Equilibrium Reconstruction without Axisymmetry

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Equilibrium Reconstruction

- Find the MHD equilibrium that is most consistent with the experimental data

$$\chi^2(\mathbf{p}) \equiv \sum_i \left(\frac{S_i^{observed}(\mathbf{d}) - S_i^{model}(\mathbf{p})}{\sigma_i} \right)^2$$

  - Reconstruction parameters ***p***
  - Signals S
  - Square root of the variance σ
- Minimize $\chi^2(\mathbf{p})$ by changing the parameters of the model (equilibrium)
- At minimizing parameter values, the model is a good representation of the experiment
- Axisymmetric Equilibrium Reconstruction
  - Equilibrium comes from a Grad-Shafranov solution
  - EFIT and other codes
- Non-axisymmetric Equilibrium Reconstruction
  - V3FIT code – This talk
  - STELLOPT code – Sam Lazerson's talk at this workshop

V3FIT Code

- Design Goal - Fast
  - Want reconstructions between shots
  - Design Choice: one reconstruction uses one CPU
  - Design Choice: single executable – to allow tighter coupling to equilibrium solver
- Design Goal - Flexible
  - Easy to understand, maintain, and modify
  - Written in Fortran 95
  - Clear and consistent data flow – modular coding
- Design Goal – Extensible
  - Initial equilibrium solver - VMEC
  - Localize VMEC code assumptions, so that could use a different equilibrium solver in the future
  - Signals – magnetic diagnostics (original), limiters, soft x-rays, Thomson scattering, combinations_of_other_signals (coosig).
- "V3FIT: a code for three-dimensional equilibrium reconstruction"
  - J D Hanson, S P Hirshman, S F Knowlton, L L Lao, E A Lazarus, and J M Shields
  - Nucl. Fusion **49**, 075031 (2009).

Current and Planned Usage

- CTH – Compact Toroidal Hybrid -Auburn University
  - Steve Knowlton's talk, this workshop
- HSX – Helically Symmetric Experiment – U. Wisconsin
  - Invited talk at 2011 APS-DPP, John Schmitt
- LHD – Large Helical Device – Japan
  - Aaron Sontag (ORNL)
- RFX – Reversed Field Pinch – Padova, Italy
  - David Terranova et al., EPS 2011
- Recent Discussions
  - MST – Madison Symmetric Torus– U. Wisconsin
  - Alcator C-Mod – MIT
  - DIII-D – General Atomics

V3FIT/VMEC Features

- VMEC – flexible and extensible mechanism for specifying profiles
- V3FIT – posterior parameter variance computation
- V3FIT – Signal Effectiveness computation
- V3FIT – New diagnostics - Soft x-ray, Thomson scattering
- V3FIT – flexible mechanism for model extension
- MAKEGRID – rotate and shift external coils

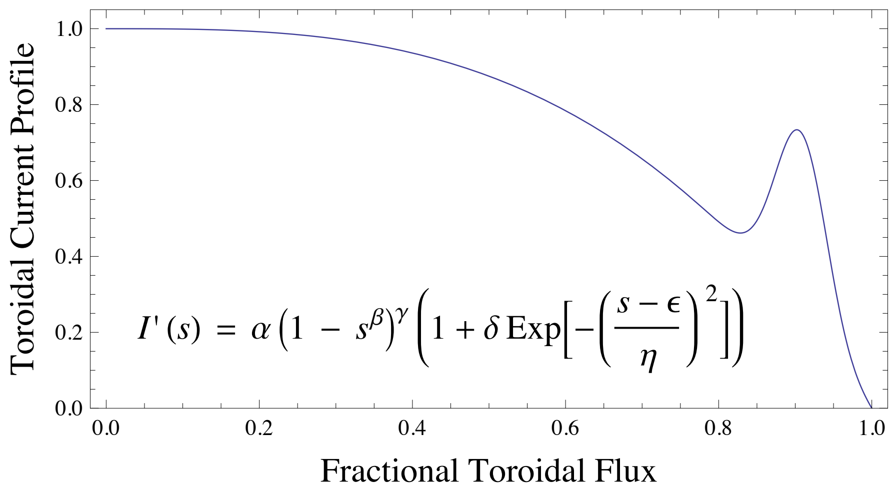
Pressure and Current Profiles in VMEC

- VMEC requires a pressure profile and either a current or iota profile
- Original specification – coefficients of power series

$$p(s) = a_0 + a_1 s + a_2 s^2 + \dots$$

  - Radial coordinate s – fractional toroidal (poloidal) flux
- Now – also specify a parameterization type
- Different types cause reinterpretation of coefficient array
  - pmass_type = 'two_power' $p(s) = a_0 \left(1 - s^{a_1}\right)^{a_2}$

- Simple code change to add new parameterization type
- Can also specify using spline positions and amplitudes



Probability and Inverse Problems

$$\chi^2 = \sum_i \left(\frac{S_i^{model}(p_j) - S_i^{observe}}{\sigma_i} \right)^2$$

$$= \left(\vec{S}^{model}(\vec{p}) - \vec{S}^{observe} \right)^T \cdot \underline{\underline{C}}^{-1} \cdot \left(\vec{S}^{model}(\vec{p}) - \vec{S}^{observe} \right)$$

$$\underline{\underline{C}}^{-1} = \begin{pmatrix} \sigma_1^{-2} & 0 & 0 \\ 0 & \sigma_2^{-2} & 0 \\ 0 & 0 & \dots \end{pmatrix} \qquad \left(\underline{\underline{C}} \right)_{ij} = \sigma_i \sigma_j \delta_{ij}$$

- C – Covariance matrix of the signal noise
- Specifies a probability distribution on space of signals.
- Assumption that signal noise is uncorrelated ⇔ C diagonal

Jacobian and Posterior Parameter Variance

- The Jacobian relates small changes in reconstruction parameters to small changes in signals: $\left(\underline{\underline{J}}\right)_{ij} = \frac{\partial S_i^{\text{model}}(\vec{p})}{\partial p_j}$ $\qquad \delta\vec{S} = \underline{\underline{J}} \cdot \delta\vec{p}$
- V3FIT computes a finite-difference approximation to the Jacobian in the course of minimizing chi-squared
- The Jacobian propagates a (Gaussian) probability distribution from *signal* space to *parameter* space

$$\underline{\underline{C_p^{-1}}} = \underline{\underline{J}}^T \cdot \underline{\underline{C^{-1}}} \cdot \underline{\underline{J}}$$

- C_p is the posterior parameter covariance matrix.
- The diagonal elements of C_p are the posterior parameter variances.

$$\sigma_{pj} = \sqrt{\left(\underline{\underline{C_p}}\right)_{jj}}$$

- C_p need not be diagonal – the reconstructed parameters may be correlated.
- V3FIT computes and prints the posterior parameter variances.

Signal Effectiveness

- Proposed measure of the effectiveness of a signal:

$$R_{ji} = \frac{d \ln \sigma_{p\,j}}{d \ln \sigma_i} = \frac{\sigma_i}{\sigma_{p\,j}} \frac{d\sigma_{p\,j}}{d\sigma_i}$$

  - Logarithmic derivative of the *j*th posterior parameter σ_p with respect to the *i*th signal σ
  - How much will the *j*th posterior σ_p improve if the noise level on the *i*th signal is reduced?
- Motivation – Signal Effectiveness can help answer important questions:
  - I wish to improve the measurement of the current profile. What diagnostics should I add?
  - Magnetic diagnostics break. For which magnetic diagnostics do I need a spare, ready and waiting to put on the machine?
  - I'm building a new stellarator. What diagnostics should I build?

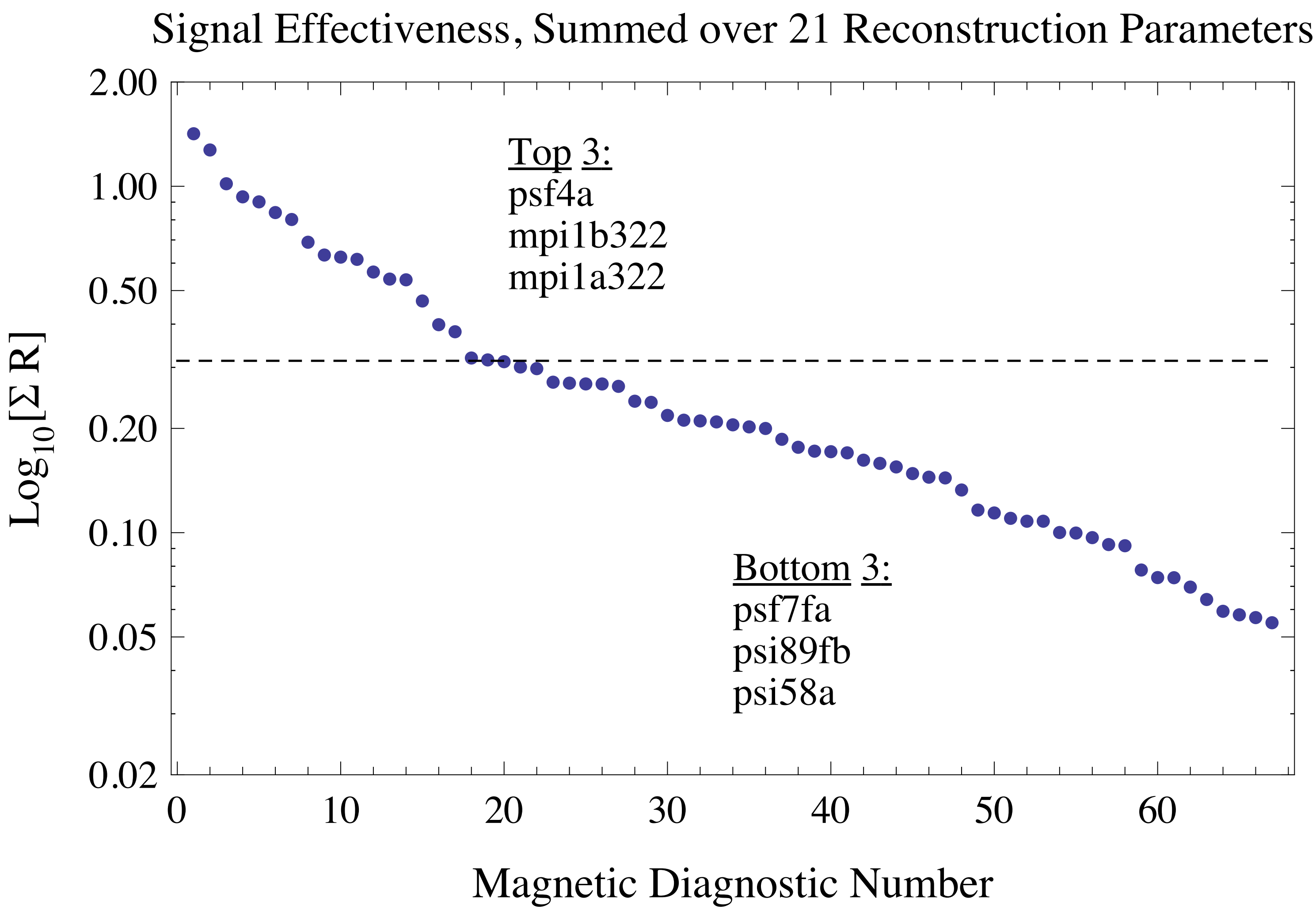
Signal Effectiveness R_{ji}

- With diagonal signal covariance C, the signal effectiveness R_{ji} is readily computable from the Jacobian $J_{ij} = \partial S_i^m / \partial p_j$.
- R_{ji} is dimensionless and non-negative.
- R_{ji} is normalized, $\sum_{i(signals)} R_{ji} = 1$
- R_{ji} is *local*. It only contains information about what happens near a particular point in parameter space.
  - May want an appropriate average over parameter space.
- Signal Effectiveness depends on the whole set of diagnostics and choices of reconstruction parameters.
- V3FIT computes and prints the signal effectiveness matrix.
- CTH – have used signal effectiveness for planning of new diagnostics.
- HSX – Enrico Chlecowitz is using the signal effectiveness for planning new magnetic diagnostics.

Signal Effectiveness Example from DIII-D

- Reconstruction detailed in Nucl. Fusion 2009 paper
- Signals – magnetic diagnostics – 67 signals
  - 31 partial Rogowsi coils
  - 36 flux loops
- Reconstruction Parameters – 21 parameters
  - 18 F-coil currents
  - Total plasma current
  - Pressure scale
  - One current profile shape parameter
- For each diagnostic, sum the Signal Effectiveness over all 21 reconstruction parameters
- Max - Average - Min SumSE is 1.42 – 0.313 – 0.055
- Factor of 25 between least and most effective diagnostic

DIII-D



Signal Effectiveness, Summed over 21 Reconstruction Parameters
2.00
1.00
0.50
0.20
0.10
0.05
0.02
0
10
20
30
40
50
60
Log10[Σ R]
Magnetic Diagnostic Number
Top 3:
psf4a
mpi1b322
mpi1a322
Bottom 3:
psf7fa
psi89fb
psi58a

Soft X-rays

$$S = \int \varepsilon_{sxr}(\underline{x}')\,d\ell = \int \left[\int A(E)\tau(E) \frac{n_e^2(x')}{\sqrt{T_e(x')}} \exp\left(-\frac{E}{T_e(x')} \right) \right] d\ell$$

- Line integral is straight path through plasma region
- Soft X-ray emissivity ε_{sxr} is *assumed* to be constant on a flux surface
- The model signal depends on *more* than just the MHD equilibrium
  - Radial profile of the emissivity
  - Radial profiles of the electron density and electron temperature

Thomson Scattering

$$S = T_e(\underline{x}_{diagnostic})$$

- Position of the temperature measurement is known
- To tie this in to the equilibrium *assume* that the electron temperature is constant on a flux surface.
- The model signal depends on *more* than just the MHD equilibrium
  - Radial profile of the electron temperature

Plasma Model and Radial Profiles

- For magnetic diagnostics, the only plasma information needed comes from the MHD equilibrium
- For other diagnostics, we need to *augment* the MHD equilibrium.
- The V3FIT model of the plasma now consists of the MHD equilibrium, and *parameterized-profiles* of
  - Soft x-ray emissivity
  - Electron density
  - Electron temperature
- Other *parameterized-profiles* can easily be added (n_i, T_i, p_{hot}).
- *Parameterized-profiles* have a type specification ("two-power", "truncated-Gaussian", "spline") that determines the meaning of the parameterized-profile parameters.
- The parameters of a *parameterized-profile* can be used as reconstruction parameters.

Choices are Explicit in V3FIT

- Soft X-rays
  - model_sxrem_type = ‘pp_sxrem’

    Parameterized profile of the soft x-ray emissivity

$$S = \int \varepsilon_{sxr}(\underline{x}')\,d\ell$$

  - model_sxrem_type = ‘pp_ne_vmec_p’

    Parameterized profile of the electron density, use the MHD pressure

$$\varepsilon_{sxr} = \alpha\, n_e^2 \sqrt{T_e} \qquad n_e T_e = f_e p_{VMEC}$$

- Thomson Scattering
  - Model_te_type = ‘pp_te’

    Parameterized profile of the electron temperature
  - model_te_type = ‘pp_ne_vmec_p’

    Parameterized profile of the electron density, use the MHD pressure

$$n_e T_e = f_e p_{VMEC}$$

External Magnetic Fields

- Free Boundary equilibrium in VMEC – Need External B

  1. Geometric model of the external coils
  2. Rotate and shift coils from their nominal positions
  3. Biot-Savart to compute B on a grid, B from each coil separately
  4. VMEC reads B information from file, uses linear combination of B fields

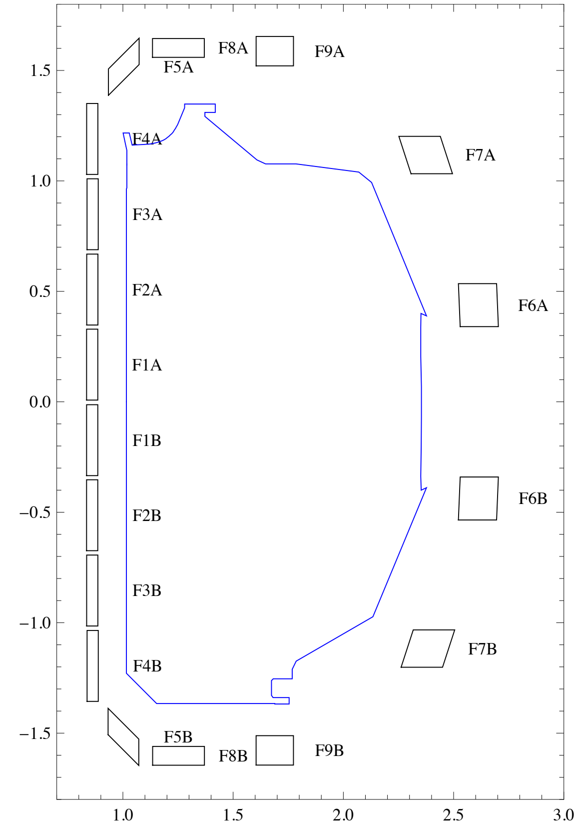
$$\vec{B}(R,\phi,Z) = \sum_i I^i_{extcur} \vec{B}_i(R,\phi,Z)$$

- Step 2 – added recently
- Shifts and rotations for DIII-D:
  - "*Anomalies in the applied magnetic fields in DIII-D and their implications for the understanding of stability experiments*" J.L. Luxon, M.J. Schaffer, G.L. Jackson et al., Nuclear Fusion **43** 1183 (2003).

Shift-Rotate Data for DIII-D

Coil	d (mm)	Angle (˚)	Tilt (˚)	Angle (˚)
F1A	1.46	348.1	0.112	177.7
F2A	2.36	336.9	0.121	113.4
F3A	3.15	0.8	0.051	148.2
F4A	2.49	2.6	0.138	185.5
F5A	4.11	290.6	0.194	102.2
F8A	4.96	277.7	0.241	98.5
F9A	4.19	271.4	0.125	126.6
F7A	12.33	257.9	0.100	103.5
F6A	5.86	216.2	0.086	119.3
F6B	7.24	243.2	0.074	123.0
F7B	7.59	219.2	0.013	177.5
F9B	3.76	231.9	0.090	39.4
F8B	6.27	183.8	0.125	21.3
F5B	1.69	265.9	0.074	76.8
F4B	0.57	303.0	0.136	168.9
F3B	1.15	286.8	0.048	253.9
F2B	2.38	311.3	0.192	182.6
F1B	2.02	277.0	0.157	146.6
Apparatus	4.64	267.8	0.058	114.1

Table 2. The poloidal field coil positions measured from the toroidal coil axis. Right-handed coordinates measured from DIII-D zero.



- Reconstruction with shifted-tilted coils gives VMEC $n = \pm 1$ R and Z expansion coefficients on the order of 0.1 to 0.8 mm.

Conclusions and Future Prospects

- V3FIT is an effective tool for equilibrium reconstruction of highly non-axisymmetric plasmas
  - Stellarators
  - RFP Quasi-Helical States,

- V3FIT's flexibility and feature set may prove useful for nearly-axisymmetric equilibrium reconstruction

- New diagnostics for reconstruction are under development:
  - Microwave interferometry and polarimetry
  - Motional Stark Effect

- Future development of V3FIT will be driven by the code users

Acknowledgements

- Auburn University
  - Steve Knowlton
  - Greg Hartwell
  - Adam Stevenson
  - Mark Cianciosa
  - Jonathan Hebert
  - John Shields
- Oak Ridge National Laboratory
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  - Aaron Sontag
  - Jeff Harris
- General Atomics
  - Lang Lao
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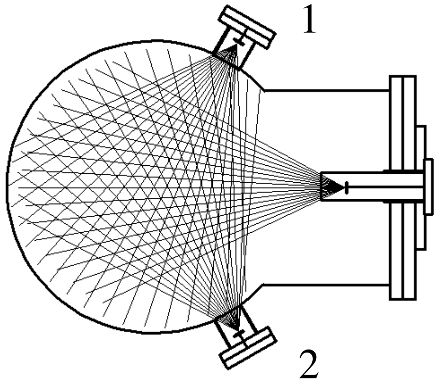
  - Joachim Geiger
- Los Alamos National Laboratory
  - John Finn
  - Chris Jones
- Princeton Plasma Physics Laboratory
  - Sam Lazerson
  - David Gates
  - Ali Zolfaghari
- College of William and Mary
  - Gene Tracy
- Tech-X Corporation
- Monetary Support - USDOE

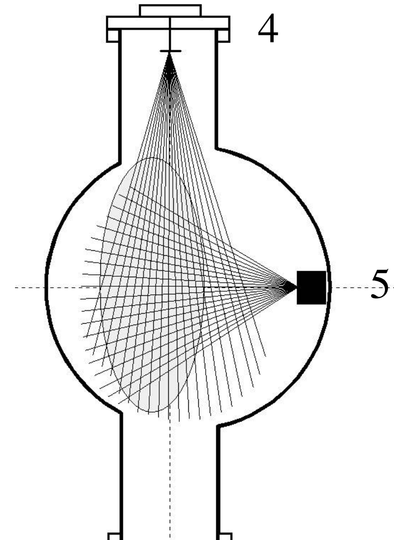
Extra Slides follow

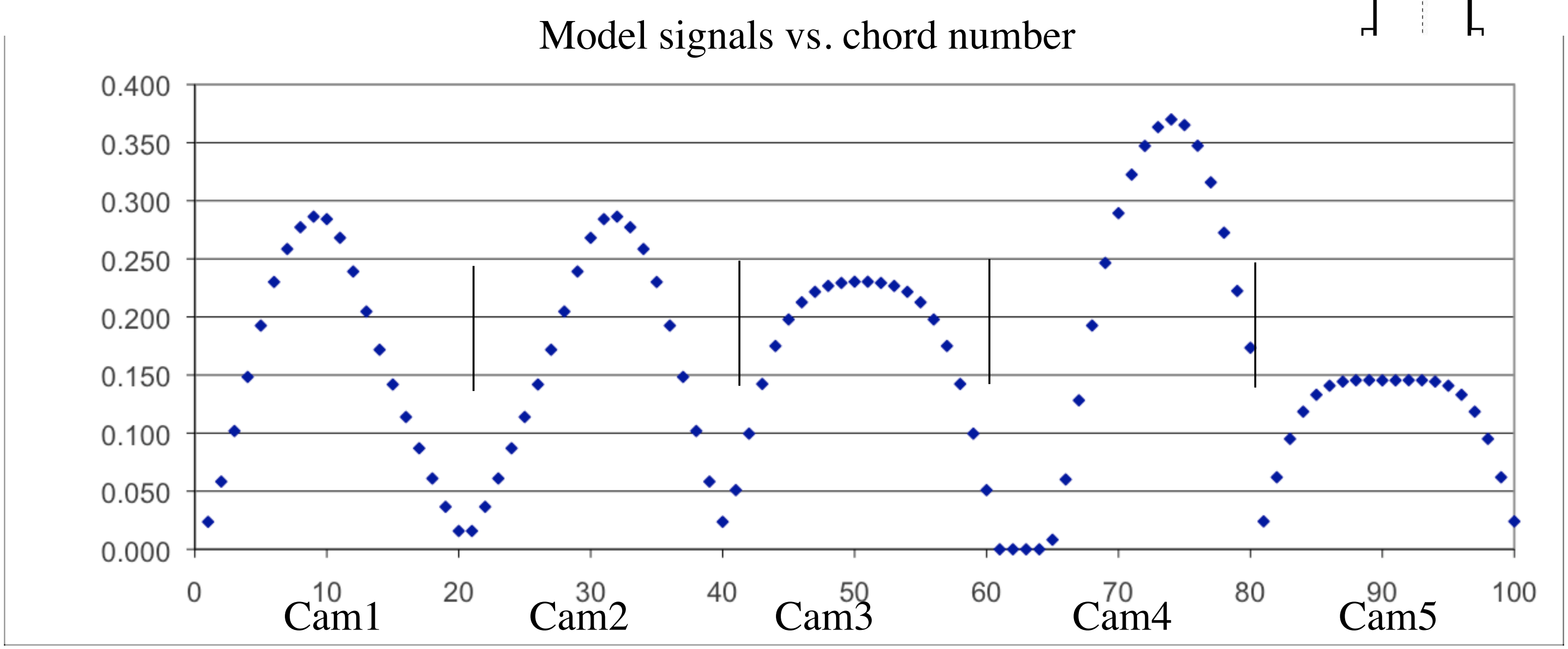
V3FIT History

- ~ 2001, NCSX (National Compact Stellarator eXperiment) on the horizon
  - Steve Hirshman, Steve Knowlton, Ed Lazarus, Lang Lao
  - Discussions about stellarator equilibrium reconstruction
- January 2002
  - First technical discussion
  - Jim Hanson joins the discussions
- "Magnetic diagnostic responses for compact stellarators",
  - S P Hirshman, E A Lazarus, J D Hanson, S F Knowlton, and L L Lao
  - Phys. Plasmas, **11**, 595 (2004)
- "V3FIT: a code for three-dimensional equilibrium reconstruction"
  - J D Hanson, S P Hirshman, S F Knowlton, L L Lao, E A Lazarus, and J M Shields
  - Nucl. Fusion **49**, 075031 (2009).
- "Equilibrium and Stability of Current-Carrying Discharges in the Non-Axisymmetric CTH Experiment"
  - J D Hanson, S F Knowlton, B A Stevenson, and G J Hartwell
  - Contrib. Plasma Phys. **50**, 724 (2010)

Soft X-ray Chords
Model Signals

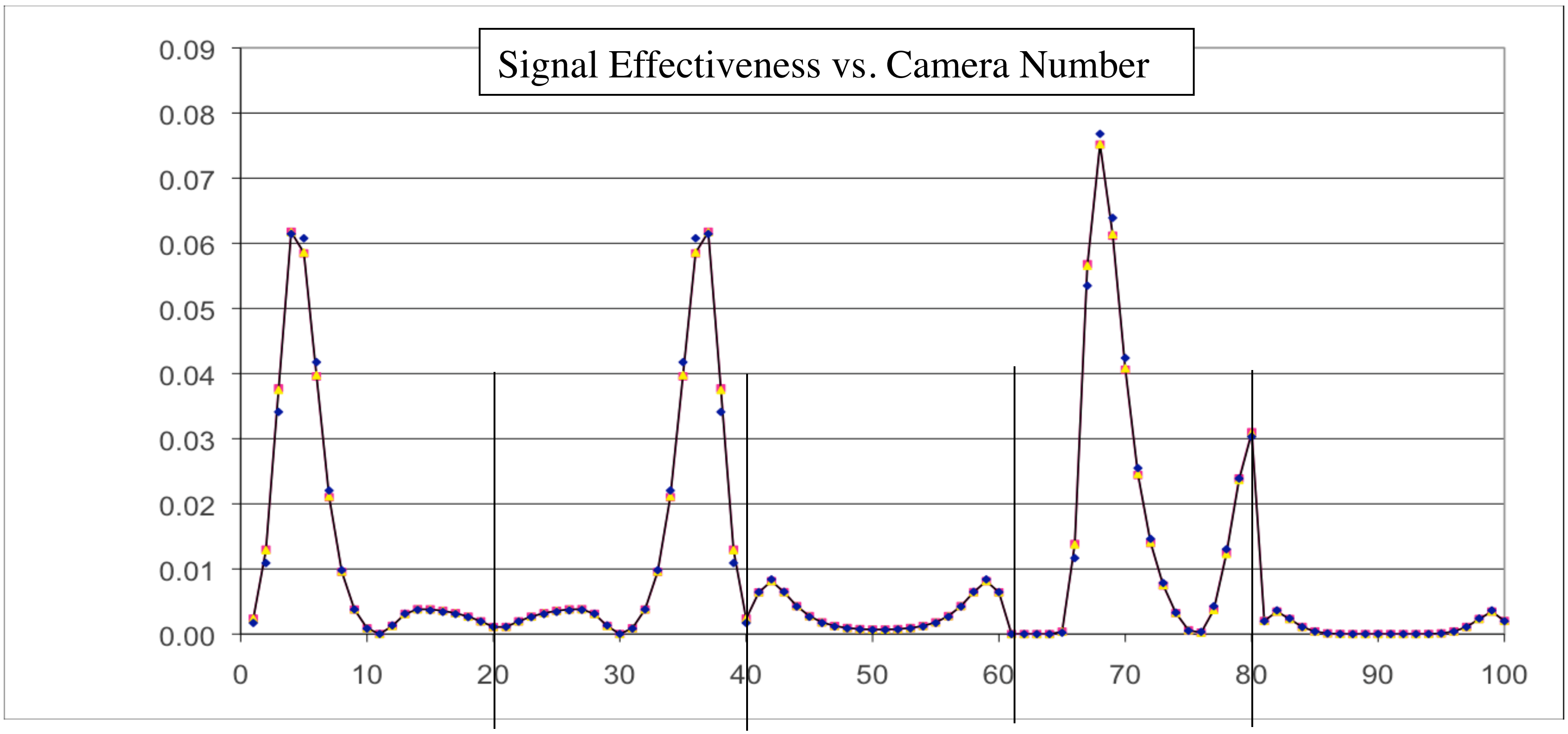






Signal Effectiveness - Soft X-Ray

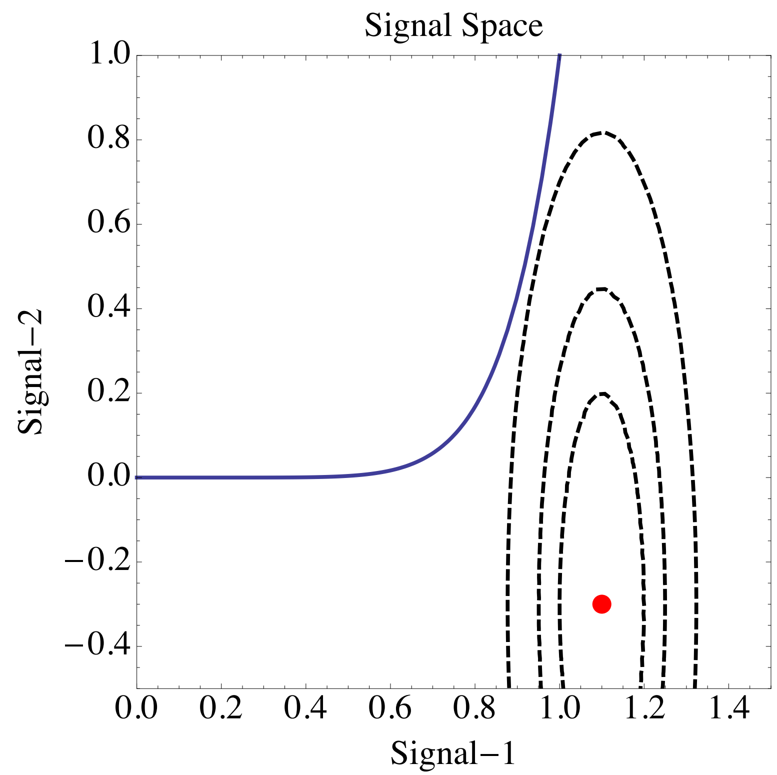
- Signal Effectiveness shown is for measuring the Trim Vertical Field reconstruction parameter.
- Effect of Trim Vertical Field is mainly to move the plasma radially outward.



Signal Space

- The model function $\vec{S}^{\text{model}}(\vec{p})$ takes a point in parameter space to a point in signal space.
- Usually, the signal space has much higher dimensionality that the parameter space.

- Solid curve – Image of a one-dimensional parameter space, $\vec{S}^{\text{model}}(p)$
- Red dot – observed signals $\vec{S}^{observe}$
- Dashed lines – probability distribution of observations



VMEC

- Three-dimensional MHD equilibrium, *assumes* closed, nested flux surfaces
  - Can *not* resolve islands and chaotic regions
  - Uses inverse-coordinate representation
  - Spectral representation for angle coordinates
  - Grid representation for radial coordinate
  - Variational principle – minimizes radial forces on flux surfaces
  - Both free-boundary and fixed-boundary equilibria
- Fast, robust, widely used throughout the world.
  - S. P. Hirshman and J. C. Whitson, *Physics of Fluids* **26**, 3553 (1983).
- VMEC parameters to use for reconstruction:

– Current and pressure profile parameters	am(i), ac(i)
– Pressure scale factor	pres_scale
– Total toroidal current	curtor
– External currents	extcur(i)
– Total toroidal flux within last closed flux surface	phiedge

Algorithm

- Minimize deviation between observed and model signals

$$\chi^2(\mathbf{p}) \equiv \sum_i \left(\frac{S_i^o(\mathbf{d},\mathbf{p}) - S_i^m(\mathbf{p})}{\sigma_i} \right)^2$$

  - Minimize $\chi^2(\mathbf{p})$. Parameters $\mathbf{p}$, Observed signals $S_i^o(\mathbf{d},\mathbf{p})$.
  - Model-computed signals $S_i^m(\mathbf{p})$, uncertainties in signals σ_i .
- V3FIT uses Newton algorithm for new parameters

$$\mathbf{A}^T \cdot \mathbf{A} \cdot \delta \mathbf{a} = \mathbf{A}^T \cdot \mathbf{e}$$

  - Jacobian (normalized) $\quad A_{ij} = \frac{\pi_j}{\sigma_i} \left(\frac{\partial S_i^m}{\partial p_j} - \frac{\partial S_i^o}{\partial p_j} \right)$
  - Error vector $\quad e_i = \left(S_i^o(\mathbf{d},\mathbf{p}) - S_i^m(\mathbf{p}) \right) / \sigma_i$
  - Normalized Parameters $\quad a_j = p_j / \pi_j$

$$\chi^2(\mathbf{p}) = \mathbf{e} \cdot \mathbf{e}$$

$$\mathbf{A} = -\nabla_{\mathbf{a}} \mathbf{e}$$

Algorithm (2)

- Jacobian Calculation
  - Finite difference approximation, $J_{i\,j} \approx \Delta S_i / \Delta p_j$
  - Small Δp in parameter space – VMEC converges rapidly
  - Need moderate accuracy in S_i^m
  - Needs well-converged VMEC
  - Does *not* need high radial resolution – improves speed
  - Use SVD on Jacobian to help avoid large steps in parameter space
- Posterior Sigmas – Confidence Limits on Parameters
  - Assume uncorrelated signals – diagonal signal covariance matrix
  - Parameter covariance matrix (also called posterior covariance)

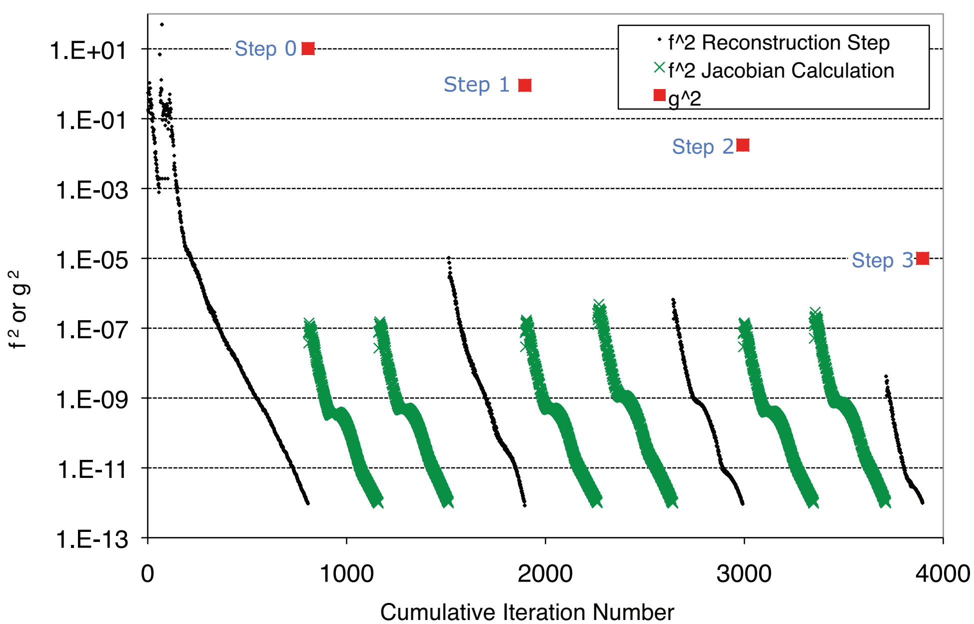
$$\mathbf{C}_p = (\mathbf{J}^T \cdot \mathbf{C}^{-1} \cdot \mathbf{J})^{-1} \qquad C_{ij} = \sigma_i^2 \delta_{ij}$$

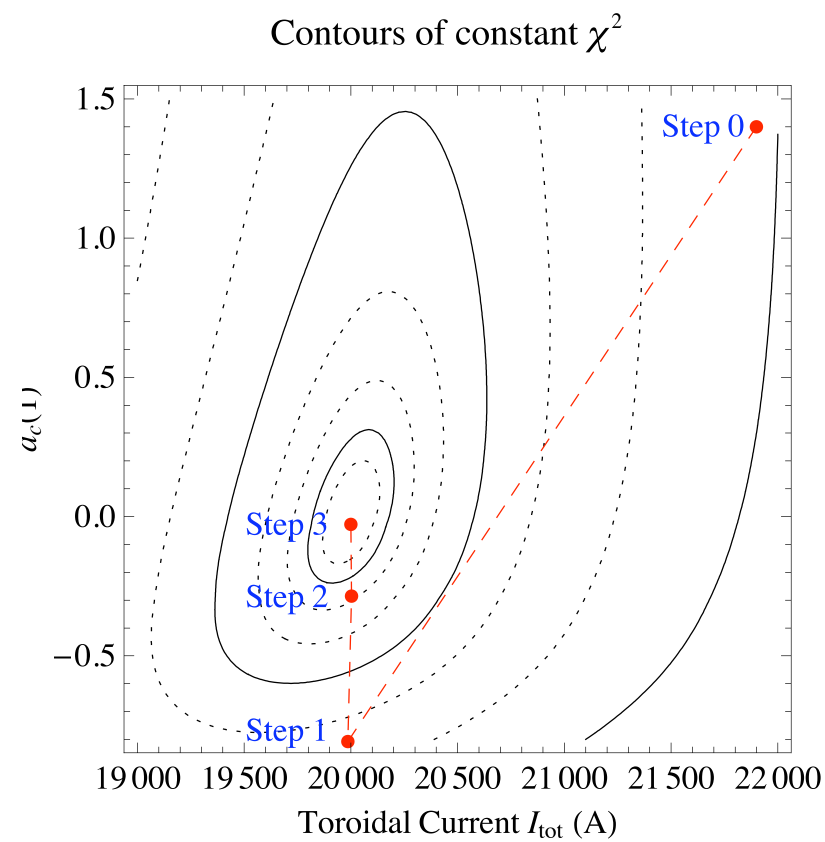
  - Confidence limit on parameter value - Measures how accurately these signals determine the *j*th reconstruction parameter.

$$\sigma_{p\,j} = \sqrt{\left(\mathbf{C}_p\right)_{jj}}$$

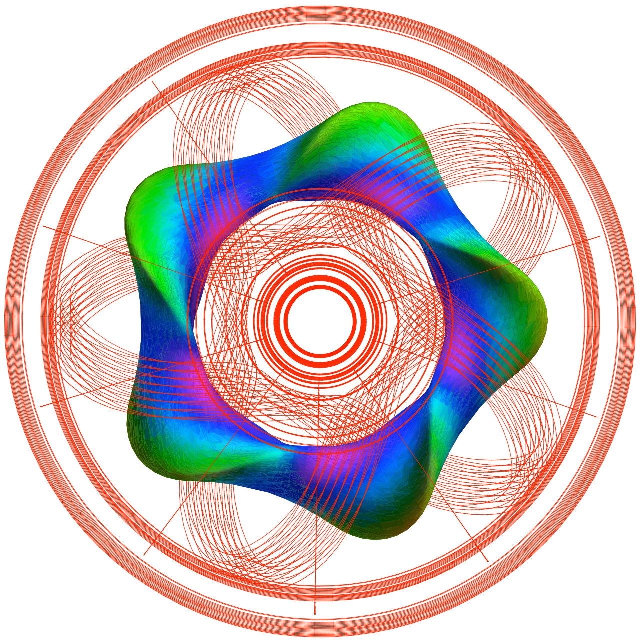
Reconstruction Illustration

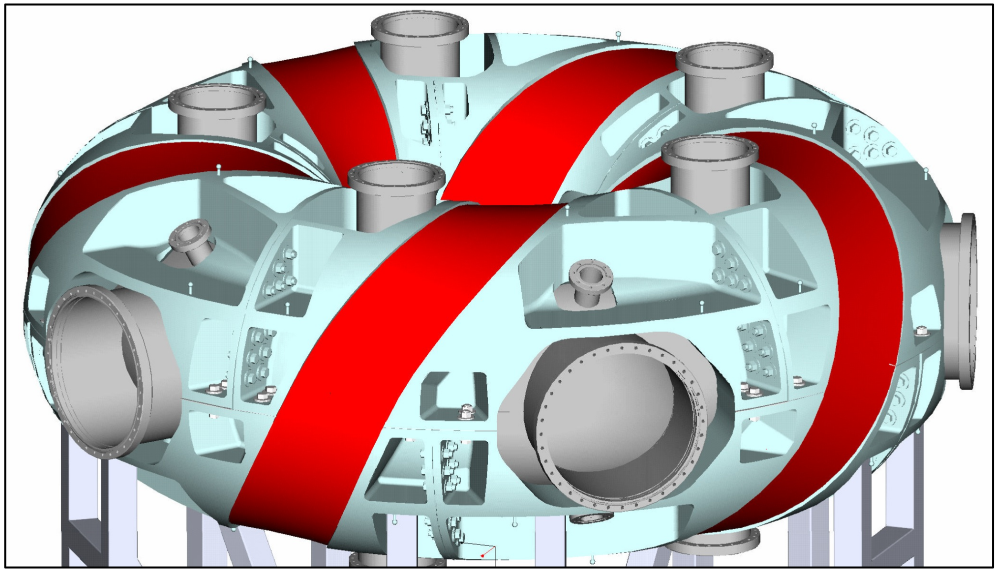
- CTH Equilibrium, Simulated observations
- 2 Parameters
  - Total toroidal plasma current I_{tot} [curtor]
  - Toroidal current profile shape $a_c(1)$
- 12 Magnetic Diagnostic signals
  - Rogowski, 8-part Rogowski
  - Two flux loops, one magnetic probe





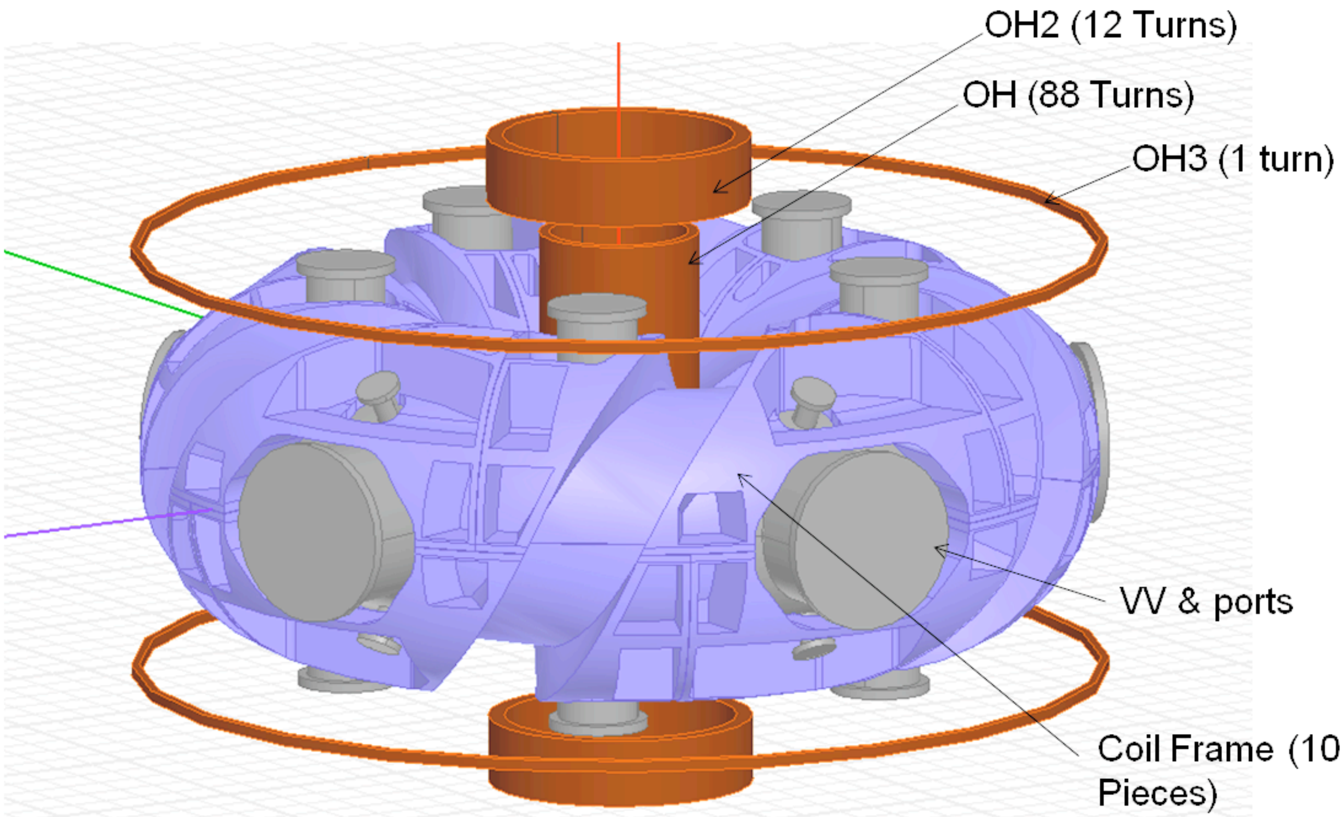
CTH – Compact Toroidal Hybrid





- Auburn University, Steve Knowlton – Principal Investigator
- Torsatron with Ohmic heating coils
- Major radius 0.75 m

CTH – Compact Toroidal Hybrid



Magnetic Diagnostics

$$\Phi_i = \sum_j M_{ij} I_j^{external} + \int_{plasma} \vec{J}(\vec{r}\,') \cdot \left(\frac{\mu_0}{4\pi} \oint \frac{d\vec{\ell}_i}{\left| \vec{\ell}_i - \vec{r}\,' \right|} \right) d\vec{r}\,'$$

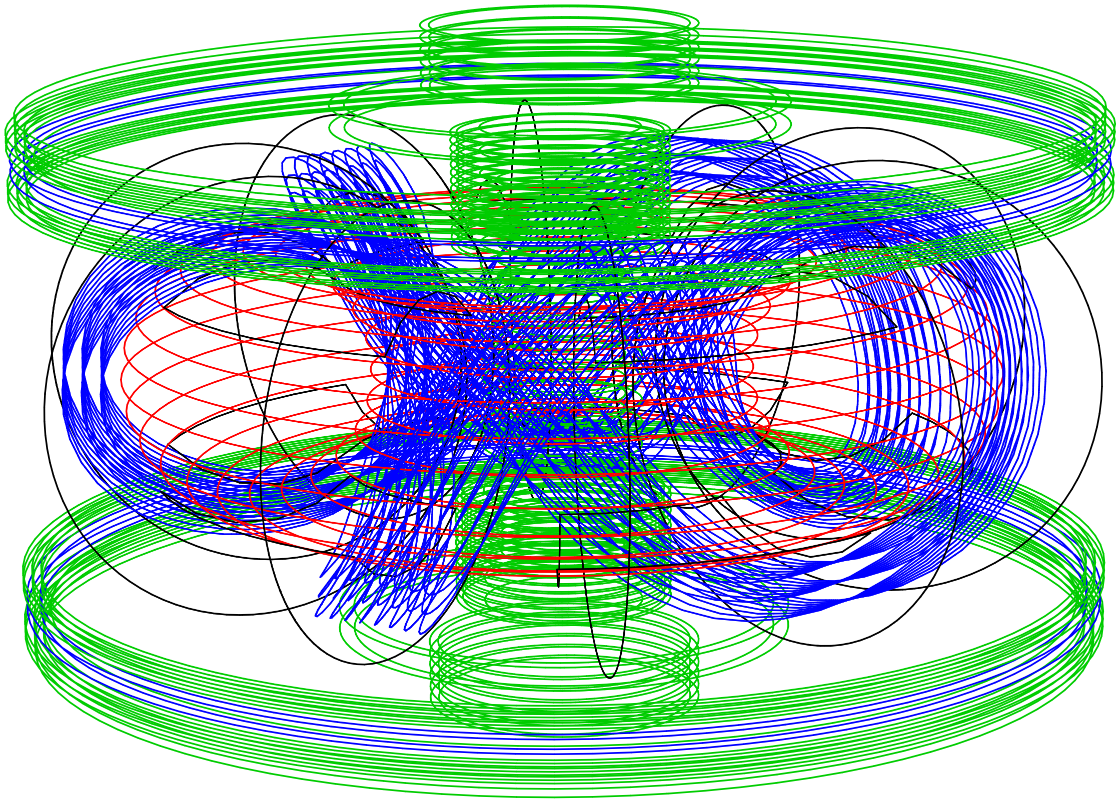
- Signal depends on details of external currents
  - Mutual inductance matrix *M* depends on geometry of both external coil and diagnostic coil
- Plasma contribution is a *volume* integral over the whole plasma
- The plasma contribution *only* depends on equilibrium quantities – no other plasma information is needed.

Details of implementation, see "*Magnetic diagnostic responses for compact stellarators*", S. P Hirshman, E. A. Lazarus, J. D. Hanson, S. F. Knowlton, and L. L. Lao, Phys. Plasmas **11**, 595 (2004).

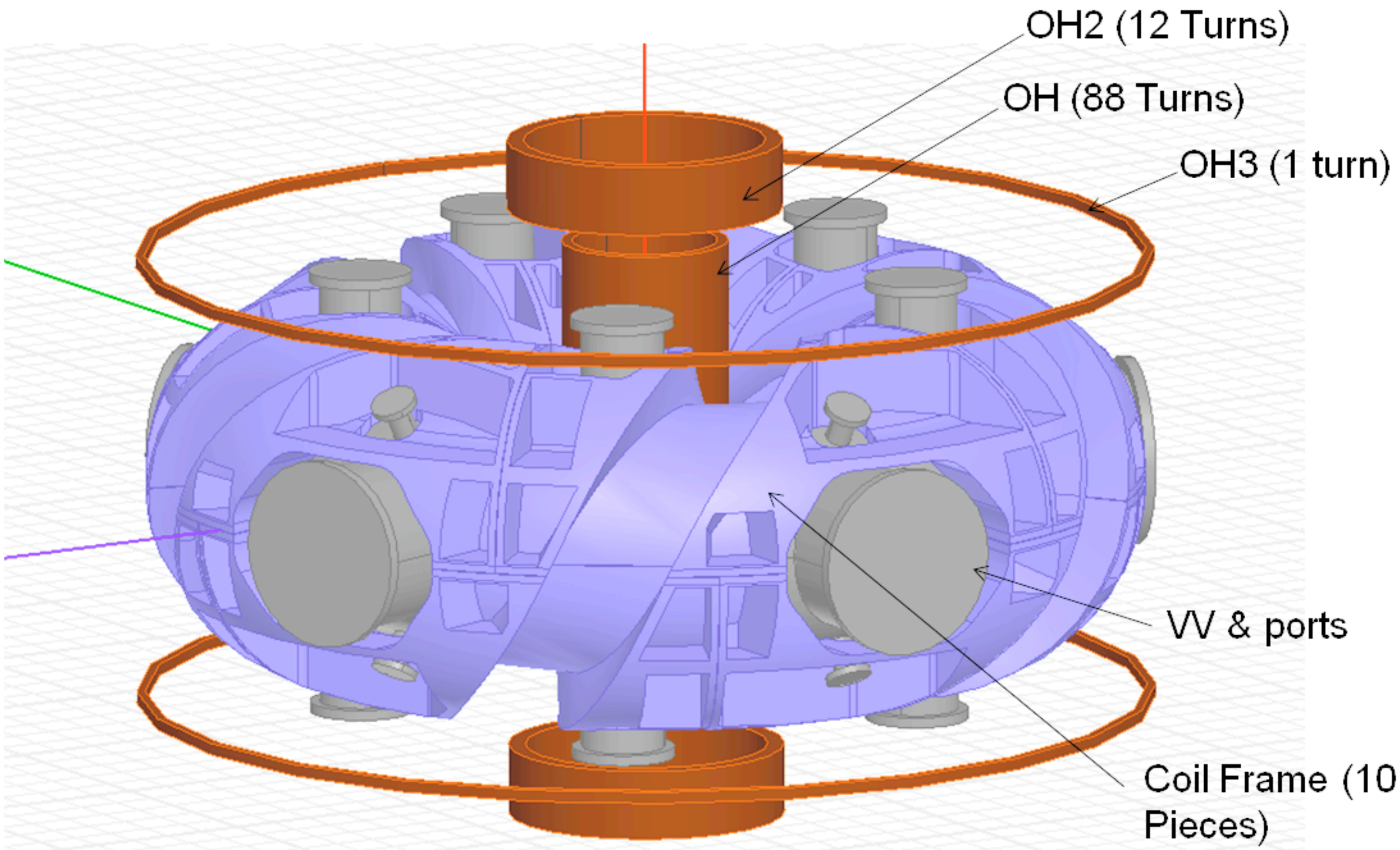
Eddy Current Modeling

- Eddy currents in the CTH coil frame and vacuum vessel can be included in the MHD equilibrium as external magnetic fields – included in the coils_dot file
- Measured currents in these structures are significant, induced by the ohmic loop voltage (and other time-changing magnetic fields)
- Auburn University requested assistance from Princeton Plasma Physics Laboratory "Offsite University Research Program" for help modeling the eddy currents in these complicated structures.
- Ali Zolfaghari and Dave Gates from PPPL.
- CAD models of structures -> eddy current modeling code SPARK.
- Translate results in format appropriate for VMEC input.

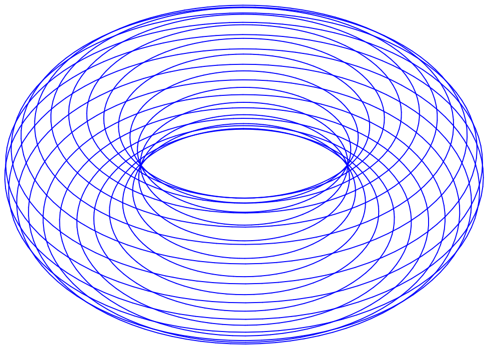
CTH External Coils

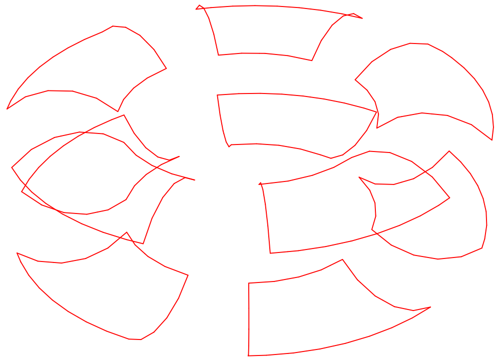


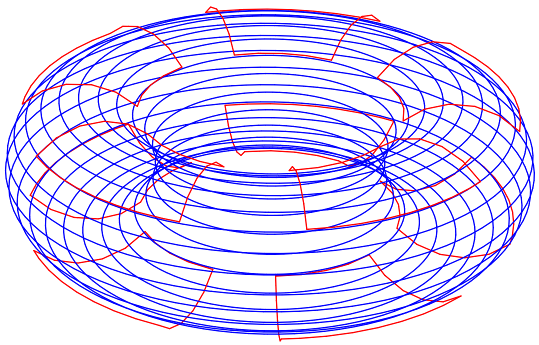
CTH


OH2 (12 Turns)
OH (88 Turns)
OH3 (1 turn)
VV & ports
Coil Frame (10 Pieces)

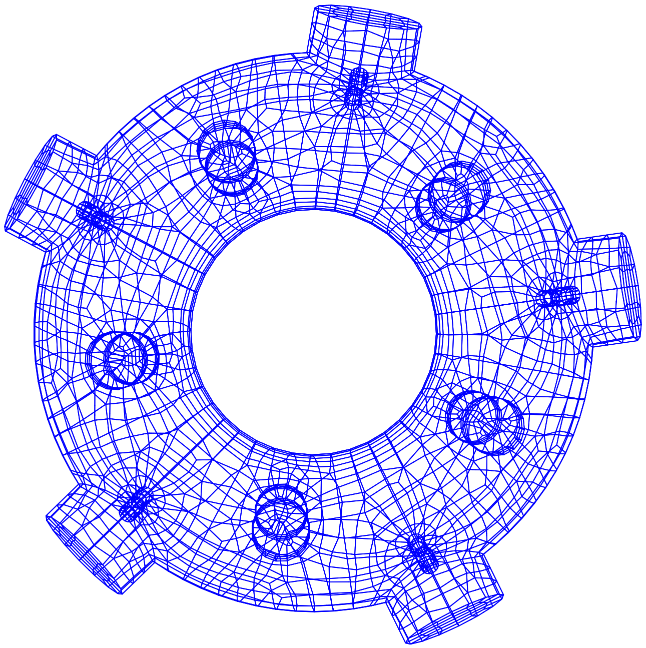
Vaccum Vessel and Coil Frame



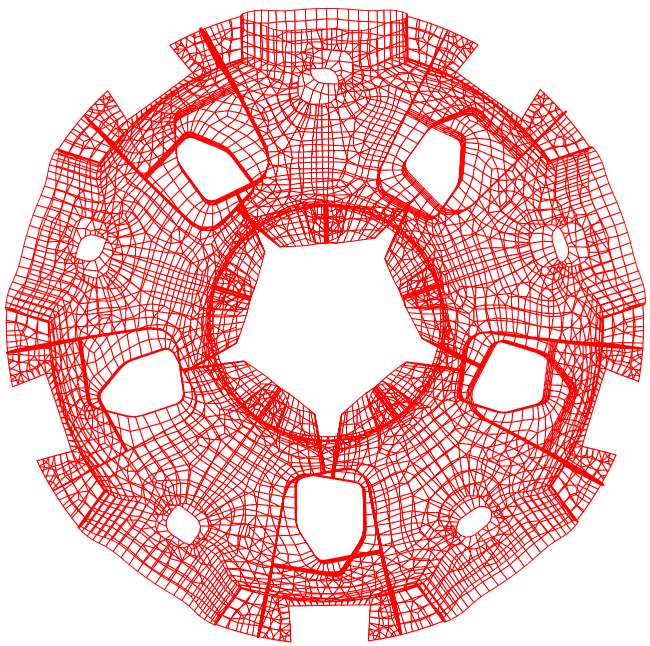




PPPL Results Translated to VMEC Format



Vacuum Vessel



Coil Frame

- Status – Checking results from PPPL
- Expect V3FIT reconstruction comparisons soon.
- See 3D versions on iPad