Beam particle distribution modification by low amplitude Modes

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Motivation

- Recent observations on DIIID show a strong modification of the high energy beam particle distribution from that predicted by TRANSP.
- TAE and RSAE modes are present, but with very low amplitude, $dB/B = 2 \times 10^{-4}$. The high energy beam profile is flattened significantly.
- This effect could result in TAE modification of the alpha profile in ITER. Ripple loss is very sensitive to alpha density profile.

Previous attempts to simulate this failed. We have examined the effects of

- Time dependence including mode frequency chirping and q profile change
- Polarization potential due to rapid electron response
- Compression effects, including $\delta B_\|$
- Mode spectrum and amplitude variation

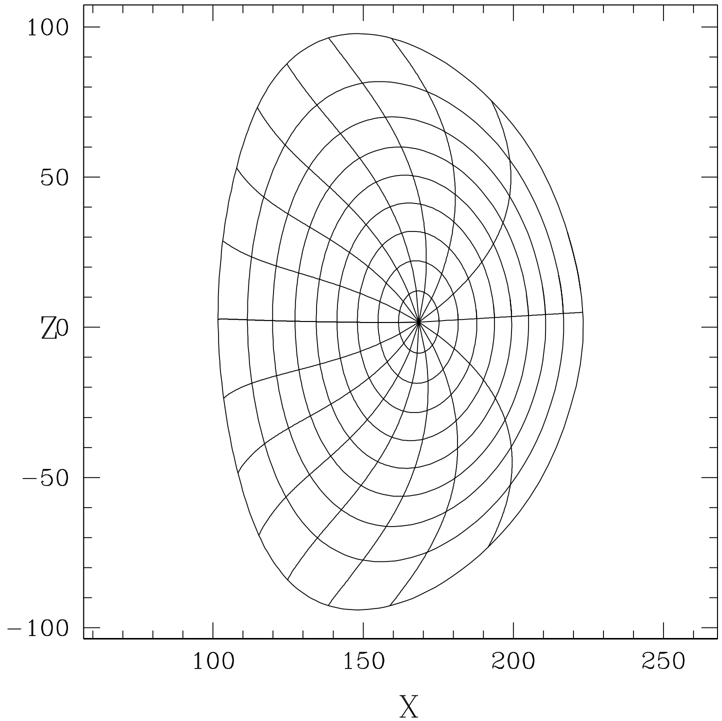
Tools for the study included

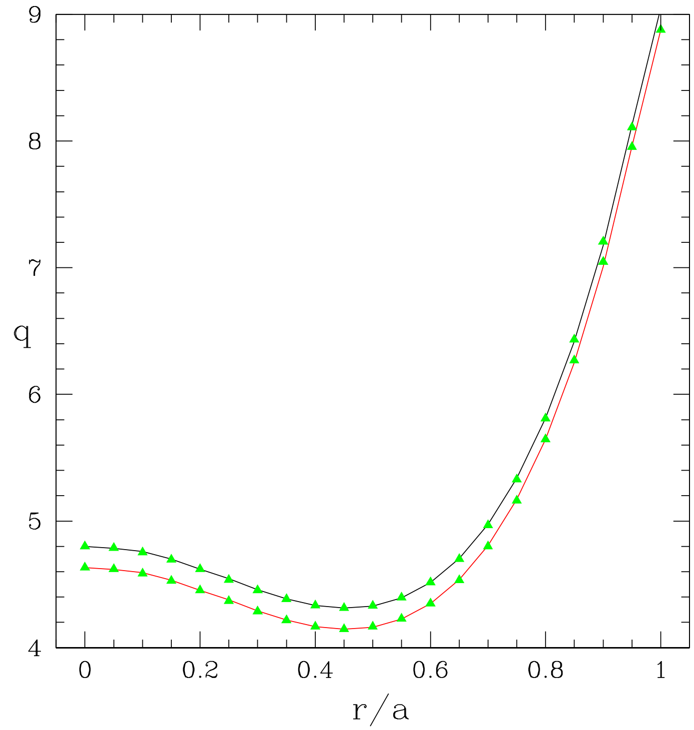
- Kinetic Poincaré plots showing resonances in ψ, θ
- Energy transfer plots in the plane of P_ζ, μ, E.
- Distribution modification calculations using full spectrum of modes

Simulation must include the polarization potential and complete spectrum

DIIID Reversed Shear Equilibrium

$B_0 = 20.3Kg$, q profile decreasing slowly over 30 $msec$, shot 122117.

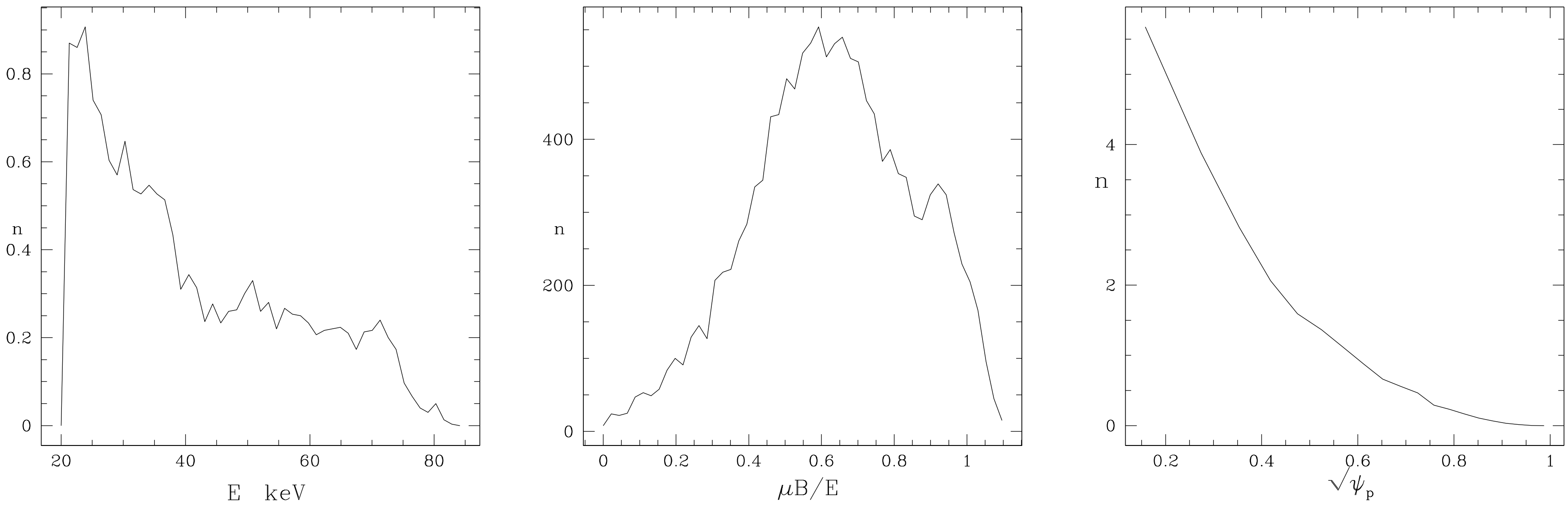




$$\vec{B} = g(\psi_p)\nabla\zeta + I(\psi_p)\nabla\theta + \delta(\psi_p, \theta)\nabla\psi_p$$

Straight field line coordinates ψ_p, ζ, θ

Beam Distribution from TRANSP in E, $\mu B_0/E$, ψ_p



Beam consists of mostly co-passing particles.

Guiding center equations including flute modes $\delta\vec{B} = \nabla\times\alpha(\psi_p,\theta,\phi)\vec{B}$

$$\dot{\rho_\|} = \frac{C}{D}\left[(\mu+\rho_\|^2 B)\frac{\partial B}{\partial\theta} + \frac{\partial\Phi}{\partial\theta}\right] - \frac{K}{D}\left[(\mu+\rho_\|^2 B)\frac{\partial B}{\partial\psi_p} + \frac{\partial\Phi}{\partial\psi_p}\right]$$
$$-\frac{F}{D}\left[(\mu+\rho_\|^2 B)\frac{\partial B}{\partial\zeta} + \frac{\partial\Phi}{\partial\zeta}\right] - \frac{\partial\alpha}{\partial t}$$

$$\dot{\psi}_p = \frac{K\rho_\| B^2}{D} - \frac{g}{D}\left[(\mu+\rho_\|^2 B)\frac{\partial B}{\partial\theta} + \frac{\partial\Phi}{\partial\theta}\right] + \frac{I}{D}\left[(\mu+\rho_\|^2 B)\frac{\partial B}{\partial\zeta} + \frac{\partial\Phi}{\partial\zeta}\right]$$

$$\dot{\theta} = \frac{-C\rho_\| B^2}{D} + \frac{g}{D}\left[(\mu+\rho_\|^2 B)\frac{\partial B}{\partial\psi_p} + \frac{\partial\Phi}{\partial\psi_p}\right]$$

$$\dot{\zeta} = \frac{F\rho_\| B^2}{D} - \frac{I}{D}\left[(\mu+\rho_\|^2 B)\frac{\partial B}{\partial\psi_p} + \frac{\partial\Phi}{\partial\psi_p}\right]$$

where $D = gq + I + (\rho_\| + \alpha)(gI'_{\psi_p} - Ig'_{\psi_p})$, $C = -1 + (\rho_\| + \alpha)g'_{\psi_p} + g\alpha'_{\psi_p}$, $K = g\alpha'_\theta - I\alpha'_\zeta$, and $F = q + (\rho_\| + \alpha)I'_{\psi_p} + I\alpha'_{\psi_p}$, and $\rho_\| = v_\|/B$

Flute like perturbations, no compression

$$\delta\vec{B} = \nabla \times \alpha(\psi_p, \theta, \phi)\vec{B}_0, \qquad \alpha = \sum_{m,n} \alpha_{mn}(\psi_p, \theta) sin(n\phi - m\theta - \omega t).$$

Resonance using large aspect ratio circular approximation

$$[n - m'/ql]\omega_t = \omega, \qquad q = \frac{m'/l}{n - \omega/\omega_t},$$

ω_t is the transit frequency.

Resonance appears when there exist integers m', l such that this relation is satisfied.

For co-moving ($\omega_t > 0$)

- **increasing E gives resonance at smaller q**
- **increasing ω gives resonance at larger q**

Kinetic Poincaré Plot

If the Hamiltonian is a function of the combination $n\zeta - \omega t$,

then $\dot{P}_\zeta = -\frac{\partial H}{\partial \zeta}$ and $\frac{dE}{dt} = \frac{\partial H}{\partial t}$

gives for fixed n that $\omega P_\zeta - nE = constant$ in time

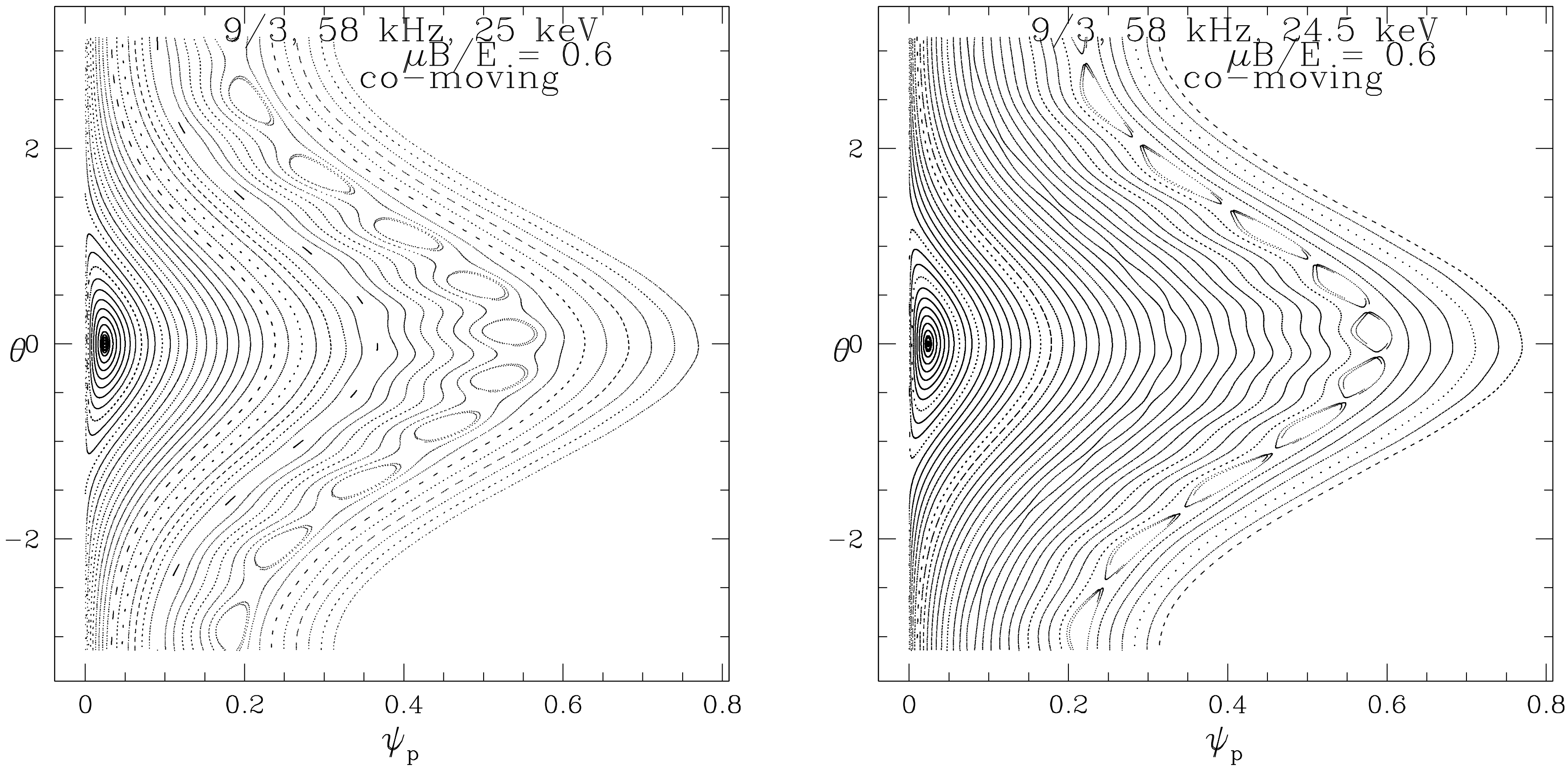
Kinetic Poincaré plot - plot of positions of orbits in the ψ_p, θ plane

at times when $n\zeta - \omega t = 2\pi k$, k integer

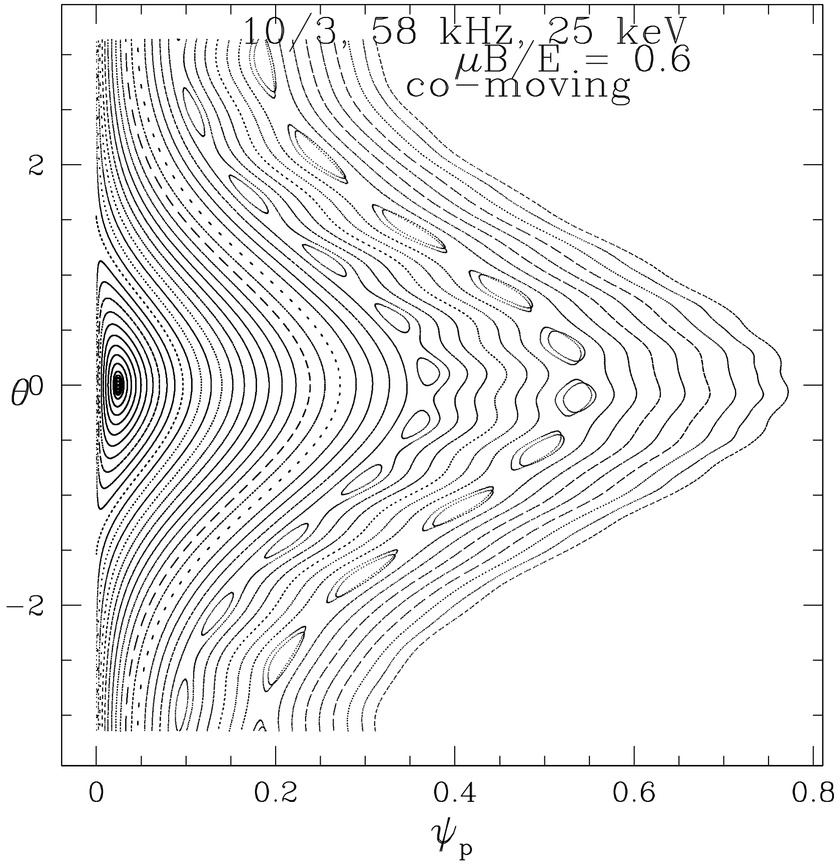
All particles having the same value of $\omega P_\zeta - nE$ and μ

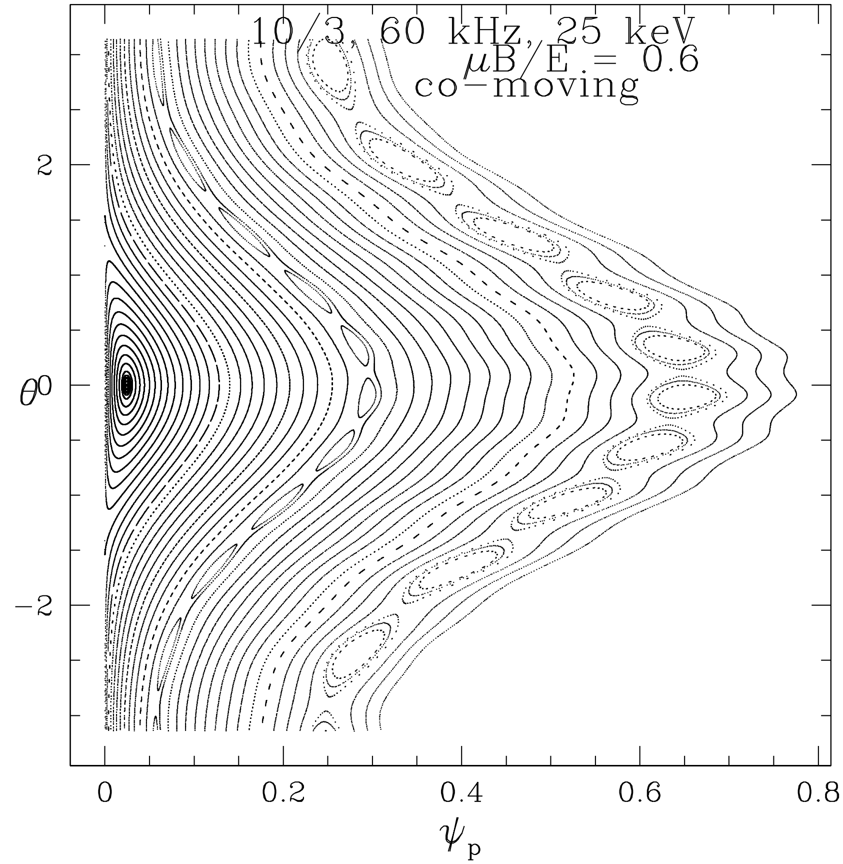
This plot shows mode-particle resonances

Kinetic Poincaré plot - Fixed values of μ and $\omega P_\zeta - nE$.



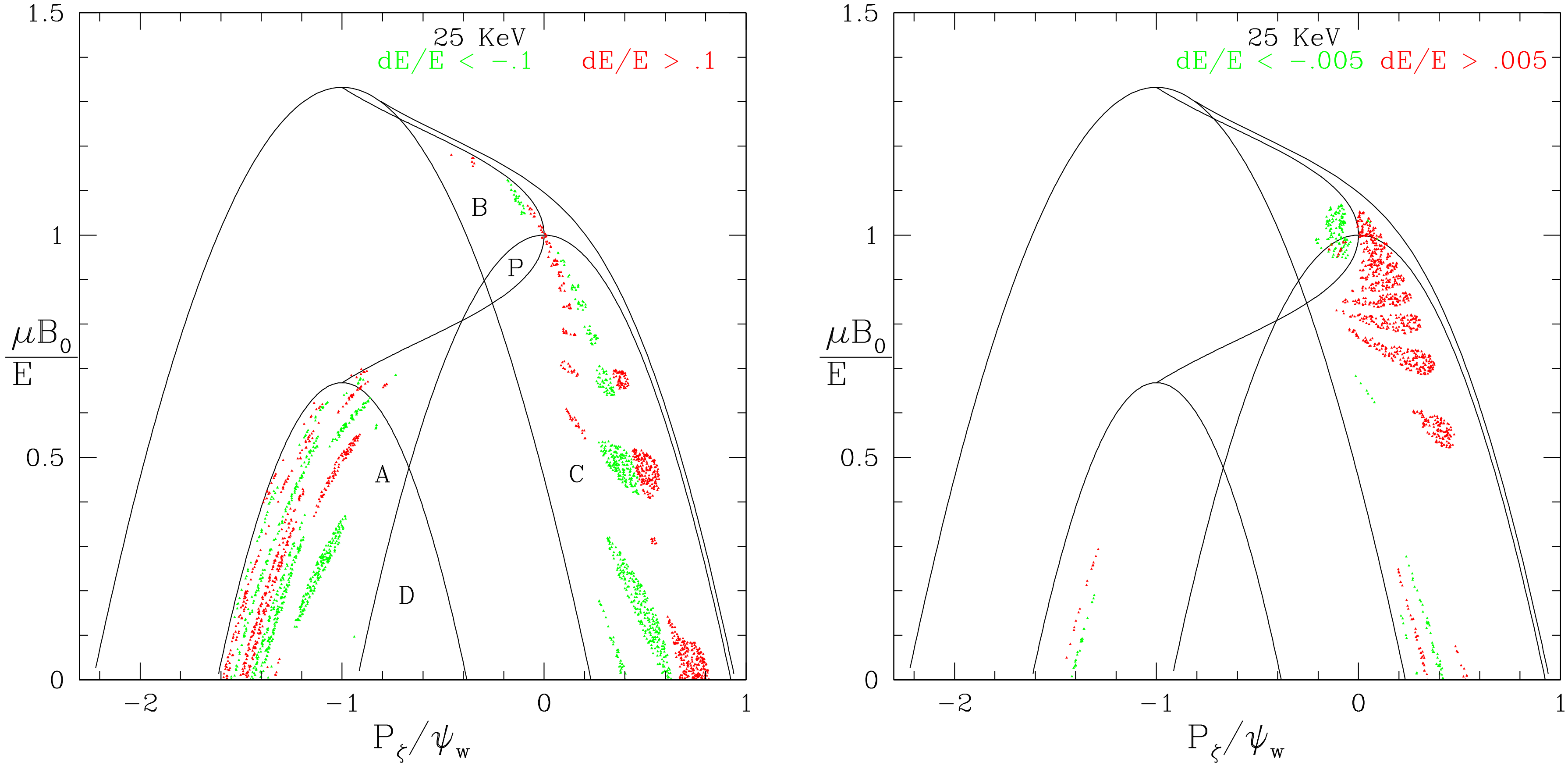
Kinetic Poincaré plots for mode $m/n = 9/3$, showing energy dependence of the $m' = 10$ resonances. Increasing E moves to smaller q





Kinetic Poincaré plots for mode $m/n = 10/3$, showing frequency dependence of $m' = 10$ resonances. Increasing ω moves to larger q

Note resonant surface unchanged from the $m = 9$ mode for 58 kHz.



A plot of the P_ζ, μ plane, showing only 25 keV particle orbits with energy loss or gain due to the mode, for a $m/n = 9/2$, mode at 90 kHz (left, showing 10 percent change) and a $m/n = 10/3$ mode at 90 kHz (right, showing only 0.5 percent change).
Both the co and counter moving parts of the distribution are shown.
A= counter passing, B= trapped, C= co-Passing, D = counter stagnation, P= potato

Electric polarization potential

Rapid mobility of the electrons makes the electric field experienced by the ions parallel to the magnetic field equal to zero.

There is a unique electric potential Φ that cancels the parallel field induced by $d\vec{B}/dt$ for each harmonic

$$\sum_{m,n} \omega B \alpha_{m,n} e^{i(n\zeta - m\theta - \omega t)} - \vec{B} \cdot \nabla \Phi / B = 0.$$

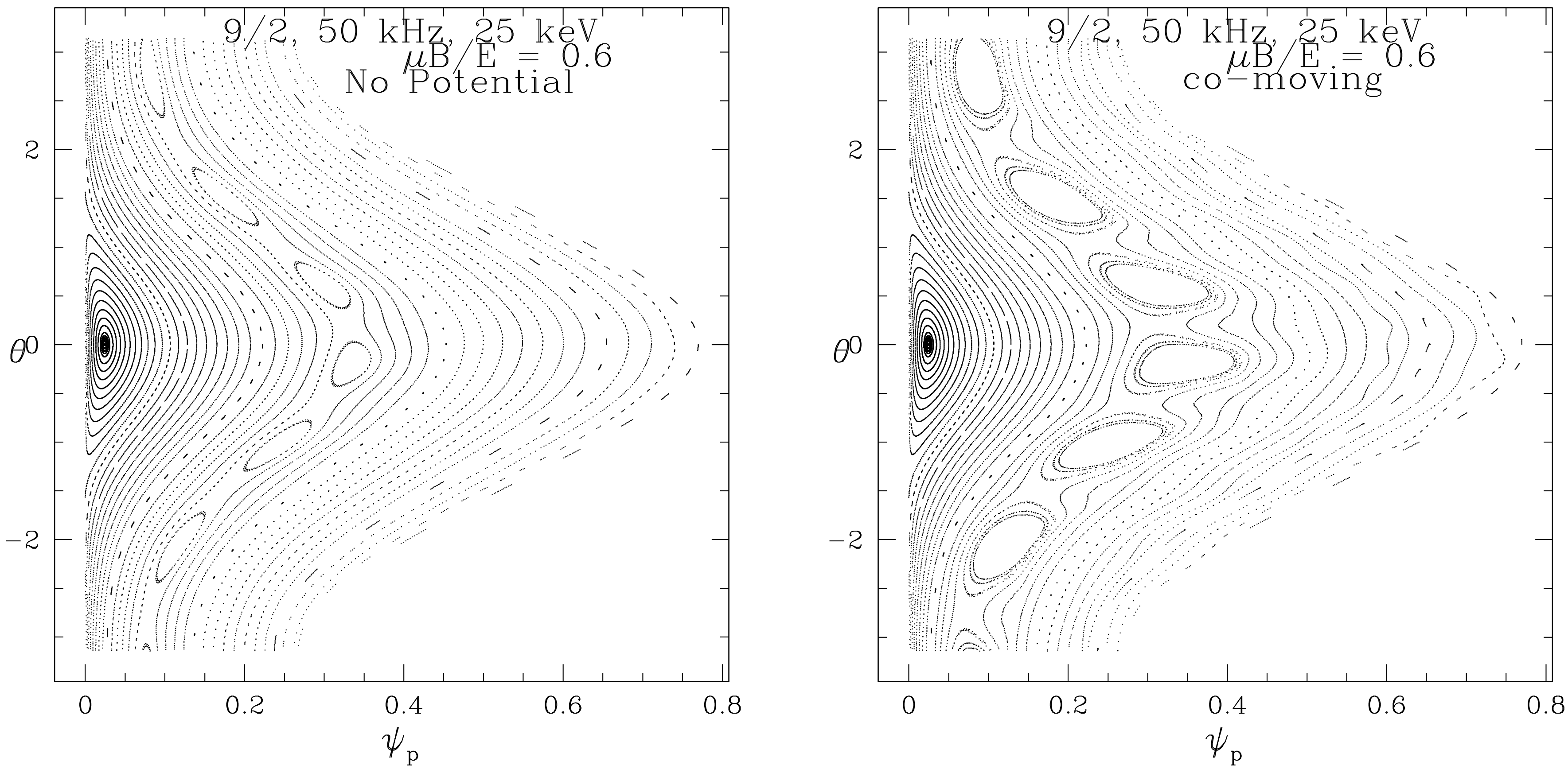
In Boozer coordinates, taking $\Phi = \sum_{m,n} \Phi_{m,n} e^{i(n\zeta - m\theta - \omega t)}$

$$(gq + I)\omega \alpha_{m,n} = (nq - m)\Phi_{m,n},$$

In general coordinates where $I = I(\psi, \theta)$ different harmonics couple

- Energy change $\sim (\rho/R)\alpha_{m,n}$ from the magnetic perturbation

$\sim (\omega/\omega_0) q \alpha_{m,n}/(nq - m)$ from potential, $\alpha_{m,n}(\psi_p)$ is zero at $nq = m$.

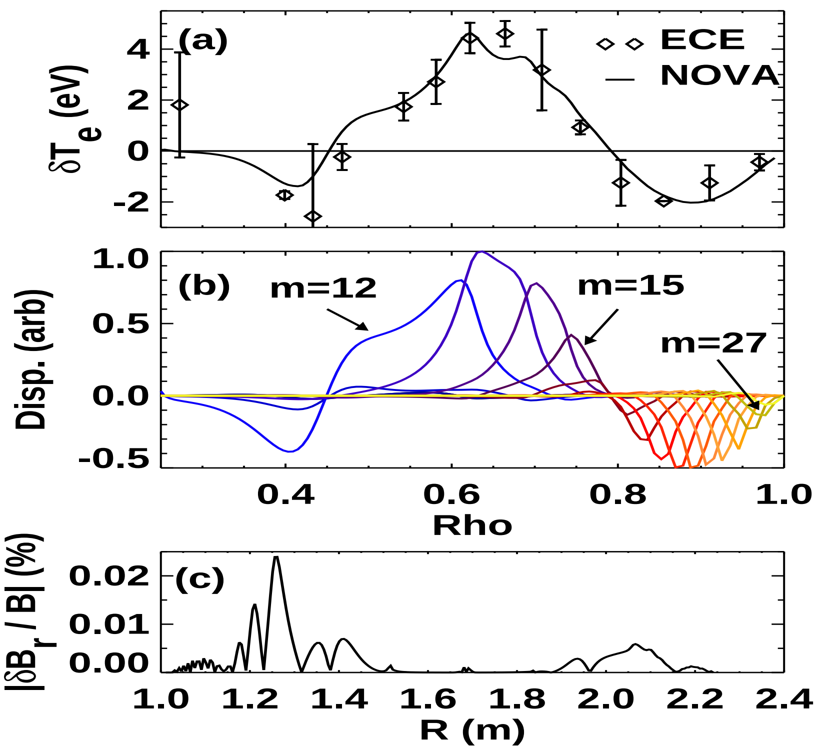


Kinetic Poincaré plots for mode $m/n = 9/2$, showing the effect of the potential on 23 Kev beam particles for a 50 kHz mode.

Polarization can be neglected only if $\omega/\omega_0 \ll \rho/R$
typical for fishbone but not for TAE

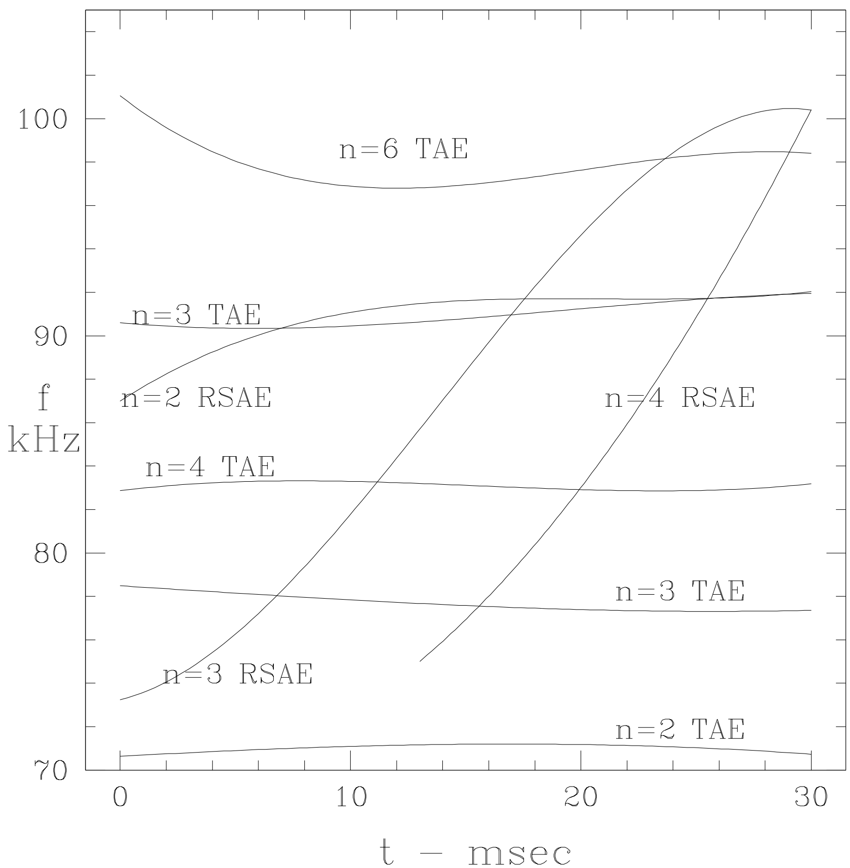
For 50 kHz and 25 keV DIIID beam $\omega/\omega_0 = 3 \times 10^{-3}$, $\rho/R = 10^{-2}$

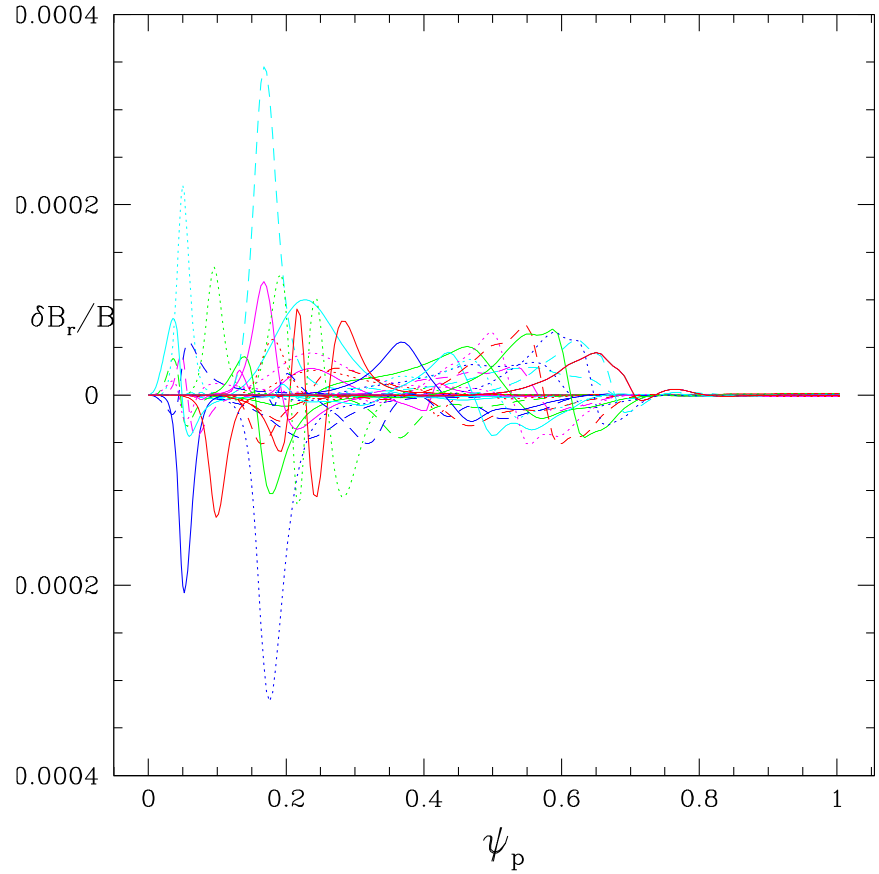
Experimental Confirmation of Mode Structure



Synthetic ECE diagnostic prediction (solid) using NOVA calculated $f = 78$ kHz global TAE. NOVA prediction scaled by single constant to match ECE data.(b) Poloidal harmonics comprising TAE.
(c) Calculated radial component of magnetic field fluctuation along device midplane.

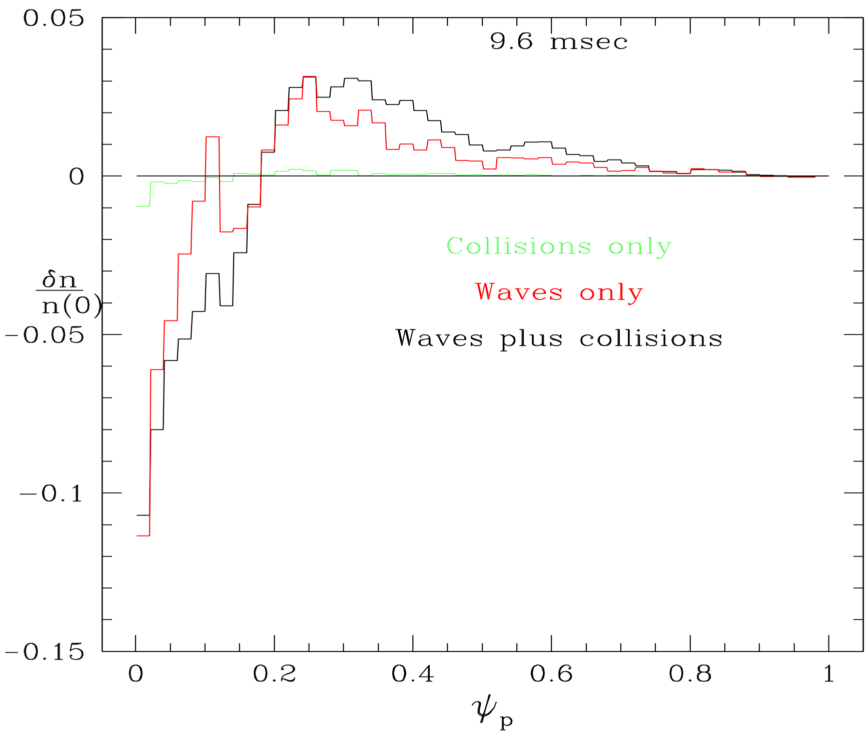
Mode time dependence

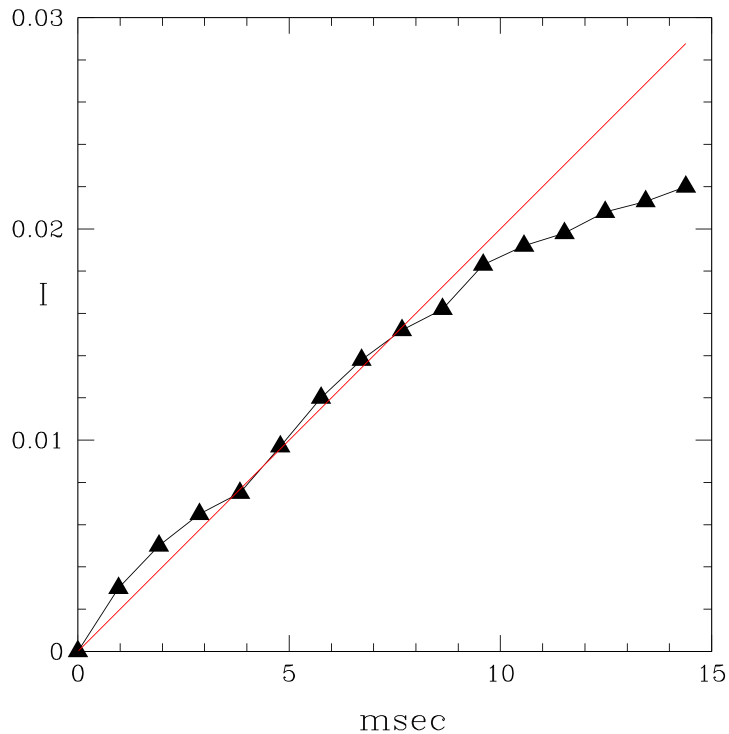




Experimentally observed modes, showing time dependence of frequencies
133 harmonics used in the simulation with $dB_r/B \simeq 2 \times 10^{-4}$.
Eigenfunctions produced by NOVA-K, compared with ECE measurements

Effect of modes on Distribution, with and without collisions. Magnitude and Location Correct, giving central flattening

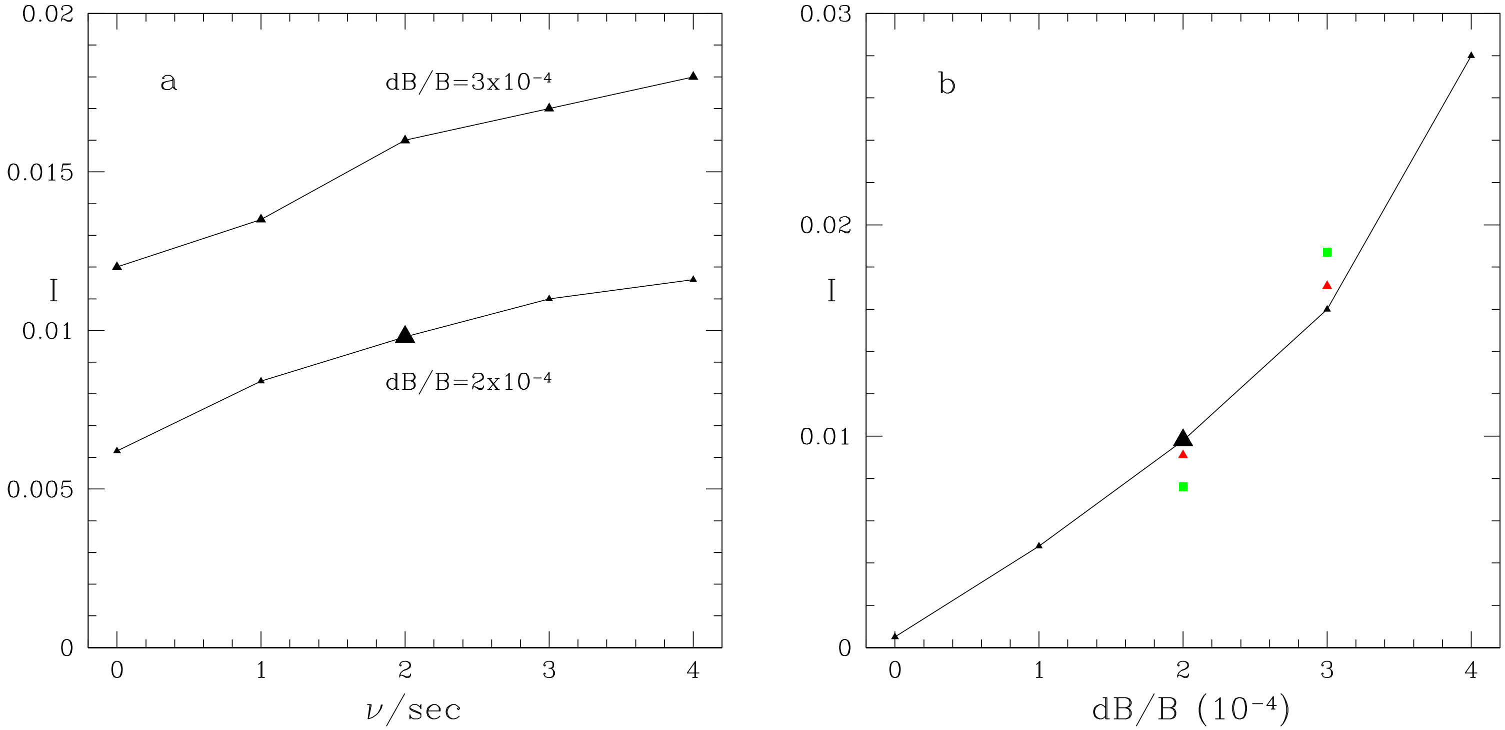




Introduce the mean total distribution shift, through $I = \frac{\int |n(\psi_p) - n_0(\psi_p)| d\psi_p}{n_0(0)}$.

For all values of collisionality and mode amplitude, the beam distribution modification is similar in shape, and changes are approximately linear in time.

Scaling with mode amplitude and collisions



Scaling with collisions and mode amplitude at a time of 4.8 $msec$

Note island width is $w \sim \sqrt{\delta B}$ so should produce collisional diffusion $D \sim \delta B$
But transport is stronger than linear. As islands begin to overlap D increases rapidly.

Colored points give the small effect of (two RSAE) mode sweeping.

Compressional effects, $\delta B_{\|}$ not important in this case, but could be at high beta values.

Stochastic Threshold

Launch distribution of 2000 particles all on the same drift surface

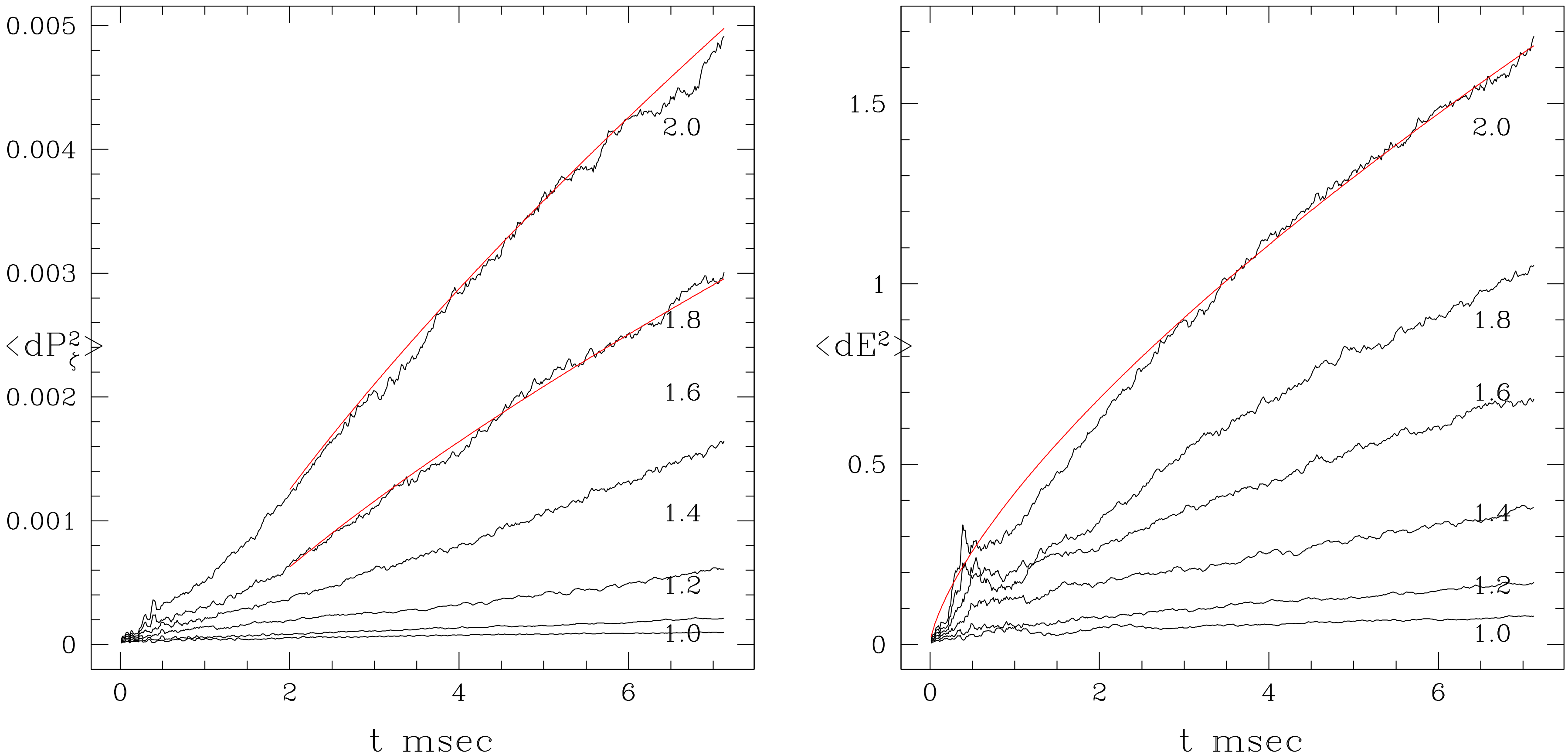
25 keV and pitch $\lambda = 0.6$ at the outer midplane, randomly toroidally.

Find time dependence of $< dP_\zeta^2 >$ and $< dE^2 >$

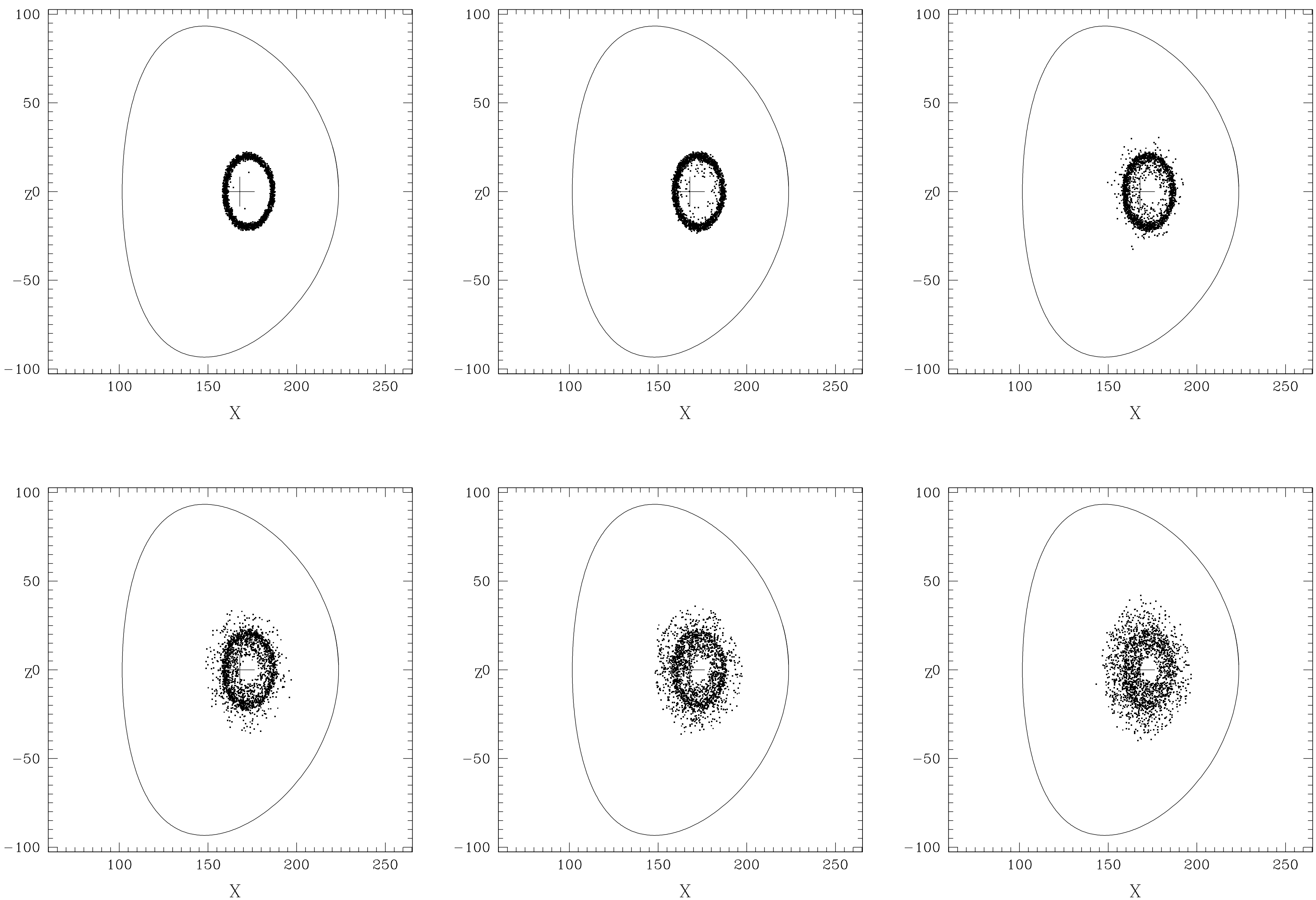
final particle positions after 7 $msec$

Scale amplitudes using whole spectrum of 133 harminics

Time dependence of $< dP_\zeta^2 >$ and $< dE^2 >$ for particles at 25 keV



$dB/B \simeq 1.0, 1.2, 1.6, 1.8, 2.0 \times 10^{-4}$
Red lines are least square fit to $t^{.7}$ and 0.7 exponent also found by least square fit



Final positions at t = 7 $msec$, $dB/B \simeq 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 \times 10^{-4}$

Conclusion

- A spectrum of TAE and RSAE modes can significantly modify a high energy particle distribution, even with amplitudes of the level of $dB/B \simeq 2 \times 10^{-4}$. The simulated rate of profile modification is roughly linear in time and agrees with the experimentally observed changes in DIIID.

- The relevant factor is the presence of resonance islands in the particle phase space, for particles at energies and pitches characteristic of the distribution.

- There is a stochastic threshold for flattening of the distribution, with a value very near the experimental amplitudes.

- Decreasing the spectrum even by eliminating some of the smallest harmonics moves the system below stochastic threshold, and significantly decreases transport.

- The time evolution of the mode frequencies can assist by causing a resonance surfaces to move through the plasma volume but bucket transport (strong time dependent frequencies) is not significantly operative in the case studied.

- Modification of the q profile in time is not an important factor in this case.