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Modeling of the Neoclassical Toroidal Plasma Viscosity Torque in Tokamaks

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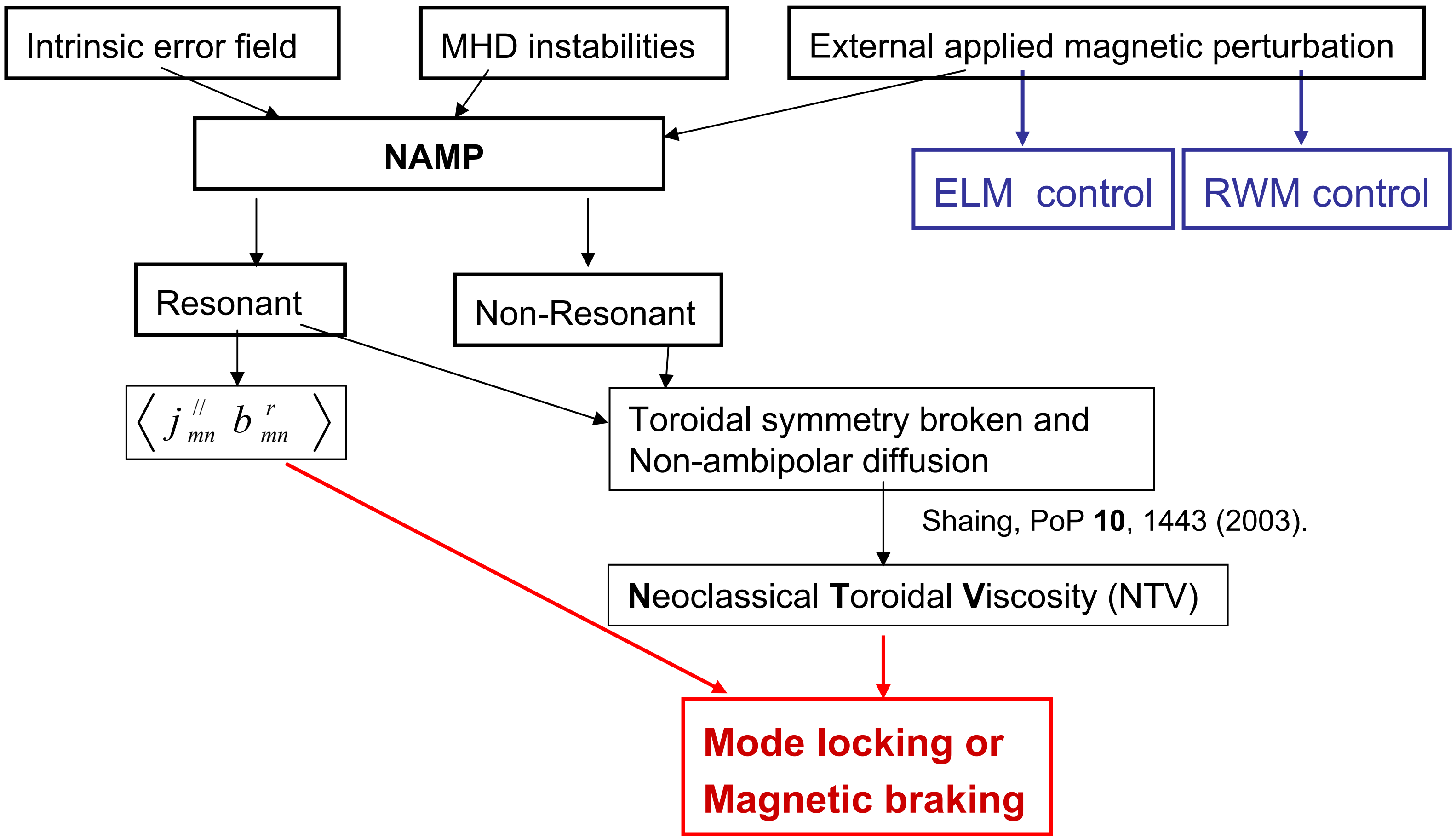
Non-Axisymmetric Magnetic Perturbation (NAMP)



Background

- **Non-resonant magnetic braking observations :**
  - ✓ NSTX [Zhu *et al.*, PRL **96**, 225002 (2006), Park *et al.*, PoP 16, 056115 (2009) and Sabbagh *et al.*, NF **50**, 025020 (2010)]
  - ✓ DIII-D [Garofalo et al., PRL **101**, 195005 (2008)]
  - ✓ MAST [Hua *et al.*, PPCF **52**, 035009 (2010)]
  - ✓ JET [Sun *et al.*, PPCF **52**, 105007 (2010), and EXS/P3-06, 23th IAEA FEC]
  - ✓ TEXTOR [Sun *et al.*, EXS/P3-06, 23th IAEA FEC]

- **Experimental regime:**
  - ✓ Covering all the $1/\nu$, $\nu-\sqrt{\nu}$ and Sb-p regimes
  - ✓ Dependence on the energy of the particles

- **General solution of NTV**:
  - ✓ Connected formula [Shaing *et al.*, NF **50**, 025022 (2010)]
  - ✓ Krook operator [Park *et al.*, PRL **102**, 065002 (2009)]
  - ✓ Numerical solution [Sun et al. PRL**105**, 145002 (2010)]

Neoclassical Toroidal Plasma Viscosity (NTV)

Magnetic field:

$$B = B_0\left[1 - \epsilon\cos\theta - \sum_n b_n(\theta) e^{\mathrm{i}n\alpha}\right]$$

$$\vec{B} = \psi_p' \nabla\hat{V} \times \nabla\alpha$$

$$\alpha = q\theta - \zeta$$

BDKE:

$$\boxed{\omega_{d\alpha}\partial_\alpha f_1 + \omega_{d\hat{V}}\partial_{\hat{V}} f_M = \frac{\nu_d}{2\epsilon}\langle L(f_1)\rangle_b}$$

Flux:

$$\Gamma = \left\langle \int d\vec{v}(\omega_{d\hat{V}} f)\right\rangle_\psi$$

$$\vec{v}_d = \frac{v_\parallel}{B}\nabla \times \frac{v_\parallel}{\Omega_g}\vec{B}$$

$$\omega_{d\hat{V}} = \vec{v}_d \cdot \nabla\hat{V}$$

$$\omega_{d\alpha} = \vec{v}_d \cdot \nabla\alpha$$

- The ambipolar condition requires the modification of the radial electric field, and hence, the toroidal plasma rotation.

NTV torque

$$\boxed{T_{NTV} = -\langle R^2\rangle_\psi \tau_{NTV}^{-1} \rho_i \omega_\phi}$$

$$\tau_{NTV}^{-1} = \langle 1/R^2\rangle_\psi R_0^2 \sum_{i=i,e} \sum_n \frac{\sqrt{\epsilon} q^2 \omega_{ti}^2}{2\sqrt{2}\pi^{3/2}} \left|\frac{e_i}{e_j}\right| \lambda_{1,n}(1 - \omega_{nc,n}/\omega_\phi)$$

$$\omega_{nc,n}^j \equiv q(\omega_\theta + \omega_{*,j} - \omega_{*,i} + \frac{\lambda_{2,n}}{\lambda_{1,n}}\omega_{*T,j})$$

$$\lambda_{l,n} \equiv \frac{1}{2}\int_0^\infty I_{\kappa n}(x)(x - 5/2)^{l-1} x^{5/2} e^{-x} dx$$

$$I_{\kappa n} \equiv \frac{|n\langle b_n\rangle_b|^2_{\kappa^2=0}}{\nu_d/(2\epsilon)} \int_0^1 4KF(I_1|\partial_{\kappa^2} f_{1n}|)^2 d\kappa^2$$

$$\boxed{I_1\langle L(f_{1n})\rangle_b - iI_2 f_{1n} + iI_3 = 0}$$

$$I_1 = [\nu_d/(2\epsilon)]/I_0 \qquad I_0 = \sqrt{[\nu_d/(2\epsilon)]^2 + \max[(n\omega_{d\alpha})^2]}$$

$$I_2 \equiv n\omega_{d\alpha}/I_0 \qquad I_3 \equiv \frac{\langle b_n\rangle_b}{\langle b_n\rangle_b|_{\kappa^2=0}}$$

Sun *et al*. PRL**105**, 145002 (2010)

Analytic solutions

$1/\nu$ regime $\nu_{*d} \equiv \frac{\nu_d/(2\epsilon)}{|n\omega_{d\alpha}|} \approx \frac{\nu_d/(2\epsilon)}{|nq\omega_E|} \gg 1$

$$\boxed{I_{\kappa n,1/\nu} = \frac{I_1^2}{\nu_d/(2\epsilon)} \int_0^1 16KF^{-1}|n\langle[\kappa^2 - \sin^2(\theta/2)]b_n\rangle_b|^2 d\kappa^2}$$

$\nu - \sqrt{\nu}$ regime $\nu_{*d} \ll 1$ and $x_{min} \equiv |q\omega_E/\omega_{B0}| \gg 1$

$$\boxed{I_{\kappa n,\nu-\sqrt{\nu}} = \frac{I_1^2}{\nu_d/(2\epsilon)} \int_0^1 4KF|n\partial_{\kappa^2}\langle b_n(1 - e^{-(1+i\sigma_E)y})\rangle_b|^2 d\kappa^2}$$

$$y \equiv (1-\kappa^2)/\Delta\kappa^2 \qquad \Delta\kappa^2 \approx \sqrt{\frac{8\nu_{*d}}{\ln(16/\sqrt{8\nu_{*d}})}} \qquad x_{min} \equiv |q\omega_E/\omega_{B0}|$$

Solution from Krook operator

$$\boxed{I_{\kappa n,K} = \int_0^1 4K \frac{\nu_d/(2\epsilon)}{[\nu_d/(2\epsilon)]^2 + |n\omega_{d\alpha}|^2} |n\langle b_n\rangle_b|^2 d\kappa^2}$$

Superbanana plateau regime $x > x_{min}$ and $\nu_{*d,sbp} \equiv [\nu_d/(2\epsilon)]/|n\frac{d\omega_{d\alpha}}{d\kappa^2}| \approx (x_{min}/x)\nu_{*d} \ll 1$

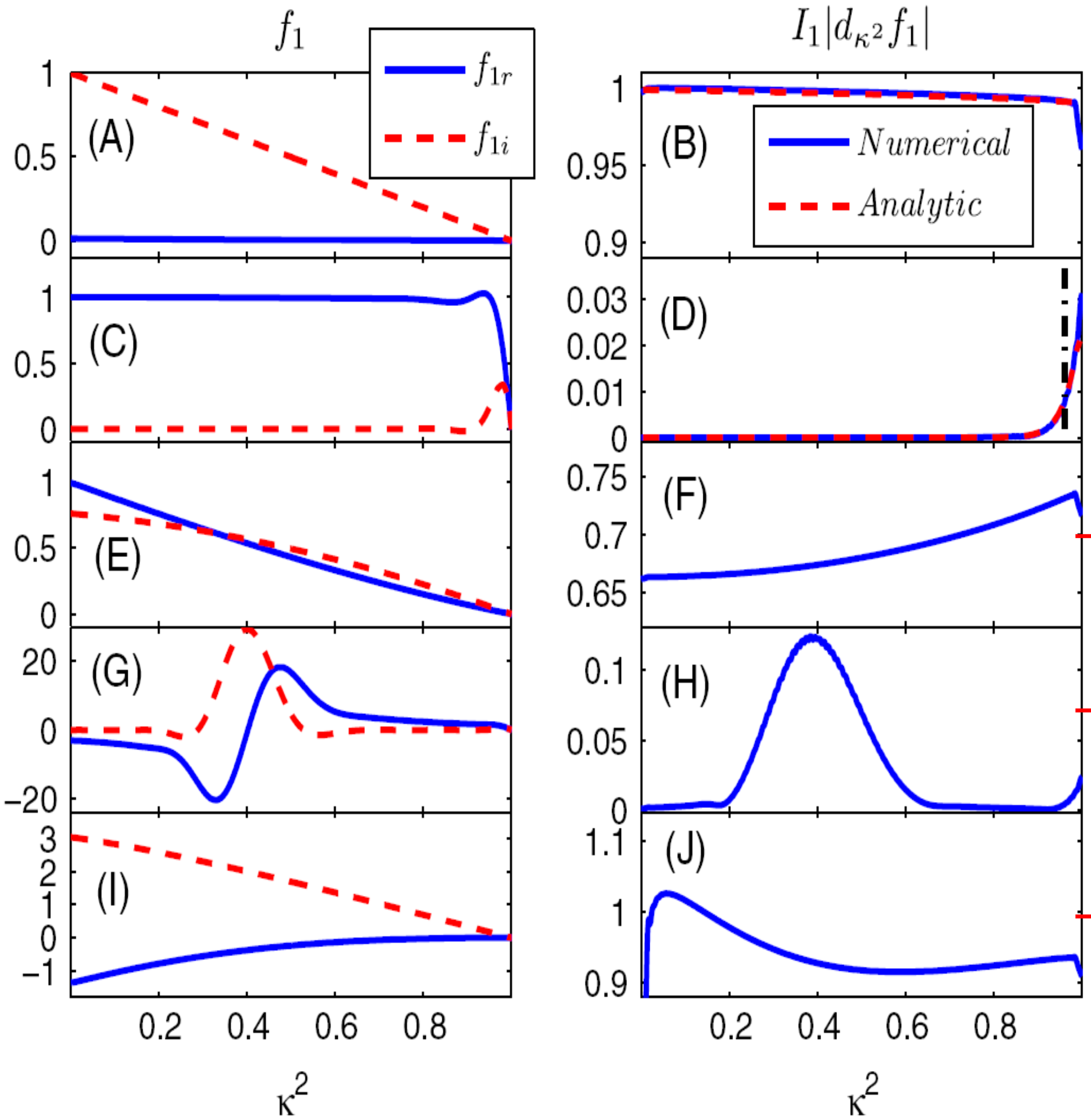
$$\boxed{I_{\kappa n,sbp} = \frac{4\pi K|n\langle b_n\rangle_b|^2}{|nd_{\kappa^2}(\omega_{d\alpha})|}\Big|_{\kappa^2=\kappa_0^2}}$$

Connected formula

$$\boxed{I_{\kappa n,C} = H(x_{min} - x)\frac{I_{\kappa n,\nu-\sqrt{\nu}}}{1 + I_{\kappa n,\nu-\sqrt{\nu}}/I_{\kappa n,1/\nu}} + H(x - x_{min})\frac{I_{\kappa n,sbp}}{1 + I_{\kappa n,sbp}/I_{\kappa n,1/\nu}}}$$

Sun *et al.*, 23th IAEA FEC(2010), Park *et al.*, PRL (2009) and Shaing *et al.* NF (2010)

Numerical solutions



$1/\nu$ regime

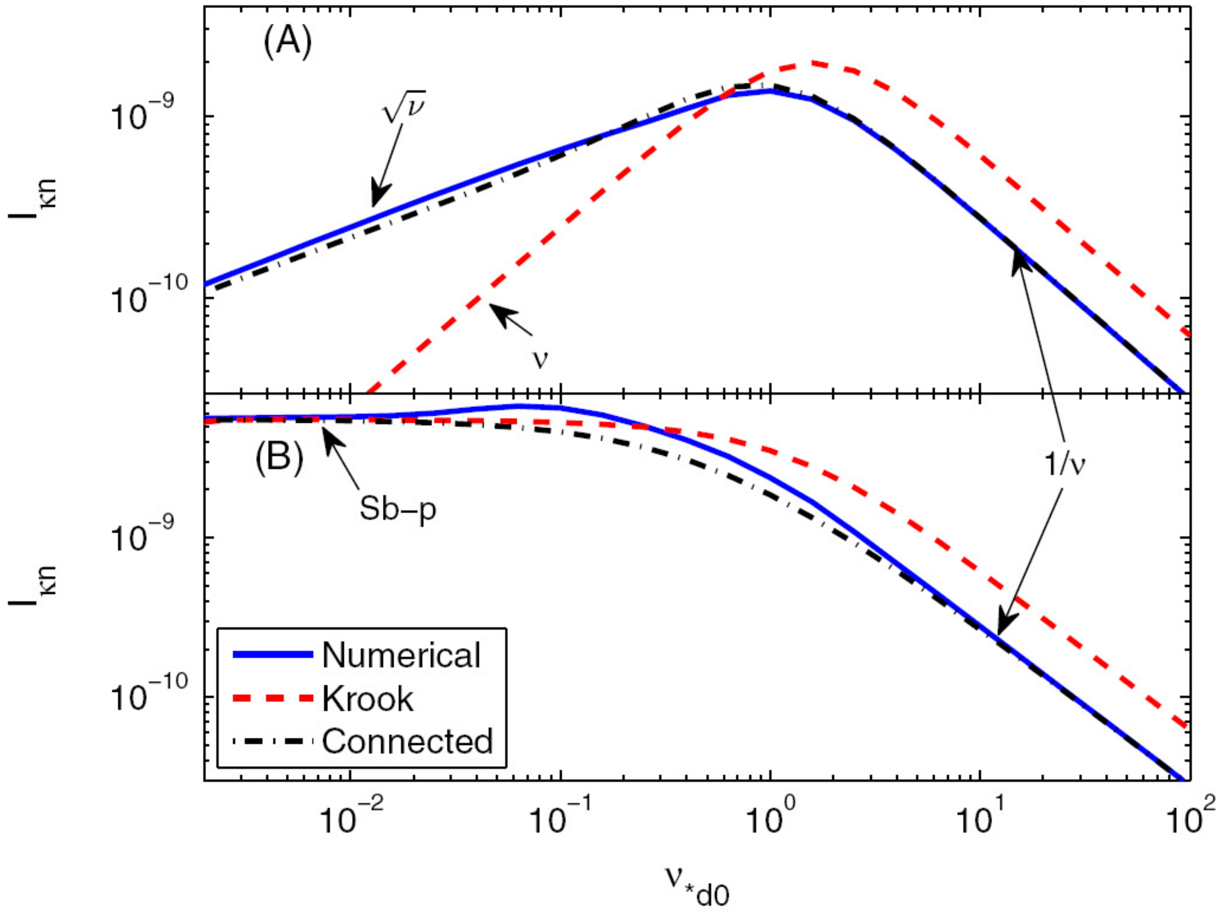
$\nu - \sqrt{\nu}$ regime

Transition between $1/\nu$ and $\nu - \sqrt{\nu}$

Sb-p

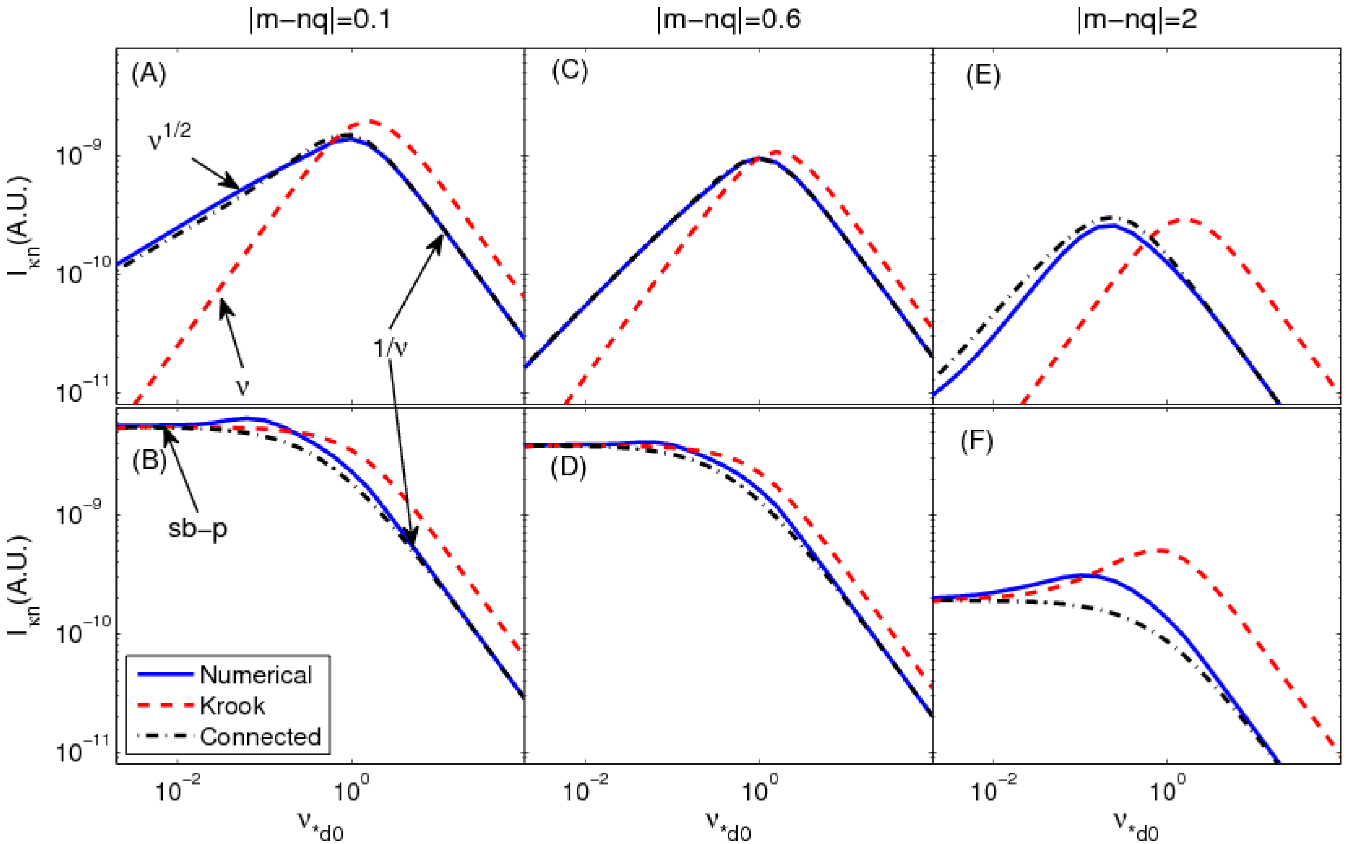
Transition between Sb-p and $1/\nu$

Comparison



The numerical solutions are in good agreement with the analytic results In different asymptotic limits of the collisionless regimes, while there are some difference in the transitions of these regimes.

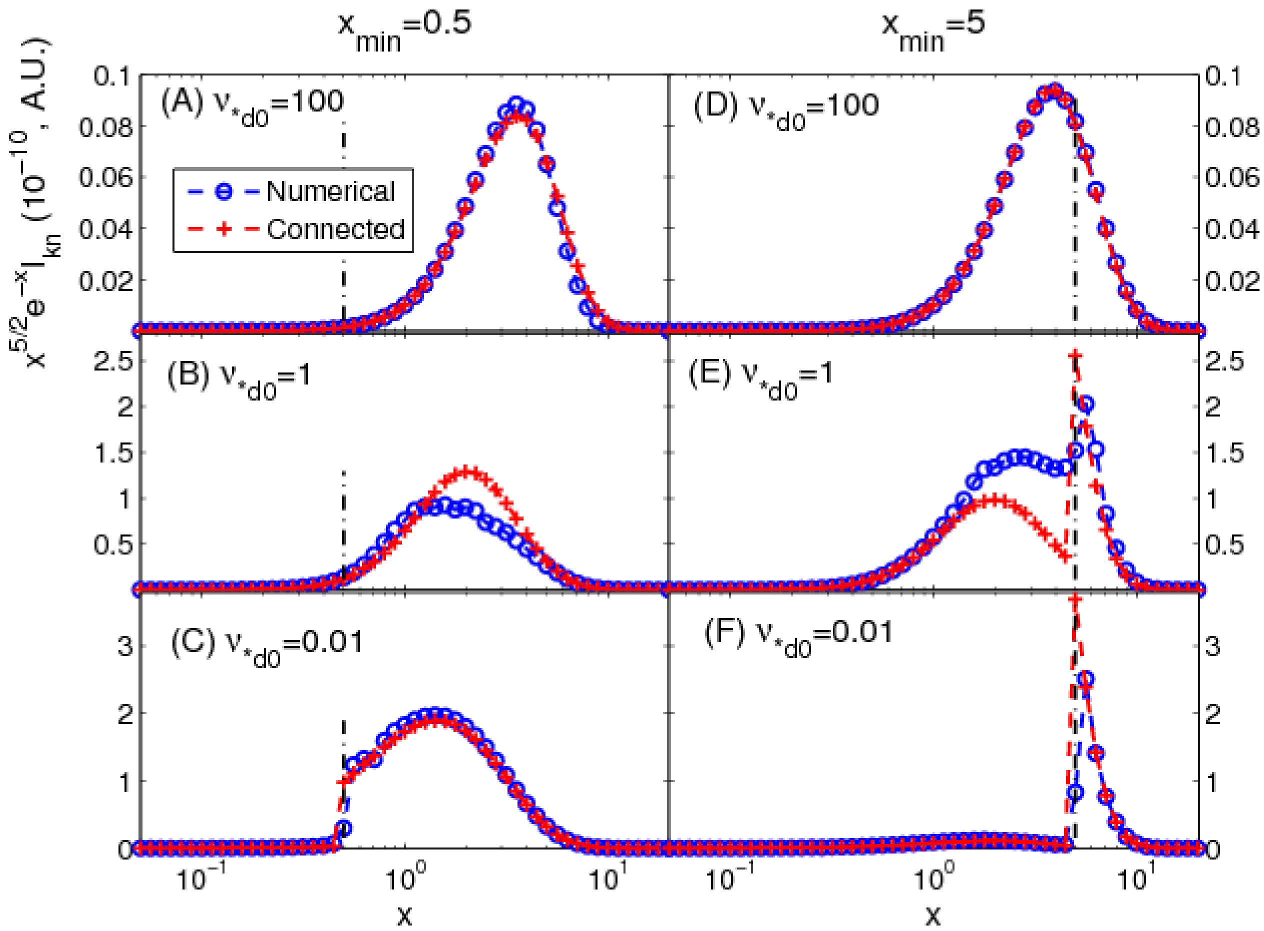
Dependence on the distance to the rational surface



It reaches maximal value at each rational surface and decays with increasing distance for both resonant and non-resonant cases

$$\langle b_n \rangle_b \propto \langle \cos(\delta\theta) \rangle_b$$
$$\delta \equiv m - nq$$

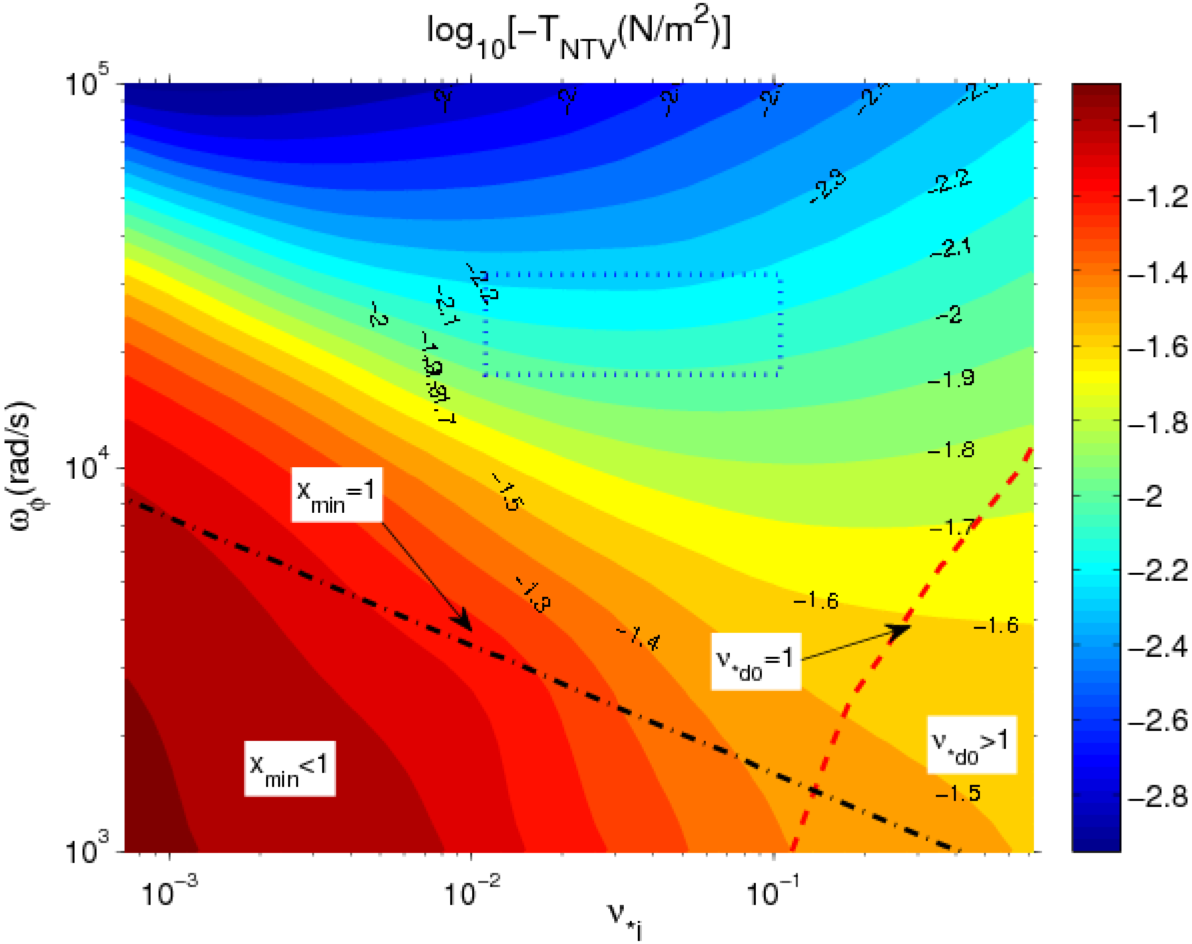
Energy dependence



- Good agreement in different asymptotic limits.
- Some differences at the transitions.

$$x_{min} \equiv |q\omega_E/\omega_{B0}|$$

NTV Torque dependence



- The NTV torque strongly depends on plasma collisionality and rotation.
- The resonant particle effect makes the NTV be important for the lower collisionality and lower rotation cases, which are the ITER relevant conditions.

Summary

- The NTV torque induced by **N**on-**A**xisymmetric **M**agnetic **P**erturbation (NAMP) in the collisionless regimes in tokamaks is obtained by numerically solving the bounce-averaged drift kinetic equation.

- This numerical method can be applied for modeling the NTV torque in different collisionality regimes and their transitions in tokamaks without additional approximations.

- In different asymptotic limits of the collisionless regimes, the numerical solutions are in good agreement with the analytic results. The analytic results are different from the numerical results in the transient regimes.

- It shows that the effect of the resonant particles makes the NTV torque to be more important for the lower collisionality and lower rotation cases, which are the ITER relevant conditions.