

Role of plasma edge in global stability and control*

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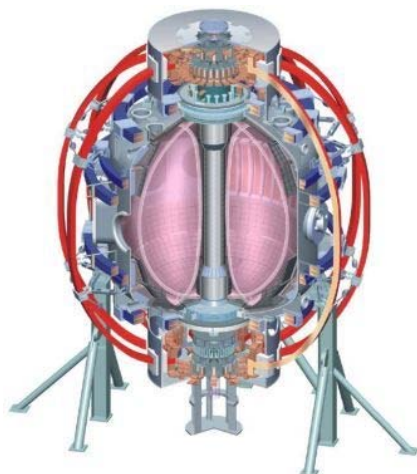
and the NSTX Research Team

MHD Control Workshop

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University of Wisconsin - Madison

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U Quebec

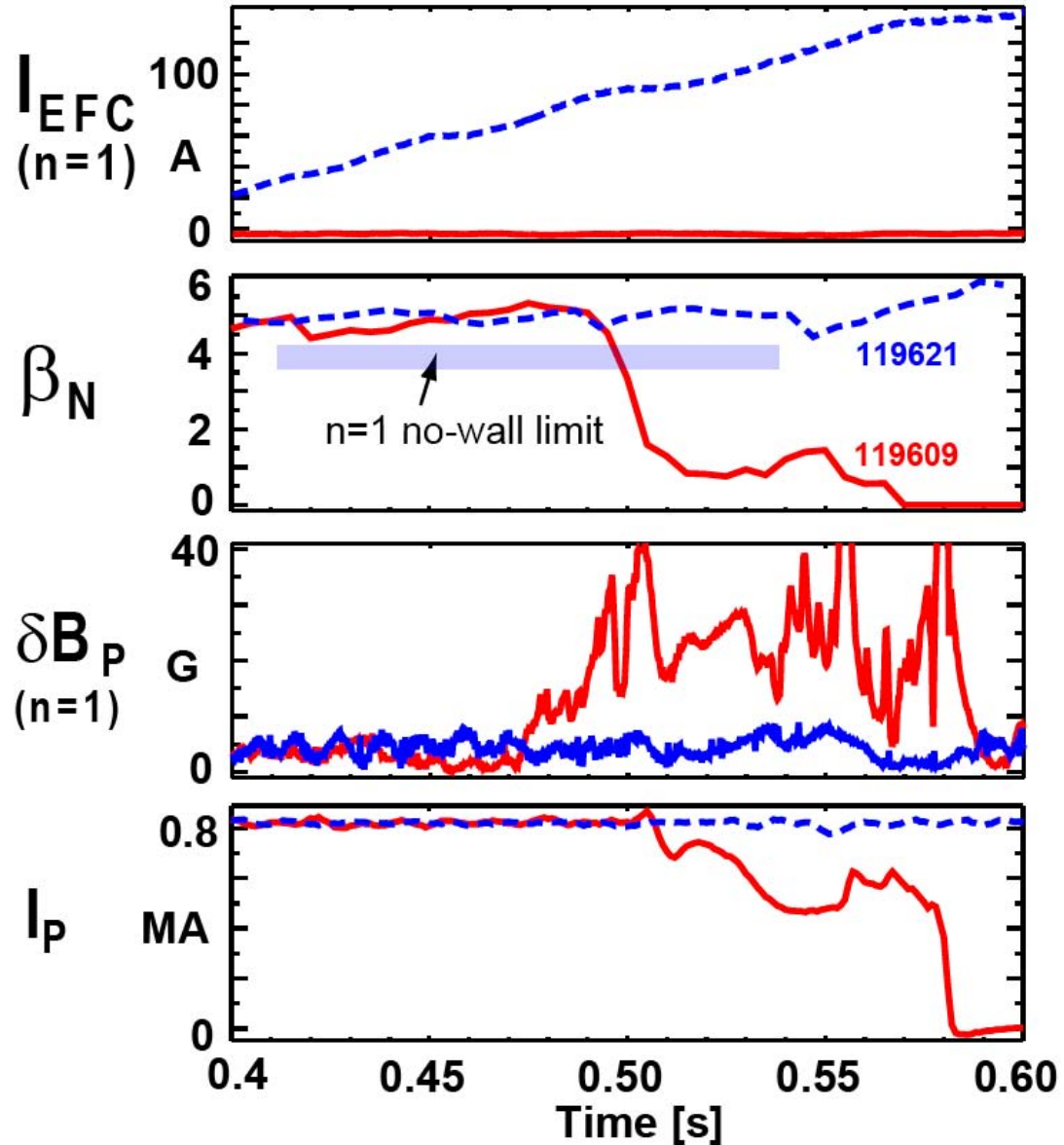
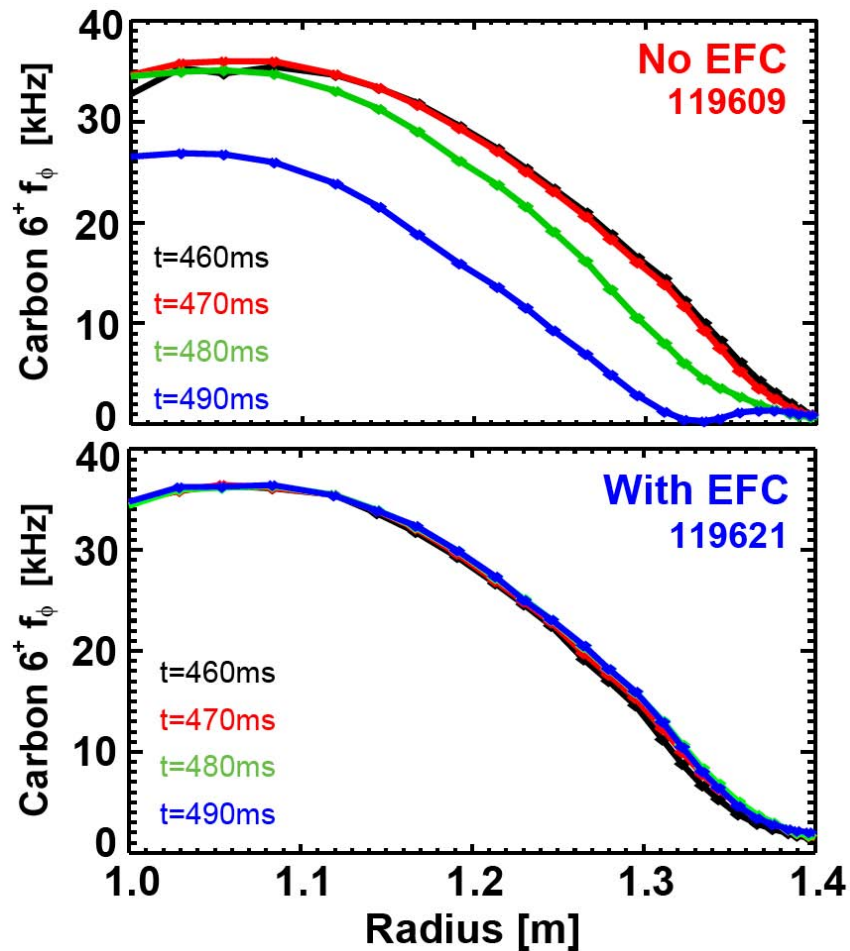
College W&M
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Outline

1. Experimental motivation
2. Role of $E \times B$ drift frequency profile
3. Kinetic stability analysis using MARS code
 - Comparisons with experiment
 - Some control considerations
 - Self-consistent vs. perturbative approach
4. Summary, future work, questions

Error field correction (EFC) often necessary to maintain rotation, stabilize n=1 resistive wall mode (RWM) at high β_N

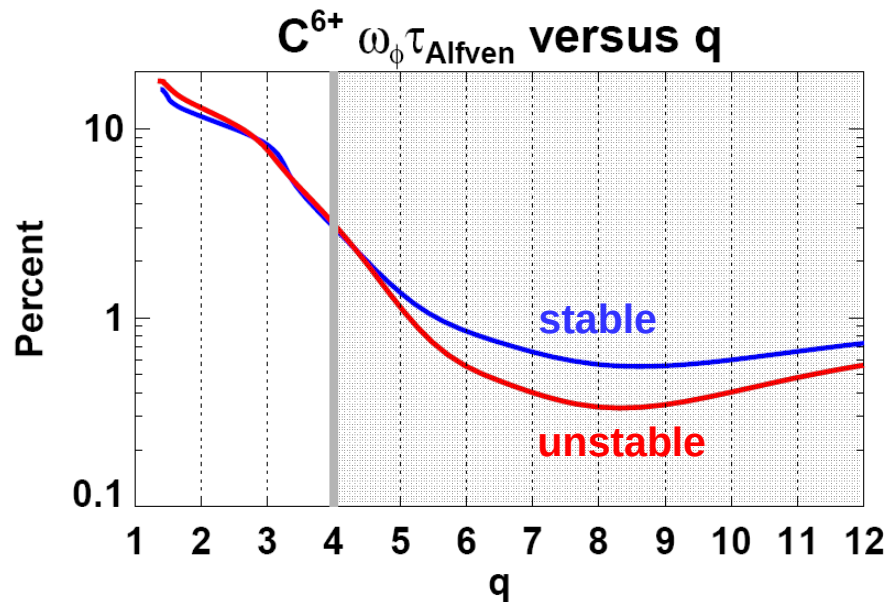
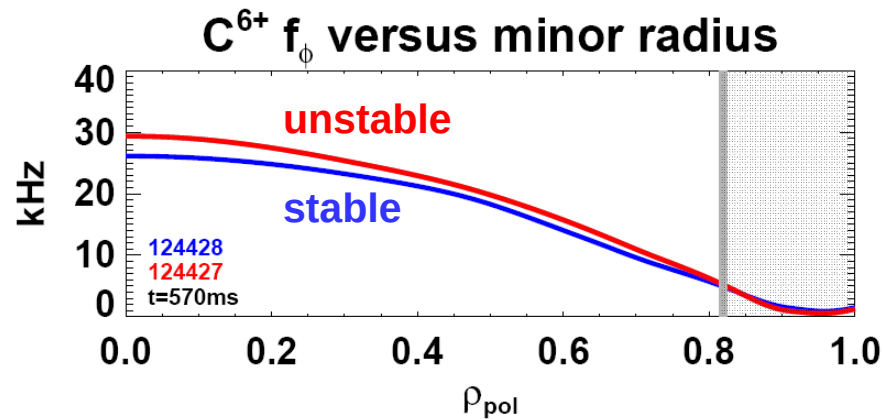
- No EFC → n=1 RWM unstable
- With EFC → n=1 RWM stable



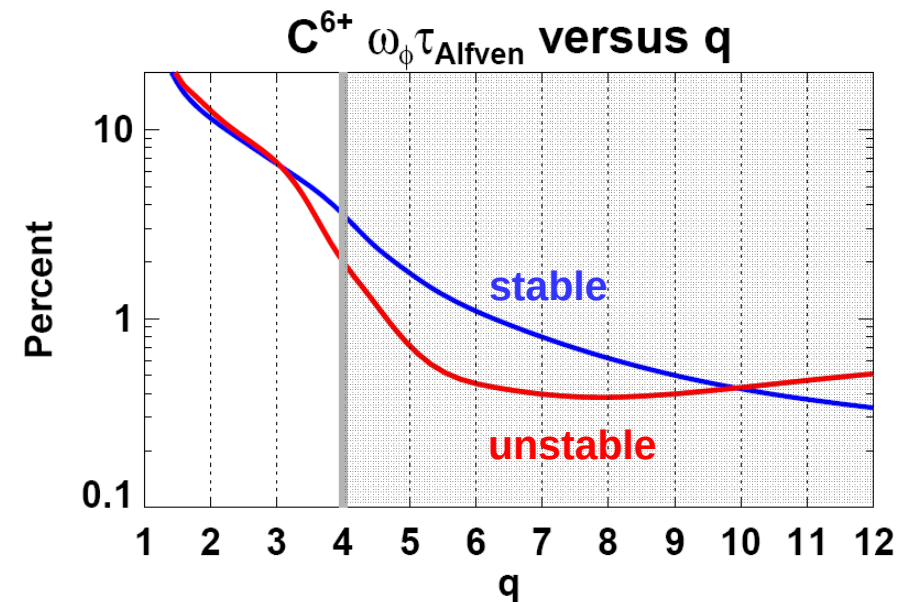
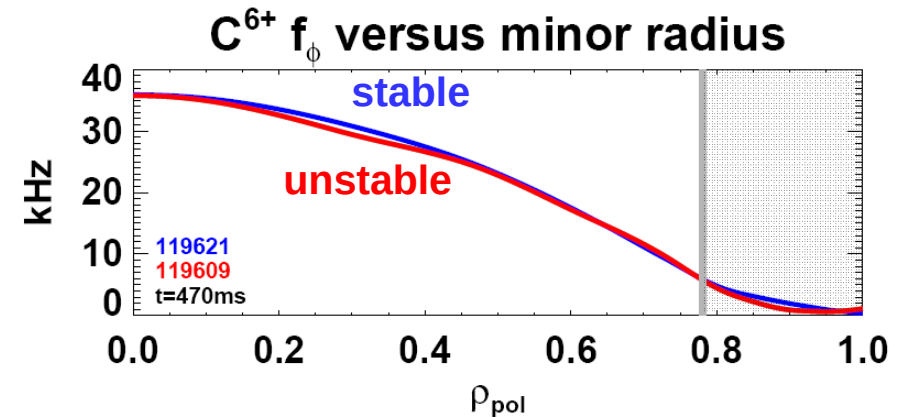
J.E. Menard et al, Nucl. Fusion 50 (2010) 045008

EFC experiments show edge region with $q \geq 4$ and $r/a \geq 0.8$ apparently determine stability

- $n=3$ EFC $\rightarrow$ stable
- No EFC $\rightarrow n=1$ RWM unstable



- $n=1$ EFC $\rightarrow$ stable
- No EFC $\rightarrow n=1$ RWM unstable



MARS is linear MHD stability code that includes toroidal rotation and drift-kinetic effects

- Single-fluid linear MHD

$$\begin{aligned}
 (\gamma + in\Omega)\xi &= \mathbf{v} + (\xi \cdot \nabla \Omega) R^2 \nabla \phi \\
 \rho(\gamma + in\Omega)\mathbf{v} &= -\nabla \cdot \mathbf{p} + \mathbf{j} \times \mathbf{B} + \mathbf{J} \times \mathbf{Q} \\
 &\quad - \rho[2\Omega \hat{\mathbf{Z}} \times \mathbf{v} + (\mathbf{v} \cdot \nabla \Omega) R^2 \nabla \phi] \\
 (\gamma + in\Omega)\mathbf{Q} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + (\mathbf{Q} \cdot \nabla \Omega) R^2 \nabla \phi \\
 (\gamma + in\Omega)p &= -\mathbf{v} \cdot \nabla P, \quad \mathbf{j} = \nabla \times \mathbf{Q}
 \end{aligned}$$

Y.Q. Liu, et al., Phys. Plasmas 15, 112503 2008

- Kinetic effects in perturbed p :

$$\begin{aligned}
 \mathbf{p} &= p\mathbf{I} + p_{\parallel} \hat{\mathbf{b}}\hat{\mathbf{b}} + p_{\perp}(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}) \\
 p_{\parallel} e^{-i\omega t + in\phi} &= \sum_{e,i} \int d\Gamma M v_{\parallel}^2 f_L^1 \\
 p_{\perp} e^{-i\omega t + in\phi} &= \sum_{e,i} \int d\Gamma \frac{1}{2} M v_{\perp}^2 f_L^1 \\
 f_L^1 &= -f_{\epsilon}^0 \epsilon_k e^{-i\omega t + in\phi} \sum X_m^u H_{ml}^u \lambda_{ml} e^{-in\tilde{\phi}(t) + im\langle \dot{\chi} \rangle t + il\omega_b t} \\
 H_L &= \frac{1}{\epsilon_k} [M v_{\parallel}^2 \vec{k} \cdot \xi_{\perp} + \mu(Q_{L\parallel} + \nabla B \cdot \xi_{\perp})]
 \end{aligned}$$

- Mode-particle resonance operator:

MARS-K:

$$\lambda_{ml} = \frac{n[\omega_{*N} + (\hat{\epsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n(\langle \omega_d \rangle + \omega_E) + [\alpha(m + nq) + l]\omega_b - i\nu_{\text{eff}} - \omega}$$

MARS-F:

$$\lambda_{ml} = \frac{n[\cancel{\omega_{*N}} + (\hat{\epsilon}_k - 3/2)\cancel{\omega_{*T}} + \omega_E] - \omega}{n(\cancel{\langle \omega_d \rangle} + \omega_E) + [\alpha(m + nq) + l]\omega_b - i\nu_{\text{eff}} - \omega}$$

+ additional approximations/simplifications in f_L^1

- Fast ions: **MARS-K:** slowing-down $f(v)$, **MARS-F:** lumped with thermal

Sensitivity of stability to rotation motivates study of all components of $\mathbf{E} \times \mathbf{B}$ drift frequency $\omega_E(\psi)$

- Decompose flow of species j into poloidal + toroidal components:

$$\vec{u}_j = u_{\theta j}(\psi) \vec{B}_P + \Omega_{\phi j}(\psi, \theta) R^2 \nabla \phi \quad \text{satisfying} \quad \nabla \cdot \vec{u}_j = 0$$
- Orbit-average $\mathbf{E} \times \mathbf{B}$ drift frequency: $\omega_E \equiv \langle \langle \vec{v}_E \cdot \nabla(\phi - q\theta) \rangle \rangle$
 Bounce average: $\langle \langle X \rangle \rangle \equiv \frac{1}{\tau_b} \oint X d\tau$ $\vec{v}_E = \mathbf{E} \times \mathbf{B}$ drift velocity
F. Porcelli, et al., Phys. Plasmas 1 (1994) 470
- Ignoring centrifugal effects (ok in plasma edge), ω_E reduces to:

1. parallel/toroidal
2. diamagnetic
3. poloidal

$$\omega_E(\psi) = \frac{\langle \vec{u}_j \cdot \vec{B} \rangle}{F} - \omega_{*j} - u_{\theta j}(\psi) \frac{\langle B^2 \rangle}{F}$$

$\frac{\langle \vec{u}_j \cdot \vec{B} \rangle}{F}$
 $\updownarrow$
 $\frac{\langle \vec{u}_j \cdot \vec{B} \rangle}{F} = \Omega_{\phi j}(\psi, \theta) + \frac{u_{\theta j}(\psi)}{F} \left[\langle B^2 \rangle - \frac{F^2}{R^2} \right]$
 $\uparrow$
 flux function

ω_{*j}
 $\updownarrow$
 measured

$u_{\theta j}(\psi)$
 $\updownarrow$
 measured or neoclassical theory

$\Omega_{\phi j}(\psi, \theta)$
 $\uparrow$
 measured

$\left[\langle B^2 \rangle - \frac{F^2}{R^2} \right]$
 $\underbrace{\hspace{10em}}$
 reconstructions

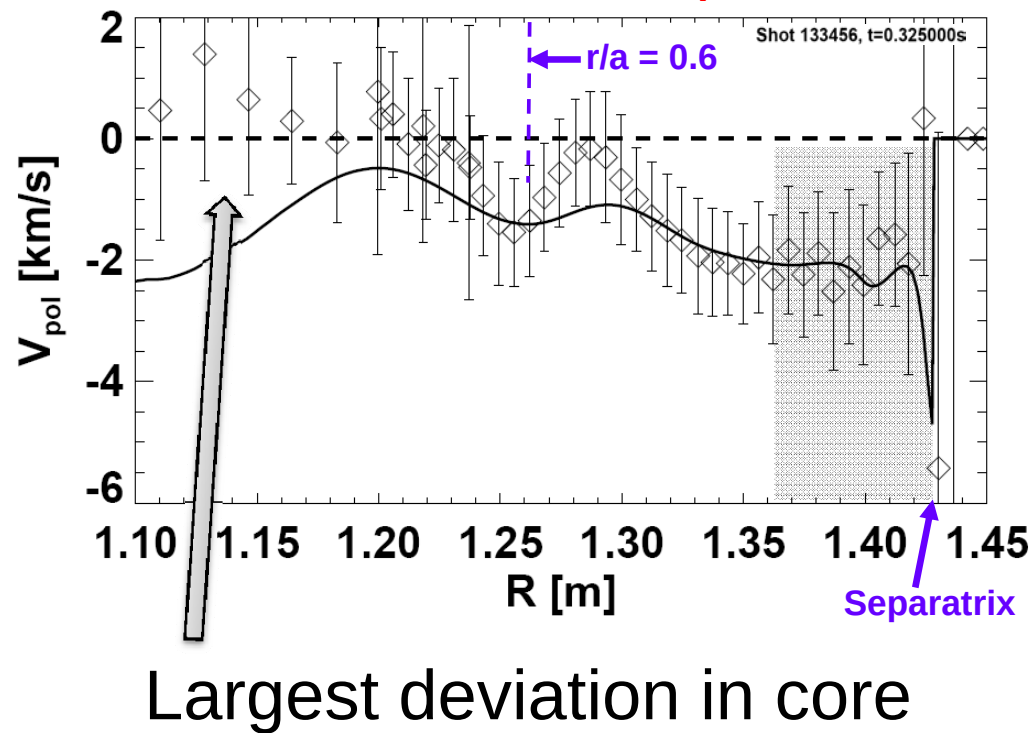
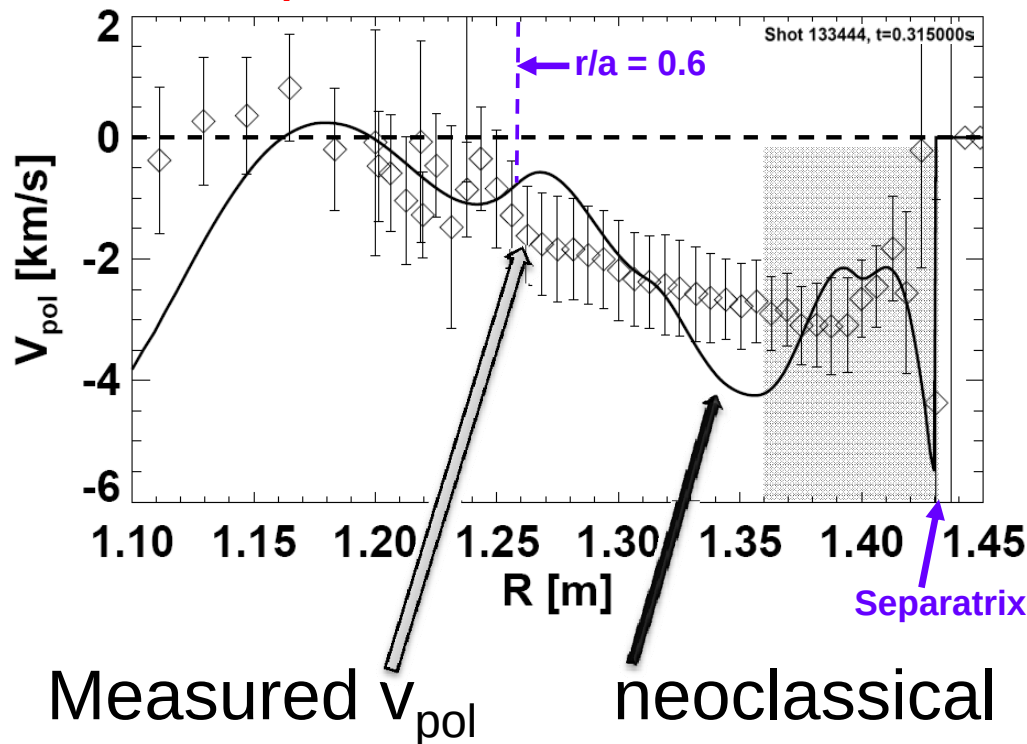
Y.B. Kim, et al., Phys. Fluids B 3 (1991) 2050
 Flux-surface average:
 $\langle X \rangle \equiv \oint JX d\theta / \oint J d\theta$
 $F(\psi) \equiv RB_\phi$

NSTX edge $v_{pol} \approx$ neoclassical (within factor of ~ 2)

$B_T = 0.34T$

$V_{pol} \propto 1/B$ – trend
consistent with neoclassical

$B_T = 0.54T$



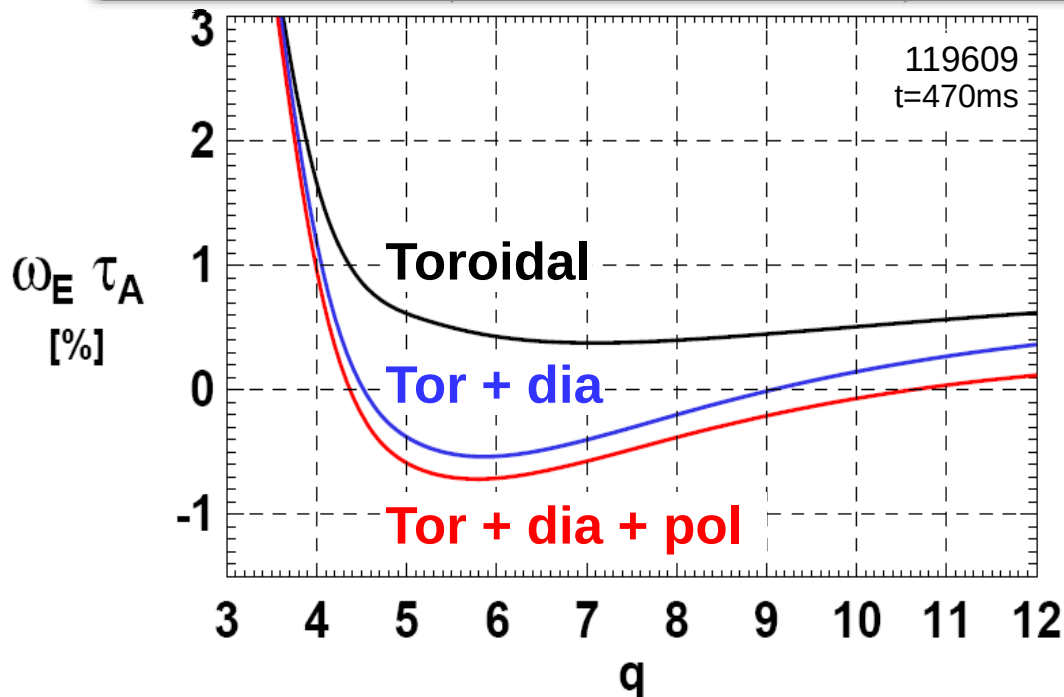
Subsequent MARS calculations use
neoclassical v_{pol} , but $v_{pol} = 0$ for $r/a < 0.6$

NSTX results: R. E. Bell, et al., Phys. Plasmas **17**, 082507 (2010)

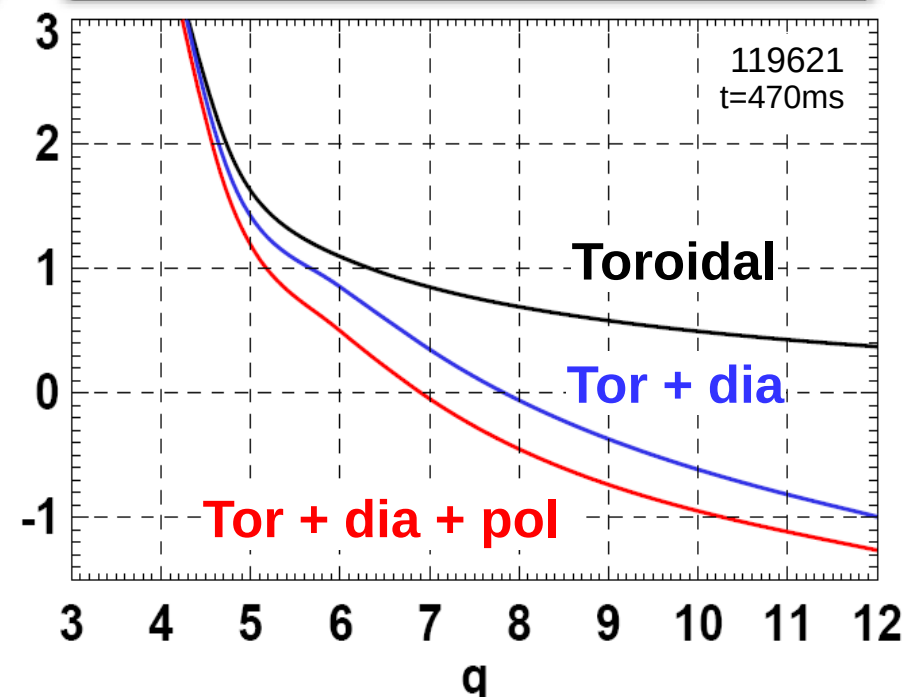
Neoclassical: W. Houlberg, et al., Phys. Plasmas **4**, 3230 (1997)

$n=1$ EFC profiles show impurity C diamagnetic and poloidal rotation modify $|\omega_E \tau_A| \sim 1\%$ in edge $\rightarrow$ potentially important

$n=1$ RWM unstable (no $n=1$ EFC)



$n=1$ RWM stable ($n=1$ EFC)



C^{6+} Toroidal rotation only: $\omega_E = \Omega_{\phi-C}(\psi)$

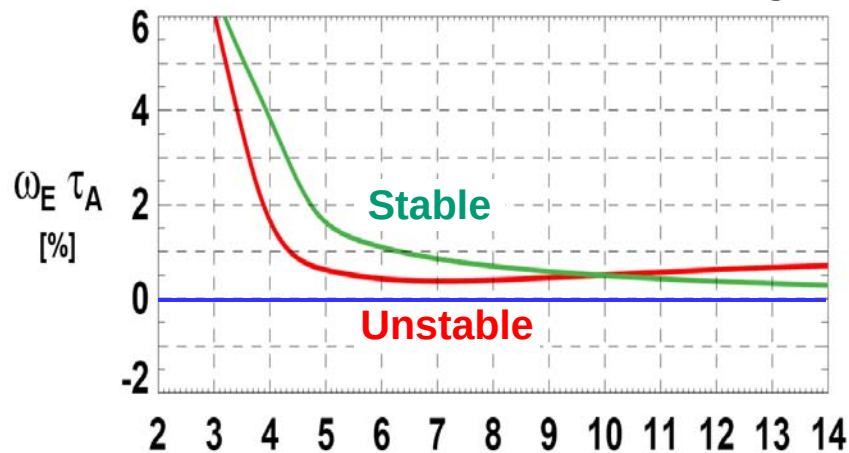
Toroidal + diamagnetic: $\omega_E = \Omega_{\phi-C}(\psi) - \omega_{*C}$

Toroidal + diamagnetic + poloidal: $\omega_E = \langle u_C \cdot B \rangle / F - \omega_{*C} - u_{\theta-C} \langle B^2 \rangle / F$

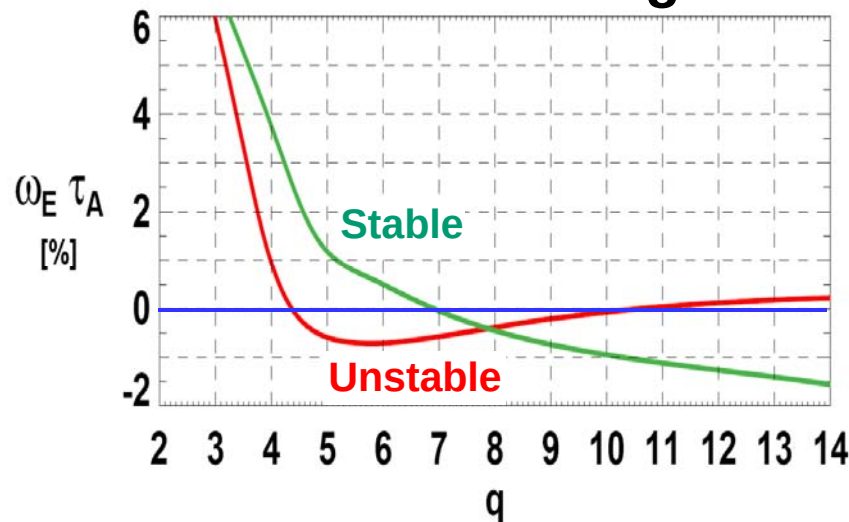
- Diamagnetic contribution to $\omega_E \tau_A \approx -0.5$ to -1.0%
- Neoclassical v_{pol} contribution to $\omega_E \tau_A \approx -0.2$ to -0.4%

Inclusion of ω_{*C} in ω_E increases separation between stable and unstable $\omega_E(\psi)$ and provides consistency w/ experiment

Toroidal rotation only

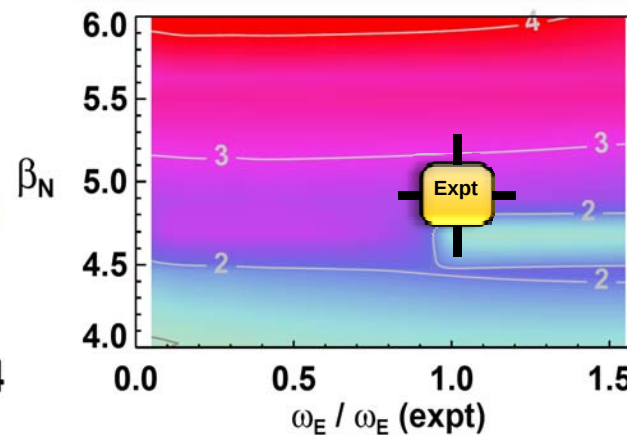


Toroidal + diamagnetic

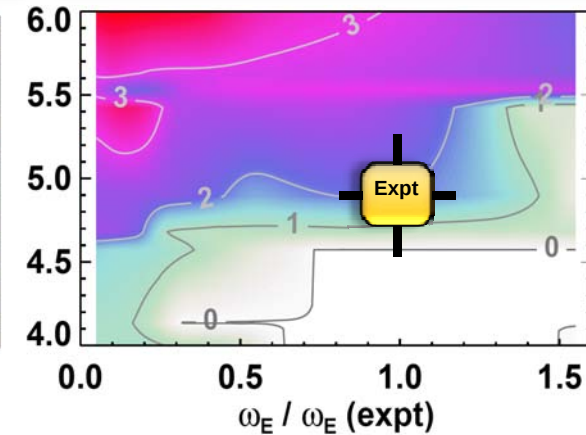


Calculated $n=1$ $\gamma\tau_{wall}$

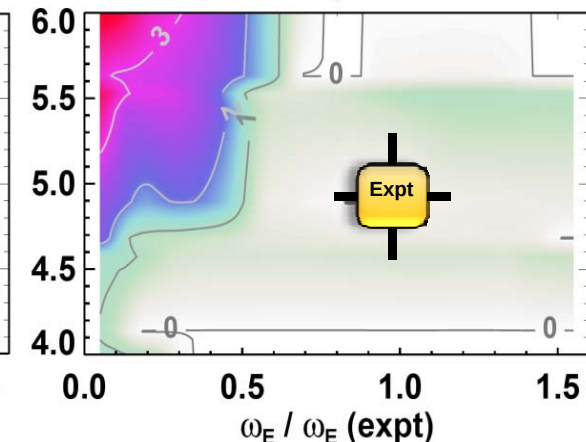
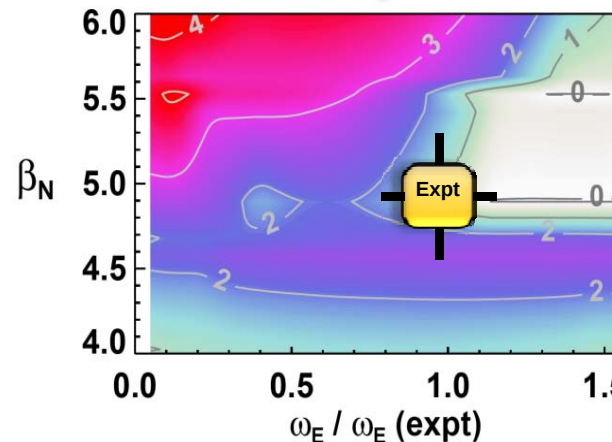
Unstable rotation profile



Stable rotation profile



Predictions inconsistent with experiment



Predictions consistent with experiment

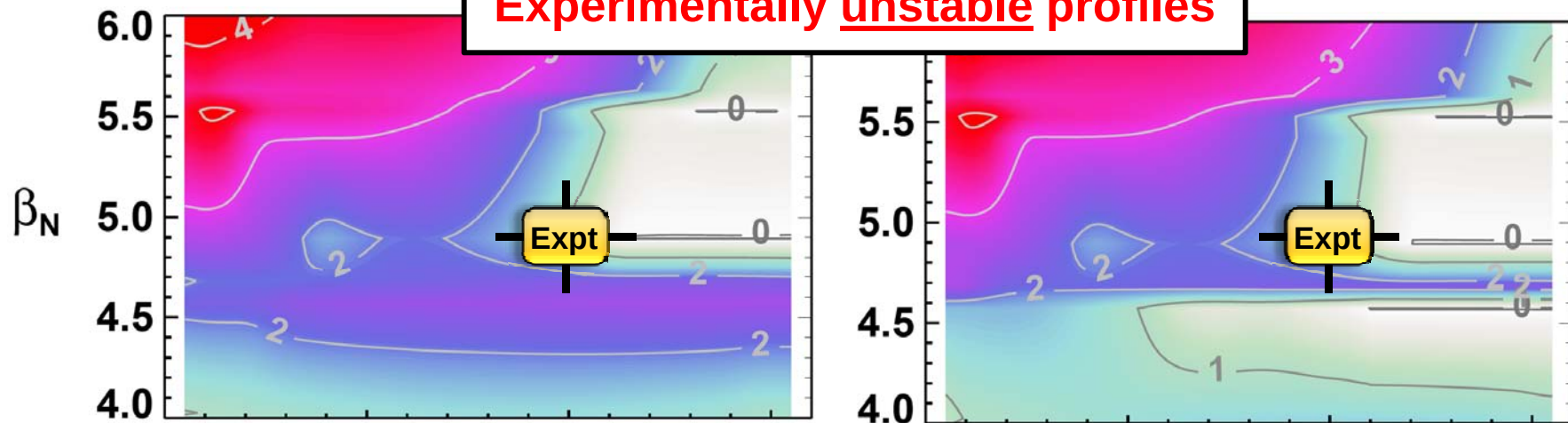
Addition of v_{pol} in ω_E modifies marginal stability

$\omega_{*C} / \omega_{*C} \text{ (expt)} = 1$
 $u_\theta = 0$

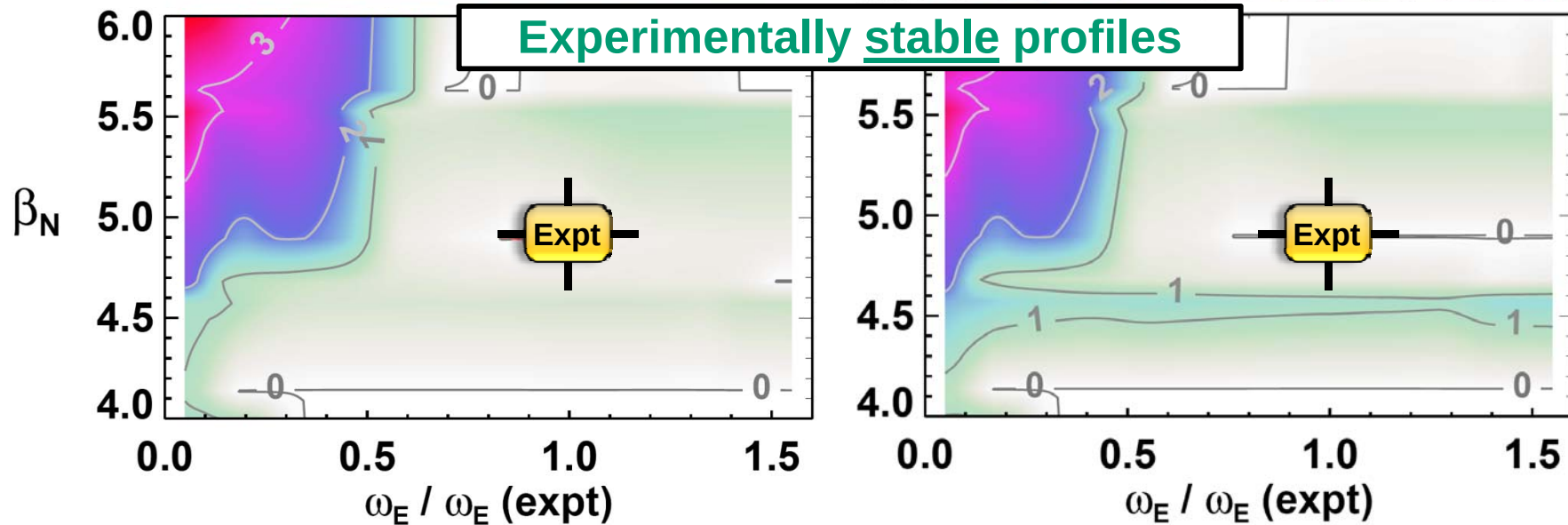
Calculated $n=1$ $\gamma\tau_{wall}$

$\omega_{*C} / \omega_{*C} \text{ (expt)} = 1$
 $u_\theta = \text{neoclassical}$

Experimentally unstable profiles



Experimentally stable profiles



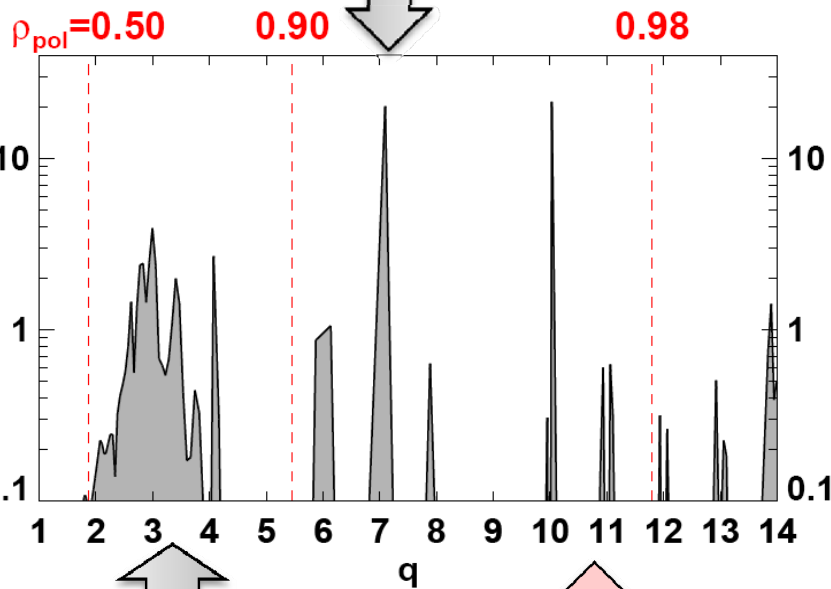
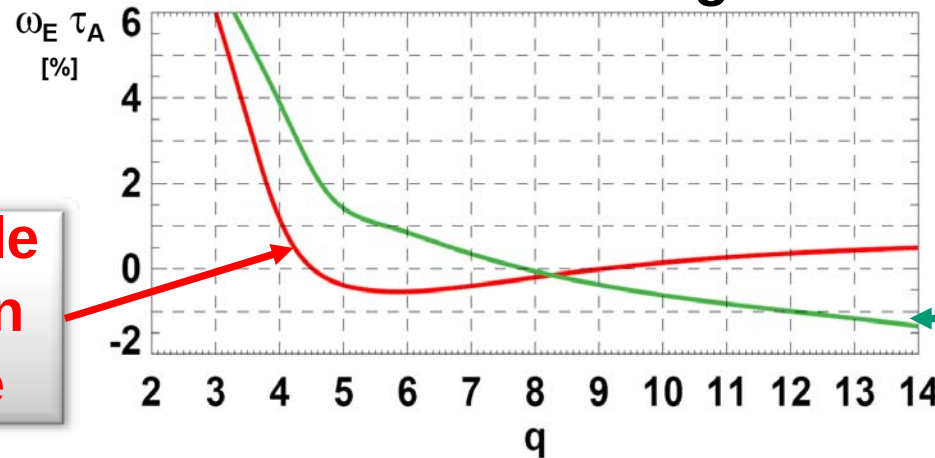
Mode damping in last 10% of minor radius calculated to determine stability of RWM in n=1 EFC experiments

Toroidal + diamagnetic

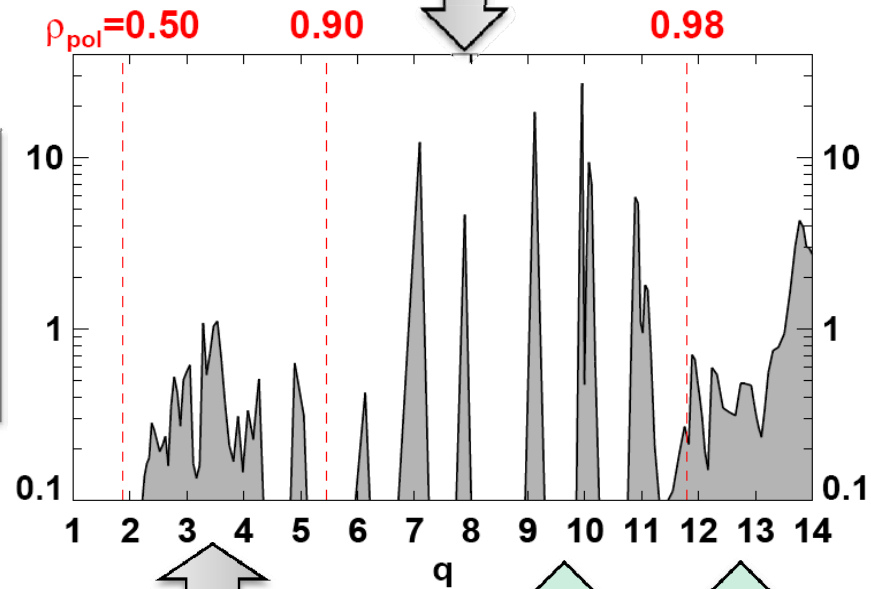
$\omega_E / \omega_E (\text{expt}) = 0.5$ used so unstable modes can be identified and local damping computed

Unstable rotation profile

Stable rotation profile



Local mode damping $\propto \frac{\partial(\delta W_{K-\text{imag}})}{\partial V}$



Higher core damping

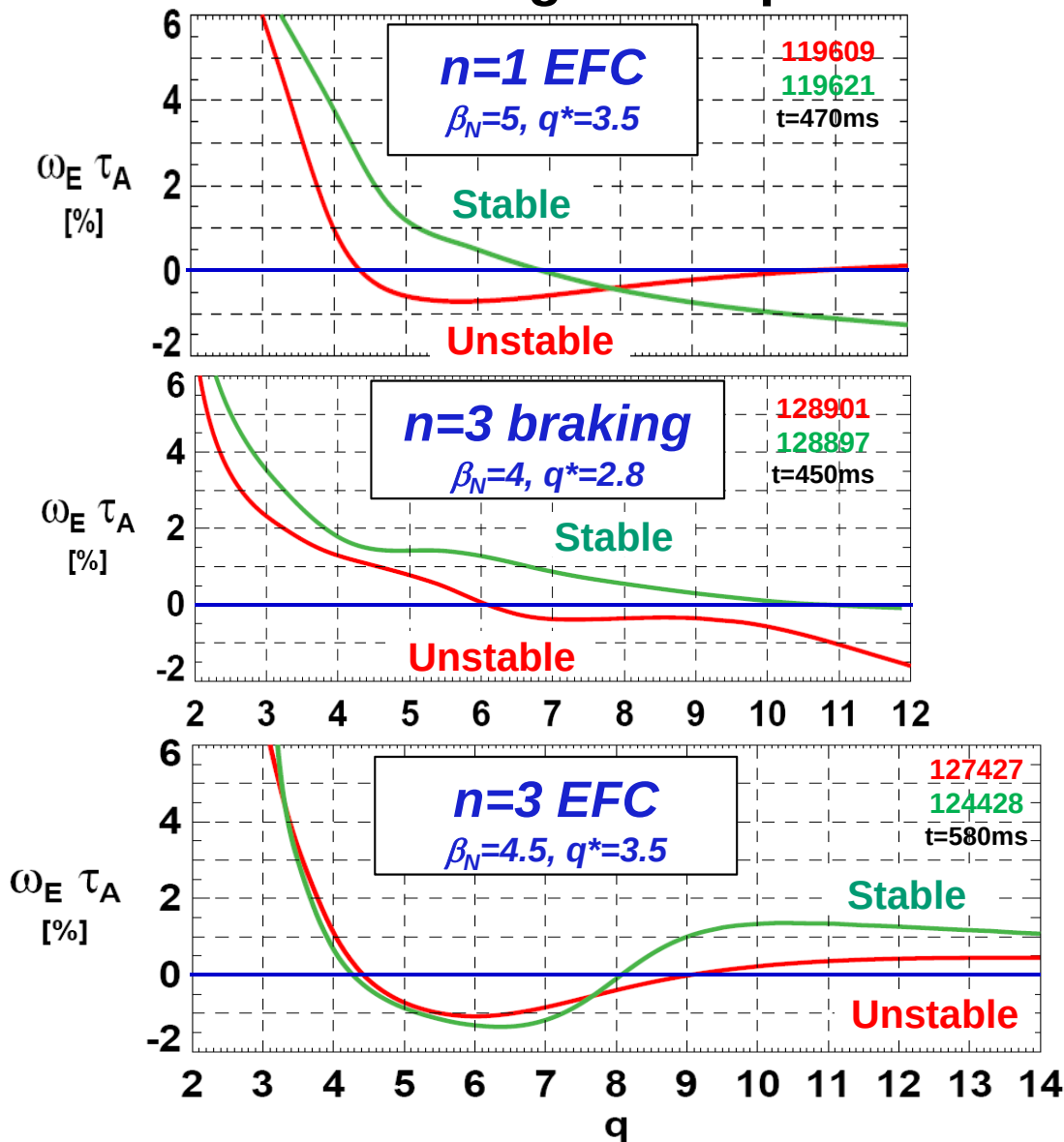
Lower edge damping

Lower core damping

Higher edge damping

Control of $\omega_E(r)$ could play important role in RWM stabilization

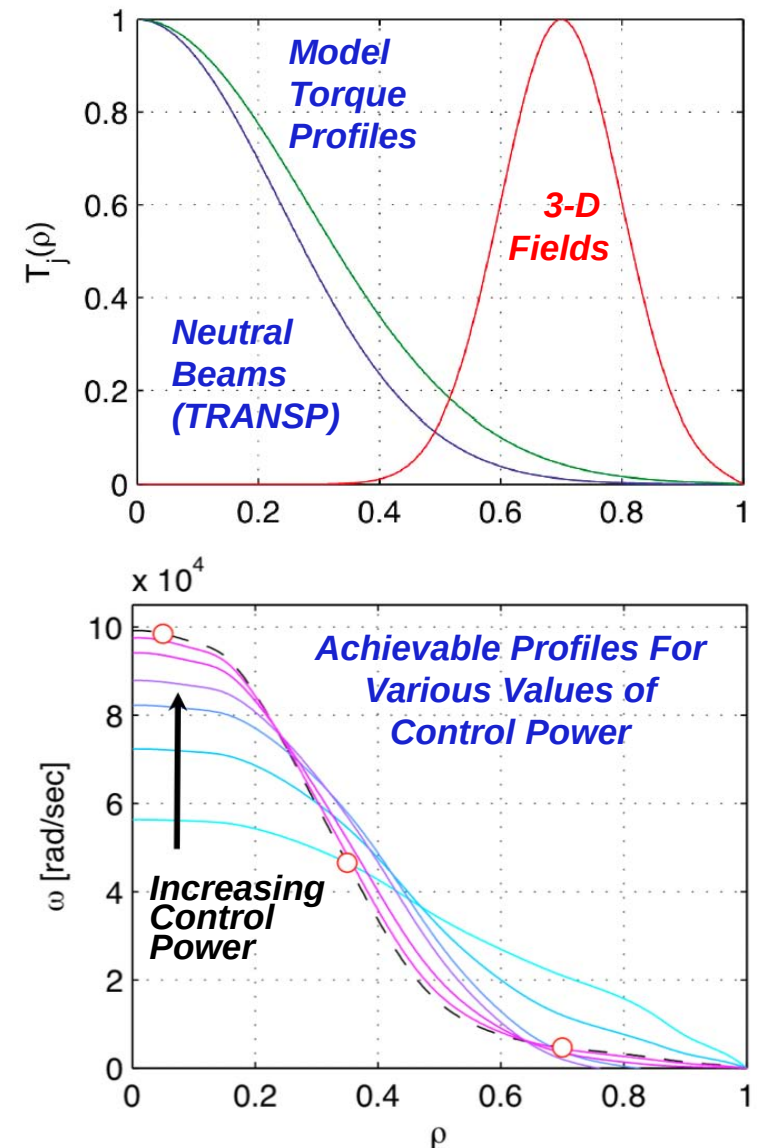
Toroidal + diamagnetic + poloidal



- Separation between **stable** and **unstable** profiles typically small:
 $\Delta(\omega_E \tau_A) \leq 1\%$
- Instability may correlate with $\omega_E \approx 0$ occurring over most of edge region
 - If true, provides insight into which conditions to avoid
- Need for fast RWM active feedback control remains

NSTX preparing for real-time rotation control

- Physics studies enabled:
 - RWM & EF physics vs. rotation and β
 - Transport dynamics vs. rotation shear
 - Pedestal stability vs. edge rotation
 - What is the optimal rotation profile for integrated plasma performance?**
- Plan: Use state-space controller based on momentum balance model
 - Real-time charge-exchange for v_ϕ
 - Neutral beams provide torque
 - 3-D fields provide braking
 - Vary toroidal mode number to vary magnetic braking profiles**
 - Use 2nd Switching Power Amplifier for independent n=1,2,3 fields (2011)**
- Goal: Test in 2012



K. Taira, C.W. Rowley, N.J. Kasdin, (PU), M. Podesta, E. Kolemen, R. Bell, D. Gates (PPPL), S. Sabbagh (CU)

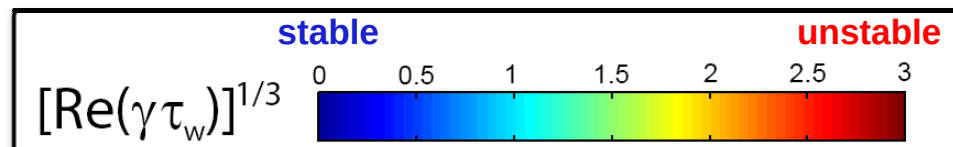
Kinetic stability analysis using MARS-K

- Comparison with experiment
- Self-consistent vs. perturbative approach
- Modifications of RWM eigenfunction

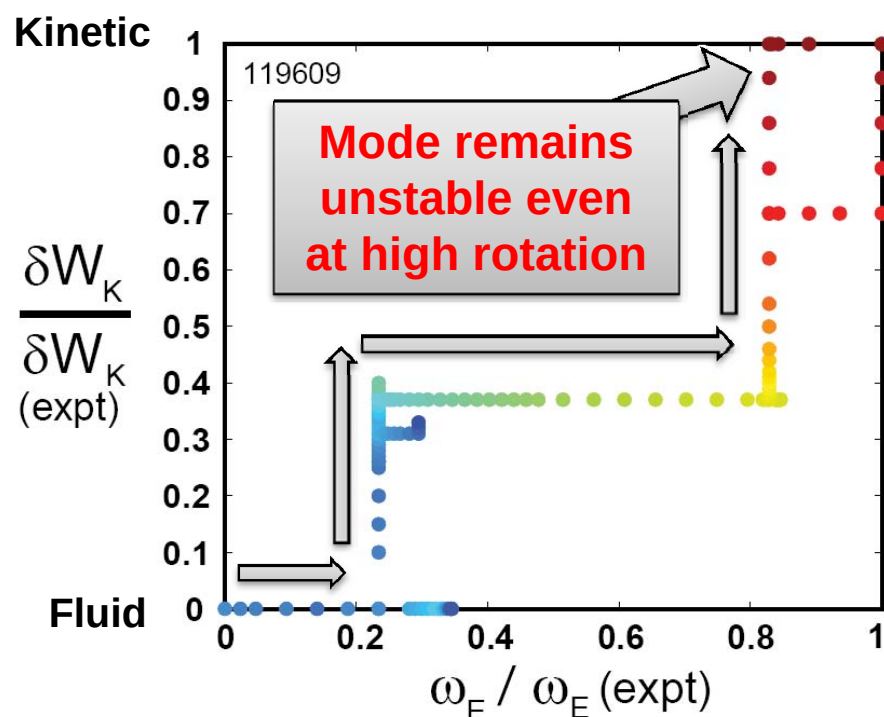
MARS-K consistent with EFC results and MARS-F trends

- MARS-K full kinetic $\delta W_K \rightarrow$ multiple modes can be present
 - Mode identification and eigenvalue tracking more challenging
- Track roots by scanning fractions of experimental ω_E and δW_K :

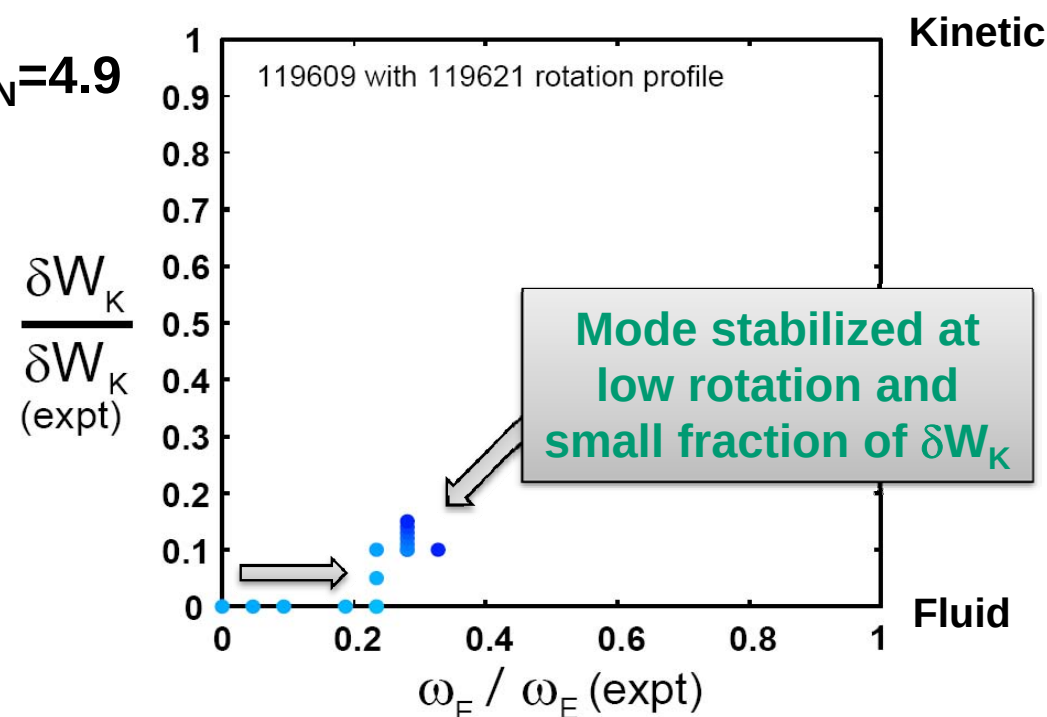
**Experimentally
unstable case**



**Experimentally
stable case**

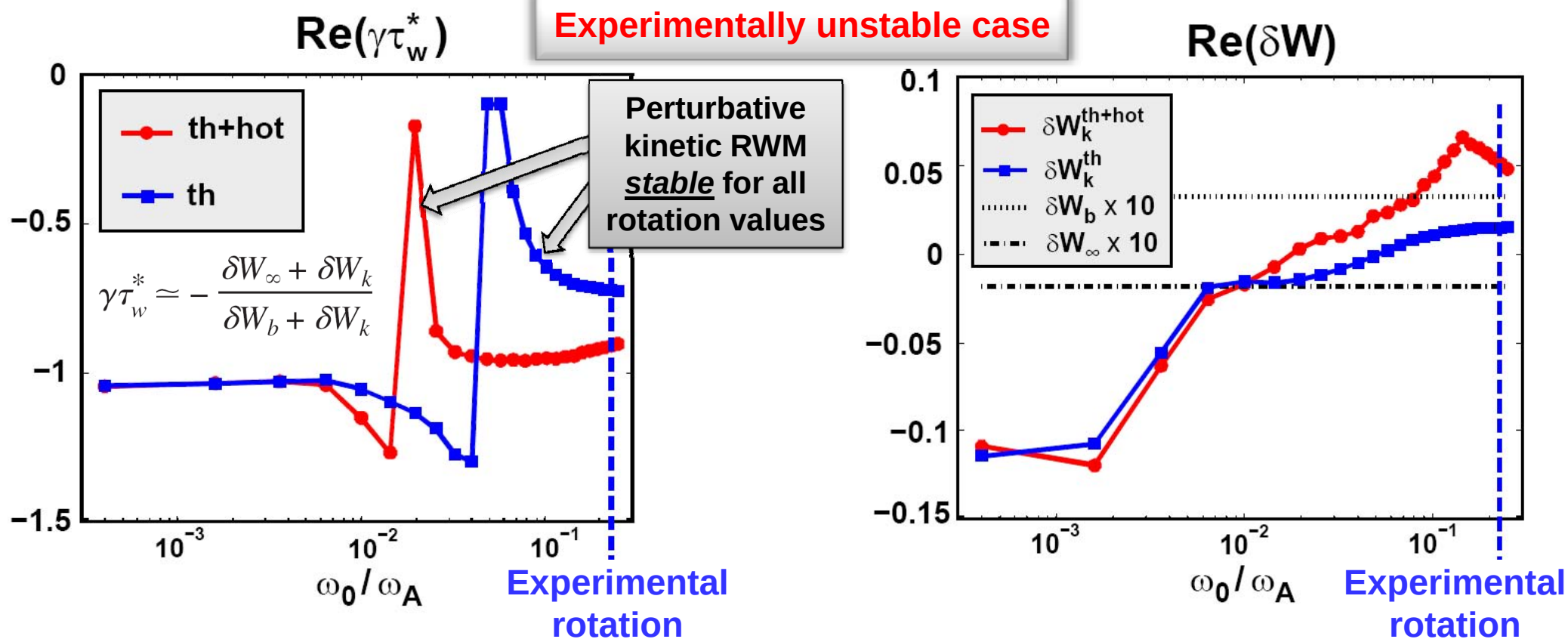


$\beta_N = 4.9$



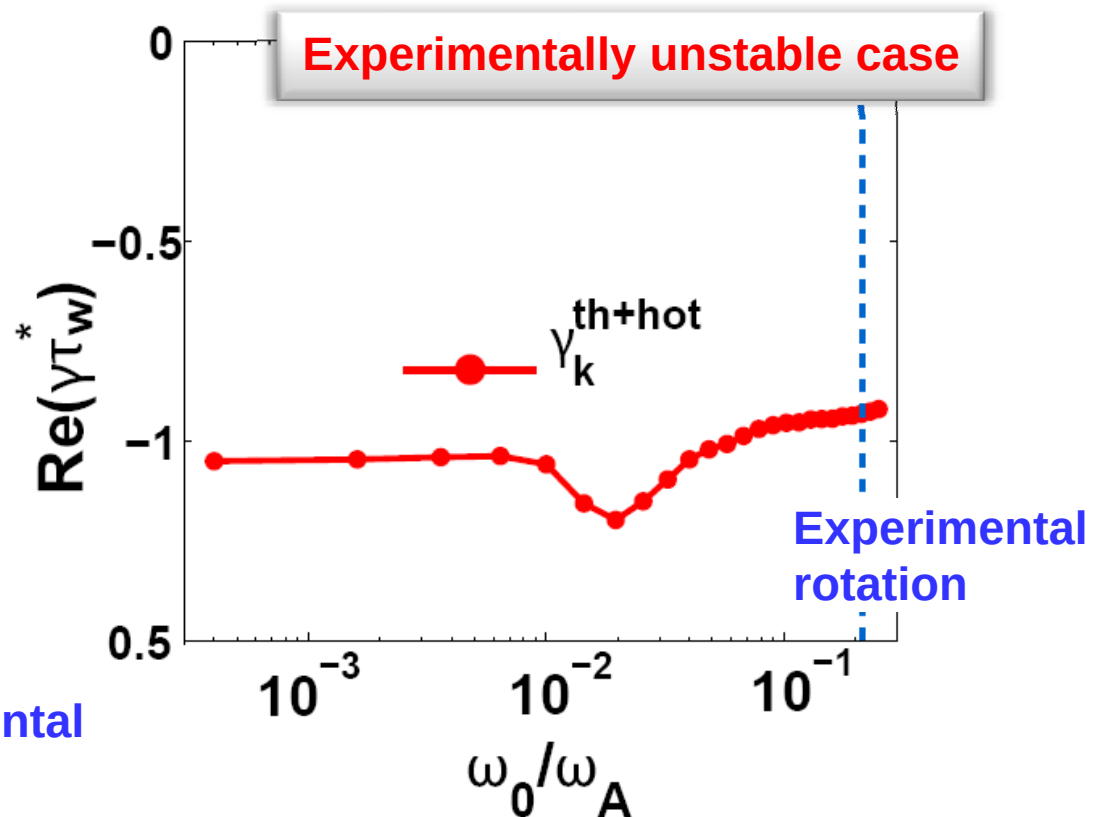
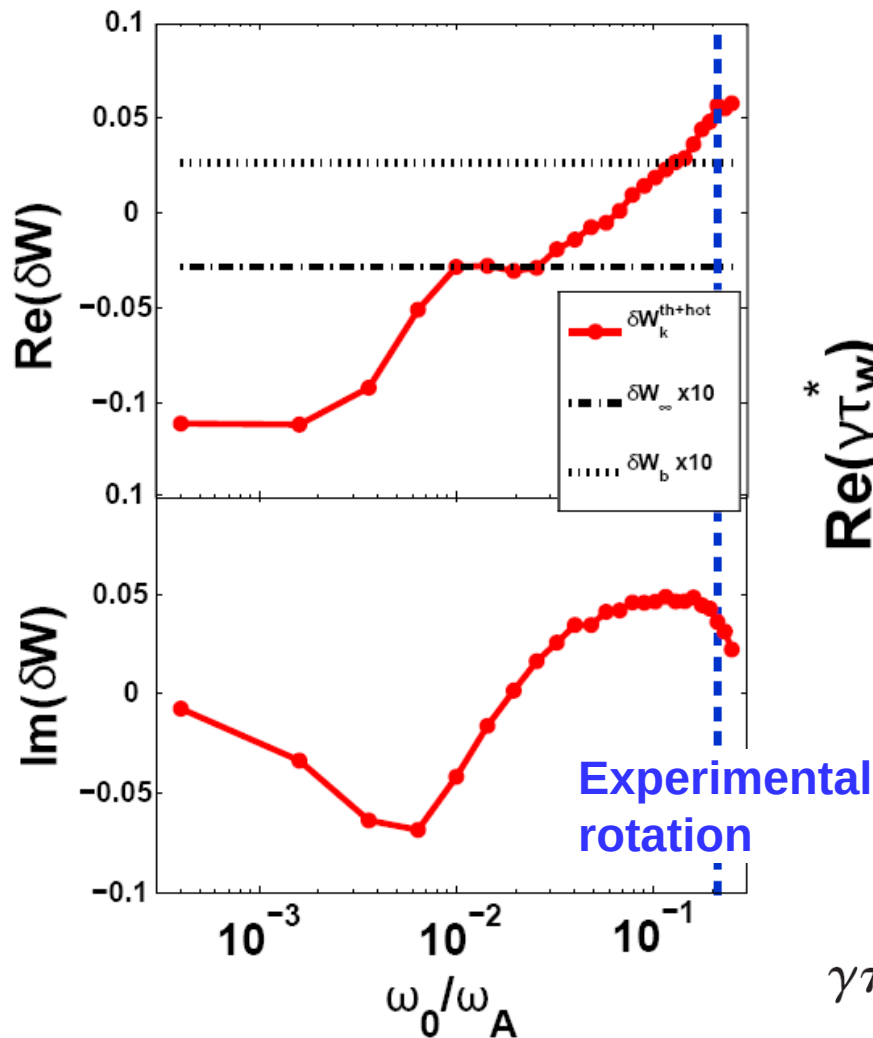
Perturbative approach predicts unstable case to be stable → inconsistent with self-consistent treatment and experiment

- Perturbative approach uses marginally unstable fluid eigenfunction at zero rotation in limit of no kinetic dissipation
- For cases treated here, $|\delta W_k|$ can be $\gg |\delta W_\infty|$ and $|\delta W_b|$
 - Possibility that rotation/dissipation can modify eigenfunction & stability



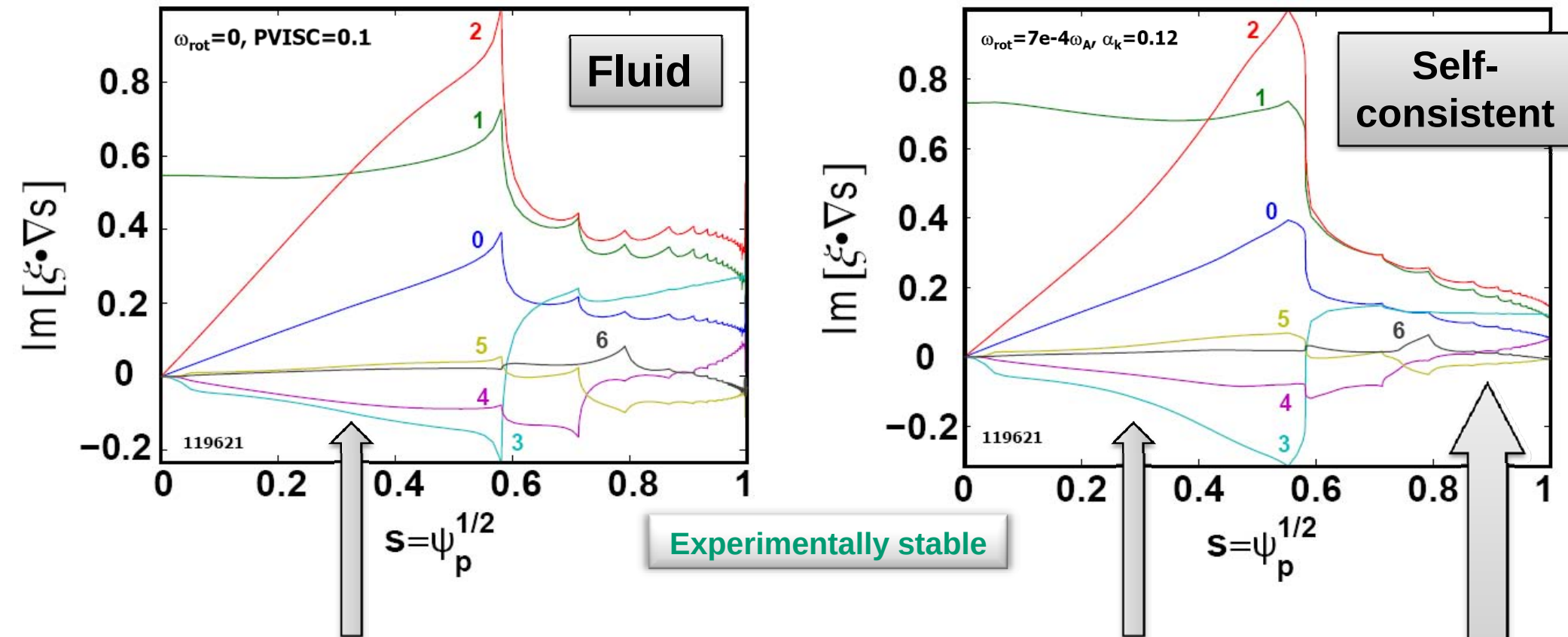
Perturbative approach results sensitive to β_N (but remain inconsistent with experiment)

- Higher $\beta_N = 5.2$ does not exhibit approach to marginal stability
- $|\delta W_K|$ does not approach 0 $\rightarrow \gamma \tau_w \approx -1$



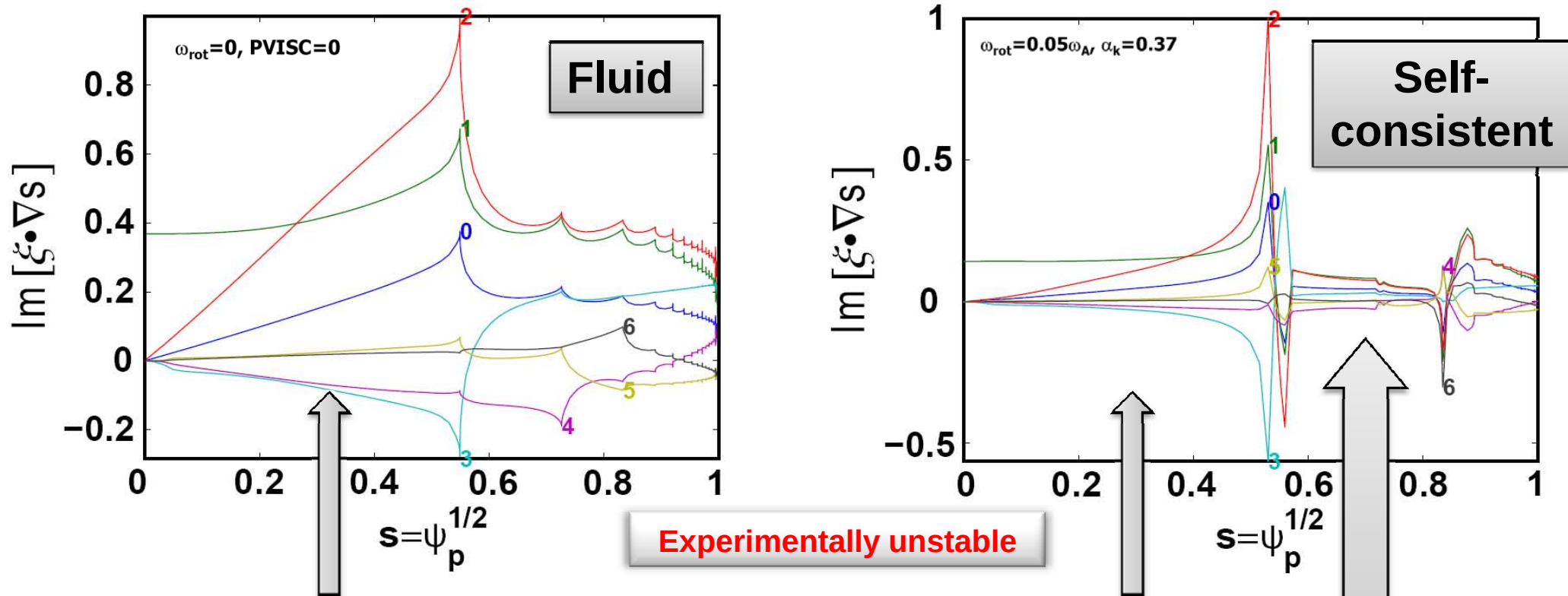
$$\gamma\tau_w^* \approx -\frac{\delta W_\infty + \delta W_k}{\delta W_b + \delta W_k}$$

MARS-K self-consistent calculations for stable case indicate modifications to eigenfunction begin to occur at low rotation



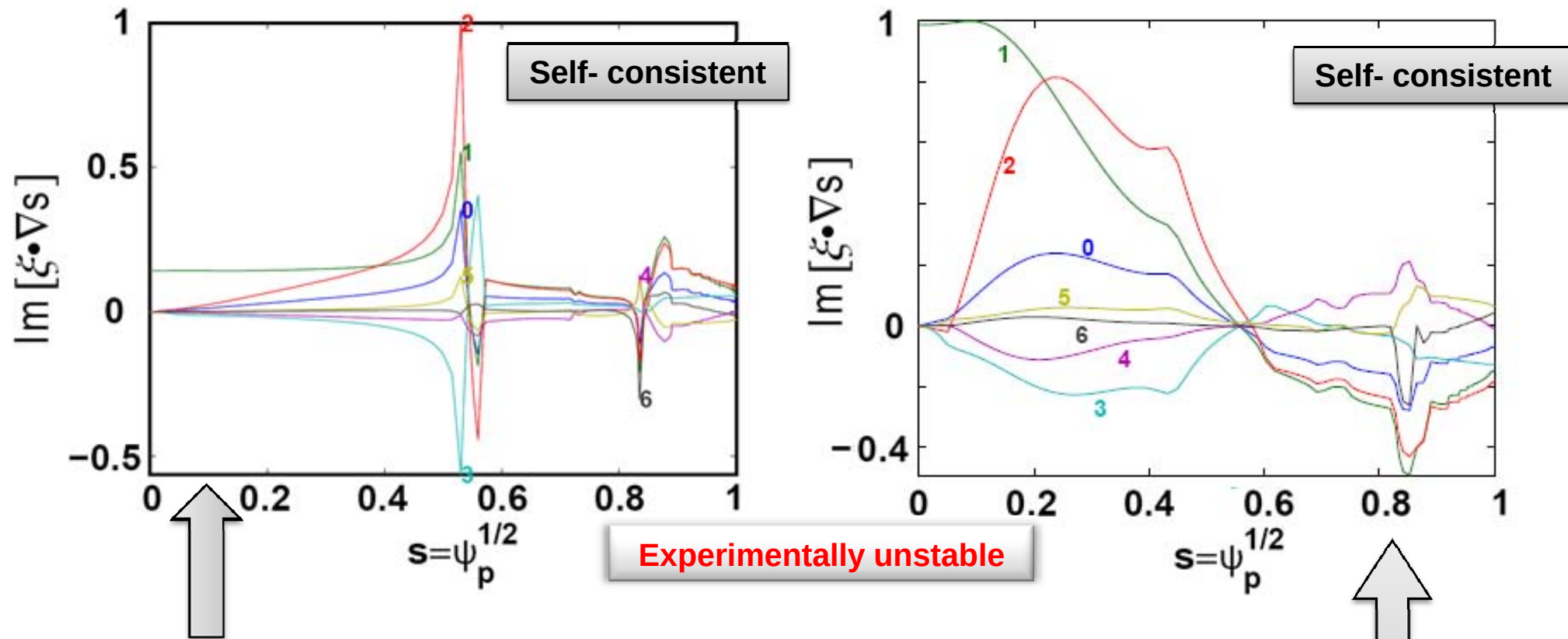
- Self-consistent (SC) eigenfunction qualitatively similar to fluid eigenfunction in plasma core
- SC RWM $\xi_{\perp}$ amplitude reduced at larger r/a
 - Low $\omega_E / \omega_E (\text{expt}) = 0.3\%$, $\delta W_K / \delta W_K (\text{expt}) = 12\%$
 - Reduced amplitude could reduce dissipation, stability

MARS-K self-consistent calculations indicate rotation and dissipation can strongly modify RWM eigenfunction



- Self-consistent (SC) eigenfunction shape **differs** from fluid eigenfunction in plasma core
- SC RWM $\xi_{\perp}$ substantially different at larger r/a
 - Moderate ω_E / ω_E (expt) = 22%, $\delta W_K / \delta W_K$ (expt) = 37%
 - Differences could be even larger at full rotation and δW_K
 - Does reduced edge $\xi_{\perp}$ amplitude explain reduced stability?

At full rotation and kinetic effects in NSTX, MARS-K indicates likely transition to 2nd unstable eigenfunction



- SC RWM $\xi_{\perp}$ at fraction of ω_E and δW_K
 - Moderate ω_E / ω_E (expt) = 22%, $\delta W_K / \delta W_K$ (expt) = 37%
- SC RWM $\xi_{\perp}$ at **full experimental ω_E and δW_K**
 - Eigenfunction shape substantially modified $\rightarrow$ transition to 2nd mode?

Summary

- Edge rotation ($q \geq 4$, $r/a \geq 0.8$) important for RWM
 - Trends consistent with stability calculations using MARS-F
- Stability quite sensitive to edge ω_E profile
 - Essential to include accurate edge ∇p in E_r profile
 - Poloidal rotation can also influence marginal stability
 - Rotation control could assist RWM stabilization (planned)
- MARS-K (full kinetic) trends (stable/unstable) using self-consistent approach consistent with experiment
- Perturbative treatment inconsistent – overly stable
 - Edge eigenfunction strongly modified by rotation/dissipation
 - Reduction/modification of $\xi_{\perp}$ reduces kinetic stabilization

Future work, remaining questions

- Future work:
 - Collisions were not included in MARS-K analysis shown
 - Adding soon - will modify electron contribution to δW_K , other?
 - Need more systematic benchmarking of fluid and kinetic δW
 - More complete comparisons to experiment: γ , ω , ξ
- Questions:
 - Are differences between perturbative & SC unique to NSTX?
 - What determines range of validity of perturbative approach?
 - How large can δW_K and dissipation be?
 - What are effects of large ω_E on dispersion?
 - Is underlying single-fluid MHD treatment sufficient?
 - Eigenfunctions, dissipation can be highly localized near rationals
 - Is continuum damping computed accurately?

$$\gamma\tau_w^* \simeq - \frac{\delta W_\infty + \delta W_k}{\delta W_b + \delta W_k}$$

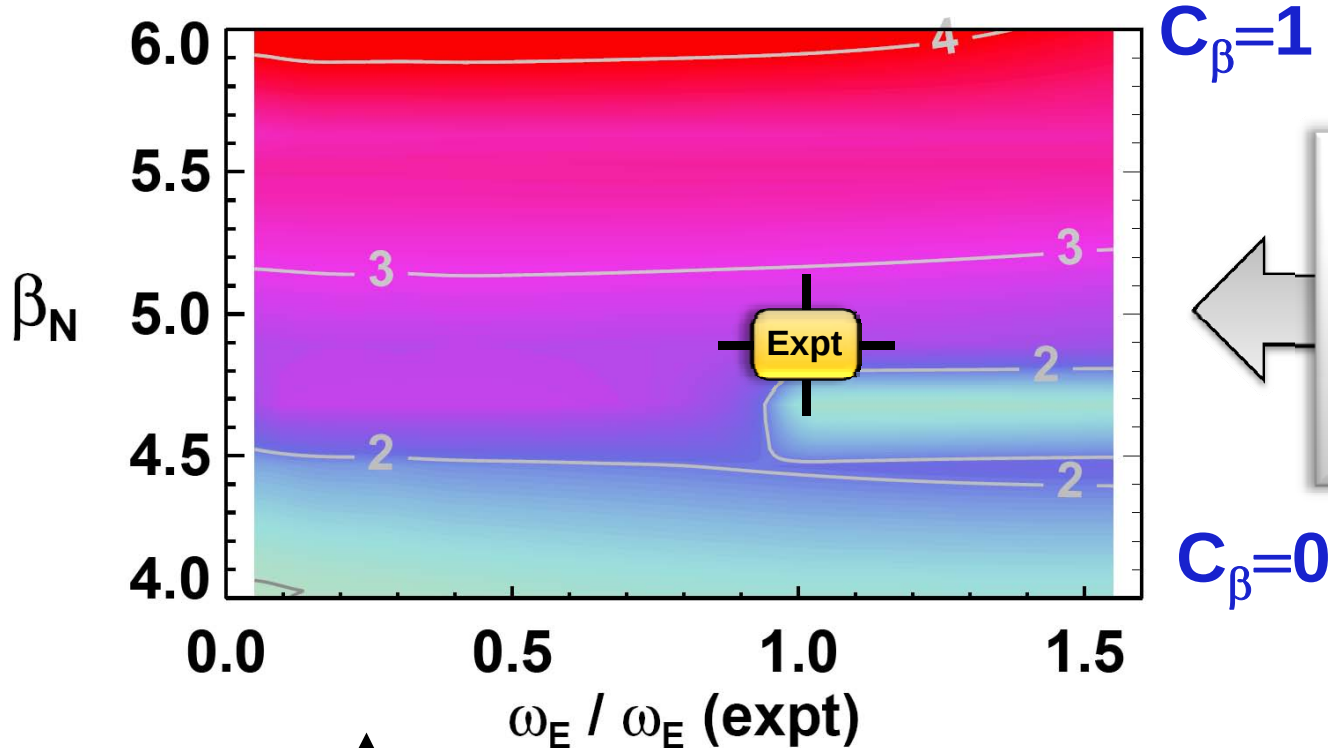
Backup

MARS-F using marginally unstable $\omega_E = \Omega_{\phi-C}$ predicts n=1 RWM to be robustly unstable $\rightarrow$ inconsistent with experiment

Calculated n=1 $\gamma\tau_{\text{wall}}$

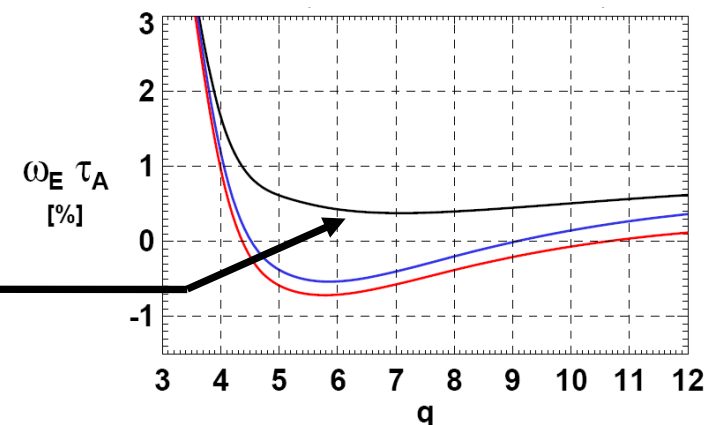
Using experimentally marginally unstable profiles

$$C_\beta \equiv \frac{\beta_N - \beta_N(\text{no-wall})}{\beta_N(\text{wall}) - \beta_N(\text{no-wall})}$$



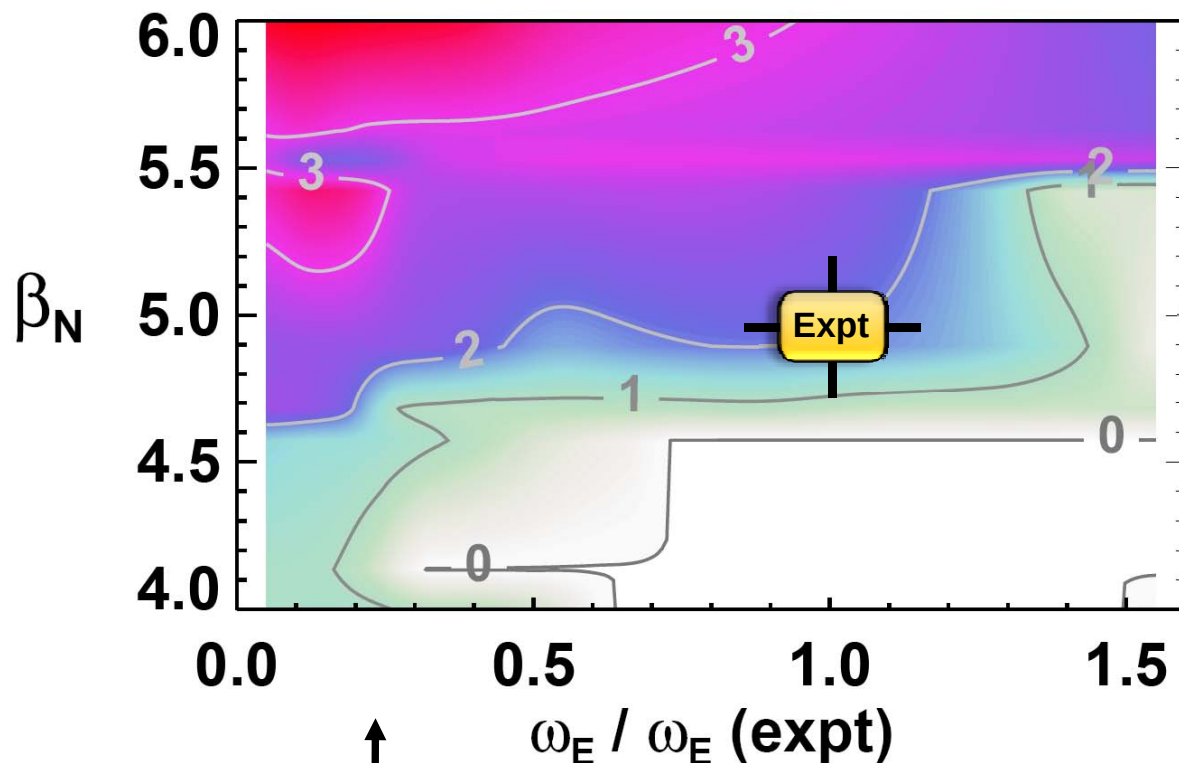
- γ depends only weakly on rotation
- γ increases with β_N

$\omega_{*C} / \omega_{*C}(\text{expt}) = 0, u_\theta = 0$



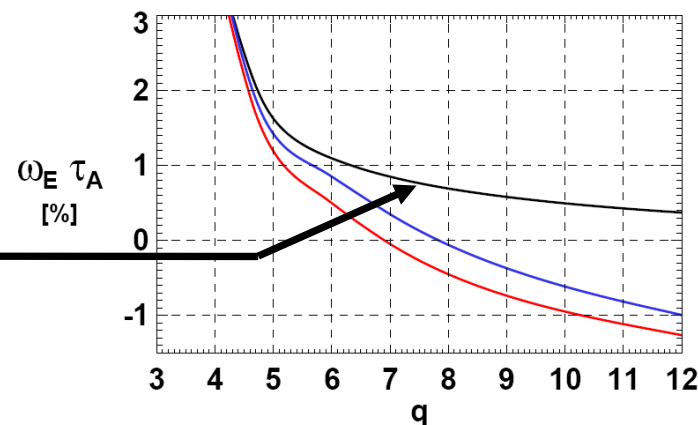
MARS-F using stable $\omega_E = \Omega_{\phi-C}$ profile predicts $n=1$ RWM to be unstable $\rightarrow$ inconsistent with experiment

Calculated $n=1$ $\gamma\tau_{\text{wall}}$
using experimentally stable profiles



- $n=1$ RWM predicted to be unstable for $\beta_N > 4.6$, but actual plasma operates stably at $\beta_N \geq 5$

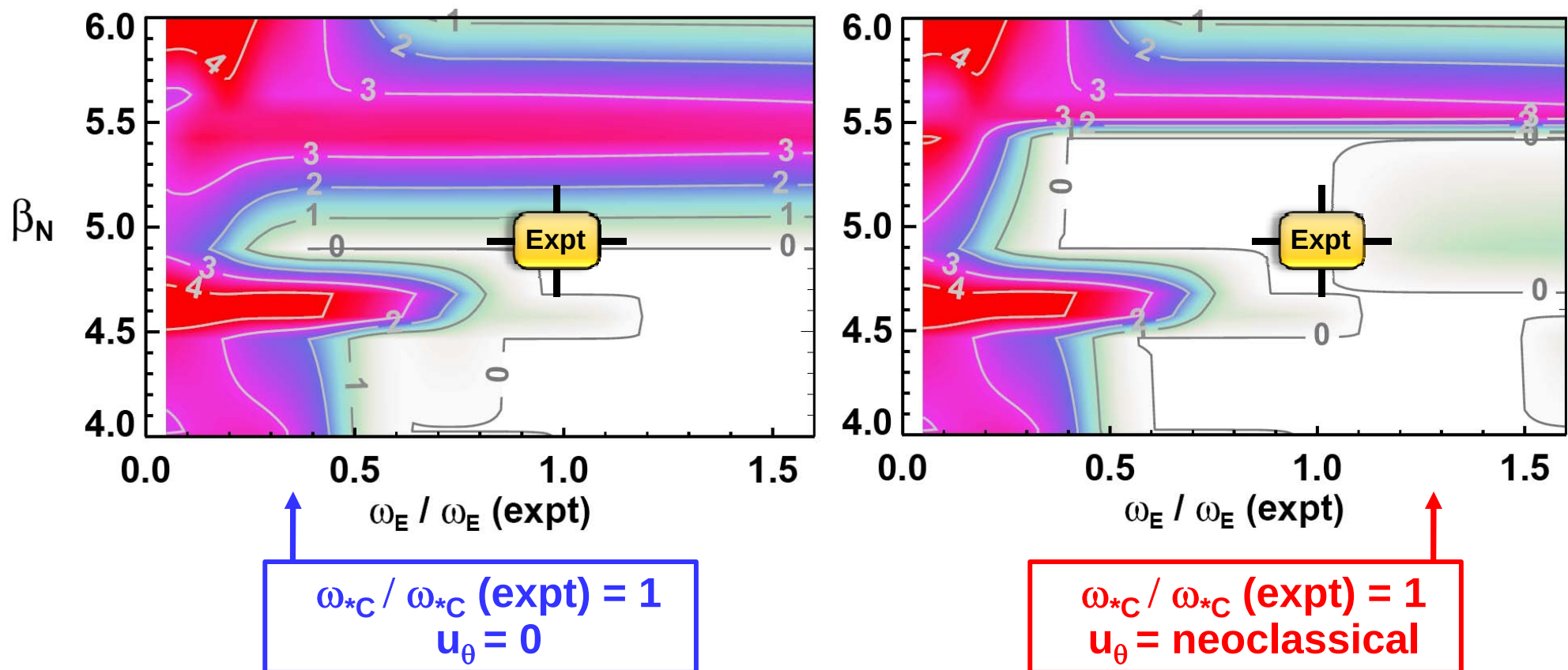
$\omega_{*C} / \omega_{*C} (\text{expt}) = 0, u_\theta = 0$



Inclusion of v_{pol} in ω_E can sometimes modify marginal stability boundary – example: wall position variation

Calculated $n=1$ $\gamma\tau_{wall}$

experimentally stable profiles and b_{wall}/a artificially increased $\times 1.1$



- Increased wall distance lowers with-wall limit to $\beta_N \sim 5.5$
- Case with $u_\theta=0$ has lower marginal stability limit $\beta_N \sim 5$