Predictor-based MIMO closed-loop system identification of the EXTRAP T2R reversed-field pinch external plasma response

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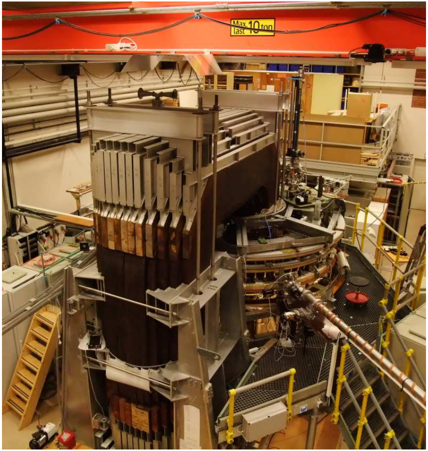
Overview

EXTRAP T2R reversed-field pinch (@KTH)



Parameters

- major radius R=1.24 m
- plasma minor radius a=18.3 cm
- shell norm minor radius $r/a = 1.08$
- shell time constant τ_{shell}=6.3 ms
- plasma current I_p=80-160 kA
- electron temperature T_e=200-400 eV
- pulse length $\tau_{pulse} \leq$ 90 ms

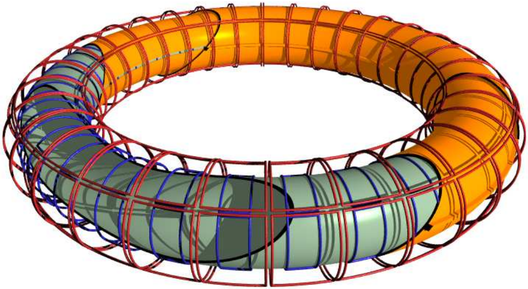
Unstable plant

Without stabilisation plasma "terminates" after $\sim$ 10-15 ms.

Resistive shell & saddle coils

Location

Sensors inside shell.
Actuators outside.
Vacuum vessel innermost.

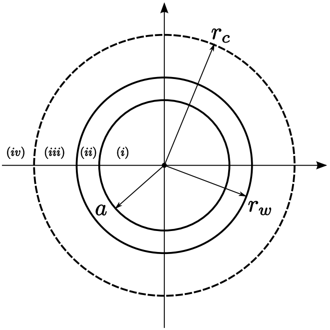


RFP RWMs

It can be shown that

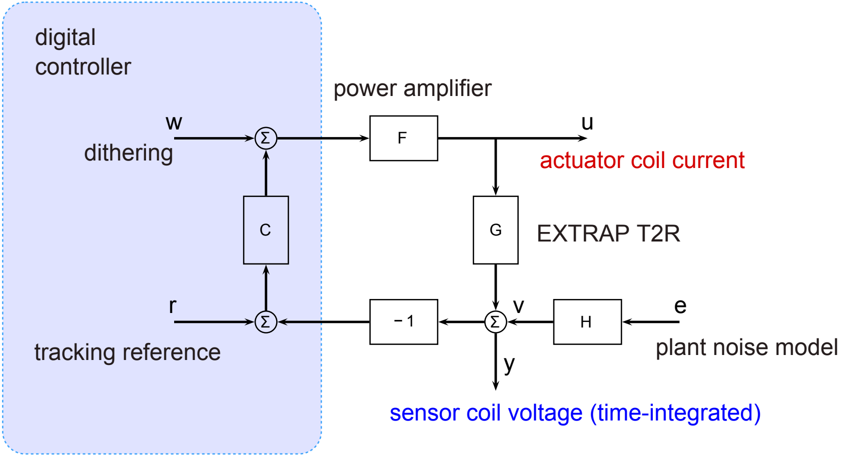
$$\tau_{m,n}\dot{b}_r^w = \hat{\gamma}_{m,n}b_r^w + a_{m,n}b_r^c \qquad (1)$$

where $\hat{\gamma}$ can be computed from an (ideal MHD) RWM dispersion relation, $\tau_{m,n} = -\tau_w \frac{I_m^{w\prime}K_m^{w\prime}}{1+\frac{m^2}{(n/R)^2 r_w^2}} > 0$, and $a_{m,n} = -d(n/R)^2 r_c I_m^{w\prime} K_m^{c\prime} > 0$. For T2R a cylindrical 1D MHD shooting code is appropiate.

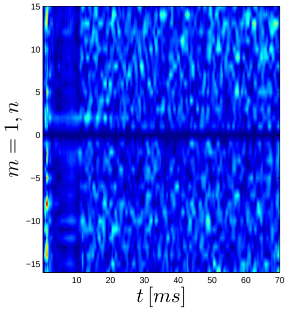


T2R stabilisation; the generalised control loop

Rack of PIDs; the control workhorse



T2R dithering; $r = 0, w \neq 0$



System identification

T2R shot #21816: injected pseudo-randomised dithering.

RFP spectrum from dithering experiments

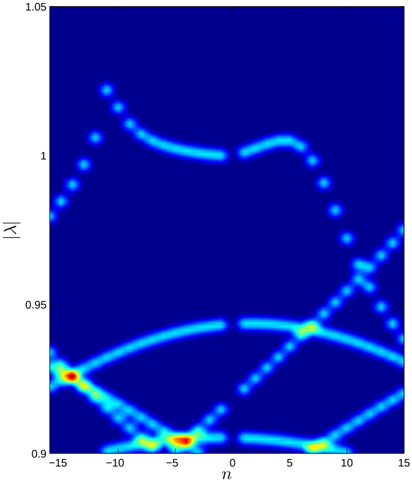
Towards RFP autodetect & autotune?

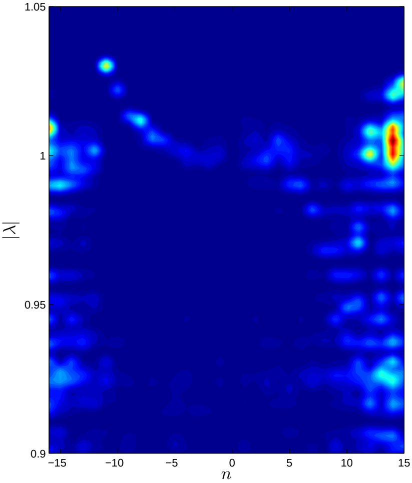
Multi-input multi-output (MIMO)

1. T2R is inherently multivariate; modes are global spatial structures
2. MIMO system identification is computationally challenging
3. An efficient approach could be to use vector autoregressive exogeneous signal models (ARXs)
4. The ARX possess some favourable asymptotic approximation properties for linear time-invariant systems (LTIs)
5. Background note: in e.g. industrial MPC projects sysid is the substantial piece (90% of cost)

Comparison of theory & dither experiment analysis

Cylindrical resistive shell theory (left); “nonregularised” system identification (right)





Some observations & remarks

Method

- Not step-response measurement; full spectrum autodetect
- More shots $\Rightarrow$ more statistics $\Rightarrow$ better precision
- LTI assumed; “truth” of this hinges on RFP equilibrium
- Model control-relevant; acquired under operating conditions

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Obtained spectrum

- Resemblance of RFP theory without regularisation
- “Unexpected” features: high-n density of unstable modes
- Possibilities?
  1. Method bias; coloured noise, closed-loop influence
  2. Real physics; pressure-driven ideal nonresonant MHD shell modes (observed in HBTX1C, P.M. Cox PPCF 1990)

The autoregressive exogeneous (ARX) model

Vector ARXs can be estimated very efficiently. But it is overparameterised.

The

ARX(n_α, n_β)

$$\mathbf{y}(k) = \sum_{i=1}^{n_\alpha} A_i \mathbf{y}(k-i) + \sum_{i=1}^{n_\beta} B_i \mathbf{u}(k-i) + \mathbf{e}(k) \quad (2)$$

is here used as a vehicle to estimate a general

Discrete-time LTI MIMO state-space system

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) \quad (3)$$

$$\mathbf{y}(k) = C\mathbf{x}(k) \quad (4)$$

Predictor form & Markov block coefficients

Various tools to go from (2) to (3)-(4).

Predictor recast of vector ARX

$$\hat{\mathbf{x}}(k+1) = A_K\hat{\mathbf{x}}(k) + B_K \begin{pmatrix} \mathbf{u}(k) \\ \mathbf{y}(k) \end{pmatrix} \quad (5)$$

$$\hat{\mathbf{y}}(k) = C_K\hat{\mathbf{x}}(k) \quad (6)$$

so that the impulse response matrices

Markov coefficient

$$H(k) = C_K A_K^k B_K \in \mathbb{R}^{n\times(n+m)}, \; k \geq 0 \quad (7)$$

are stable. Here n is the number of outputs and m the number of inputs.

The eigensystem realisation algorithm (ERA)

Creates a particular LTI state-space system from the impulse response

ERA is based on the singular value decomposition (SVD) of

Hankel matrix

$$H_0^{f,p} = \begin{pmatrix} H(0) & H(1) & H(2) & \dots & H(p-1) \\ H(1) & H(2) & H(3) & \dots & \\ H(2) & H(3) & H(4) & \dots & \\ H(3) & \dots & & & \\ \vdots & & & & \\ H(f-1) & \dots & & & H(f+p-2) \end{pmatrix} \tag{8}$$

i.e. a factorisation $H_0^{f,p} = U\Sigma V^T$. From U, Σ, V it is then possible to extract/compute an estimate of the system matrices (A, B, C) of (3)-(4). The eigenvalues/eigenvectors of A are then visualised.

What was done
(and some extrapolation)

1. Dither-injected RFP experiments performed
2. Direct MIMO LTI discrete-time state-space system estimation
3. Comparison to ideal MHD resistive shell theory
4. Autodetected model looks “physical” and is control-relevant by construction (RWM control mainly)
5. Most unstable empirical eigenmode is monochromatic and directly mapped to theory
6. Possible immediate extension: general and systematic approach to measure the in situ experimental plasma response (wanted in e.g. RMP & ELM research)

Some references

Thank you for your attendance.

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