Physics of Tearing Mode Response to 3D Fields

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General Atomics

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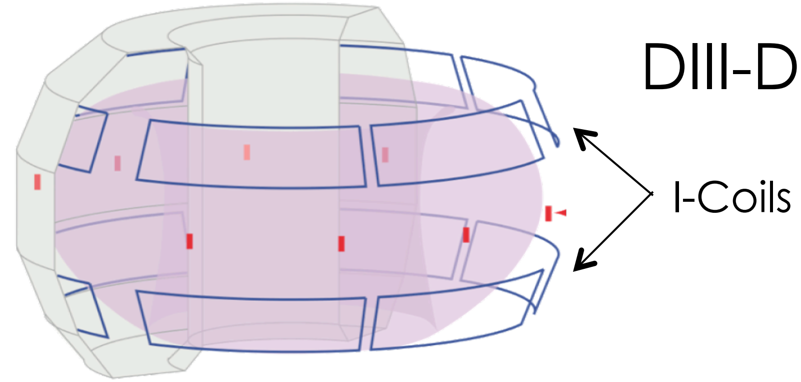
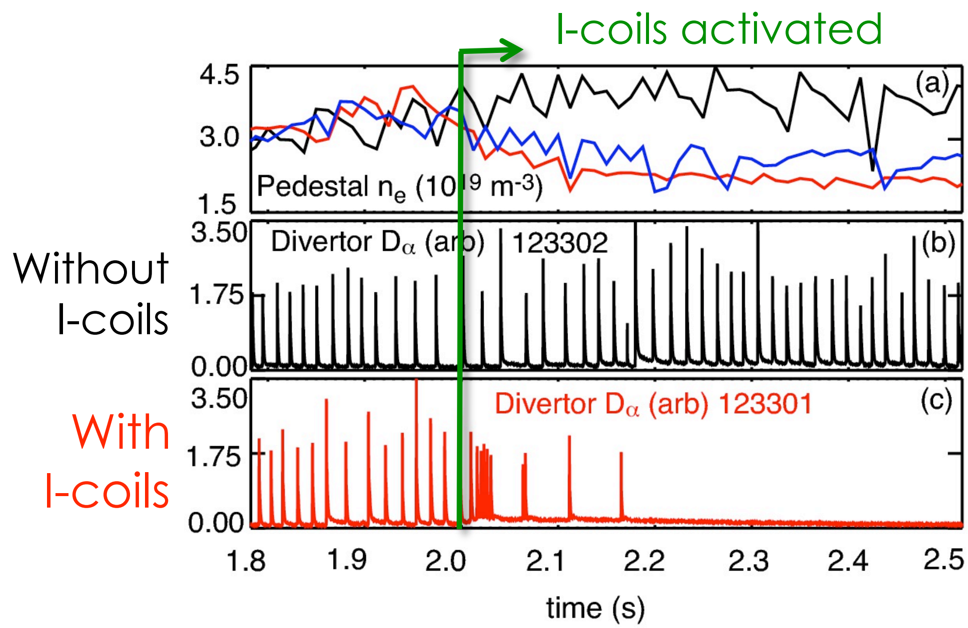
GENERAL ATOMICS

3D Fields Significantly Affect Tokamak Performance

- **Edge-Localized Modes**
  - Mitigate/suppress ELMs in H-mode
- **Transport**
  - Density pump-out
- **Drive/Brake Rotation**
  - Affects RWM stability
  - Affects tearing mode stability
  - Allow access QH-mode without NBI





Evans, *et al. Phys. Plasmas* **13** (2006)

A Predictive Capability Requires Understanding Plasma Response

- **Predictive capability is challenging because plasma response is complicated**
  - Plasma may strongly enhance/suppress spectral components of applied field
  - New fields affect transport and rotation
  - Rotation strongly affects plasma response

- **New tools are being developed and applied to gain predictive understanding (M3D-C1)**

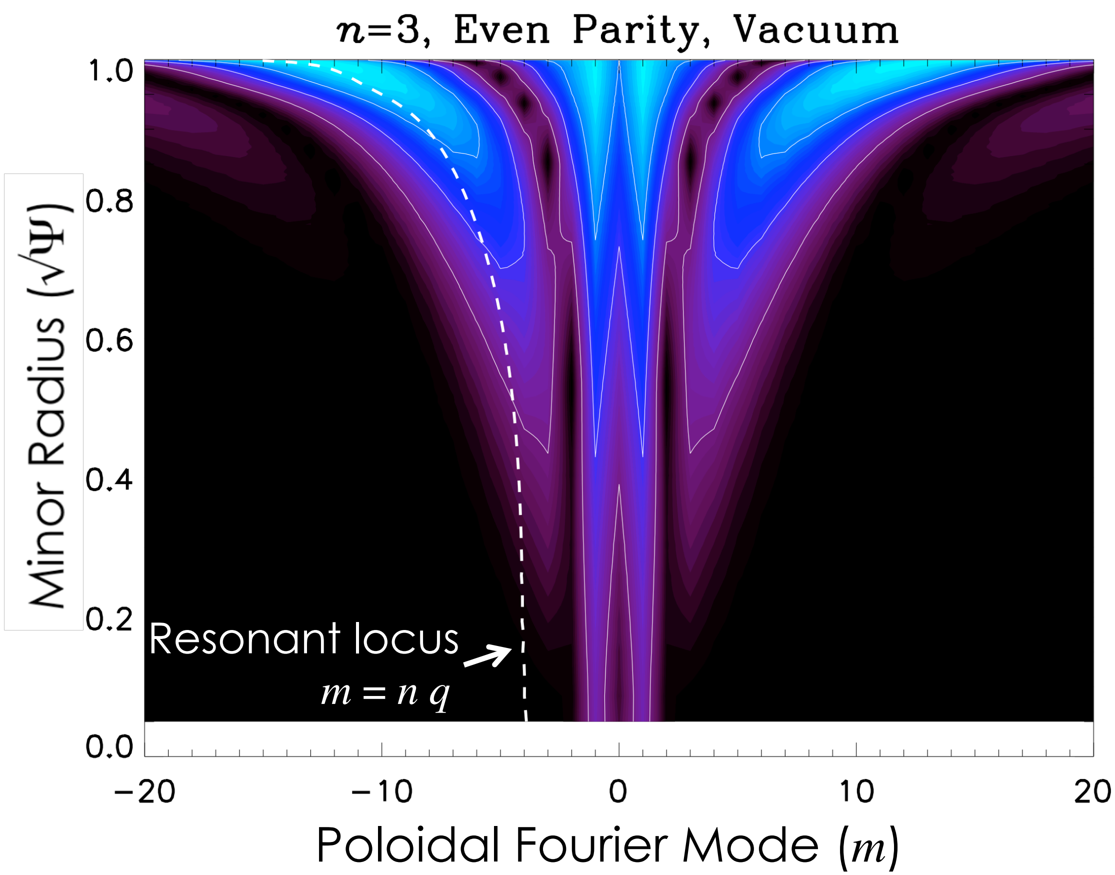
Outline

- **Basics of 3D Response**
- **Introduction to M3D-C1**
- **Linear Results**
  - Influence of rotation
  - Ion rotation vs. Electron rotation
  - When is a linear model appropriate?
- **Implications for Stability Control**

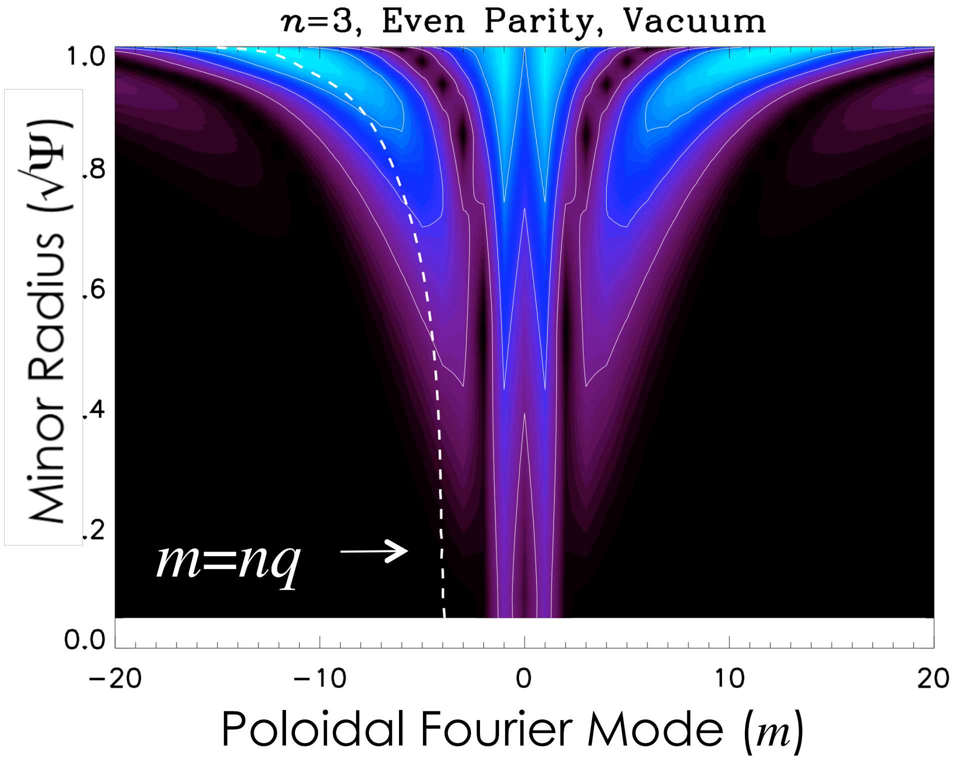
3D Response Basics

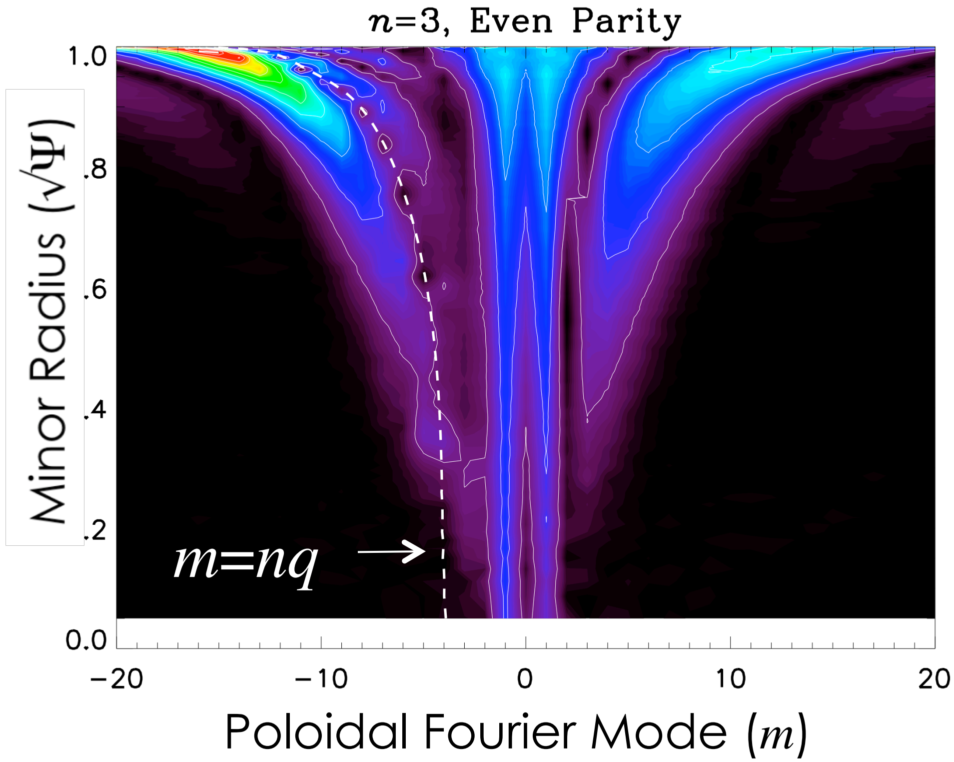
Resonant Fields Tear Surfaces; Non-Resonant Fields Bend Surfaces

- **Plot shows Fourier spectrum of B_n**
- **B_n = component of applied field normal to equilibrium magnetic surfaces**
- **Resonant components (along dashed line) cause islands**
- **Non-resonant components cause bending of surfaces**
- **Poloidal spectrum of B_n depends on Ψ**



Plasma Response Modifies Spectrum





- **Ideal response → no islands → reduction in resonant components**
- **Excited ideal modes → enhancement of non-resonant components**

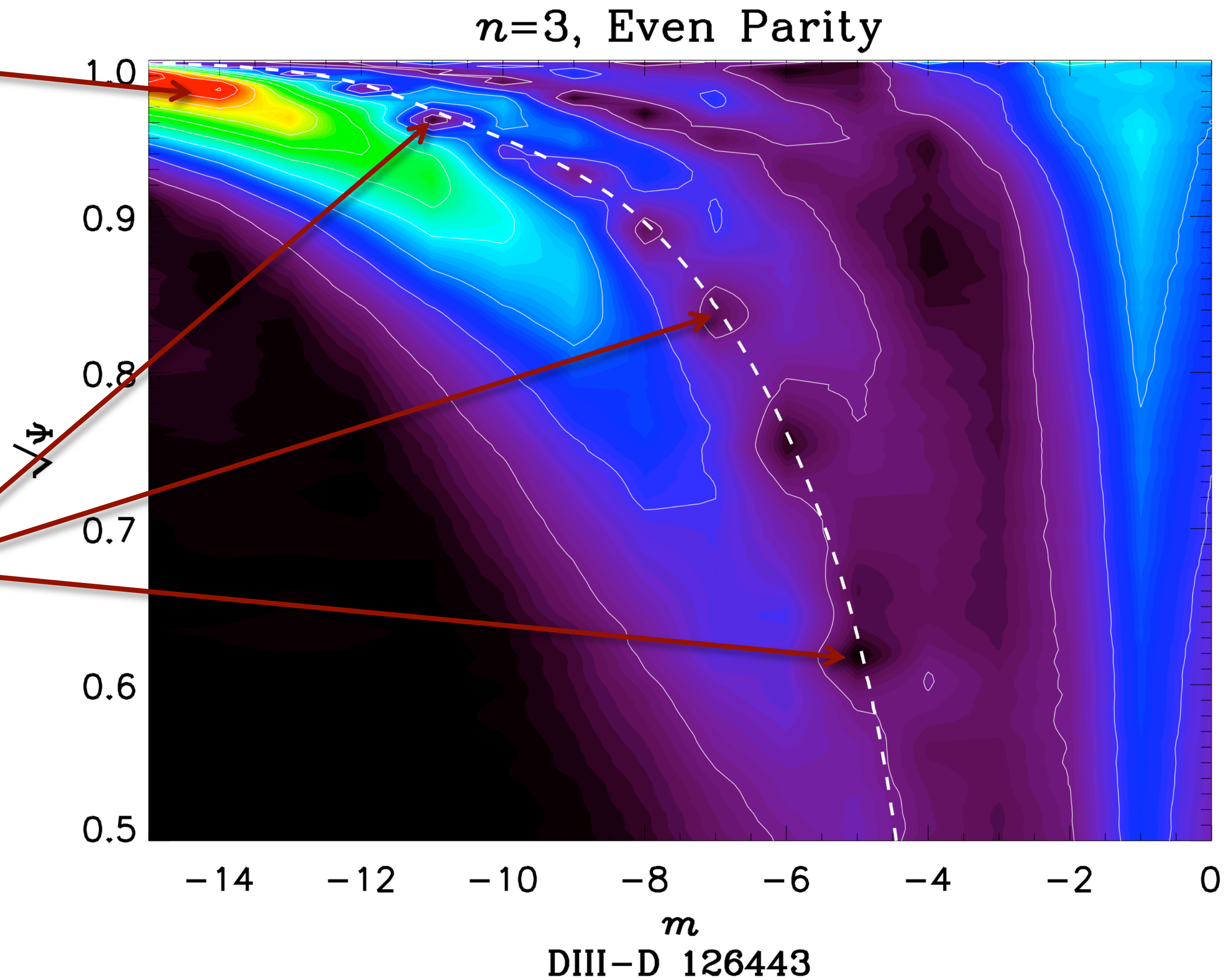
Plasma Can Kink and Screen

"Kinking"

- **Distorts surfaces**

Screening

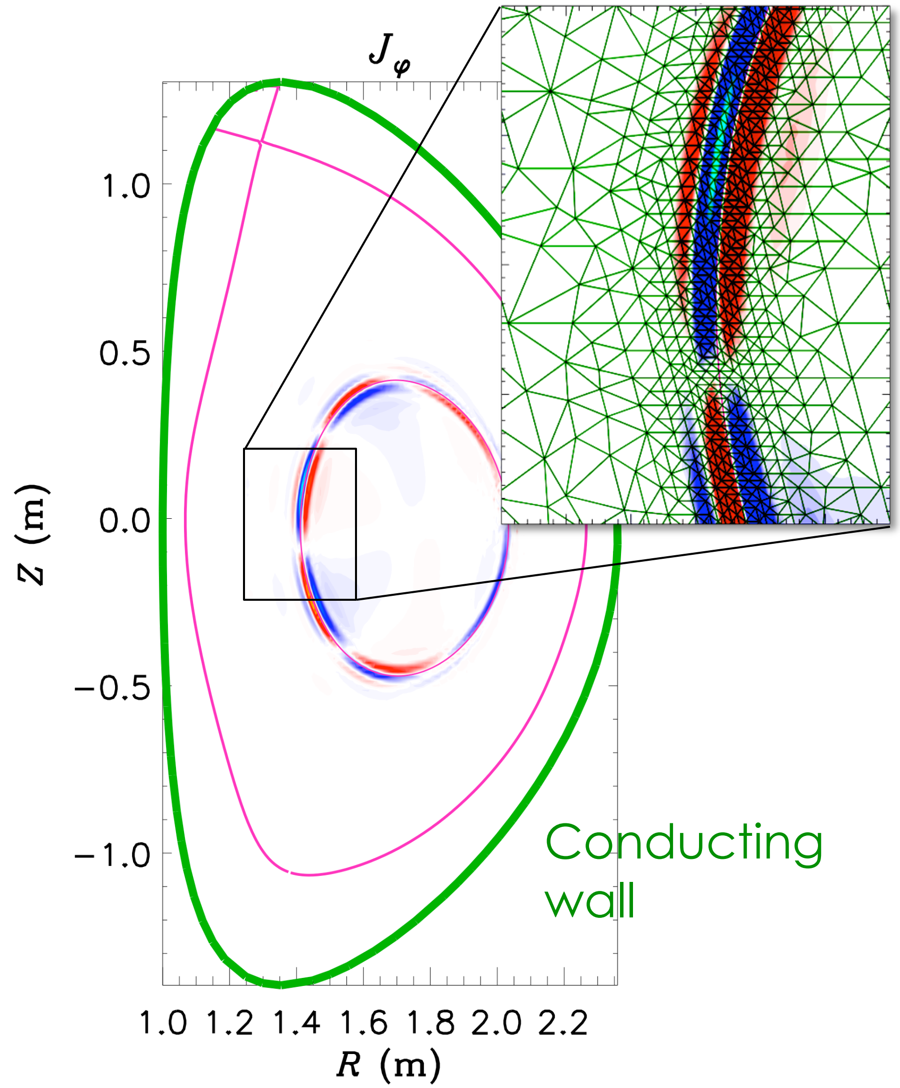
- **Eliminates islands**



Numerical Methods (M3D-C1)

M3D-C1 Can Calculate Two-Fluid Response

- **M3D-C1 is a two-fluid resistive finite element code**
  - Shares some design principles with M3D
  - (R,Z) coordinates (not spectral in poloidal angle)
- **Computational domain includes plasma, separatrix, and open field-line region**
- **Unstructured mesh allows resolution packing at rational surfaces**
- **Both linear and nonlinear models are implemented**



Two-Fluid Model

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$n\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\Gamma p \nabla \cdot \mathbf{u} - \frac{d_i}{n}\mathbf{J} \cdot \left(\Gamma p_e \frac{\nabla n}{n} - \nabla p_e\right) - (\Gamma - 1)\nabla \cdot \mathbf{q}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} + \frac{d_i}{n}(\mathbf{J} \times \mathbf{B} - \nabla p_e)$$

$$\Pi = -\mu\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right]$$

$$\mathbf{q} = -\kappa \nabla\left(\frac{p}{n}\right) - \kappa_{\parallel} \mathbf{bb} \cdot \nabla\left(\frac{p_e}{n}\right)$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

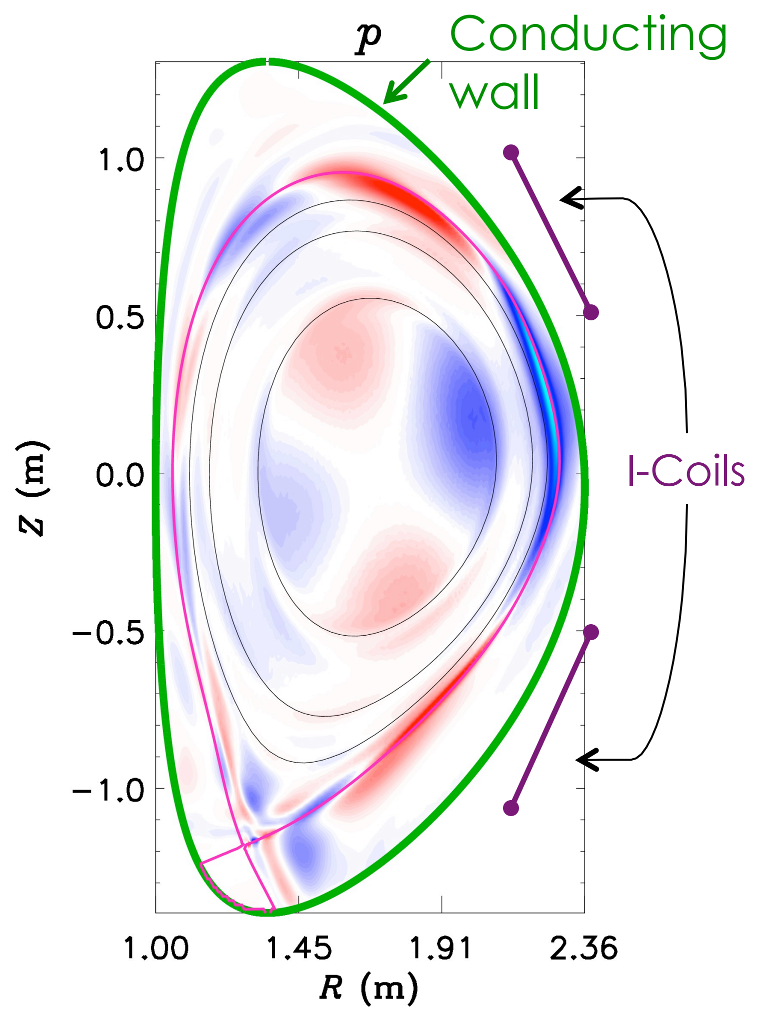
$$\Gamma = 5/3$$

$$p_e = p/2$$

- Complete (not reduced) two-fluid model is implemented
- Time-independent equations may be solved directly for linear response

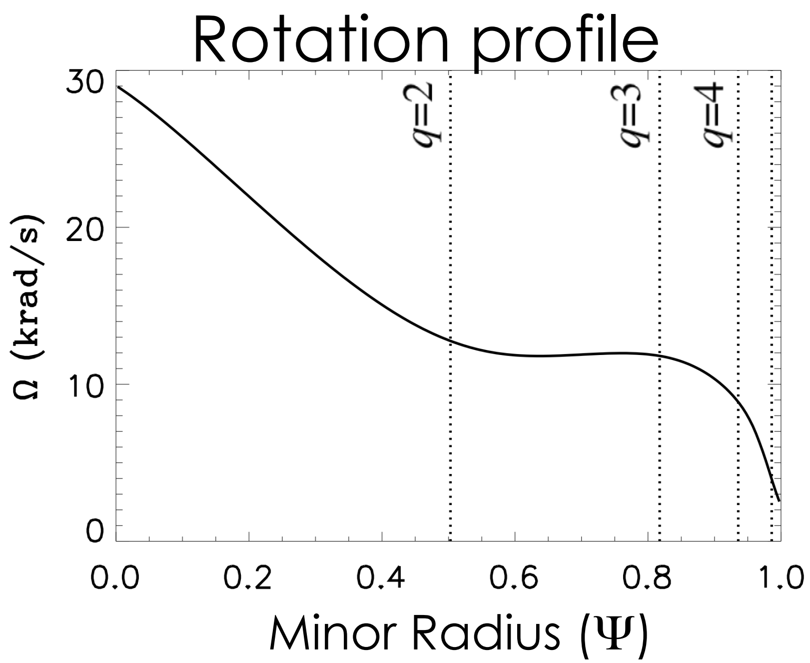
Analysis Considers Reconstructed DIII-D Equilibria

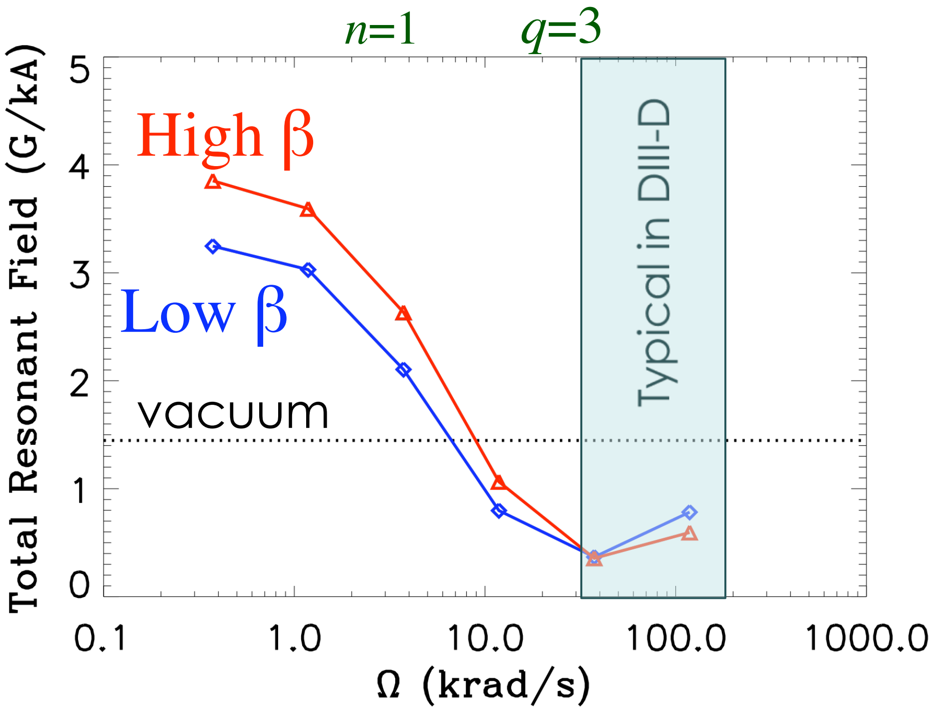
- **Vacuum fields generated by DIII-D I-coils**
- **Boundary conditions:**
  - Vacuum B_n is held constant at the boundary
  - No-slip ($\mathbf{v}=0$)
- **Realistic transport parameters**
  - Lundquist number ~ 10^9
- **Toroidal rotation**
  - Rotation is added self-consistently: $p \neq p(\psi)$



Single-Fluid Results

Single-Fluid Result:
Rotation (Usually) Improves Screening





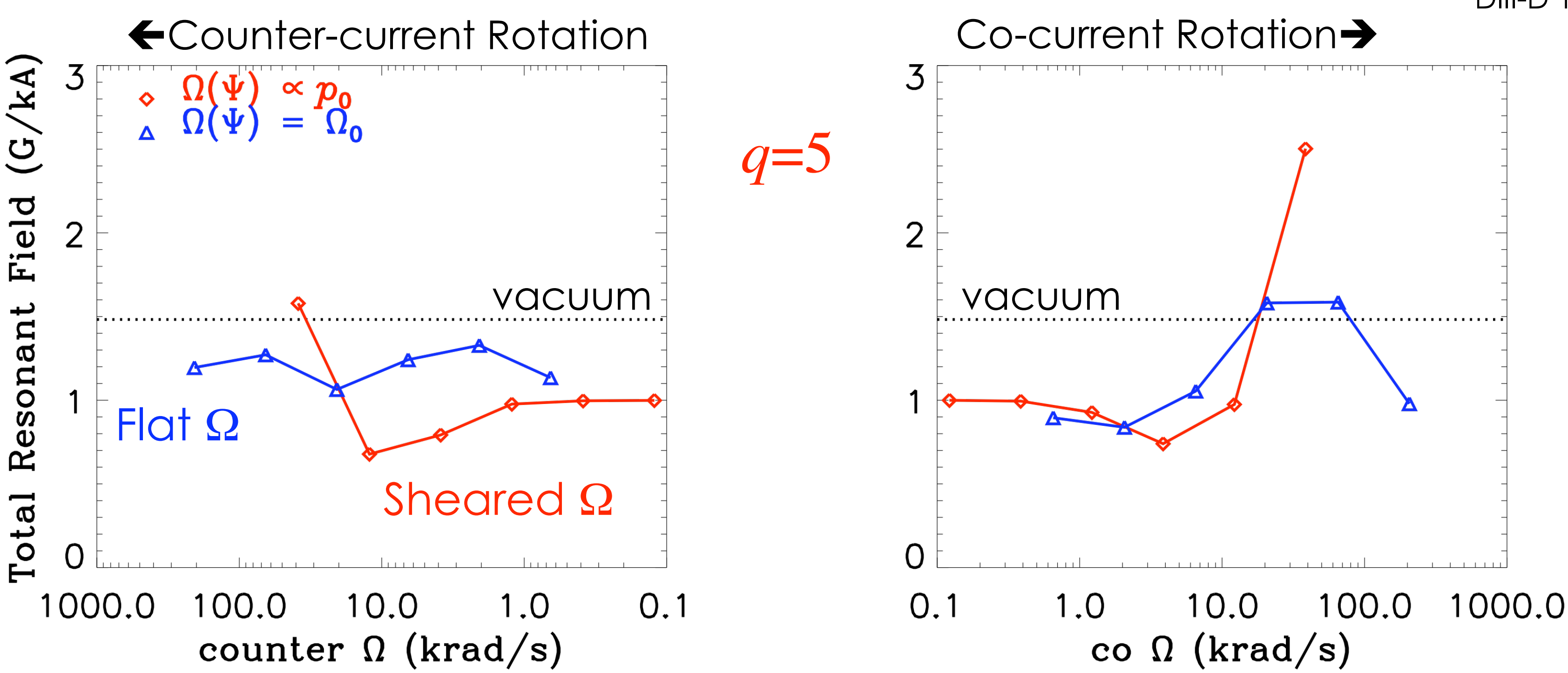
- **Plasma may enhance resonant fields at low rotation**
- **Large rotation screens resonant fields**
- **Response depends on beta**

Why Is Plasma Response Sensitive to Rotation? Why Is It Sensitive To Beta?

- **From a (rotating) plasma's perspective, the static external fields are oscillating**
  - If field is oscillating faster than tearing response, plasma won't tear
- **Rotation drives static tearing modes away from marginal stability**
- **Higher Beta moves modes closer to marginal stability**
  - At marginal stability, an infinitesimal perturbation yields an infinite response

Single-Fluid Result:
Rotation Shear Increases Edge Response

DIII-D 135762

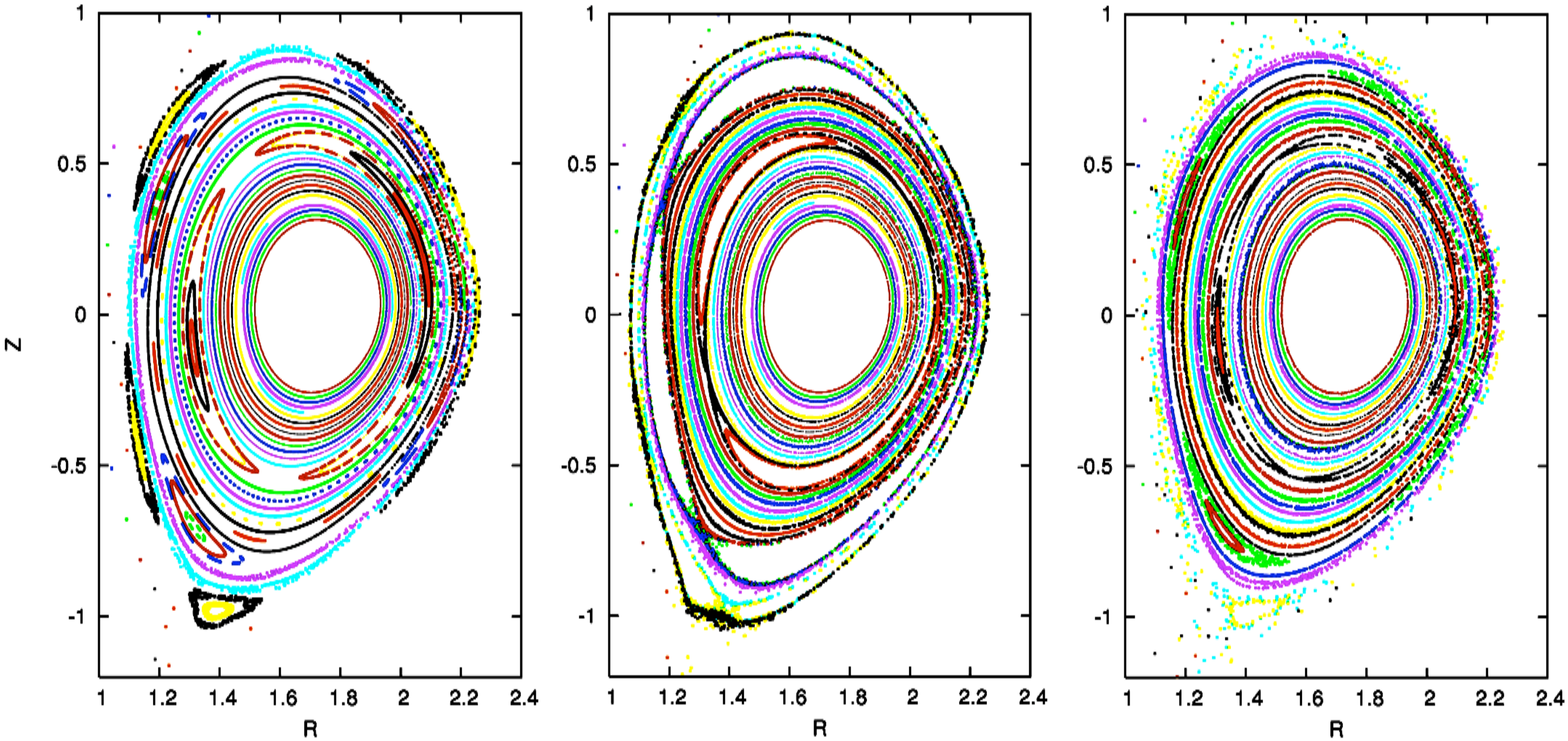


- **Large rotation shear seems to increase edge response**
- **Why? Theory predicts Ω' is destabilizing to low-*n* peeling-ballooning modes***

* Snyder, *et al.* *Nucl. Fusion* **47** (2007)
Aiba, *et al.* *Nucl. Fusion* **50** (2010)
Ferraro, *et al. Phys. Plasmas* **17** (2010)

Rotation Improves Core Screening; But Sheared Rotation Stochasticizes Edge

Vacuum | Plasma, Static | Plasma, Rotating

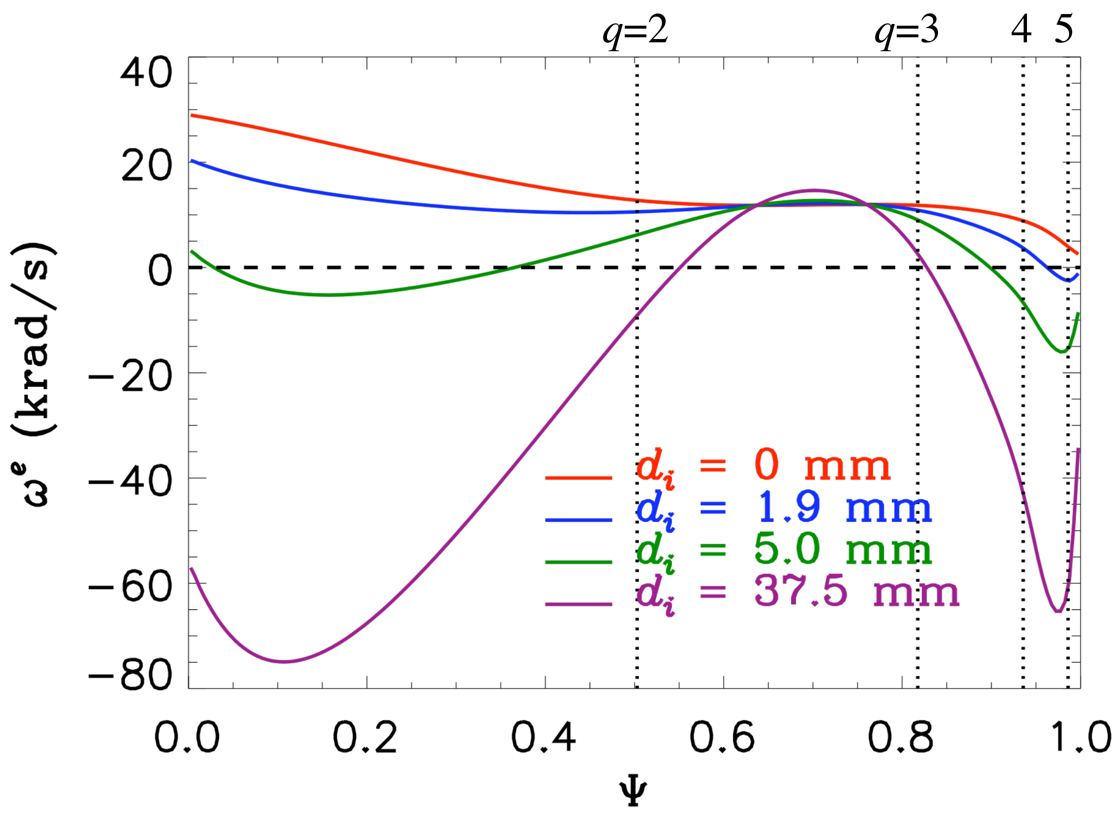


Ω_0=0 | Ω_0=300 krad/s

Two-Fluid Results

Two-Fluid Results: Ion and Electron Rotations are Distinct

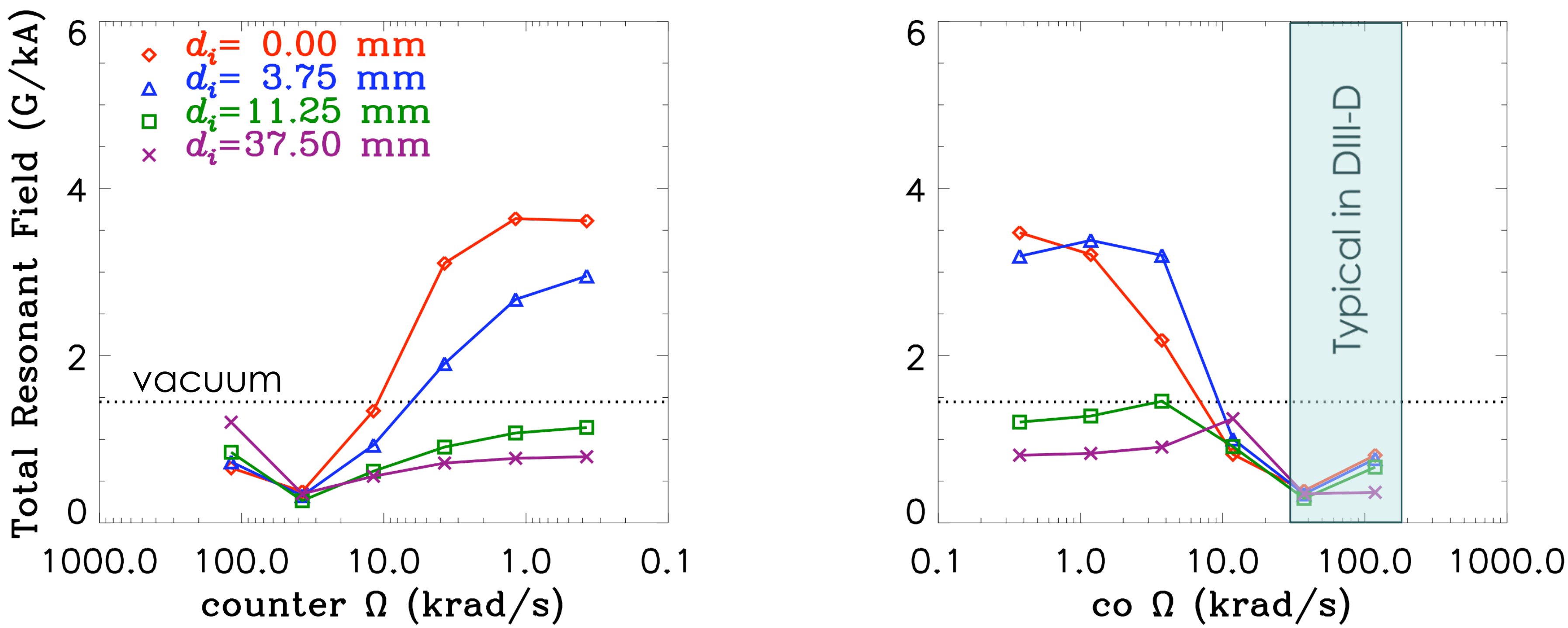
- **Given Ω, we can change $\Omega^e = \Omega + \omega_*$ by adjusting** $\omega_* = d_i\, p'/n$



For this equilbrium, $d_i = 37.5$ mm is the physical value

Two-Fluid Effects Shift Resonance

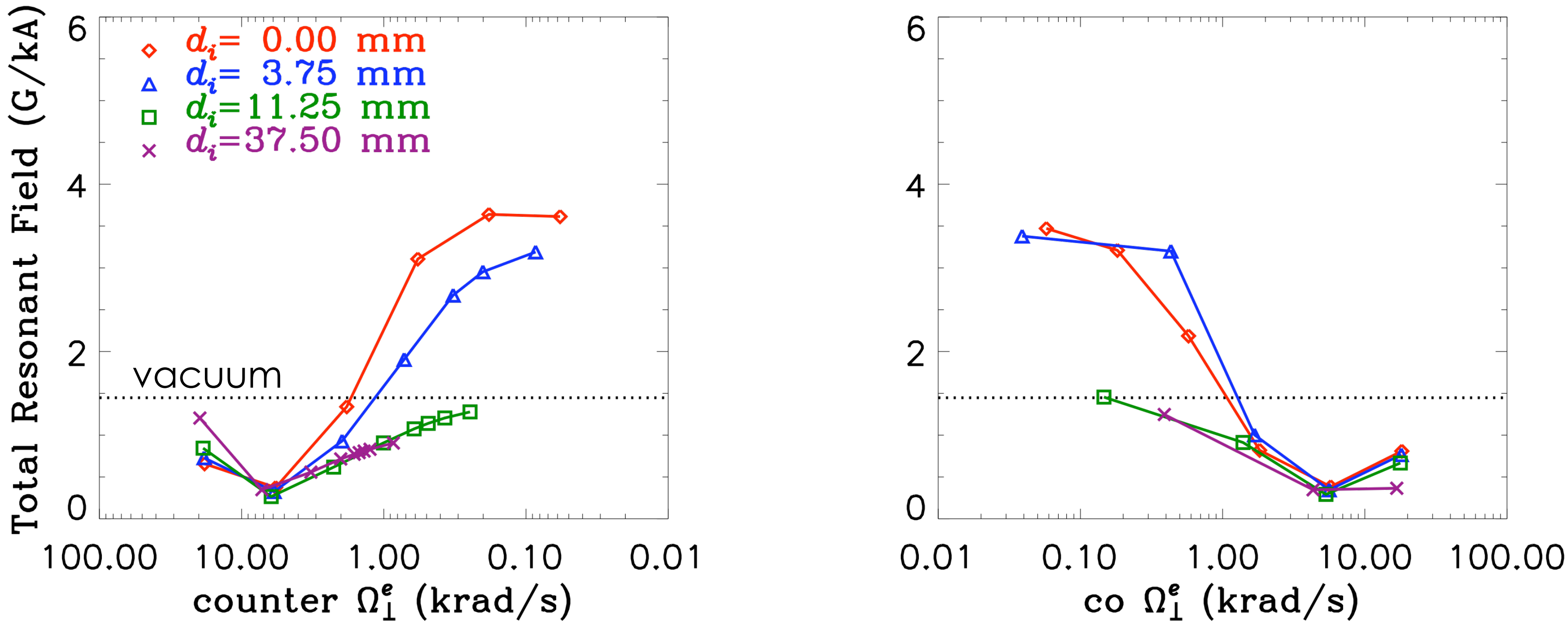
(Mass) rotation at $q=3$



- **Strongest tearing no longer occurs at $\Omega = 0$**

Penetration In Core Depends on Electron Rotation

Perpendicular electron rotation at $q = 3$



- **Screening of q=3 island clearly depends more on Ω^e than Ω**

What is "Perpendicular" Electron Velocity?

- **The perpendicular angular velocity is defined as**

$$\Omega_{\perp}^{e,i} = \frac{\mathbf{v}^{e,i}}{R} \cdot \frac{\mathbf{B} \times \nabla\psi}{|\mathbf{B} \times \nabla\psi|}$$

- **To lowest order,** $\mathbf{v}^{e,i} = R^2\omega^{e,i}(\psi)\nabla\varphi + \lambda^{e,i}\mathbf{B}$. **Thus:**

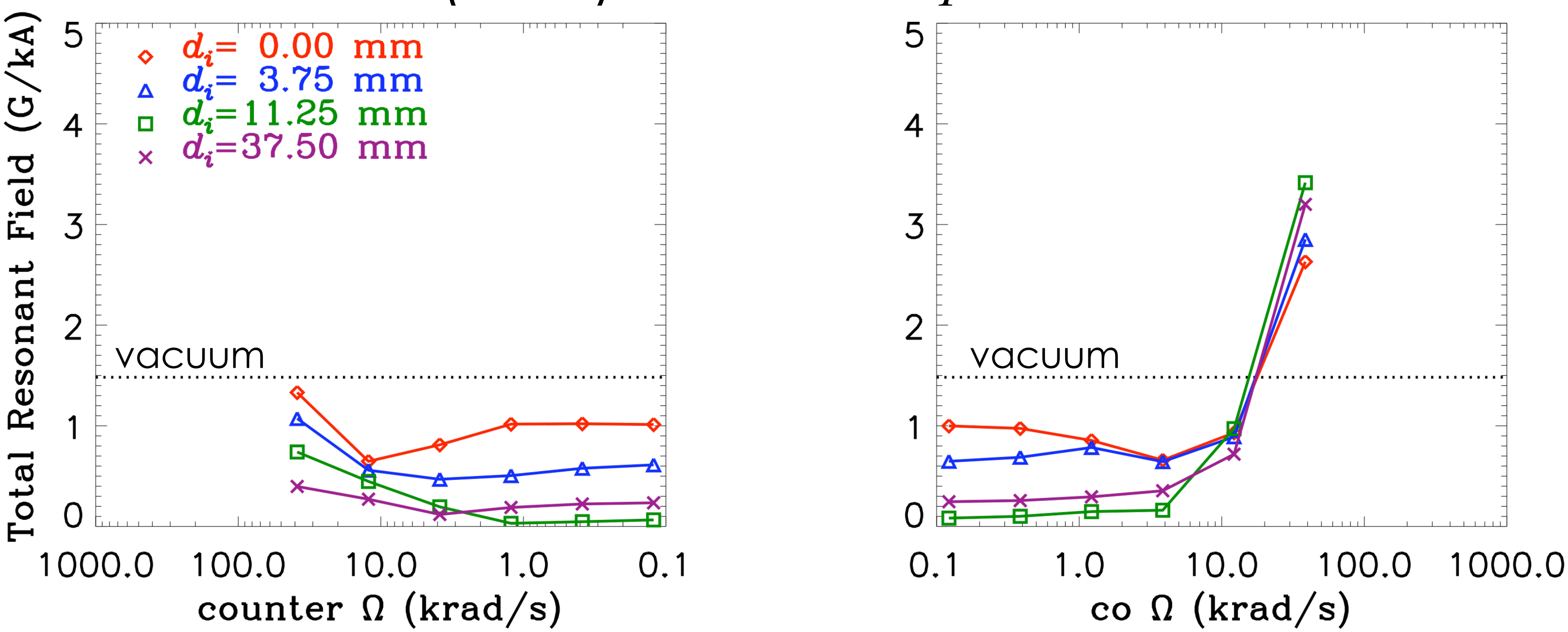
$$\Omega_{\perp}^{e,i} = \frac{|\nabla\psi \times \nabla\varphi|}{|B|}\omega^{e,i}(\psi)$$

- **From radial force balance:** $\omega^{e,i}(\psi) = \phi'(\psi) + \frac{p_{e,i}{}'(\psi)}{n_{e,i}q_{e,i}}$

Edge Response Depends on Mass Rotation Shear

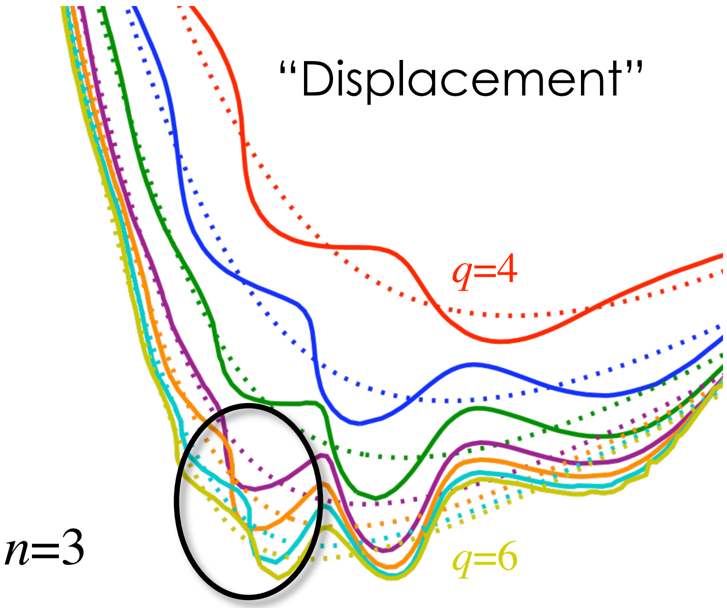
- **Tearing of edge modes is dependent on ion, not electron, rotation shear**

(Mass) rotation at q=5



When is Linear Response Appropriate?

- **For typical experimental parameters, linear response may not be strictly valid in some regions**
  - Large current density near rational surfaces
  - Back-reaction on rotation is important



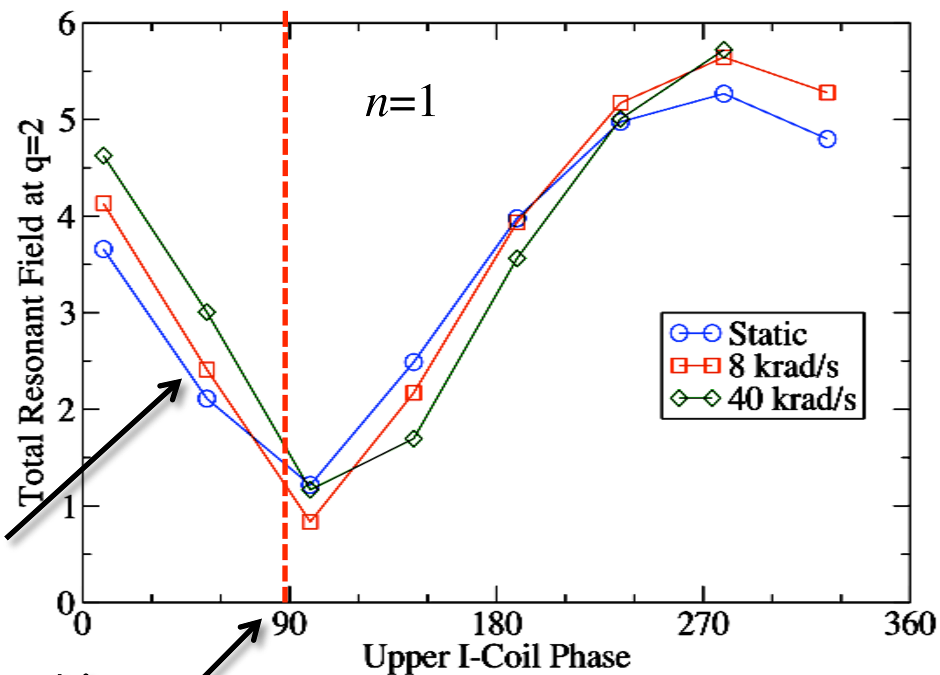
5 kAt even-parity I-Coil

  - "Displacement" shows overlapping surfaces near separatrix!

- **Quantitative predictions of island size, stochasticity from linear calculations are suspect**

Linear Response Gets Some Things Right

- **Which modes are most sensitive**
- **How parameters (rotation, viscosity, etc.) affect sensitivity**
- **How to optimize coil design**



Calculated resonant field (proxy for resonant torque)

Empirical phase least prone to locking

DIII-D 144182

Summary of Theory/Modeling Results

- We can now quantify the dependence of plasma response on parameters in experimentally relevant regimes
- Tearing response is closely correlated with rotation
- Core tearing is correlated with electron rotation
- Edge response is correlated with ion (mass) rotation shear

Confidence in Control Methods Require Model Validation

- **Linear modeling is probably sufficient for coil optimization**
- **Nonlinear (or QL) modeling is necessary for other things**
  - Changes to n=0 rotation/pressure profiles
  - Locking bifurcation threshold
  - Effect of 3D equilibrium on ELM stability
- **Modeling to test/inform ELM suppression hypotheses is underway**

Implications For ELM Suppression Control

- **Empirical q_{95} resonances strongly imply that current profile control will be necessary**

- **It is not yet clear whether tearing is a necessary condition for ELM stabilization**
  - If it is, then modeling implies **rotation control** might be necessary
    - ECCD (electron rotation control) can cause/heal tearing
  - Feedback on island size is not feasible

- **If transport (in pedestal or at pedestal top) is necessary, are there other ways of driving it?**