EXTREMUM-SEEKING STABILIZATION OF NEOCLASSICAL TEARING MODES IN TOKAMAK FUSION PLASMAS

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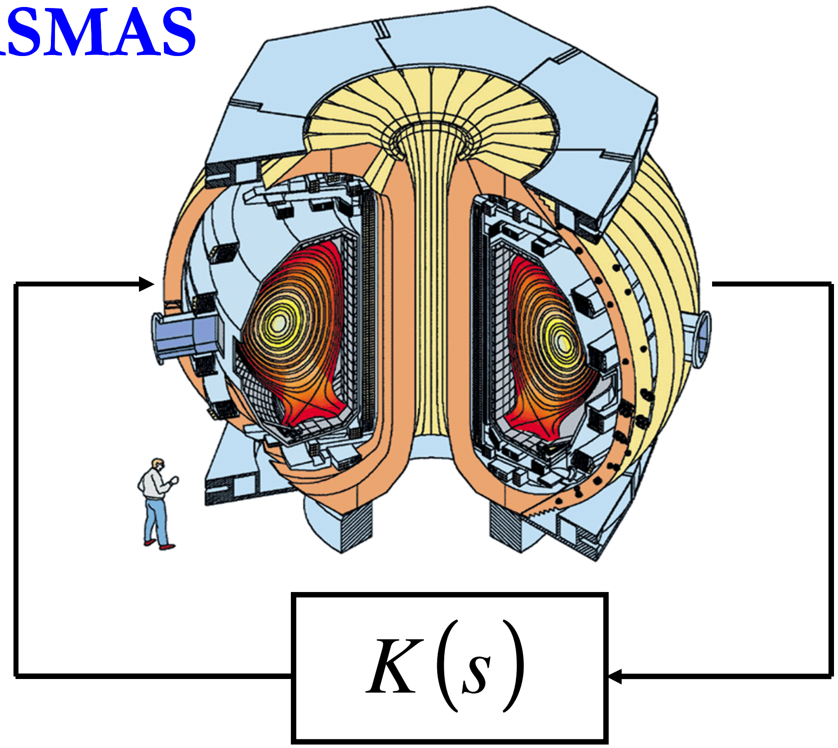

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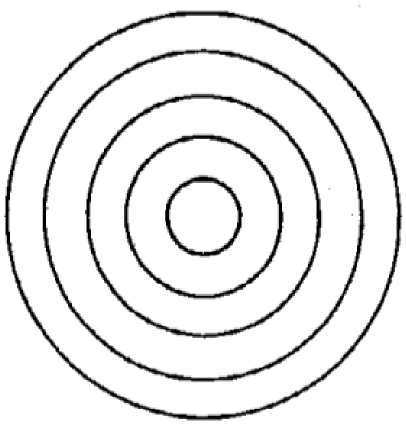
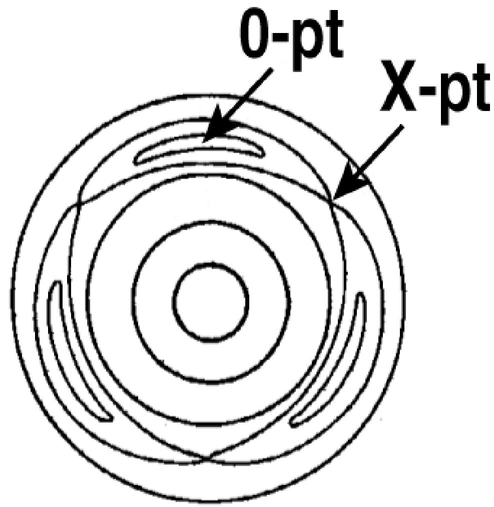
Neoclassical Tearing Modes (NTM's)

High plasma pressure can cause ideally nested magnetic flux surfaces to tear and reconnect, leading to the formation of magnetic islands. The neoclassical tearing mode (NTM) instability drives the islands to grow to their saturated widths, at which they can persist stably in the plasma.



Normal flux surfaces

(good confinement)



Helically perturbed flux surfaces

(leaky confinement)

The plasma heat is confined between the flux surfaces. Heat is conducted across the island. Hot interior is cooled.

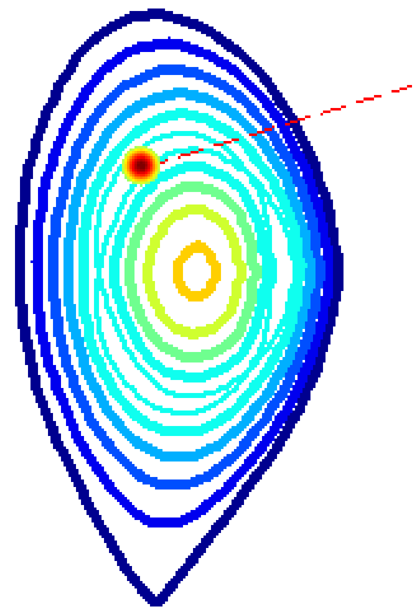
Suppression of these islands is necessary for sustained energy confinement and efficient operation in tokamaks.

How Do We Control NTM's?

A narrow microwave beam is sent into the plasma. The microwave deposits current in a spot, replacing the missing bootstrap current. If this current is inside the island, the island begins to shrink. Current-spot can be aligned to island by:

- Moving plasma
- Moving spot along beam
  (by changing magnetic field)
- Moving spot vertically
  (by changing beam direction via steerable mirror)

Electron Cyclotron Current Drive (ECCD)



The plasma rotates around the torus. As a result the island structure rotates in a fixed cross-section. Current driven in the center of the island around the O-point has the highest stabilizing effect. Current driven outside of the island or around the X-point has a destabilizing effect. Thus it is beneficial to modulate the beam in sync with island rotation to deliver current around the O-point.

Motivation

- It has been shown that increasing current within magnetic islands shrinks the islands driven by neoclassical tearing modes (NTMs)
- Localized current drive (e.g., ECCD) can be used to inject current into islands, with the strongest shrinking effect when current is injected at island center
- Experimentally, it is difficult to determine the position of magnetic islands in a plasma in real time
- The present work seeks to model the effect of off-center current drive on the island width as well as the effect of power modulation in sync with the island rotation
- Extremum Seeking is employed to improve beam efficiency using only a measurement of the island width
- The performance of Extremum Seeking is compared to current control algorithms (Search & Suppress)

Magnetic Island Dynamics

The growth dynamics of tearing mode islands in response to applied ECCD is governed by the modified Rutherford Equation

ECCD stabilizing term

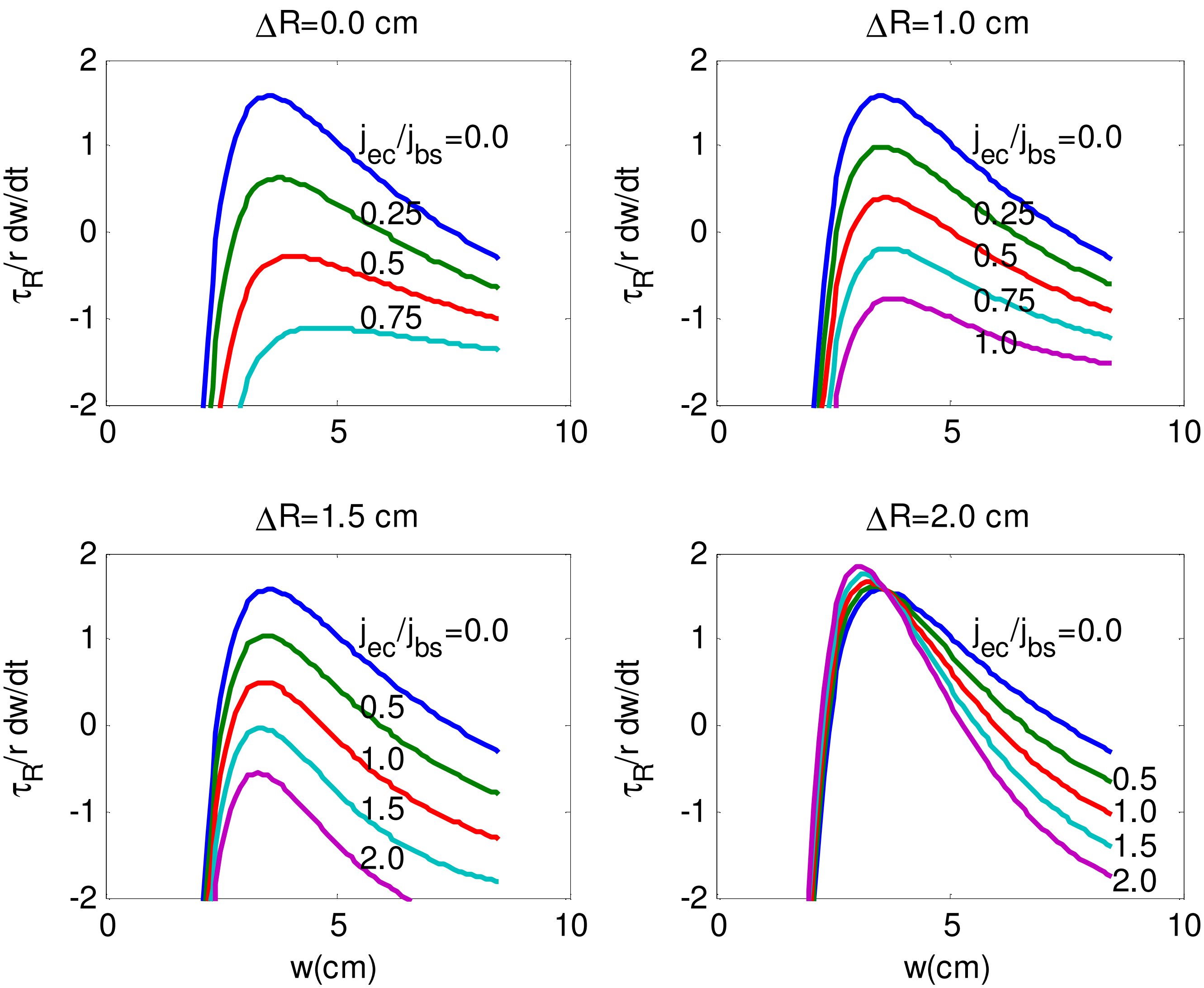
$$\frac{\tau_R}{r}\frac{dw}{dt} = \Delta' r + \varepsilon^{1/2}\left(\frac{L_q}{L_p}\right)\beta_p \frac{r}{w}\left[\frac{w^2}{w^2+w_d^2} - \frac{w_{pol}^2}{w^2} - \boxed{K_1\left(\frac{w}{w_{cd}}, \frac{\Delta R}{w_{cd}}, \tau, \xi\right)\frac{j_{ec}}{j_{bs}}}\right]$$

Here w is the full island width and K_1 is the efficiency of the ECCD suppression, which depends on the alignment error ΔR and the ECCD power modulation represented by τ and ξ

r	Radius at which the magnetic island is resonant
Δ'	Standard tearing stability index
τ_R	Current diffusion time
j_{ec} / j_{bs}	Peak ECCD current density normalized to local equilibrium bootstrap current

R.J. LaHaye, "Neoclassical tearing modes and their control," *Physics of Plasmas,* vol. 13, 055501, 2006

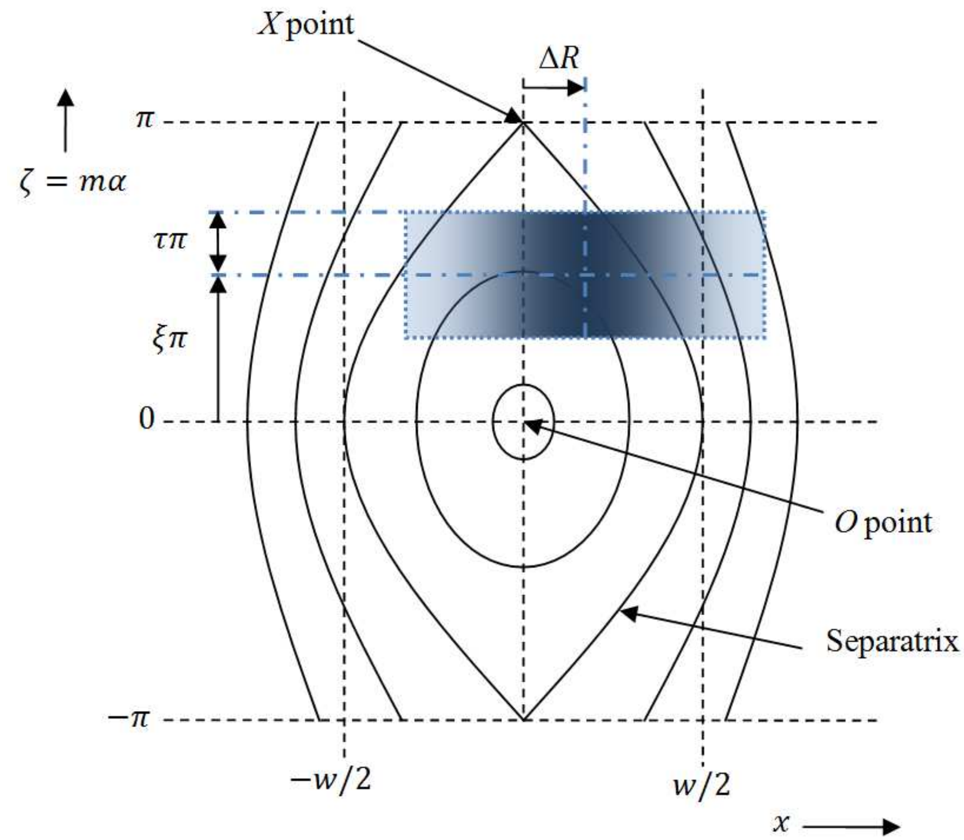
Magnetic Island Dynamics



Normalized island growth rate for parameters typical of DIII-D discharges associated with the *m/n* = *3/2* NTM and unmodulated current drive.

Magnetic Island – ECCD Interaction Parameters

Magnetic island projected on the helical angle coordinate $\zeta = m\alpha$ is shown here. The shaded area represents the island section covered by the current drive. Here ξ is the phase mismatch relative to the O-point and τ is the extension along the helical angle. ΔR is the misalignment between current drive and island center.



Nondimensional helical flux function : $\psi = \dfrac{x^2}{(w/2)^2} - \left(\dfrac{1+\cos(\zeta)}{2}\right)$

We assume a Gaussian driven current density distribution:

$$j_{cd}(x) = j_{cd0}\exp\left[\frac{-4\left(x - x_{dep}\right)^2}{w_{cd}^2}\right]$$

x_{dep}: deposition location

w_{cd}: full e^{-1} current density width.

Magnetic Island – ECCD Interaction Dynamics

$$\boxed{K_1\left(\frac{w}{w_{cd}},\frac{\Delta R}{w_{cd}},\tau,\xi\right)=\frac{1}{C}\int_{-1}^{\infty}J(\psi)W(\psi)\,d\psi}$$

F. Perkins *et al.*, "Prospects of election cyclotron current drive stabilization of neoclassical tearing modes in ITER," *Fusion Engineering, 17th IEEE/NPSS Symposium,* vol. 2, pp. 749-51, 1997

Prater *et al., "*Discharge improvement through control of NTMs by localized ECCD in DIII-D," *Nuclear Fusion,* vol. 43, no. 10, pp. 1128-1134, 2003

O. Sauter *et al.,* "On the contribution of local current density to neoclassical tearing mode stabilization," *Physics of Plasmas,* vol. 11, no. 10, pp. 4808-13, 2004

Magnetic Island – ECCD Interaction Dynamics

$$J(\psi) = \frac{1}{V(\psi)} \oint \frac{\tilde{j}_{cd}(\psi,\zeta) M(\zeta,\tau,\xi)}{\sqrt{\psi + \cos^2(\zeta/2)}} d\zeta$$

$$W(\psi) = \oint \frac{\cos(\zeta)}{\sqrt{\psi + \cos^2(\zeta/2)}} d\zeta \qquad C = \int_{-1}^{0} W d\psi = 2.67$$

$$V(\psi) = \oint \frac{1}{\sqrt{\psi + \cos^2(\zeta/2)}} d\zeta$$

where $M(\zeta,\tau,\xi)$ is the modulation function and $j_{cd}(\psi,\zeta)$ is the normalized current deposition profile:

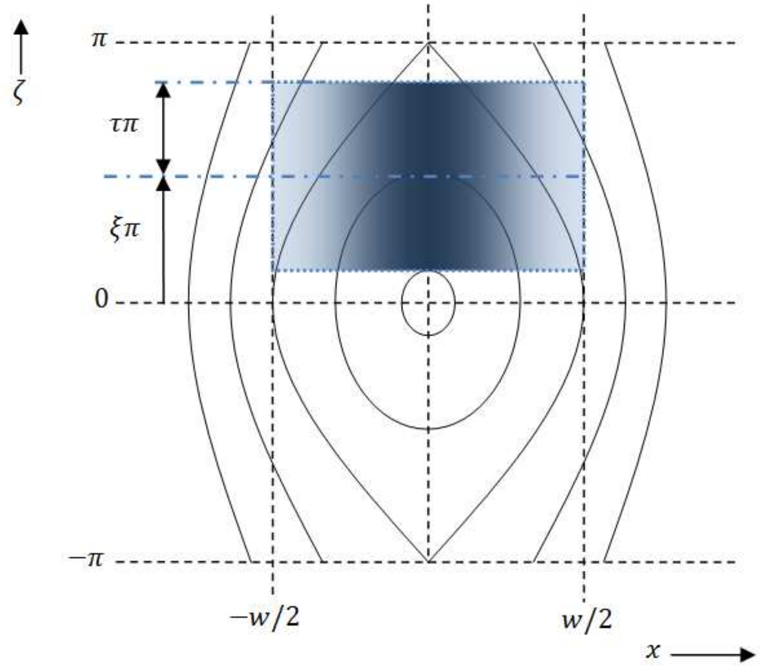
$$\tilde{j}_{cd}(\psi,\zeta) = \exp\left[-\left(\frac{\pm w}{w_{cd}} \sqrt{\psi + \cos^2(\zeta/2)} + 2\frac{\Delta R}{w_{cd}} \right)^2 \right]$$

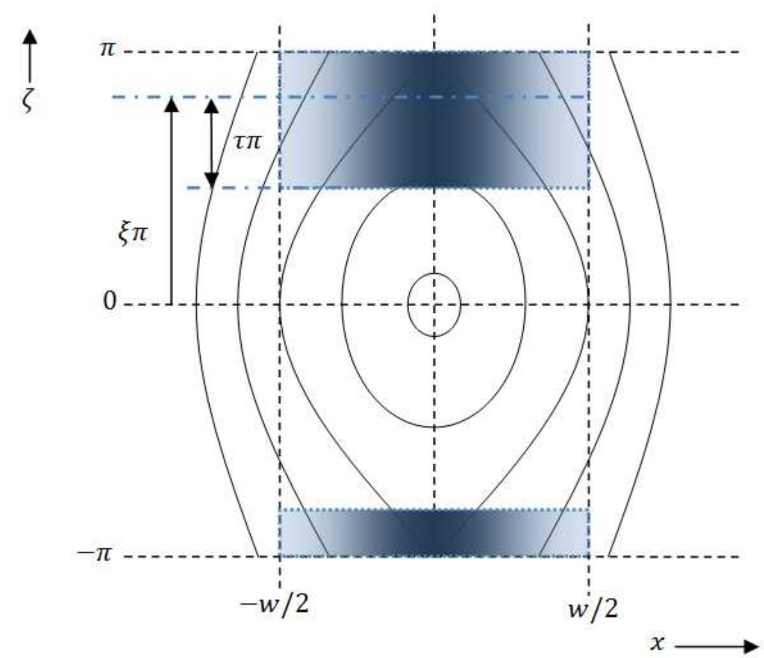
Parameterization of Modulation Function

The modulation function $M(\zeta, \tau, \xi)$ is defined as 1 during the period the ECCD is turned on and 0 when it is off. To parameterize the modulation function in terms of ξ and τ, there are two cases to be considered: *i*. $\xi\pi + \tau\pi \leq \pi$, in which the on period is entirely on one island; *ii*. $\xi\pi + \tau\pi > \pi$, in which the on period overlaps from one island into the next.

$$M = \begin{cases} 1 & \xi\pi - \tau\pi \leq \zeta \leq \xi\pi + \tau\pi \\ 0 & \xi\pi + \tau\pi < \zeta \leq \pi \\ 0 & -\pi \leq \zeta < \xi\pi - \tau\pi \end{cases}$$

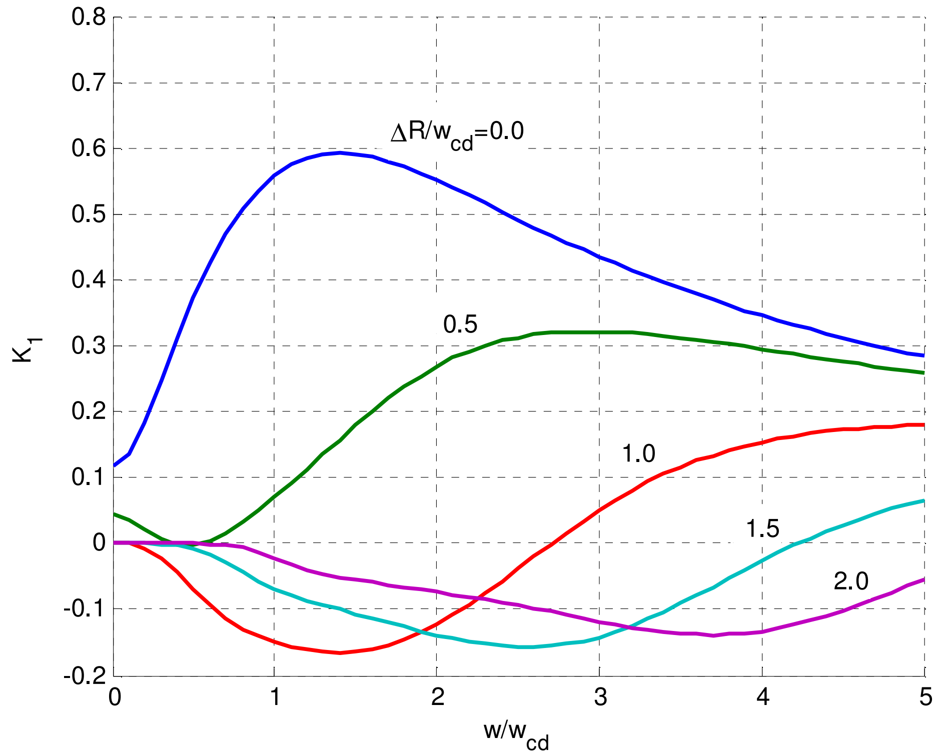
$$M = \begin{cases} 1 & \xi\pi - \tau\pi \leq \zeta \leq \pi \\ 1 & -\pi \leq \zeta \leq (\xi\pi + \tau\pi) - 2\pi \\ 0 & (\xi\pi + \tau\pi) - 2\pi < \zeta < \xi\pi - \tau\pi \end{cases}$$





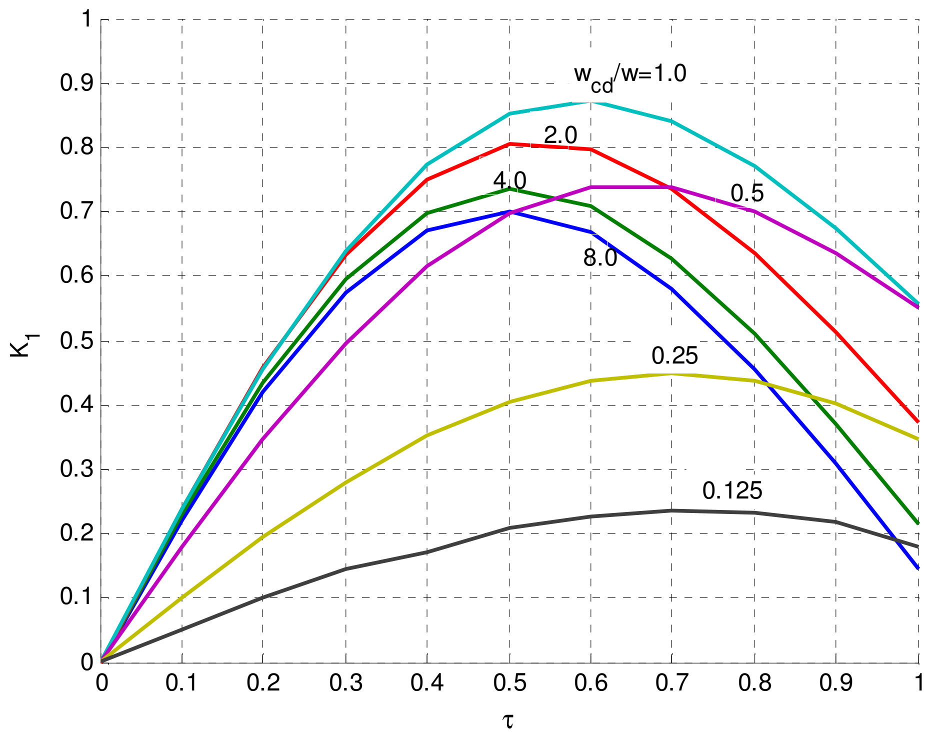
ECCD Effectiveness

- ECCD effectiveness plotted as a function of island width for <u>unmodulated</u> current drive
- The curves show a strong sensitivity to ΔR, especially around w/w_{cd}=1
- The ECCD is destabilizing K_1<0 with sufficiently large displacements for small island widths because a large portion of current is driven outside of the island
- Eventually the effect of ECCD vanishes with sufficiently large displacements



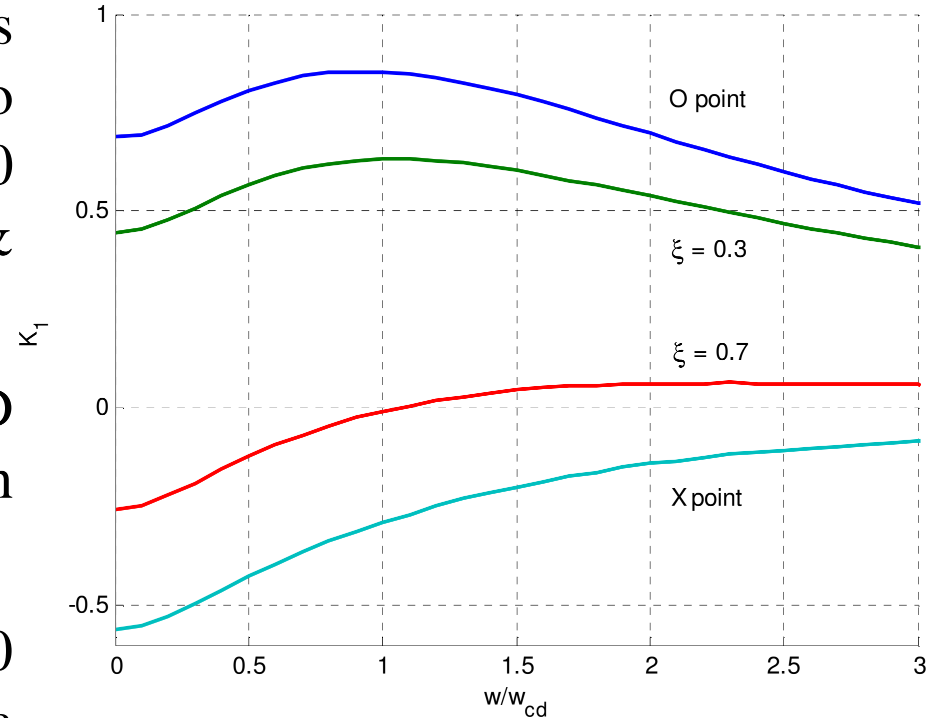
ECCD Effectiveness

- ECCD effectiveness K_1 plotted as a function of duty cycle τ for no phase mismatch (ξ=0), i.e, O-point modulation, and no misalignment (ΔR=0)
- The ECCD effectiveness increases via modulation
- The optimal duty cycle τ is a function of the current-drive width to island width ratio
- The ECCD effectiveness for O-point modulation is maximized for w/w_{cd}=1



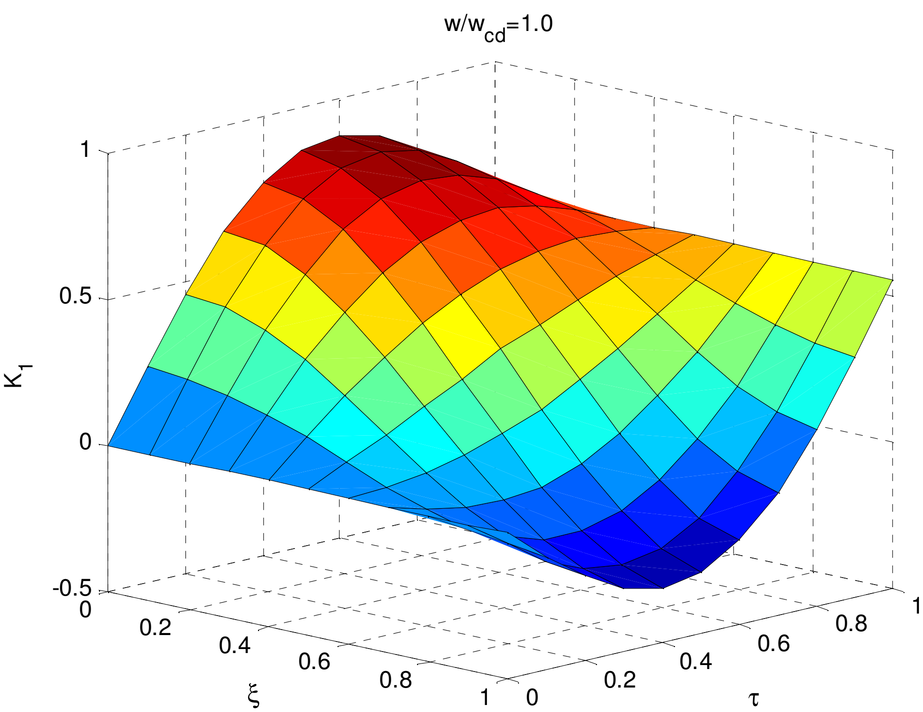
ECCD Effectiveness

- ECCD effectiveness K_1 plotted as a function of island width to current-drive width ratio for 50/50 duty-cycle modulation (τ=0.5) & no misalignment (ΔR=0)
- The improvement in ECCD effectiveness diminishes with increasing island size
- For island width ratios above 1.0 the ECCD effectiveness for a phase mismatch of ξ=0.3 matches closely to that of unmodulated current drive
- ECCD effectiveness is maximized for O-point modulation



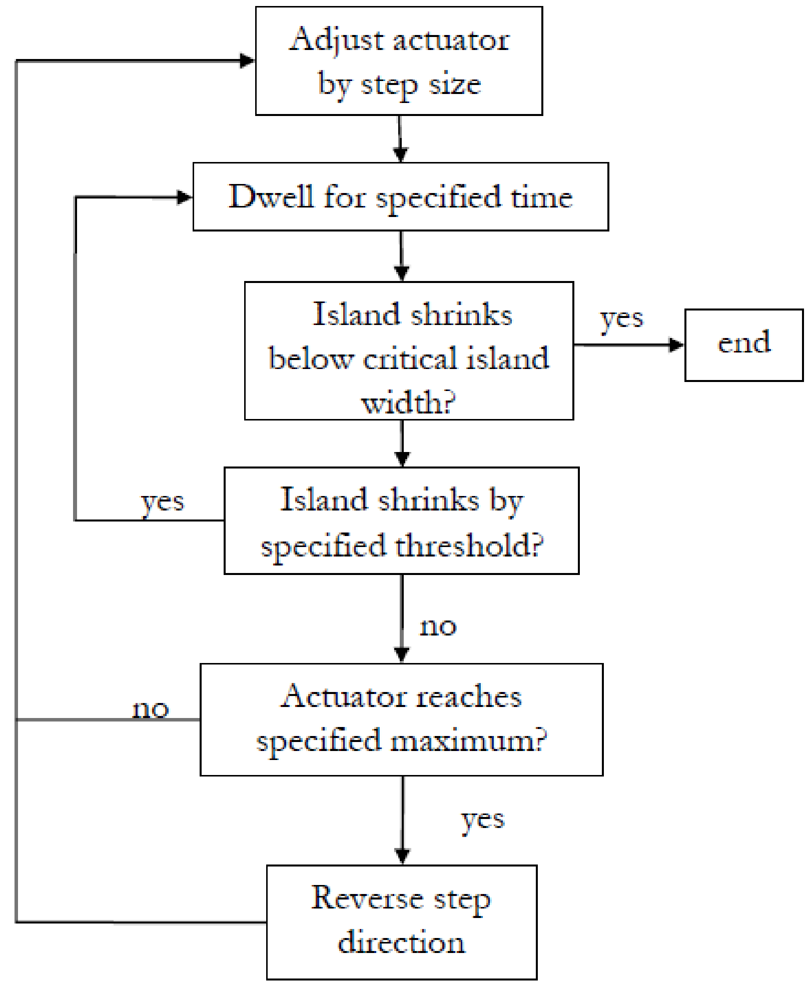
ECCD Effectiveness

- Combined effect of modulation duty-cycle and phase mismatch on ECCD effectiveness while the beam is well aligned to the island center (ΔR=0)
- Current driven near the O-point is stabilizing (K_1>0) and current driven near the X-point is destabilizing (K_1<0)
- The correct phase must be determined before attempting to apply modulated current
- It is possible to tune the phase mismatch while simultaneously tuning the misalignment via Extremum Seeking



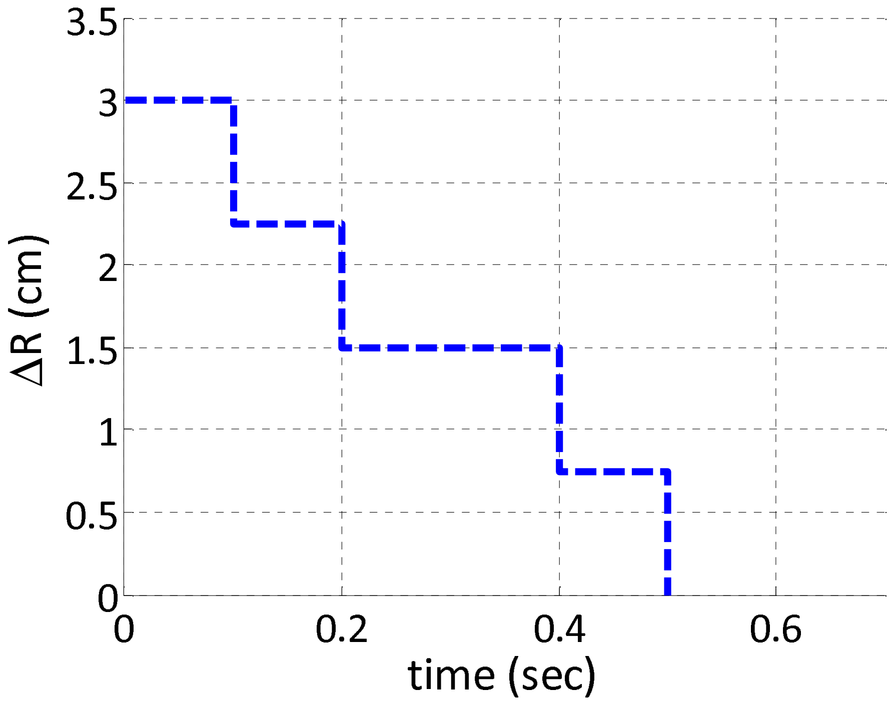
Search and Suppress Algorithm

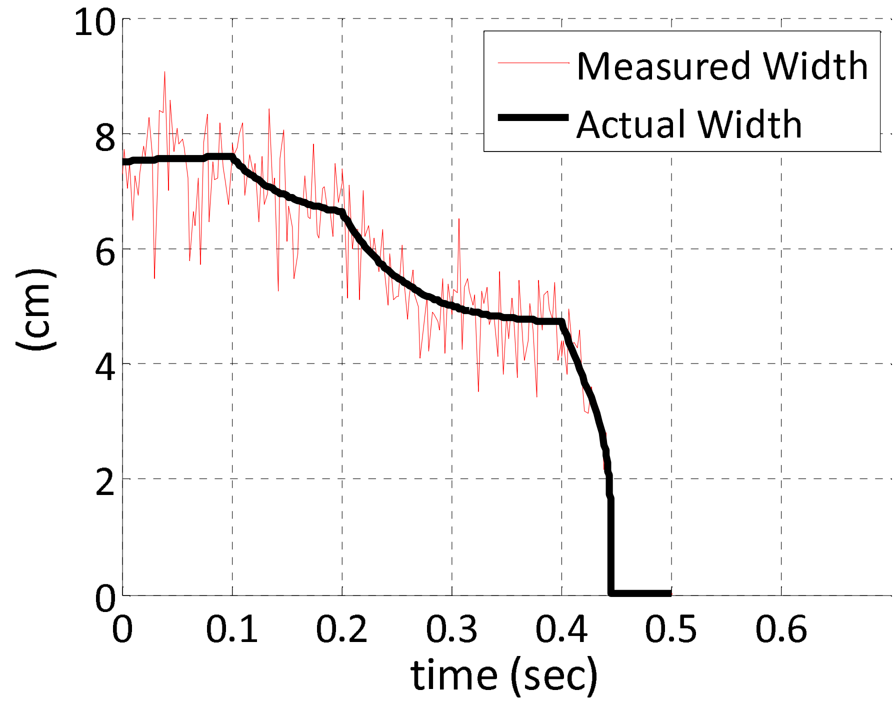
- Search and suppress with active tracking
- Control variable fixed for specified dwell time
- If mode amplitude shrinks by some pre-specified threshold, continue dwell
- Otherwise increment control variable by specified amount and freeze for another dwell time
- Continue Search dwell search until suppression is achieved



D. Humphreys et al., “Active control for stabilization of neoclassical tearing modes,” *Physics of Plasmas,* vol. 13, 2006

Search and Suppress Algorithm

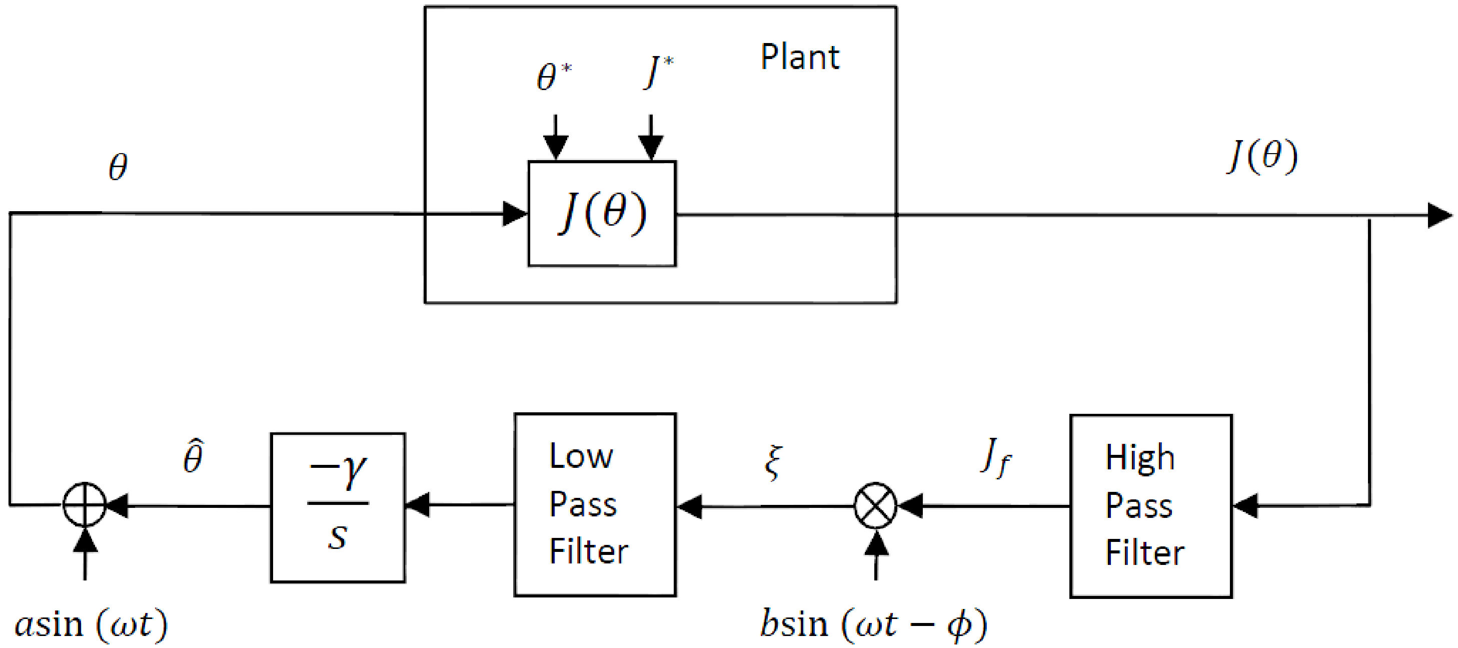




$$\Delta R_0 = 3.0\,\text{cm}, \quad w_0 = w_{sat} = 7.5\,\text{cm}$$

If the initial step direction is unknown the average suppression time is *m = 0.62s* with standard deviation *σ = 0.14s* for continuous current drive and *m = 0.50s, σ = 0.18s* for 50/50 duty-cycle modulated current drive centered at the O-point.

Extremum Seeking Algorithm



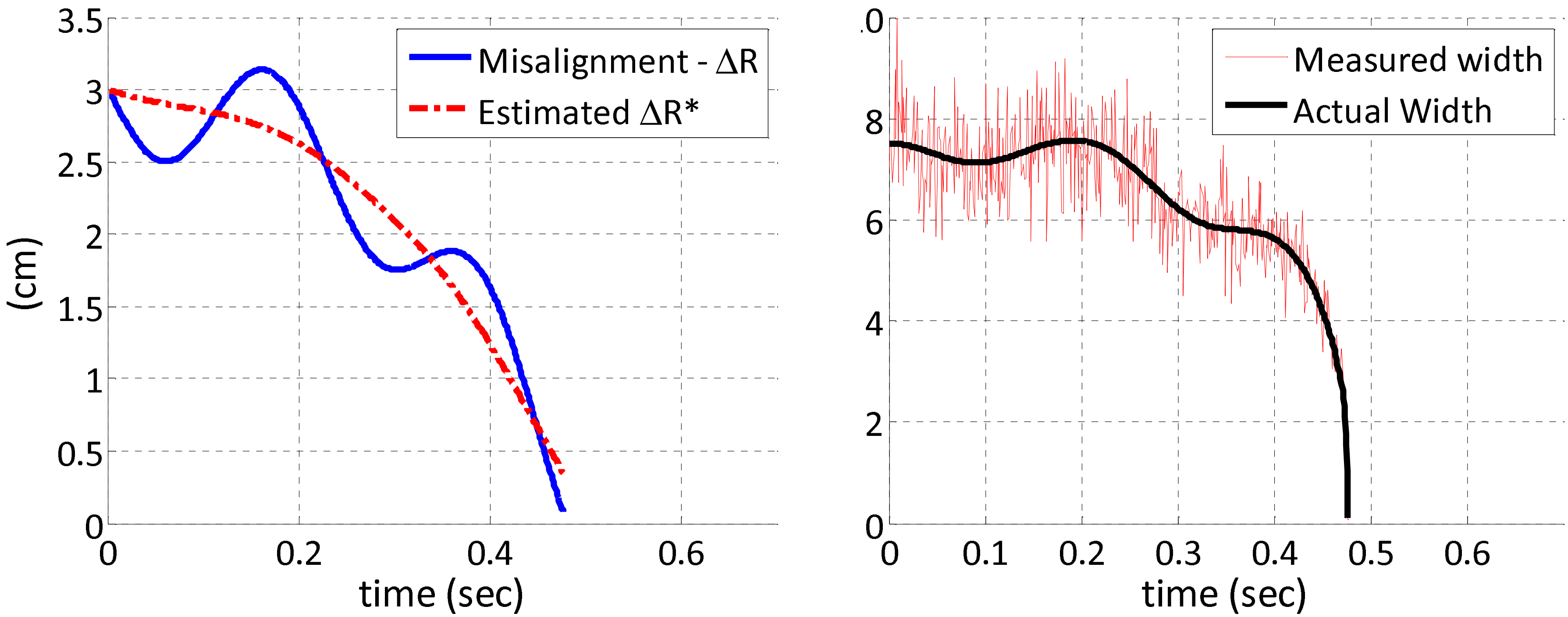
$$\dot{J}_f(t) = \dot{J}(t) - hJ_f(t)$$

$$\xi(t) = J_f(t)b\sin(\omega t - \varphi)$$

$$\dot{\hat{\theta}}(t) = -\gamma\xi(t)$$

$$\theta(t) = \hat{\theta}(t) + a\sin(\omega t)$$

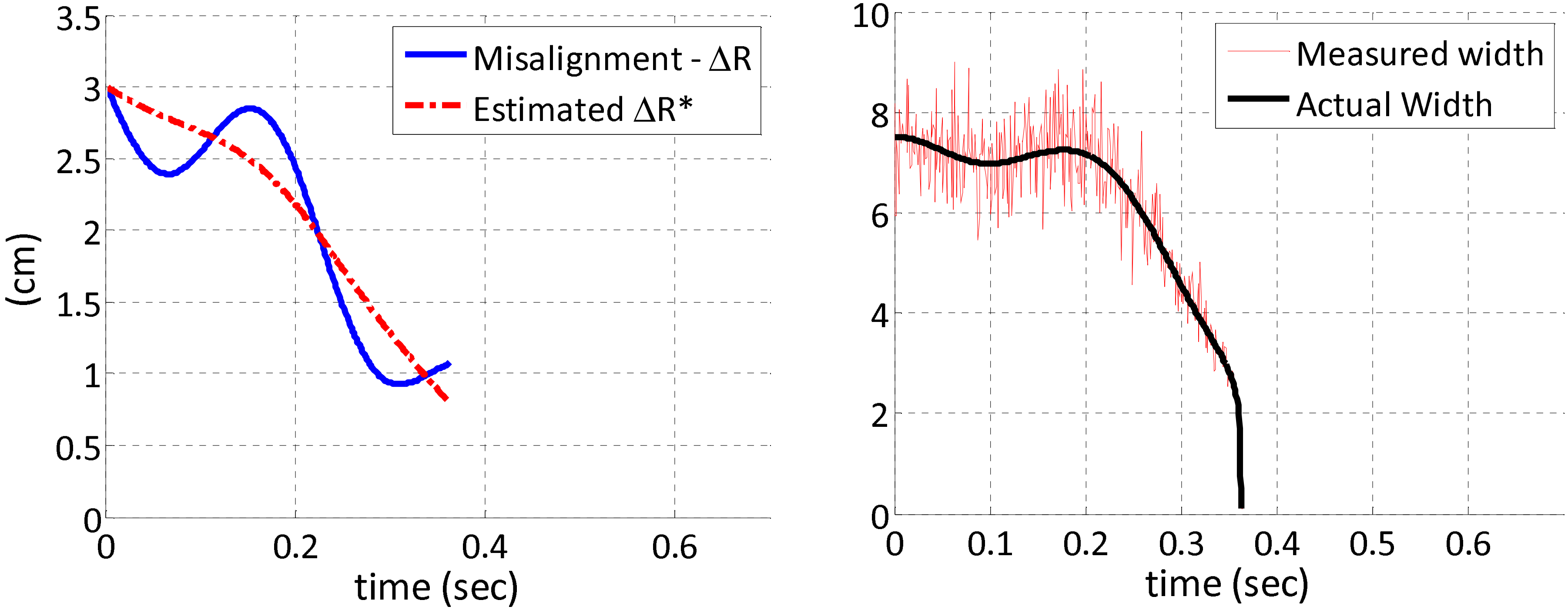
Extremum Seeking Results



$$\Delta R_0 = 3.0\,\text{cm}, \quad w_0 = w_{sat} = 7.5\,\text{cm}$$

For continuous current drive the average suppression time is $m = 0.54s$ with standard deviation $\sigma = 0.06s$ and $m = 0.44s$, $\sigma = 0.10s$ for 50/50 duty-cycle current drive centered at the O-point. There is a notable improvement in average suppression time as well as consistency over the search and suppress algorithm.

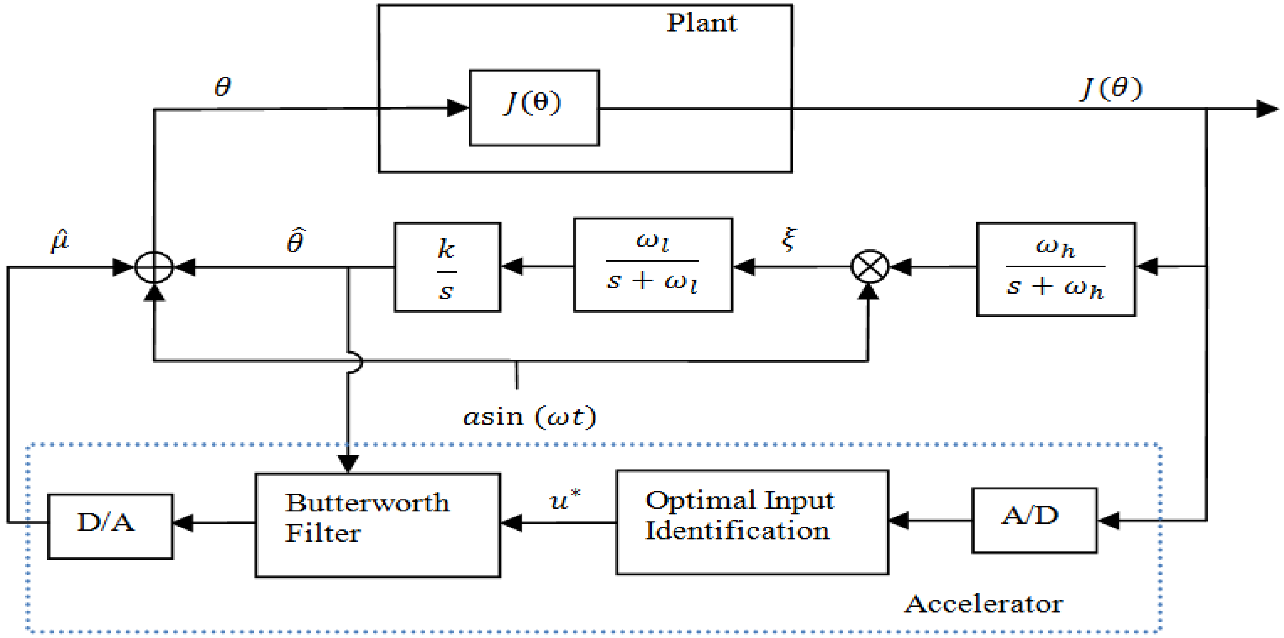
Extremum Seeking Results



$$\Delta R_0 = 3.0\,\text{cm}, \quad w_0 = w_{sat} = 7.5\,\text{cm}$$

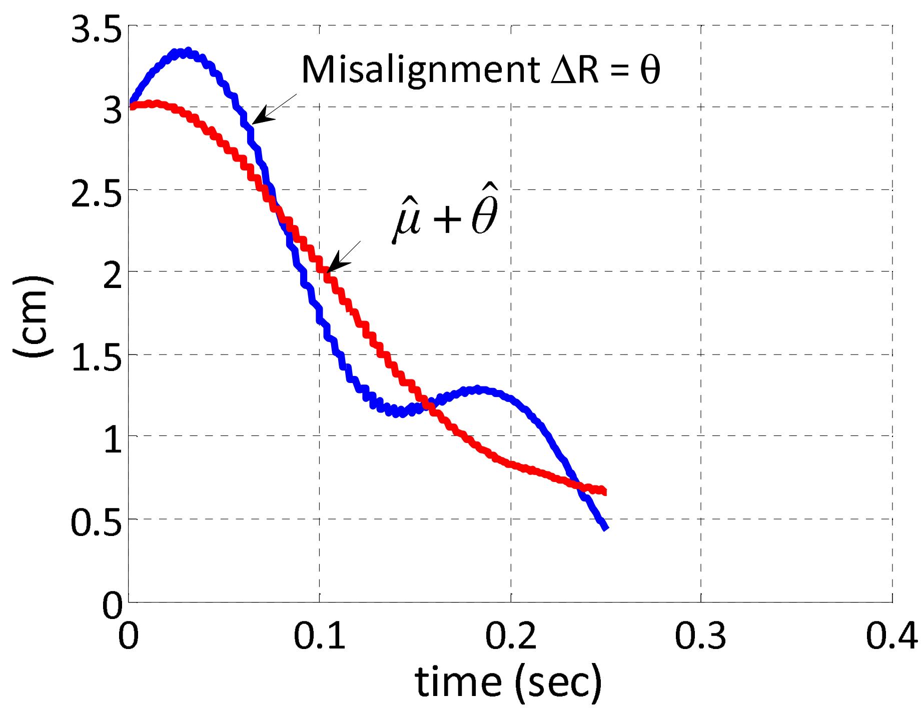
Modified Algorithm: Use gain scheduling to modify the gain γ as a function of the island width. For continuous current drive the average suppression time is *m=0.40s* with standard deviation *σ = 0.07s* and *m=0.41s, σ = 0.13s* for 50/50 duty-cycle current drive centered at the O-point.

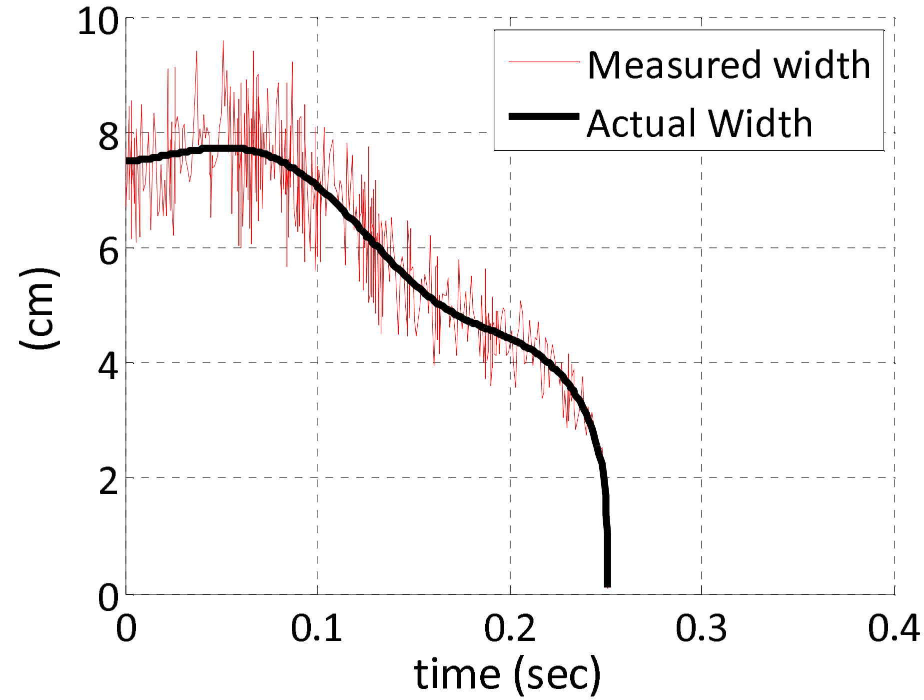
Extremum-Seeking Algorithm with Accelerator



- The model can be used to infer an approximate value of the misalignment based on measurements of the island width and instantaneous growth rate
- During control this information is used to shorten the time to the optimal operating point by jumping the actuator towards the island
- Large, coarse adjustments in control are made by the accelerator and fine adjustments by the Extremum Seeking
- The accelerator contribution is given by $\hat{\mu} = u^* - \hat{\theta}$ where u^* is the location of island center estimated by the accelerator

Extremum Seeking Results



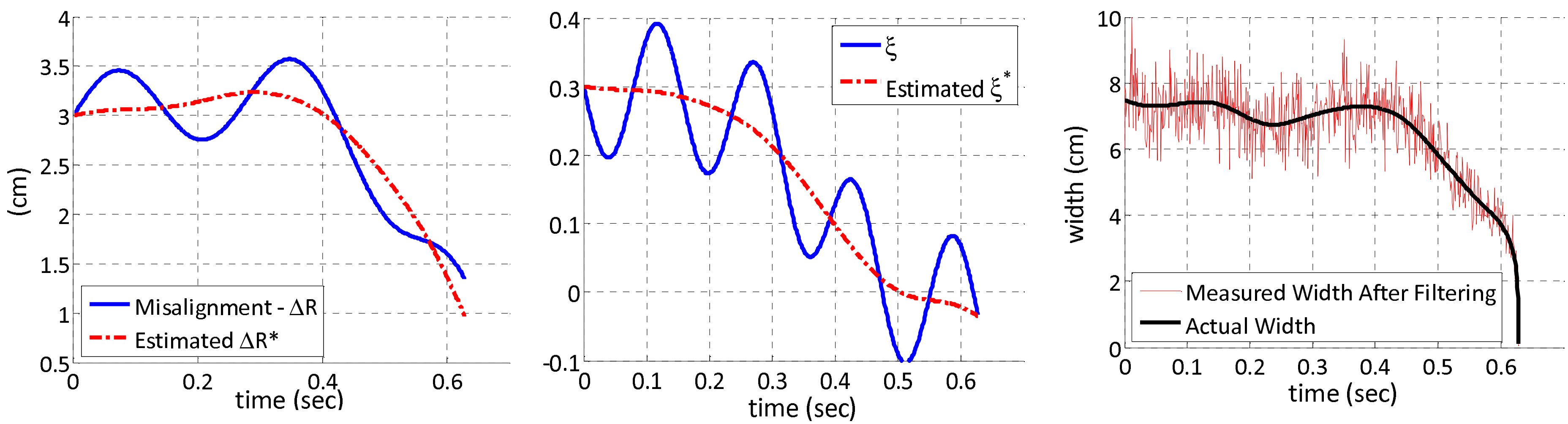


$$\Delta R_0 = 3.0\,\text{cm}, \quad w_0 = w_{sat} = 7.5\,\text{cm}$$

If the initial step direction is unknown the average suppression time is $m = 0.25s$ with standard deviation $\sigma < 0.01s$ for continuous current drive.

Extremum Seeking Results - Multiparameter

For the most efficient suppression by modulated current drive, we must align the on period of the ECCD with the island O-point. If the optimal phase cannot be measured accurately, then Extremum Seeking can be used to correct the phase mismatch.



$$\Delta R_0 = 3.0\,\text{cm}, \quad \xi_0 = 0.3, \quad w_0 = w_{sat} = 7.5\,\text{cm}$$

Standard Extremum Seeking algorithm with two parameter optimization ($\Delta R, \xi$): The average suppression time is m = *0.62s* with standard deviation σ = *0.10s.*

NTM Suppression Time Comparison

The suppression times for the various control methods. The first number in each entry represents the average suppression time and the number between parentheses represents the standard deviation.

	ES	ES (Gain Schd)	ES (Accel)	SS
No mod	0.54 (0.06)	0.40 (0.07)	0.25 (<0.01)	0.62 (0.14)
O-point	0.44 (0.10)	0.41 (0.13)	NA	0.50 (0.18)

It has been shown that the Extremum Seeking method has the potential of reducing NTM suppression times. Also, Extremum Seeking can be employed to optimize other parameters other parameters beyond island-beam alignment also affecting the effectiveness of the ECCD suppression method such as the phase mismatch between the modulation on period and the O-point.

Conclusions and Future Work

- Effect of general, Gaussian off-center current drive on magnetic island width has been derived including the effects of current drive power modulation
- The performances of Search and Suppress and Extremum Seeking in suppressing a magnetic island are examined
- Extremum Seeking has the potential for improved suppression time
- Incorporation of ECCD power modulation frequency into the model
- Simulations involving simultaneous optimization of ECCD power modulation parameters and beam steering via Extremum Seeking