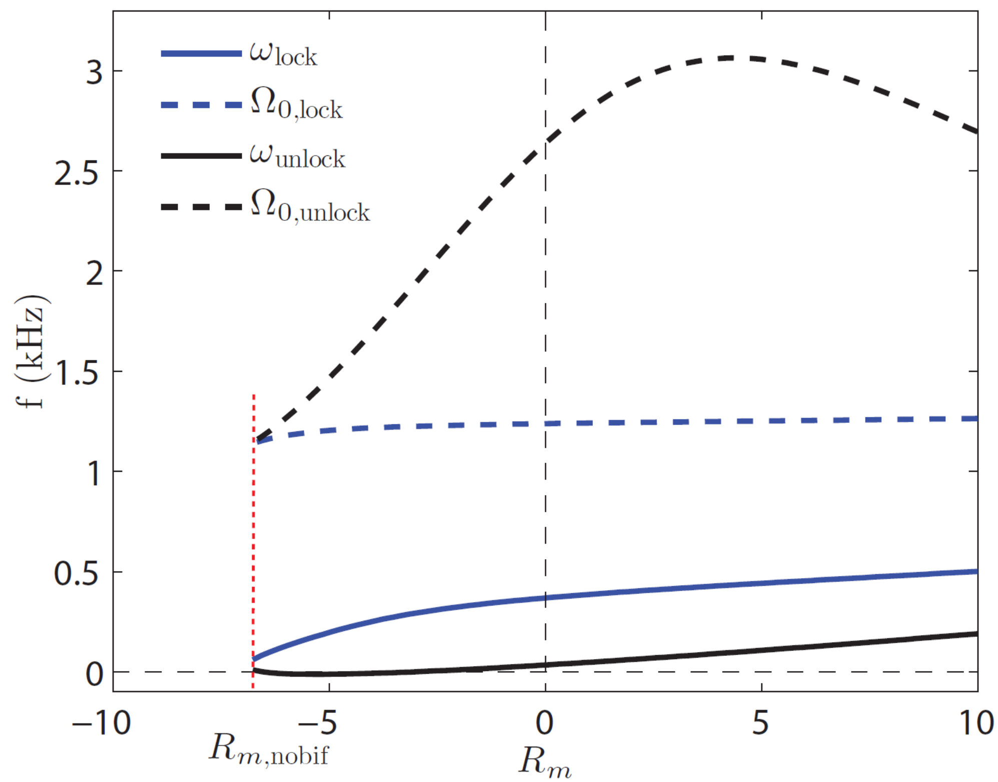




$$\hat{R}_m \equiv \hat{\omega}$$

Bifurcations altered by wall rotation





$$\hat{R}_m \equiv \hat{\omega}$$

Feedback alters the torque balance, and it depends on the phase

Conclusions

- Vertical field penetration time decreases as wall rotation increases
- RWM mode-locking in our device shows similar phenomenology to the torus
- Mode-locking modifications by wall rotation explored and demonstrated
  - Active control of ExB rotation through bias plates would further this study
  - Analogy to feedback gain will be pursued