Transport and linear stability studies for PPCD optimization in RFPs

Gian Luca Delzanno (LANL)

John M. Finn (LANL)

John Sarff (UWi)

Los Alamos NATIONAL LABORATORY EST.1943

NNSA

Pulsed Poloidal Current Drive (PPCD)

- Standard MH RFP:
  - Several unstable MHD modes generate a dynamo to maintain steady state
  - Strong transport in stochastic magnetic field → bad confinement
- PPCD improves confinement
  - System is not stationary
  - Poloidal current reduces MHD dynamo and magnetic stochasticity
  - Reduced transport

Chapman et al, POP 2002

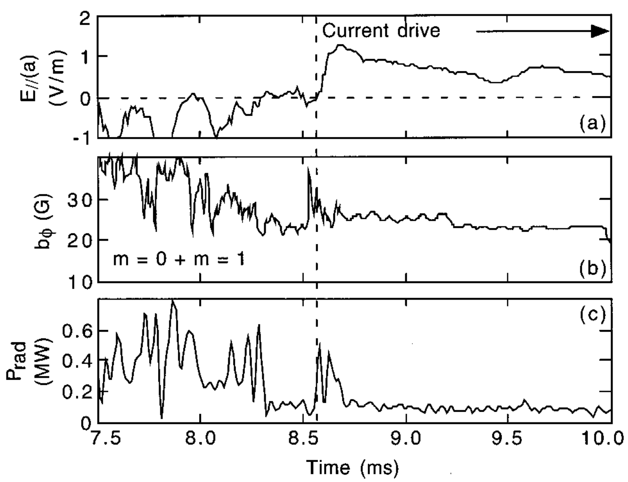


FIG. 4. From the plasma on the right in Fig. 3, (a) surface parallel electric field, (b) root-mean-square (rms) sum of $m=0$ and $m=1$ toroidal magnetic fluctuations with $n=1-15$, and (c) total radiated power (photons only).

Approach & Goal

Approach

- Perform 1D transport simulations of the fields subject to PPCD external driving
- For the profiles obtained by the transport simulations, perform MHD stability analysis of several modes

Goal

- Understand the effect of PPCD
- Optimize the PPCD waveforms to improve RFP performance and pulse length

Approach: 1D transport

- Azimuthal component of Ohm's law

$$\frac{\partial A_\theta(r,t)}{\partial t} = -\frac{u_r}{r}\frac{\partial}{\partial r}(rA_\theta) + \eta\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\right] \qquad E_\theta = -\frac{\partial A_\theta}{\partial t}$$

- Axial component of Ohm's law

$$\frac{\partial A_z(r,t)}{\partial t} = -u_r\frac{\partial A_z}{\partial r} + \frac{\eta}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) \qquad E_z = -\frac{\partial A_z}{\partial t}$$

- Force balance

$$\mathbf{J}\times\mathbf{B} = 0 \qquad B_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) \qquad B_\theta = -\frac{\partial A_z}{\partial r}$$

Approach: 1D transport (2)

- Boundary conditions

$$A_\theta(0,t) = 0 \qquad A_\theta(r_\mathrm{w},t) = \mathrm{PPCD} \qquad E_\theta = -\frac{\partial A_\theta}{\partial t}$$

$$\left.\frac{\partial A_z}{\partial r}\right|_{r=0} = 0 \qquad A_z(r_w,t) = \mathrm{PPCD} \qquad E_z = -\frac{\partial A_z}{\partial t}$$

- Examples of PPCD waveforms

$$E_\theta(r_\mathrm{w},t) = \alpha E_0 \Longrightarrow A_\theta(r_\mathrm{w},t) = A_\theta(r_\mathrm{w},0) - \alpha E_0 t$$

$$E_z(r_\mathrm{w},t) = E_0\left(1 - \frac{t}{T}\right) \Longrightarrow A_z(r_\mathrm{w},t) = A_z(r_\mathrm{w},0) - E_0 t\left(1 - \frac{t}{2T}\right)$$

Approach: MHD stability analysis

- Dimensionless MHD:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = D\nabla^2 \rho \\ \\ \rho \left[\dfrac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\, \mathbf{V} \right] = \mathbf{J} \times \mathbf{B} + \nu \rho \nabla^2 \mathbf{V} \\ \\ \dfrac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \end{cases}$$

- Zero β limit; aspect ratio R/a=4
- $\nu = \dfrac{1}{\mathrm{R_e}} \quad \eta = \dfrac{1}{\mathrm{S}}$
- Investigate m=0,1 and various n modes

Equilibrium / initial state

- Defined by

$$\lambda = \frac{\mathbf{J}\cdot\mathbf{B}}{\mathbf{B}^2} = 3.6\left[1 - \left(\frac{r}{r_{\mathrm{w}}}\right)^{2.6}\right]$$

with

$$\mathbf{J}\times\mathbf{B} = 0$$

- This is NOT an Ohmic equilibrium, therefore we modify Ohm's law (equivalent to adding a dynamo term)

$$E_\theta = u_r B_z + \eta J_\theta + F_\| \frac{B_\theta}{B}$$

$$E_z = -u_r B_\theta + \eta J_z + F_\| \frac{B_z}{B}$$

- These equations determine $u_r(r)$ and $F_\|(r)$

Equilibrium (2)

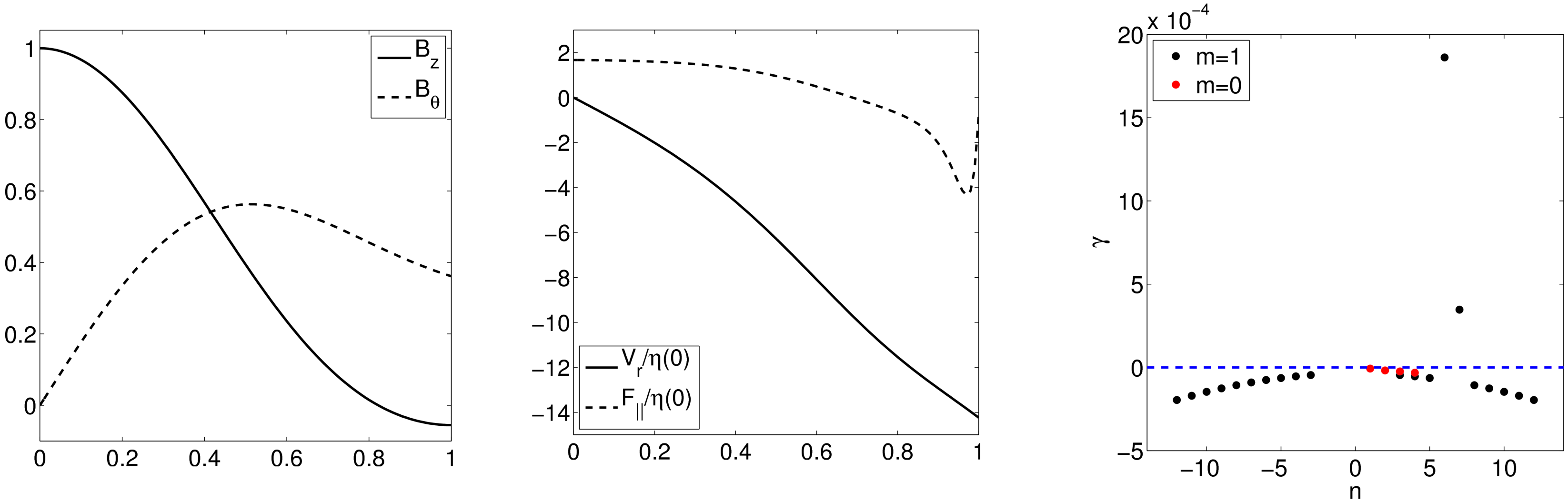
- In the initial equilibrium: E_θ=0, E_z=E_0, leading to

$$u_r = -\frac{E_0 B_\theta}{B^2} \qquad F_\| = \frac{E_0 B_z - \eta \mathbf{J}\cdot\mathbf{B}}{B}$$

E_0 determined by:

$$\int_0^{r_w} F_\| |\mathbf{B}| r dr = 0$$

- Profiles

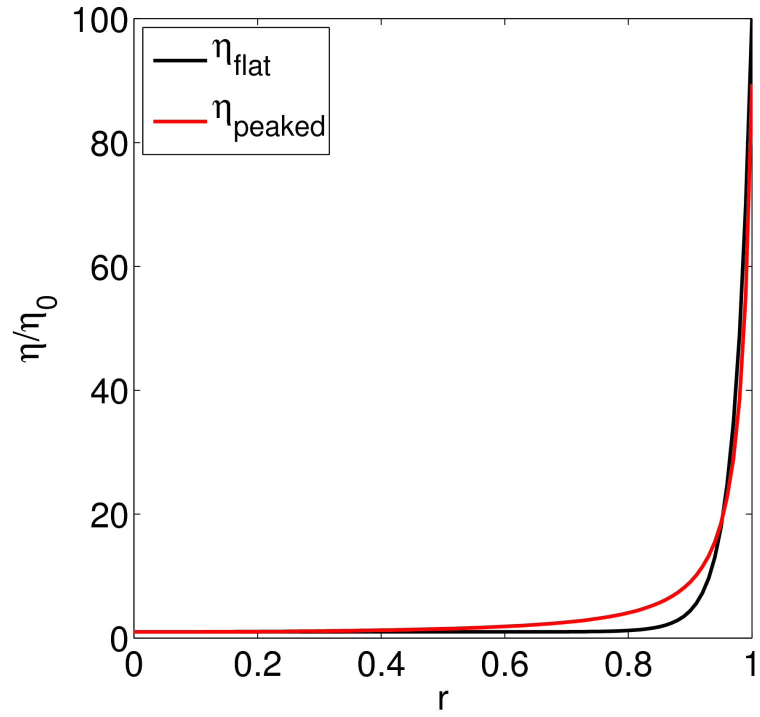


Equilibrium (3)

- In this study, we consider two profiles of resistivity

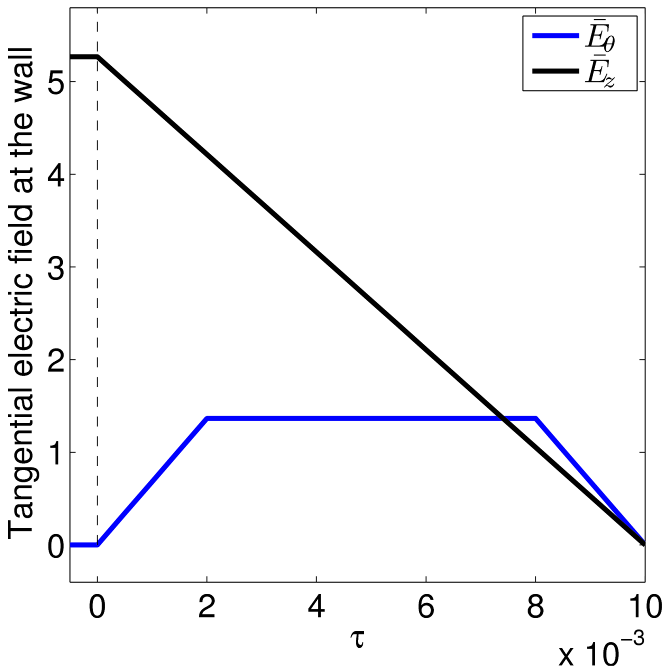
$$\eta_{\text{flat}} = \eta_0 \left[1 + 9\left(\frac{r}{r_{\text{w}}}\right)^{20}\right]^2$$

$$\eta_{\text{peaked}} = \eta_0 \left[1 - 0.95\left(\frac{r}{r_{\text{w}}}\right)^{2}\right]^{-3/2}$$



PPCD studies:

- Program E_θ and E_z with given waveforms
- Length of the pulse: T=0.01 (in resitive time units)
- $F_\parallel(r,t)=a(t)F_\parallel(r)$ ramped down linearly to zero at T/5=0.002

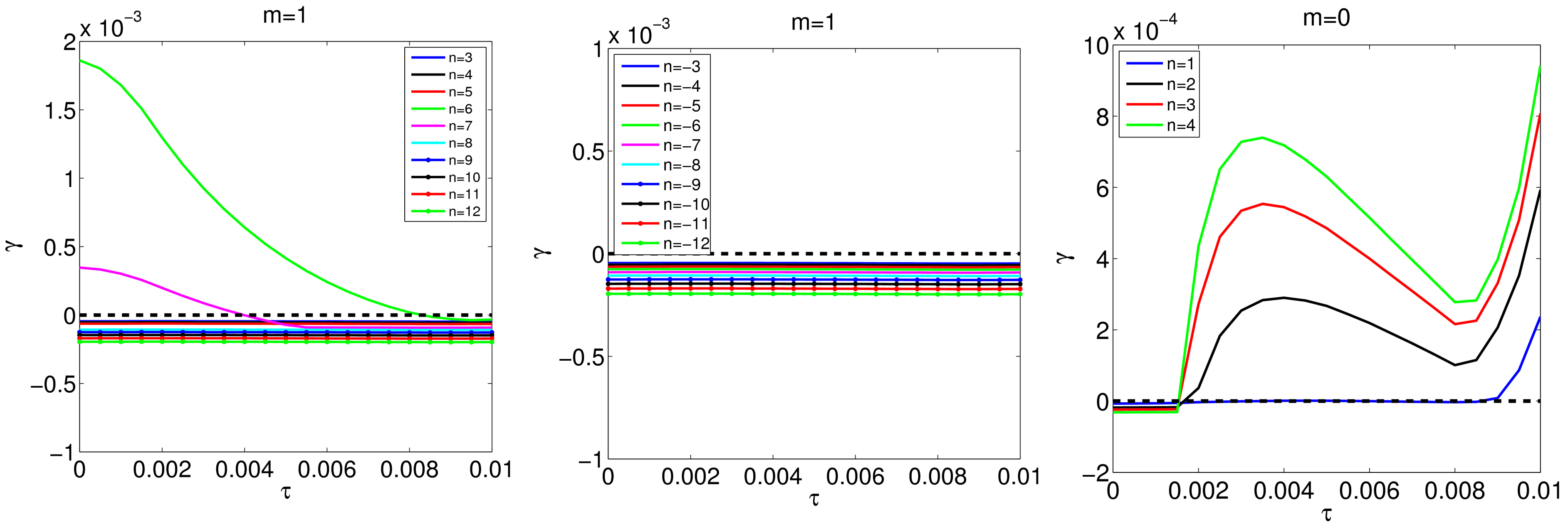


$$\Delta\Phi = \frac{\Phi(t=0) - \Phi(t=T)}{\Phi(t=0)} = 10\%$$

$$\Phi = 2\pi \int_0^{r_\mathrm{w}} B_z r dr$$

$$\eta = \eta_\mathrm{flat}$$

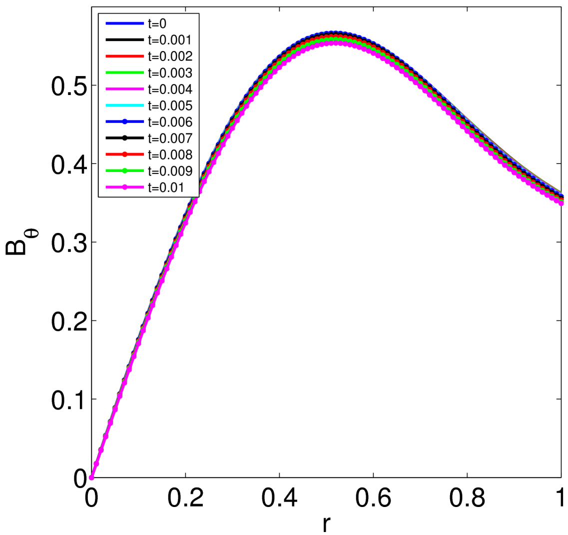
Growth rates:

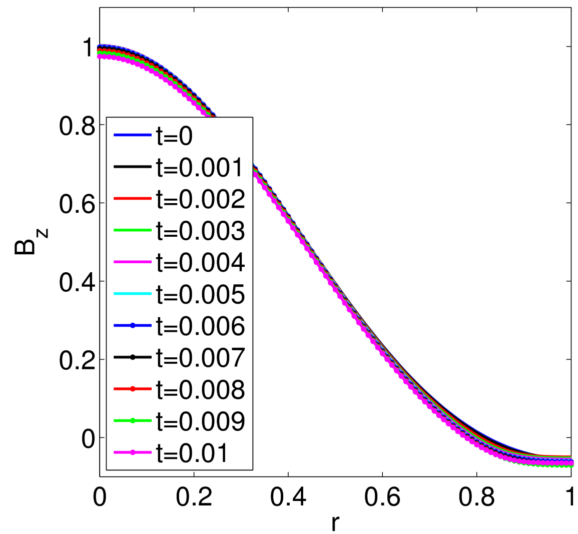


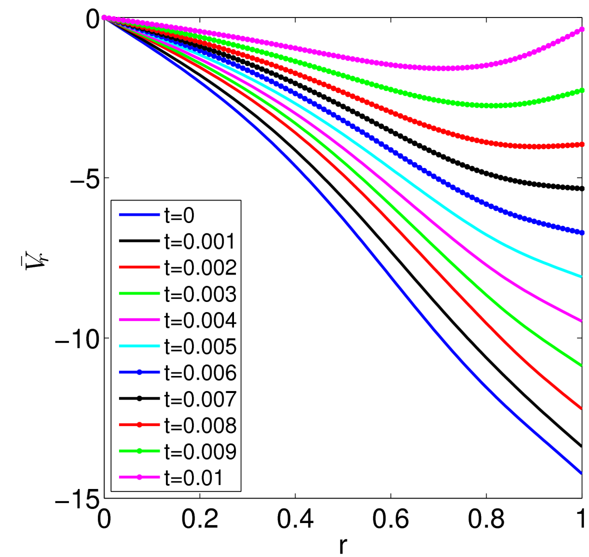
Stability window: $\Delta = 0$ $\eta = \eta_{\text{flat}}$

$$\Delta\Phi = \frac{\Phi(t=0) - \Phi(t=T)}{\Phi(t=0)} = 10\%$$

Profiles:



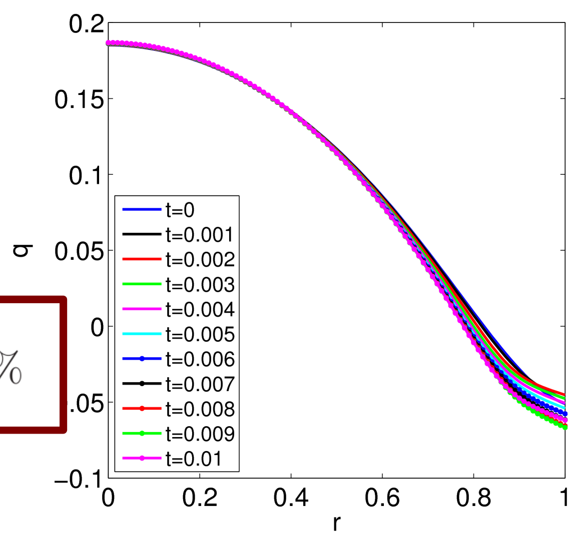


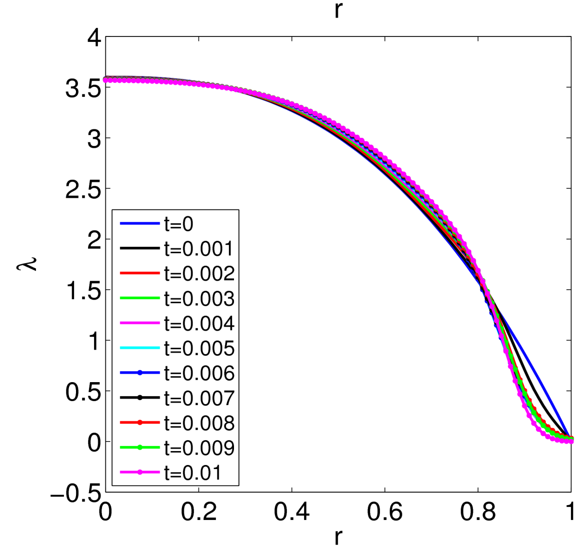


$$\Delta = 0$$

$$\eta = \eta_{\text{flat}}$$

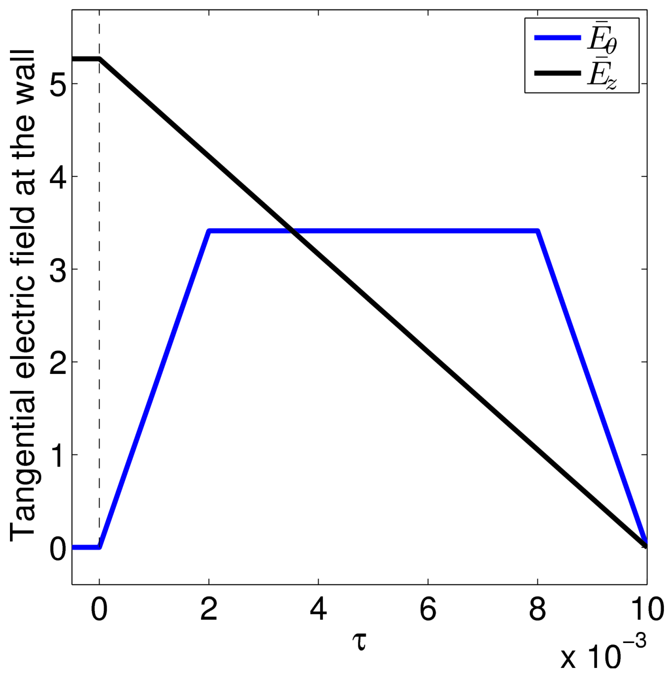
$$\Delta\Phi = \frac{\Phi(t=0) - \Phi(t=T)}{\Phi(t=0)} = 10\%$$





PPCD studies (2):

- Program E_θ and E_z with given waveforms
- Length of the pulse: T=0.01 (in resitive time units)
- $F_\parallel(r,t)=a(t)F_\parallel(r)$ ramped down linearly to zero at T/5=0.002

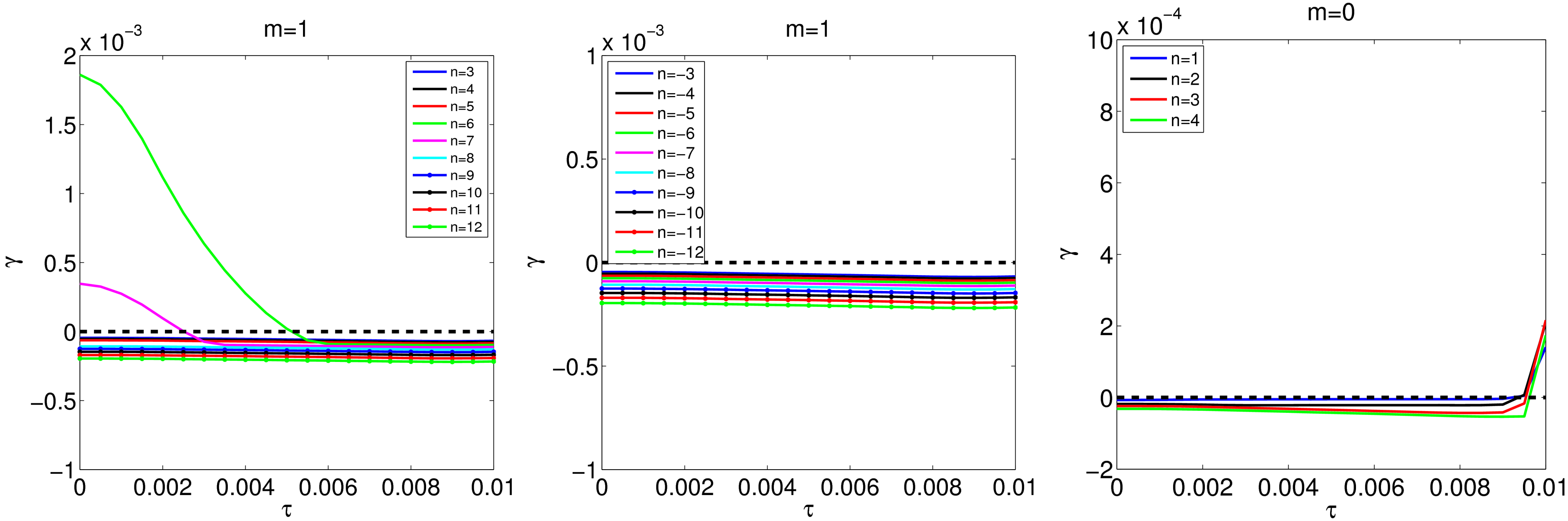


$$\Delta\Phi = 25\%$$

$$\Phi = 2\pi \int_0^{r_w} B_z r dr$$

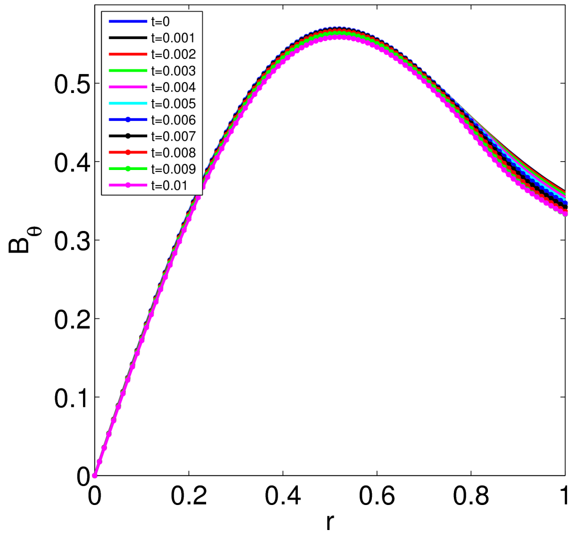
$$\eta = \eta_{\text{flat}}$$

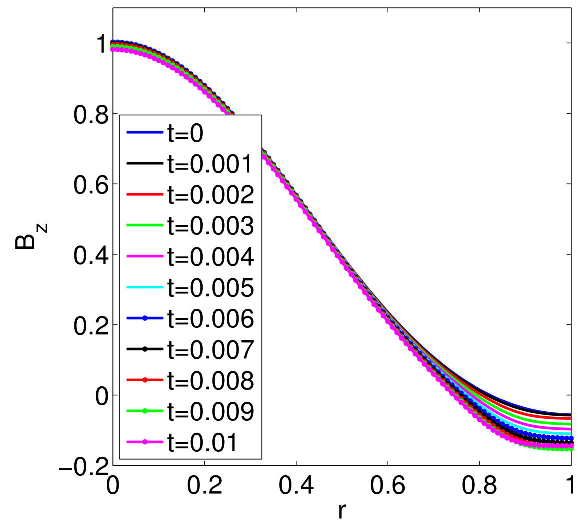
Growth rates:

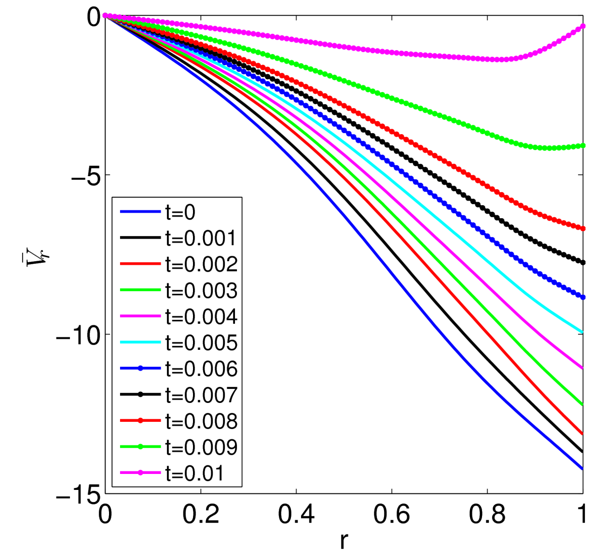


Stability window: $\Delta = 0.0044$ $\eta = \eta_{\text{flat}}$ $\Delta\Phi = 25\%$

Profiles:



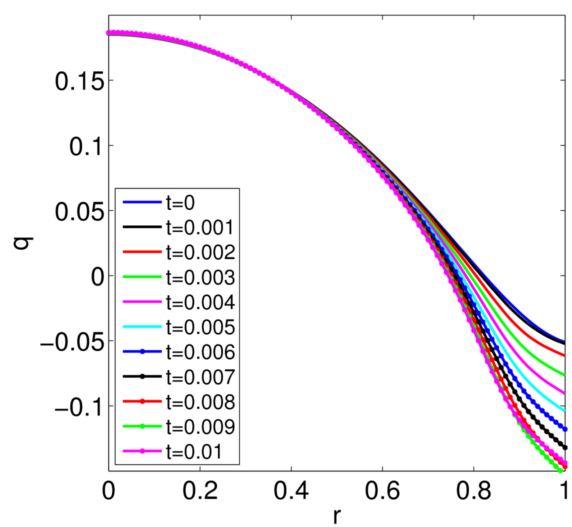


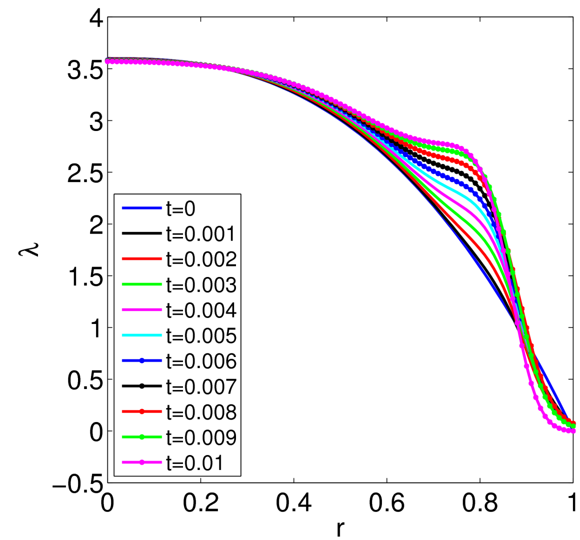


$$\eta = \eta_{\text{flat}}$$

$$\Delta = 0.0044$$

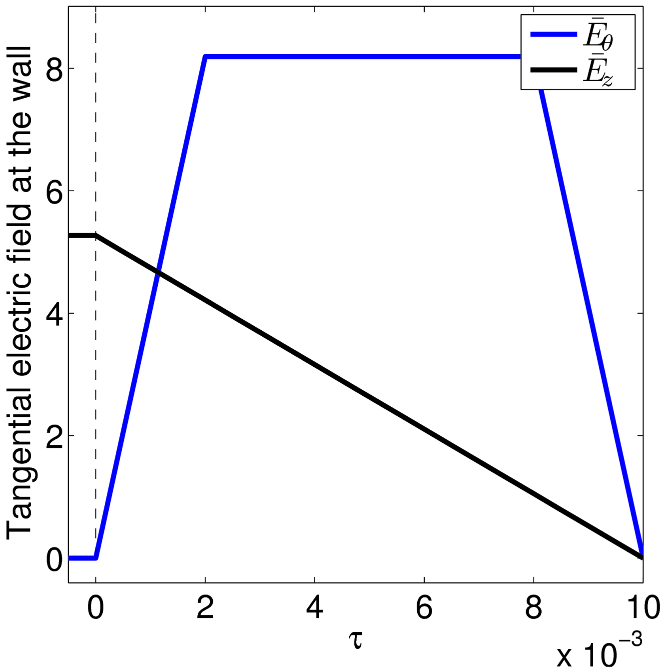
$$\Delta\Phi = 25\%$$





PPCD studies (3):

- Program E_θ and E_z with given waveforms
- Length of the pulse: T=0.01 (in resitive time units)
- $F_\parallel(r,t)=a(t)F_\parallel(r)$ ramped down linearly to zero at T/5=0.002

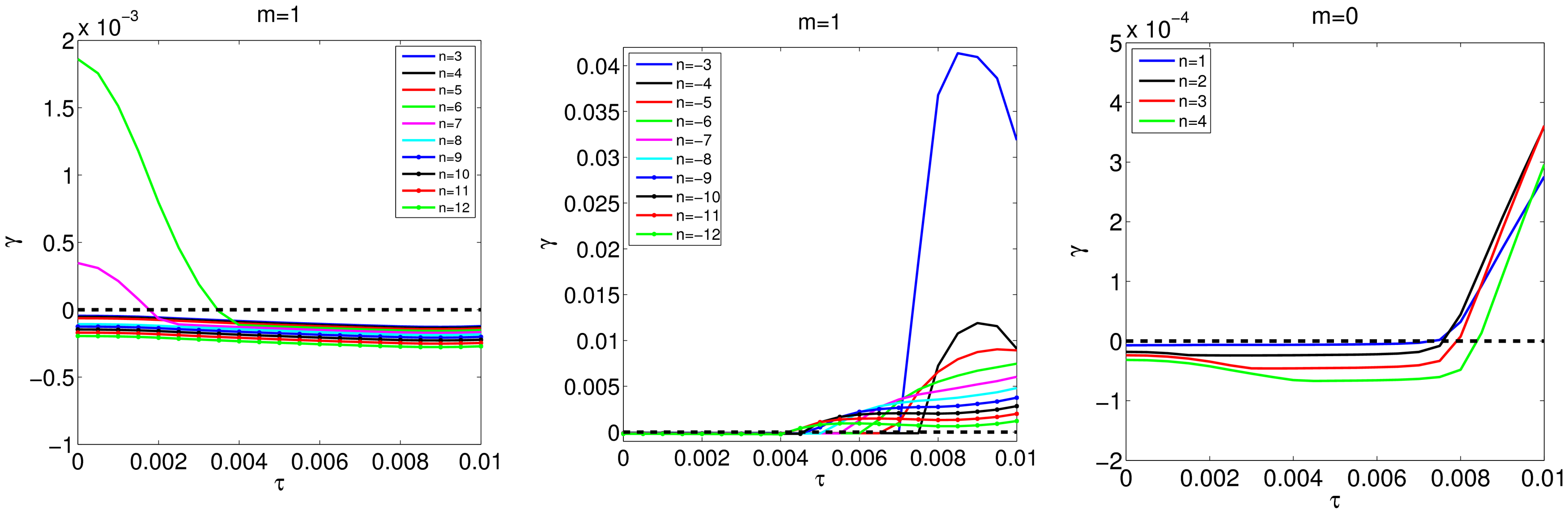


$$\Delta\Phi = 60\%$$

$$\Phi = 2\pi \int_0^{r_w} B_z r dr$$

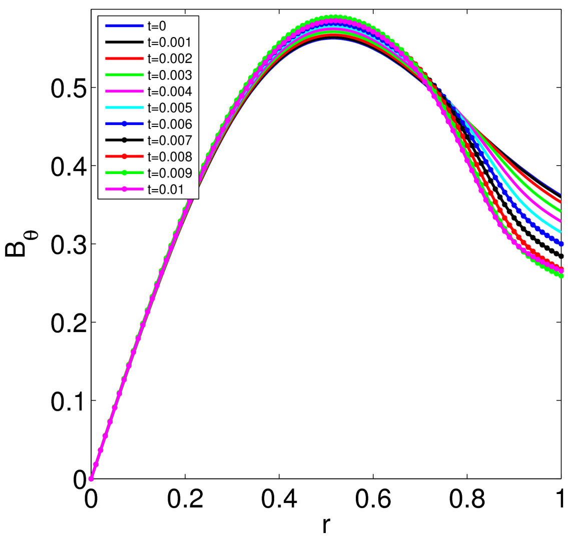
$$\eta = \eta_{\mathrm{flat}}$$

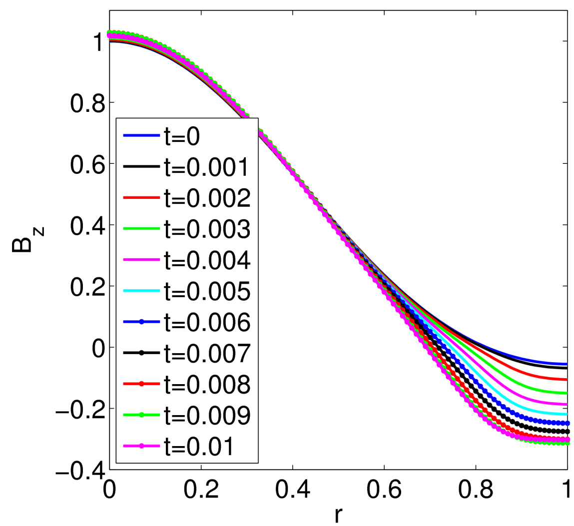
Growth rates:

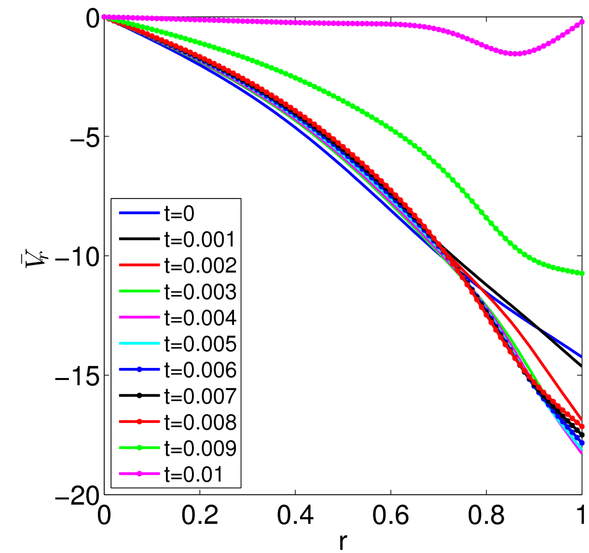


Stability window: $\Delta = 0.0007$ $\eta = \eta_{\text{flat}}$ $\Delta\Phi = 60\%$

Profiles:



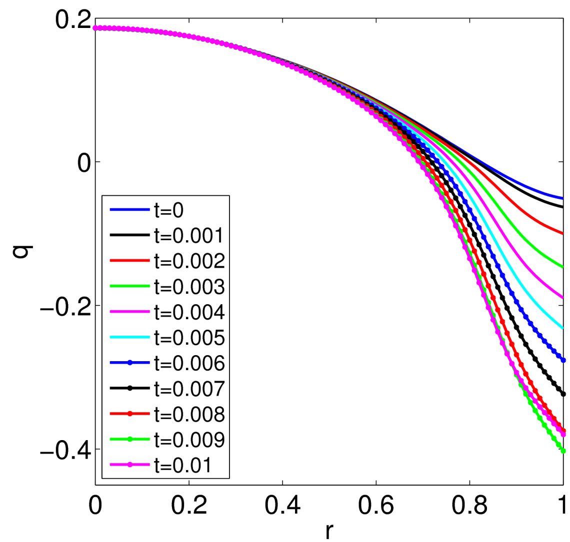


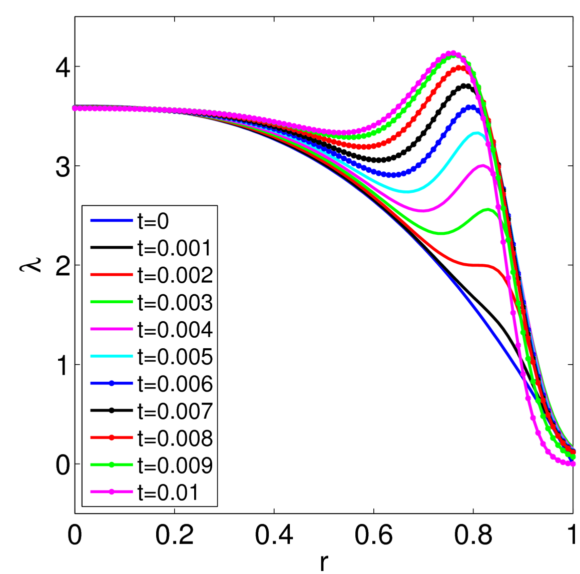


$$\eta = \eta_{\text{flat}}$$

$$\Delta = 0.0007$$

$$\Delta\Phi = 60\%$$

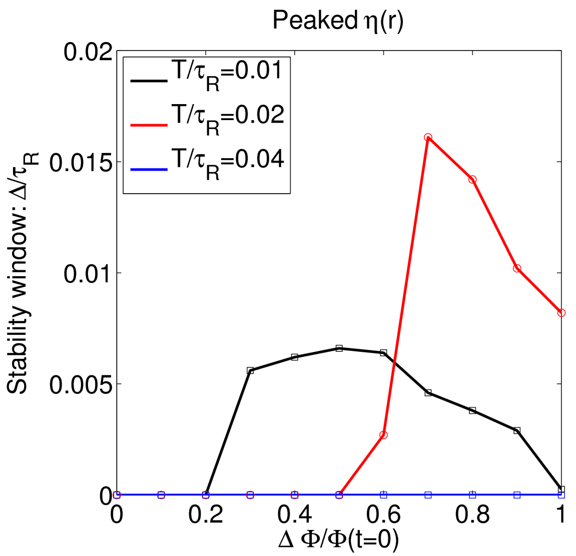


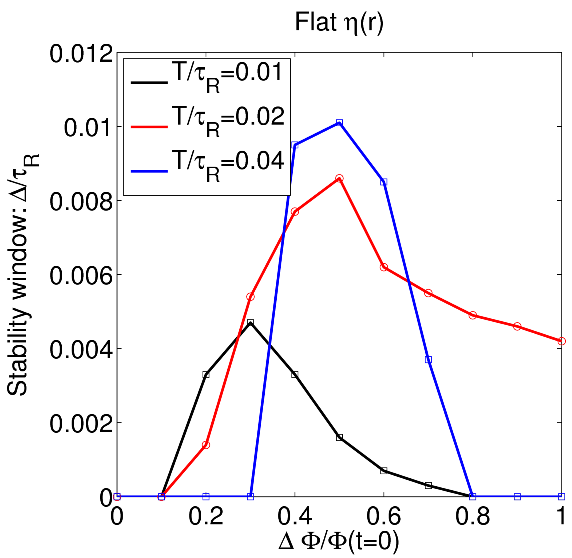


Los Alamos
NATIONAL LABORATORY
EST. 1943

PPCD studies (4): different resistivity profiles

- Similar qualitative behavior for flat and peaked resistivity: PPCD can stabilize the modes and give a stability window
- It appears that the PPCD regime of stabilization is limited to ~1/100 of the resistive time





How could we improve PPCD and extend the pulse?

- Optimize the electric field pulse at the wall to force the system to remain within some stability region
- Simplest idea: shape the electric field pulse at the wall to bring the system into a state where the q and λ profiles remain constant in time.
  - Do states like this exist? Yes, self-similar rampdown
  - They are associated with profiles that decay exponentially in time
  - If a stable state can be found, then the pulse can be extended until the fields fall below a certain threshold

Self-similar rampdown

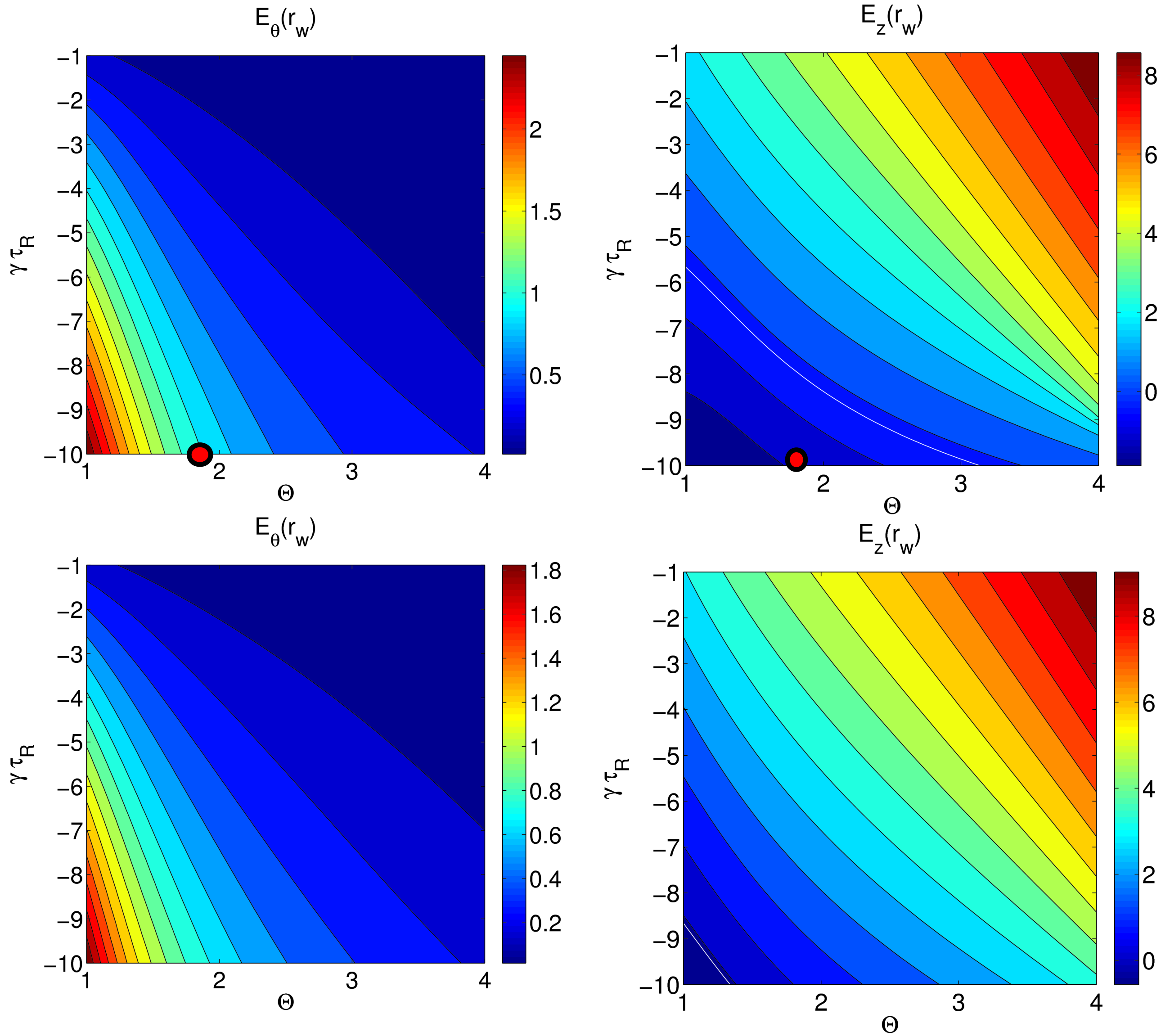
- REF: Nebel, Schnack, Gianakon, POP02
- Start from 1D time-dependent transport equations
- Space and time separable forms for the unknowns
- In simplest form, velocity and resistivity are time-independent, while the magnetic field behaves as $e^{\gamma t}$
- Fix γ and Θ to get the spatial forms of the fields consistent with the rampdown
- Main result: $\gamma=-10\tau_R$ better for stability

Self-similar rampdown: results

- Nebel, Schnack, Gianakon, POP02

$$\eta = \eta_{\text{flat}}$$

$$\eta = \eta_{\text{peaked}}$$

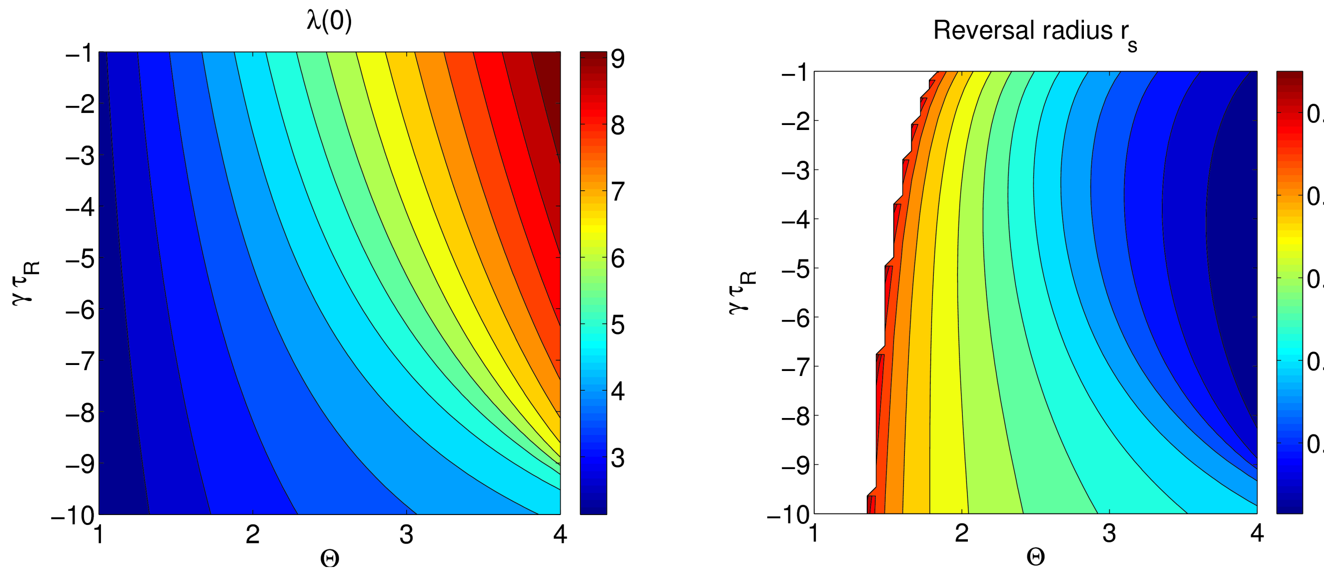


Los Alamos NATIONAL LABORATORY EST. 1943

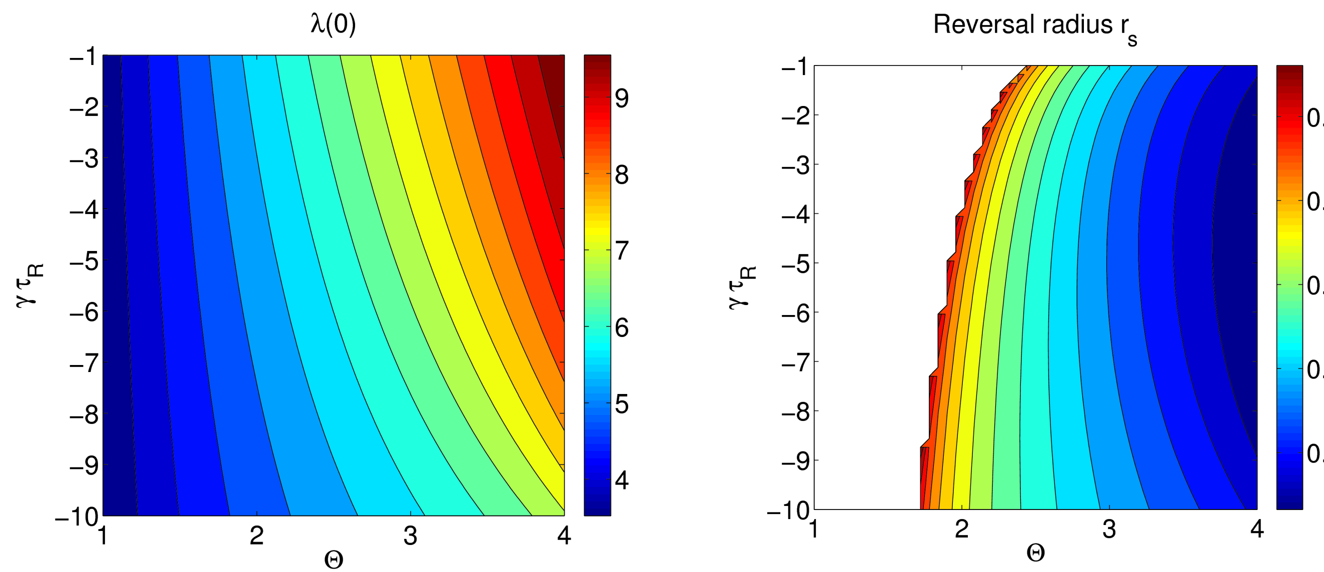
NNSA

Self-similar rampdown: results (2)

$$\eta = \eta_{\text{flat}}$$



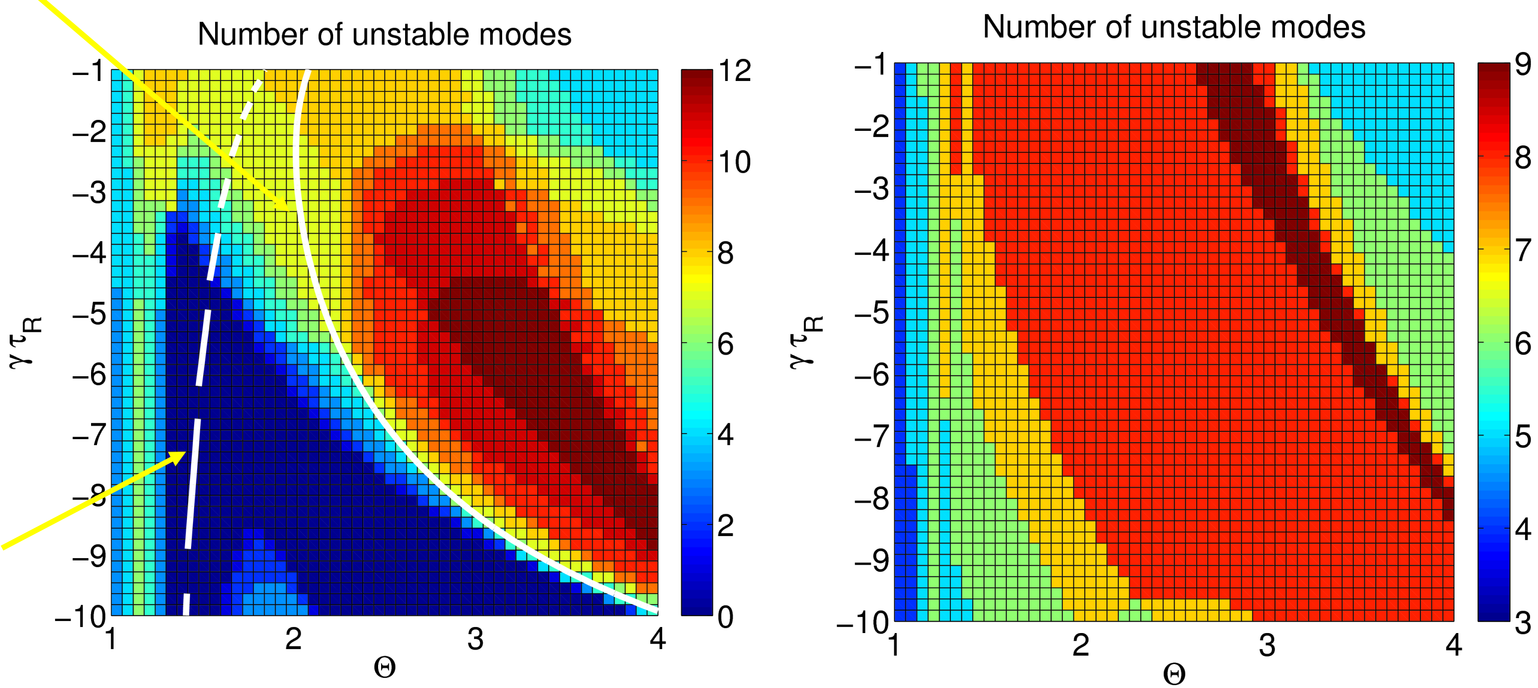
$$\eta = \eta_{\text{peaked}}$$



Self-similar rampdown: stability diagram

$\lambda(r_w)=0$

No stable SS profiles for peaked resistivity!



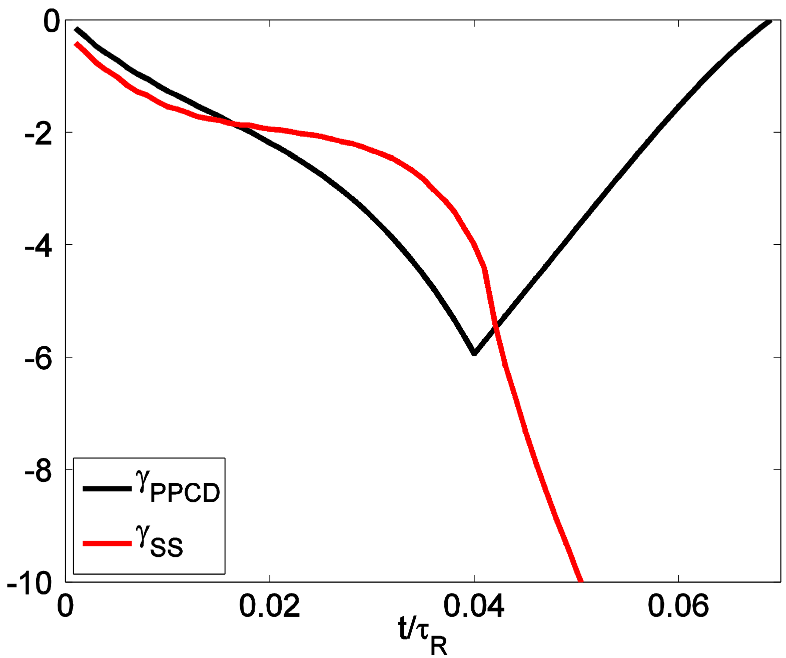
$B_z(r_w)=0$

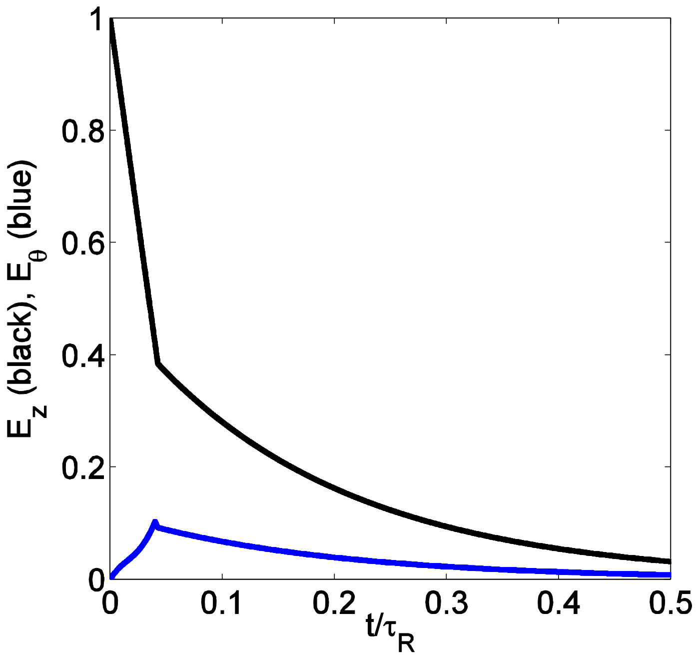
$$\eta = \eta_{\text{flat}}$$

$$\eta = \eta_{\text{peaked}}$$

Attempt to use SS stable states for flat resistivity

- Idea: PPCD to lock into a favorable selfsimilar state 
- Example:





The procedure

1. Start with α equilibrium ($\alpha = 2.2$) with E_0, $F_{||}$
2. Ramp $E_z(t) = E_0(1 - t/T)$ and $F_{||} = F_{||}(0)(1 - 2t/T)$
3. Determine $E_\theta(t)$ by $\mathbf{E} \cdot \mathbf{B} = 0$ at $r = r_w$
4. Monitor γ_{ppcd} such that $\gamma_{ppcd}\Phi = -2\pi r_w E_\theta(t)$
5. $E_\theta(t)$ and $E_z(t)$ determine a SSRD state. Monitor γ_{ssrd}
6. When these cross, $\gamma_{ppcd} = \gamma_{ssrd} \equiv \gamma$ let

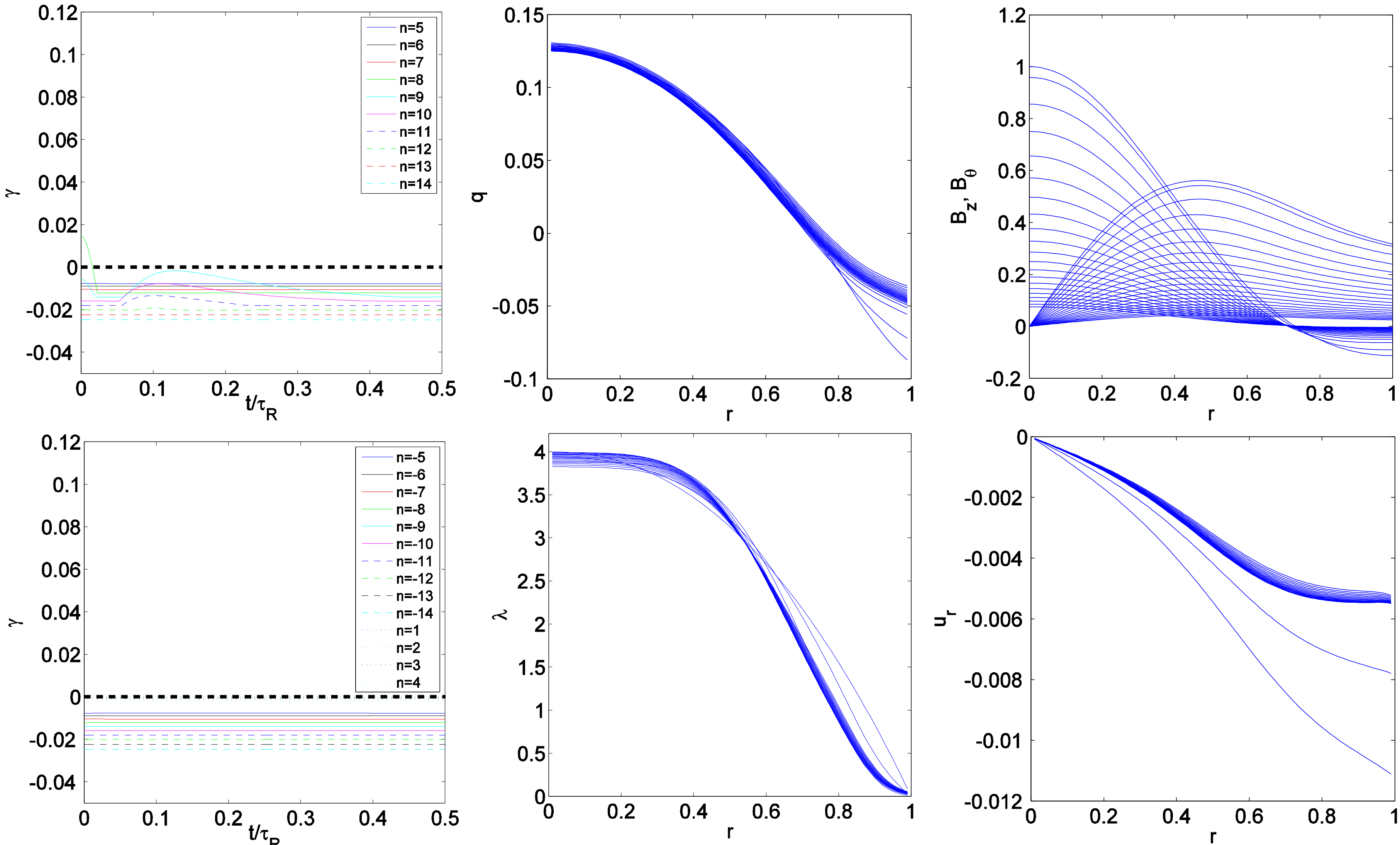
$$E_z(t) = E_z(t_1)exp(\gamma t)$$
$$E_\theta(t) = E_\theta(t_1)exp(\gamma t)$$

7. Find Θ and γ in SSRD space. Stable? Reversed?
8. After a short transient, plasma converges to a SSRD state

Parameters

- $\lambda(r)=4(1-r^{2.2})$ – $\alpha=3.5$ too stable
- $\eta(r)=10^{-3}(1+9r^{20})$
- $\gamma\tau_R=-5.5$
- $E_\theta/E_0=0.09$
- $E_z/E_0=0.39$
- $F_\parallel$ ramped down linearly to zero at $t/\tau_R=0.04$
- Selfsimilar rampdown starts at $t/\tau_R=0.045$

Results



Stability is achieved over a much longer pulse!

Conclusions

- We have studied PPCD transients by combining 1D transport simulations with stability analysis
- We have studied the PPCD stability limits:
  - PPCD is effective for both flat and peaked resistivity profiles
  - If the q profile is reversed too much, m=1,n<0 and m=0 modes can become unstable
  - The maximum pulse length appears to be of the order of 1/100 of the resistive time, comparable with what obtained in current experiments
- We have started investigating how to optimize the PPCD pulse to extend the pulse length and the stability window
- We have shown that it is possible to vary $E_\theta(t)$, $E_z(t)$ until matching with a self-similar (SS) state, then program $E_\theta(t)$, $E_z(t)$ with the correct exponential decay. This holds the q profile constant in time and so the stability properties of the system do not change.
  - This idea is effective for flatter resistivity profiles (where stable SS states exist), but does not work for peaked resistivity profiles (due to the lack of stable SS states).