Control of ideal and resistive magnetohydrodynamic modes in reversed field pinches with a resistive wall

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MHD Control Workshop
November 16, 2010
Background

- Stabilization of resistive wall modes (RWMs) is important for devices with thin walls, and some RFPs (e.g. RFX, EXTRAP T2R) use feedback to stabilize the RWMs.

- The idea is to cancel out the radial component of the magnetic perturbation, thus creating a “virtual shell” which restores the stabilizing features of a thick conducting wall.

- This feedback is “1D”, i.e., proportional to only one quantity; the radial component of the fluctuation.
Introduction

- In [Finn, Phys. Plasmas 13, 082504 (2006)], it was proposed to use feedback proportional to *two* components of the perturbed field, the radial and one tangential component.

- Since modes which are unstable with an ideal wall have zero perturbed $B_r$, this opens the possibility of stabilizing modes above the ideal wall limit.

- Stabilization up to the ideal-wall ideal-plasma limit was found in [Finn 2006], which used a very simplified plasma model.

- We have applied this feedback to a more realistic model: viscoresistive MHD, realistic RFP equilibria, cylindrical geometry.
Model & methods:

Plasma model

- **Linearized MHD equations in a cylindrical plasma with zero pressure:**

  \[ \frac{\partial \tilde{v}}{\partial t} = \left[ \nabla \times \tilde{B} - \lambda(r) \tilde{B} \right] \times B_0 + \nu \nabla^2 \tilde{v} \]

  \[ \frac{\partial \tilde{B}}{\partial t} = \nabla \times (\tilde{v} \times B_0 - \eta \nabla \times \tilde{B}) \]

  \[ \tilde{B} = \tilde{B}(r) \exp(i(m\theta + kz)) \]

  \[ n = -kR \]

- **RFP equilibria:**

  \[ j_0 = \lambda(r) B_0 \quad \lambda(r) = \frac{\lambda_0}{1 + (r/a)^2} \]
Model & methods:

Boundary conditions

- **At** \( r = 0 \): 
  \[
  \partial_r (\tilde{\varphi}_r - i\tilde{\varphi}_\theta) = (\tilde{\varphi}_r + i\tilde{\varphi}_\theta) = \tilde{\varphi}_z = 0 \\
  \partial_r (\tilde{B}_r - i\tilde{B}_\theta) = (\tilde{B}_r + i\tilde{B}_\theta) = \tilde{B}_z = 0 
  \]

- **At** \( r = r_w \):
  - **Ideal Ohm's law:** 
    \[
    i\mathbf{k} \cdot \mathbf{B}_0 \tilde{\varphi}_r = \gamma \tilde{B}_r(r_w) 
    \]
  - **No stress boundary condition:** 
    \[
    \begin{align*}
    im\tilde{\varphi}_r / r + r\partial_r (\tilde{\varphi}_\theta / r) &= 0 \\
    ik\tilde{\varphi}_r + \partial_r \tilde{\varphi}_z &= 0
    \end{align*}
    \]
  - **Thin wall approximation:** 
    \[
    \gamma \tau_w \tilde{B}_r = [\tilde{B}_r']_{r_w}
    \]
  - **Zero tangential current:** 
    \[
    \begin{align*}
    \partial_r (r\tilde{B}_\theta) - im\tilde{B}_r &= 0 \\
    \partial_r \tilde{B}_z - ik\tilde{B}_r &= 0
    \end{align*}
    \]
Model & methods: Feedback control

- Feedback applied by setting boundary condition at $r_c$ to be proportional to a linear combination of components of the fluctuation measured at the wall:

$$ \tilde{B}_r(r_c) = -(r_w / r_c) \left\{ G \tilde{B}_r(r_w) + iK [k \cdot \tilde{B}(r_w^-)] \right\} $$

- This “2D” scheme can affect modes, even if $\tilde{B}_r(r_w) = 0$

$$ k \equiv (m/r)\hat{\theta} + k\hat{z} $$

$$ \sigma \equiv \hat{r} \times k = (m/r)\hat{z} - k\hat{\theta} $$
Results without feedback:

**Spectrum and mode structure**

- Partial spectrum of $m = 1, n = 8$ MHD fluctuations ($a = 0.9, \lambda_0 = 3.5, \tau_w = 5 \times 10^4, \eta = 10^{-7}, \nu = 2 \times 10^{-5}, R = 4, r_c = 1.2$)

\[ \tau_w = 5 \times 10^4, \eta = 10^{-7}, \nu = 2 \times 10^{-5} \]
Results without feedback:

Plasma stability

- Growth rate of the fastest growing mode depends on the value of $\lambda_0$
- Four stability limits can be identified, and are associated with resistive/ideal wall, and resistive/ideal plasma

Conceptual sketch of stability limits

![Conceptual sketch of stability limits](image)

Computed stability limits

![Computed stability limits](image)
Results with feedback:

Feedback stabilization

\[ \tau_w = 5 \times 10^4, \eta = 10^{-7}, \nu = 2 \times 10^{-5} \]
Results with feedback:

Feedback stabilization

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Stabilized above the resistive-wall ideal-plasma threshold at 3.76!
Results with feedback:

**Marginally stable modes**

Simple model for MS mode stability line: $K_c = K_0 + K_1 \tau_w (\nu + \eta)$
As predicted, $K_c$ depends approximately linearly on viscosity. Also, when the MS boundary moves left, stabilization is lost at a smaller value of $\lambda_0$. 
Results with feedback:

**Dependence on resistivity**

K_c depends only weakly on resistivity, since \( \eta \ll \nu \). The lower boundary moves up with increasing resistivity, and this also causes loss of stabilization at a smaller value of \( \lambda_0 \).
Results with feedback:

Dependence on wall time

K_c depends strongly on wall time, moving to the right for larger wall time. While the stabilized region is smaller for smaller wall time, the limit of stabilization is about the same.
Summary

- Modeled RFP plasmas using viscoresistive MHD in cylindrical geometry and realistic equilibrium profiles
- Applied a new “2D” feedback scheme, and analyzed the dependence on system parameters
- Demonstrated stabilization of tearing modes and ideal plasma modes, above the ideal-wall tearing limit, and up to the ideal-wall ideal plasma limit

Future work

- Use this feedback scheme in experiments
- Add effects of plasma rotation, flow, control system delays
- Consider “3D” feedback scheme, where feedback is proportional to radial and both tangential components of the perturbation
Future work:

Preliminary results

- Analyze “3D” feedback based on the radial and both tangential components of the magnetic fluctuation