Comparison of Resistive Wall Mode Kinetic Stabilization Theory and Experiment

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Motivation

RWM stabilization is key for disruption-free operation of a low rotation, low collisionality burning plasma (ITER)

Outline

• Thermal Particles
  – Rotational resonances: can explain NSTX and DIII-D experimental results
  – Collisionality: implications for future machines

• Energetic Particles
  – Effect compatible with NSTX and DIII-D experiments
  – Provide the basis for an understanding of the difference between the devices
A scalar critical plasma rotation model cannot explain RWM stability; it depends on the $\omega_\phi$ profile

- **RWM unstable plasma**
  - Instability occurs at relatively high rotation level, and not at highest $\beta_N$ (4.7)

- **RWM stable plasma**
  - MHD spectroscopy: increased resonant field amplification (RFA) indicates reduced stability
  - Plasma moves to more stable regime (lower RFA) at lower rotation ($\beta_N$ up to 6.5)

[S. Sabbagh et al., IAEA FEC 2010, EXS/5-5]
Kinetic $\delta W_K$ term in the RWM dispersion relation provides dissipation that enables stabilization.

A momentum balance: 

$$\rho \frac{dv}{dt} = j \times B - \nabla \cdot P$$

leads to an energy balance:

$$-\frac{1}{2} \int \rho \omega^2 |\xi_\perp|^2 dV = \frac{1}{2} \int \xi_\perp^* \cdot \left[ j \times B_0 + j_0 \times \tilde{B} - \nabla \tilde{p}_F - \nabla \cdot \tilde{P}_K \right] dV$$

The change in potential energy due to the kinetic pressure is:

$$\delta W_K = -\frac{1}{2} \int \xi_\perp^* \cdot \left( \nabla \cdot \tilde{P}_K \right) dV$$

Dissipation from kinetic term enables stabilization of the RWM:

$$(\gamma - i\omega_r) \tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

Calculation of $\delta W_K$ with the MISK code includes:

- Trapped Thermal Ions and Electrons
- Circulating Thermal Ions
- Alfvén Layers at rational surfaces
- Trapped Energetic Particles

[B. Hu et al., Phys. Plasmas 12, 057301 (2005)]
Full MISK calculation shows that trapped thermal ions near the edge are the most important

\[ \delta W_K = \sum_j \sum_{i=\infty}^\infty 2\sqrt{2\pi^2} \int_0^{\Psi_a} \frac{d\Psi}{m_j^2 B} \int_{-1}^1 |\chi|d\chi \]

\[ \int_0^\infty \left[ \frac{(\omega - n\omega_E) \frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e} \frac{\partial f_j}{\partial \psi}}{n\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + n\omega_E - \omega} \right] \varepsilon^{\frac{1}{2}} d\varepsilon \]

\[ \left| \langle (3\chi^2 - 1) \kappa \cdot \xi_\perp - (\chi^2 - 1) \nabla \cdot \xi_\perp \rangle \right|^2 \]

Full \( \delta W_K \) eqn. for general \( f \)

- Frequency resonance denominator key part of \( \delta W_K \) eq.
- Examine \( \delta W_K \) from each particle type vs. \( \Psi \)
  - Thermal ions are the most important contributor to stability.
  - Entire profile is important, but \( q > 2 \) contributes \( \sim 60\% \).
  - RWM eigenfunction and temperature, density gradients are large in this region.
When the rotation is in resonance, the plasma is stable

- Stable cases in bounce (or transit) resonance at high rotation
- Stable cases in precession drift resonance at low rotation

\[ \delta W_K \sim \left[ \frac{1}{\langle \omega_D \rangle + i \omega_b - i \nu_{\text{eff}} + \omega_E} \right] \]

\[ \omega_E = \omega_{\phi} - \omega_{*i} \]
Thermal particle rotational resonances can explain NSTX and DIII-D experimental results

NSTX

DIII-D

Measured rotation dependence of the plasma response to an external $n=1$ AC field provides direct evidence of the relevance of trapped particle precession frequency resonance.
MISK calculations consistent with RWM destabilization at intermediate plasma rotation in NSTX

$\gamma \tau_w$ contours vs. $v$ and $\omega_\phi$

- Destabilization appears between precession drift resonance at low $\omega_\phi$, bounce/transit resonance at high $\omega_\phi$ [J. Berkery et al., Phys. Rev. Lett. 104, 035003 (2010)]
  [S. Sabbagh et al., Nucl. Fusion 50, 025020 (2010)]
Stability is altered by collisionality: at low $\nu$, resonance effects are enhanced

- The effect of reduced $\nu$ in future machines:
  - Lower stability region less stable, higher stability more stable
  - Destabilization at higher $\omega_\phi$

\[
\delta W_K \sim \frac{1}{\Omega - i\nu} = \left[ \frac{\Omega + i\nu}{\Omega^2 + \nu^2} \right]
\]

**On-resonance: $\Omega << \nu$**

\[
\delta W_K \sim \frac{\Omega}{\nu^2} + \frac{i}{\nu}
\]

**Off-resonance: $\Omega >> \nu$**

\[
\delta W_K \sim \frac{1}{\Omega} + \frac{i\nu}{\Omega^2}
\]
Energetic particles provide a stabilizing force that is nearly independent of rotation and collisionality.

\[ \delta W_K \sim \left[ \frac{1}{\langle \omega_D \rangle + l \omega_b - i \nu_{eff} + \omega_E} \right] \]

Significant \( \text{Re}(\delta W_K) \), but nearly independent of \( \omega_\phi \)

Energetic particles are not in mode resonance

Effect is not energy dissipation, but rather a restoring force

[J. Berkery et al., Phys. Plasmas 17 082504 (2010)]
An NSTX experiment examined the role of energetic particles in RWM stability

Illustrative example

- Kinetic model can explain experiments
  - NSTX: low EP: unstable more often, rotation dependence seen
  - DIII-D: higher EP: mode stable except when triggered by fishbones
MISK calculations are consistent with DIII-D stability when EPs are included

[H. Reimerdes et al., IAEA FEC 2010, EXS/5-4]

- Kinetic model has to include EPs to explain experimental stability
- One can see why the loss of EPs might lead to instability
Measured amplitude and phase of the plasma response reveals the characteristics of kinetic stabilization in DIII-D

Single mode model links $\gamma$ and $\omega_r$ from MISK to amplitude and phase of the plasma response:

$$\delta B^\text{plas} = \frac{M^* ((\gamma + i\omega_r)\tau_w + 1)}{(i\omega_{\text{ext}}\tau_w - ((\gamma + i\omega_r)\tau_w)(i\omega_{\text{ext}}\tau_w + 1))} I_c$$

- **Coupling coefficient**
- **Rotation frequency of external field**
- **MISK calculation (with EPs)**

[H. Reimerdes et al., IAEA FEC 2010, EXS/5-4]
Model of kinetic modifications to ideal stability can unify RWM stability results between devices

- Observed, NSTX:
  - RWM can cross marginal point as $\omega_\phi$ is varied
  - RFA can increase in stable plasmas at intermediate $\omega_\phi$

- Observed, DIII-D:
  - RWM stable at all $\omega_\phi$, but RFA can increase in at intermediate $\omega_\phi$
  - RWM destabilized by events that reduce EP population

- Explanation:
  - Both have rotational resonances between the mode and thermal ion precession drift.
  - DIII-D has a higher EP fraction than NSTX, leading to higher stability.

[M. Okabayashi et al., Nucl. Fusion (2009)]
[S. Sabbagh et al., IAEA FEC 2010, EXS/5-5]
MISK computed RWM stability of ITER scenario 4 including energetic particles near marginal at $\beta_N = 3$

- **ITER advanced scenario 4**
  - With $\beta_N = 3$ (20% above $n = 1$ no-wall limit)
  - Polevoi rotation profile

- **Plasma rotation effect**
  - Stabilizing precession drift resonance $\omega_\phi = 0.8 \omega_\phi^{\text{Polevoi}}$

- **Energetic particle effect**
  - Isotropic slowing down distribution of alphas
  - Near RWM marginal stability at expected $\beta_\alpha/\beta_{\text{total}}$
Anisotropic distribution function of beam ions impacts stability; work continues on improving model

Real (from TRANSP)

\[ f(\varepsilon, \Psi, \chi) = \frac{C(\Psi)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}} \frac{\varepsilon^{(\chi-\chi_0)^2/\delta\chi^2}}{\delta\chi} \]

Isotropic

Simple anisotropic test case

\( \chi_0 = 0.75, \delta\chi = 0.25 \)

Work towards an analytical model of the TRANSP energetic particle distribution function continues
Advancements in the theoretical model continue

**Electrostatic effect**

The electrostatic component of the perturbed distribution function contributes to $\delta W$. This effect is likely to be small, however.

\[
\delta W_{\Phi} = -\frac{1}{2} \int e^2 \left| \tilde{\Phi} + \xi_{\perp} \cdot \nabla \Phi_0 \right|^2 \sum_j Z_j^2 \frac{n_j}{T_j} d\mathbf{V}
\]

[B. Hu et al., Phys. Plasmas 12, 057301 (2005)]

**Additional anisotropic term**

In addition to the effect of anisotropy on $\delta W$, when $f$ is anisotropic an additional term arises that is proportional to $\tilde{\mathbf{B}}_\parallel$:

\[
\delta W_{\tilde{B}} = \sum_j \frac{1}{2} \int \int \langle HT_j \rangle^* \frac{\tilde{\mathbf{B}}_\parallel}{B} \frac{\partial f_j}{\partial \mu} d^3 \mathbf{v} d\mathbf{V}.
\]

**Centrifugal destabilization**

This fluid force term is usually neglected, but it is always destabilizing, and could be important if the plasma rotation Mach number is significant, or for alpha particles rotating at higher frequency $\sim \omega_\alpha$.

\[
\delta W_C = -\frac{1}{2} \sum_j \int \xi_{\perp}^* \cdot [\tilde{\rho} \mathbf{v}_0 \cdot \nabla \mathbf{v}_0] d\mathbf{V}
\]

**Other possibilities:**

- Inclusion of plasma inertia term in the dispersion relation.
- Effect of poloidal rotation on $\omega_E$.
- Use of a Lorentz collisionality model instead of current ad-hoc inclusion of collisionality.

\[
C \left( \tilde{f} \right) = \frac{1}{2} \nu \Pi_\varepsilon \frac{\partial}{\partial \chi} \left( 1 - \chi \right)^2 \frac{\partial \tilde{f}}{\partial \chi}
\]
Kinetic RWM stability model can unify results between machines

- The MISK code is used to calculate RWM stability with kinetic effects, including EPs, for NSTX and DIII-D.
- Thermal ion resonances can explain the complex experimental relationship between plasma rotation and stability.
- This effect will be enhanced by low collisionality in future machines.
- Computations indicate that energetic particles have a stabilizing effect, consistent with experiments.

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