Error field threshold in NBI heated H-modes

By

Holger Reimerdes[1]

In collaboration with

R.J. Buttery,[2] A.M. Garofalo,[2]
R.J. La Haye,[2] M.J. Lanctot,[1] J-K. Park[3],
M. Schaffer,[2] W.M. Solomon[3] and E.J. Strait,[3]

[1]Columbia University, New York, NY, USA
[2]General Atomics, San Diego, CA, USA
[3]Princeton Plasma Physics Laboratory, Princeton, NJ, USA

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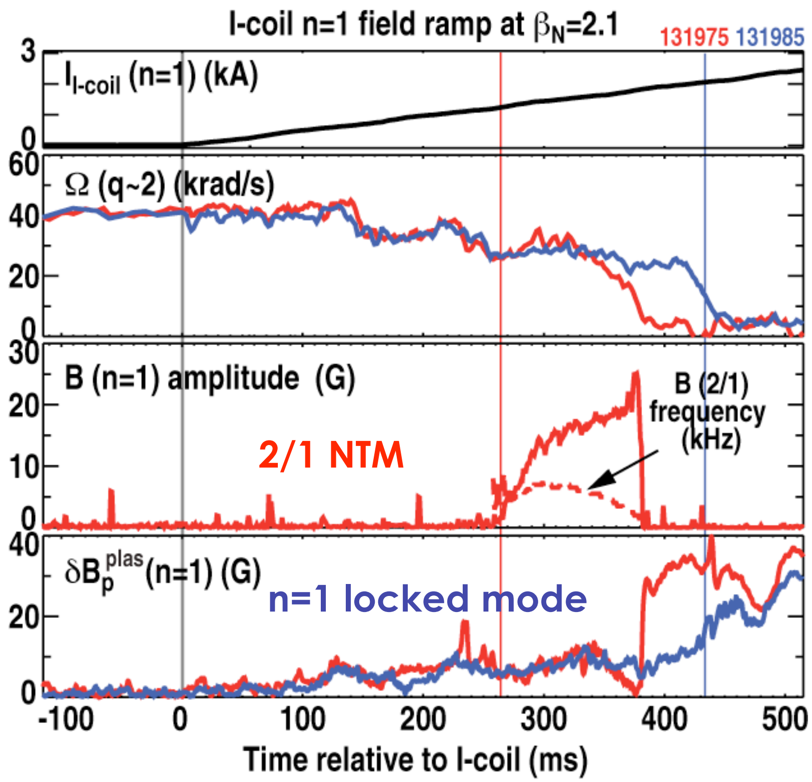
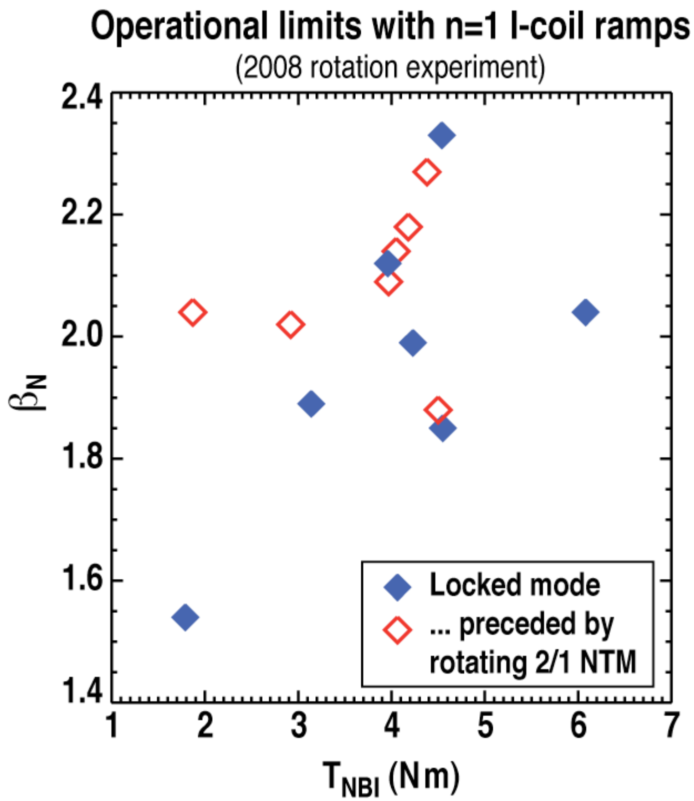
Outline/Main results

- Resonant braking leading to a loss of torque balance determines the "ultimate" *n*=1 error field threshold in NBI heated H-modes
- Decrease of *n*=1 error field tolerance with increasing β is caused by an increasing plasma amplification
- Plasma is sensitive to kink-resonant rather than pitch-resonant external fields
- Plasma response leads to a resonant and non-resonant magnetic torque

External non-axisymmetric "error" fields limit NBI heated H-modes by inducing locked modes or by "triggering" NTMs





- **NTM is born in the plasma frame, i.e. it rotates with the plasma**
  - NTM onset predominantly observed at higher β_N and lower NBI torque
    ➔ R. Buttery "Effect of 3D fields near the tearing beta limit" on Tuesday
- **Locked mode is always born locked to the vessel** ➔ This talk

At low β, NBI heating generally increases the error field tolerance

- **Beneficial effect has been attributed to the toroidal torque associated with NBI heating leading to higher rotation** [R.J. La Haye, et al., *Nucl. Fusion* (1992)]
  - Error field threshold (JET L-modes) consistent with

$$\delta B_{crit}^{ext} \propto \Omega_0^{0.5}$$

scaling (Ω_0 is the unperturbed rotation before the external field is applied) [E. Lazzaro, et al., *Phys. Plasmas* (2002)]

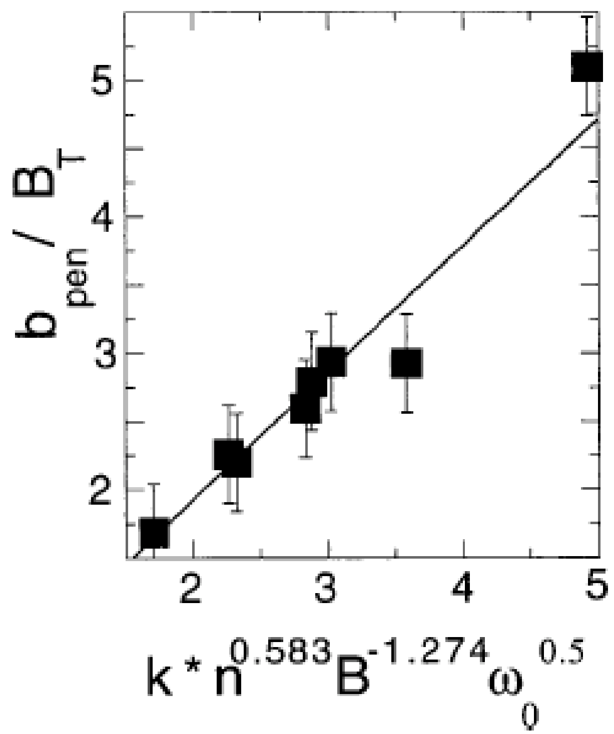


FIG. 7. Scaling of the penetration threshold vs power law $\sim n^{0.58}B^{-1.274}\omega_0^{1/2}$ where ω_0 the local angular rotation frequency at the $q=2$ surface.

Figure from Lazzaro, et al., Phys. Plasmas (2002)

At intermediate values of β, the increase of the error field tolerance with NBI heating (and hence β) roles over

- **Early high β experiments in DIII-D have already revealed a strong reduction of the error field tolerance with β**
  [R.J. La Haye, et al., *Nucl. Fusion* (1992)]

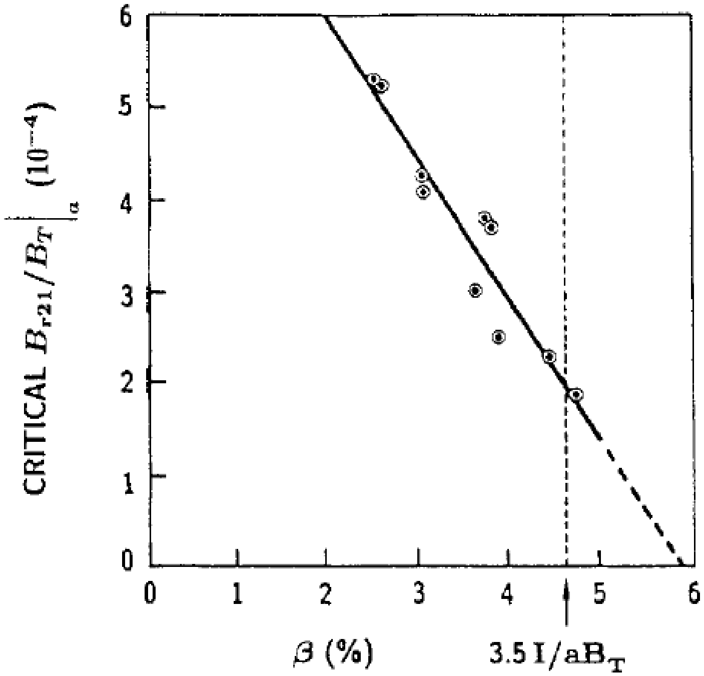


FIG. 10. Critical 2,1 relative error field for instability in H-mode plasmas as a function of beta (left or left/right beams).

Figure from La Haye, et al., Nucl. Fusion (1992)

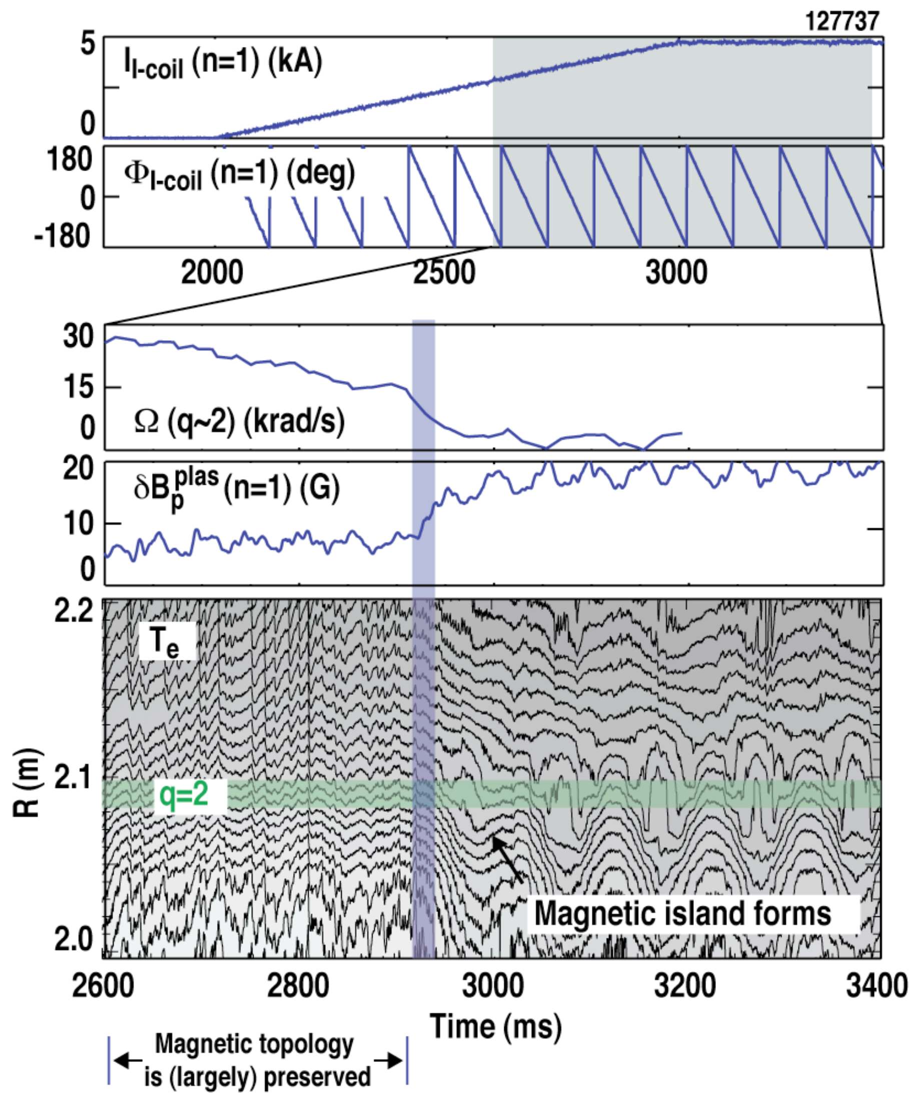
Error field tolerance in NBI heated H-modes is determined by resonant braking leading to a loss of torque balance

- **Increase the amplitude of a slowly rotating (10Hz) externally applied $n = 1$ "*error*" field $\delta B^{ext} \propto I_{\text{I-coil}}$**

↓

- **Rotation evolution is described by resonant braking** [Garofalo, et al., *Nucl. Fusion* **47** (2007) 1121]
  - At high rotation the resonant field is shielded, but exerts a torque
  - Rotation decrease is followed by a loss of torque balance

- **Magnetic island observed only after rotation collapses**



Resonant magnetic braking torque leads to a bifurcation of the plasma rotation

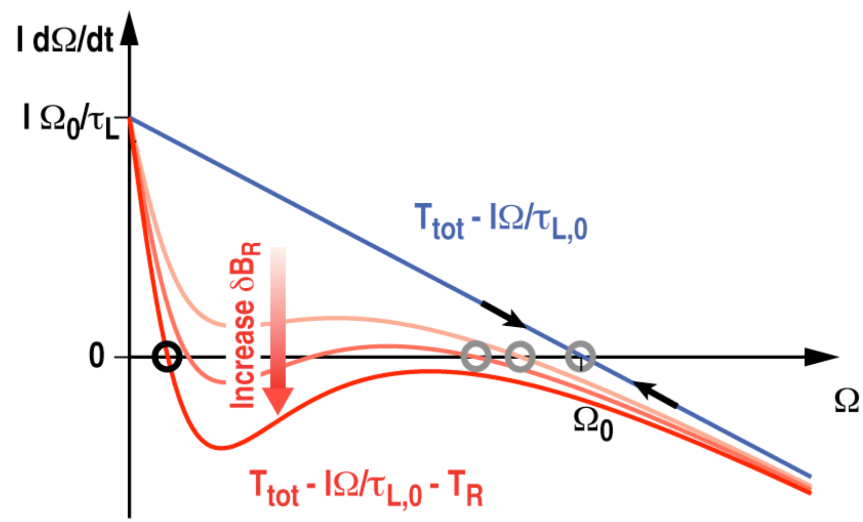
- **0D-model for plasma rotation**

$$I\frac{d\Omega}{dt} = T_{in} - \frac{I\Omega}{\tau_{L,0}} - T_{MB}$$

with $\tau_{L,0}$ being the momentum confinement time without braking

- **Assume a resonant magnetic braking torque with $T_R \propto \Omega^{-1}$**
[R. Fitzpatrick, *Nucl. Fusion* (1993)]

$$T_{MB} \rightarrow T_R = K_R\, \delta B_R^2\, \Omega^{-1}$$



- **Torque balance $d\Omega/dt = 0$ when** $$\Omega = \frac{T_{in}\tau_{L,0}}{2I} \pm \sqrt{\left(\frac{T_{in}\tau_{L,0}}{2I}\right)^2 - \frac{K_R\tau_{L,0}}{I}\delta B_R^2}$$

  – Bifurcation at $\Omega = \frac{\Omega_0}{2}$ (with $\Omega_0 = \frac{T_{in}\tau_{L,0}}{I}$ being the unperturbed rotation), when resonant perturbation exceeds $$\delta B_{R,crit} = T_{in}\sqrt{\frac{\tau_{L,0}}{4IK_R}}$$

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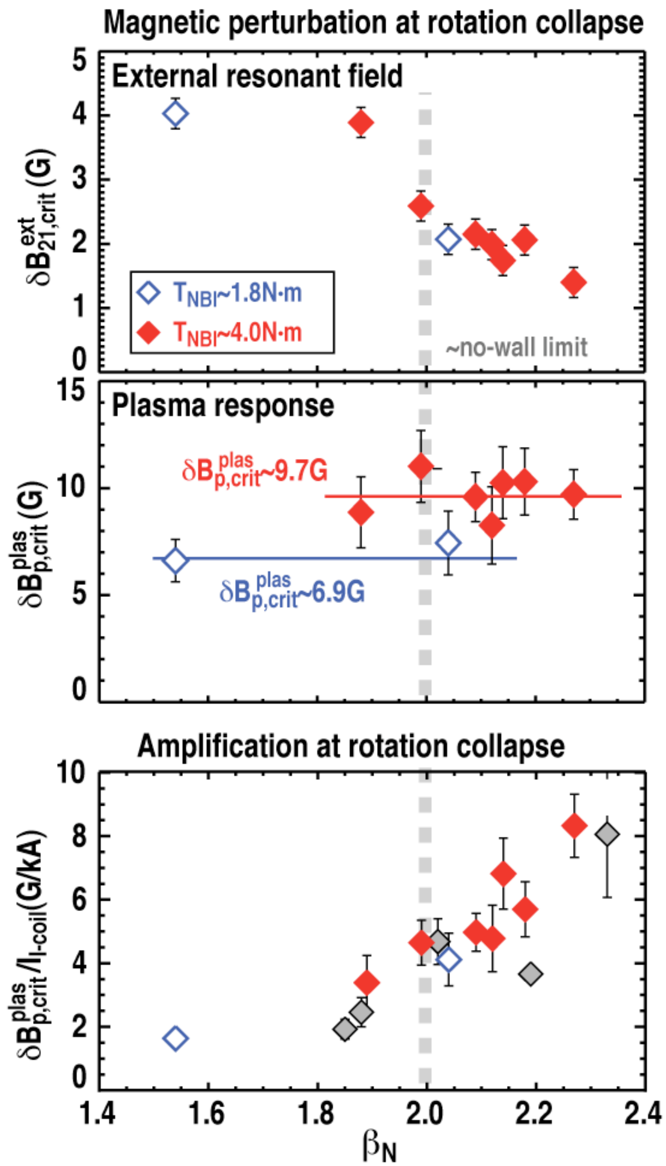
Decrease of *n*=1 error field tolerance with increasing β_N is attributed to plasma amplification

[Reimerdes, et al., *Nucl. Fusion* (2009)]

- **Decrease of critical external field** $\delta B^{ext}_{21,crit}$ **(SURFMN) at** T_{NBI} = const. **is particularly strong above the no-wall limit**
  - External field is also increasingly amplified

- **Rotation collapse occurs at a fixed plasma response** $\delta B^{plas}_{p,crit}$

- **Critical plasma response** $\delta B^{plas}_{p,crit}$ **increases (as expected) with NBI torque** T_{NBI}



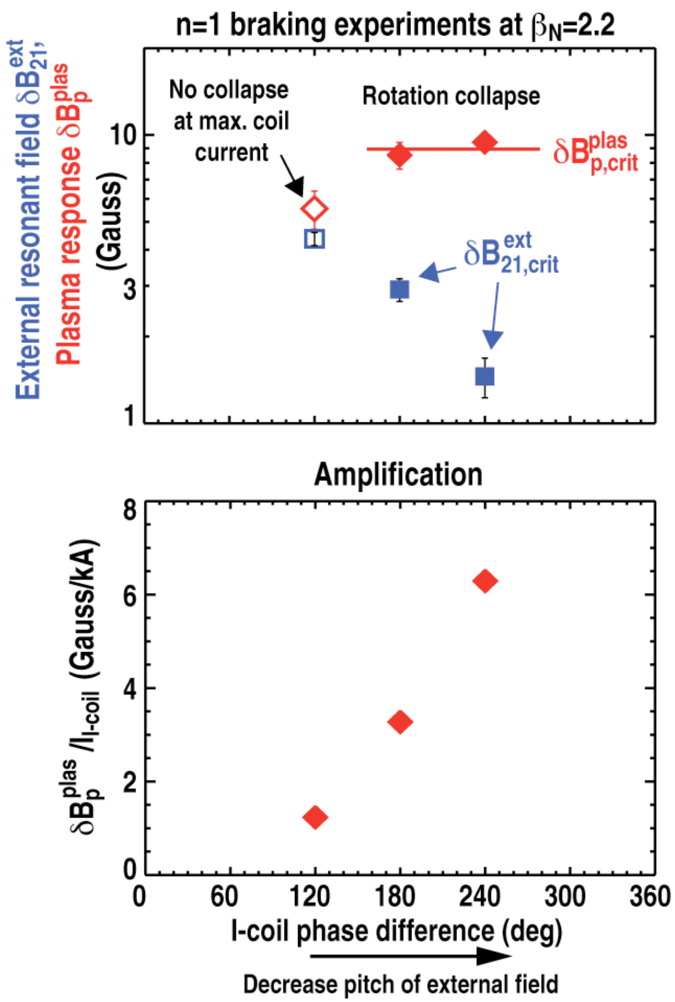
Dependence on the poloidal spectrum of the external field is also attributed to plasma amplification

- **Critical external field $\delta B^{ext}_{21,crit}$ at const. NBI torque and const. β_N changes with I-coil phasing (by more than a factor of 3)**
  - Amplification changes, too
- **Rotation collapse occurs at a fixed plasma response $\delta B^{plas}_{p,crit}$**

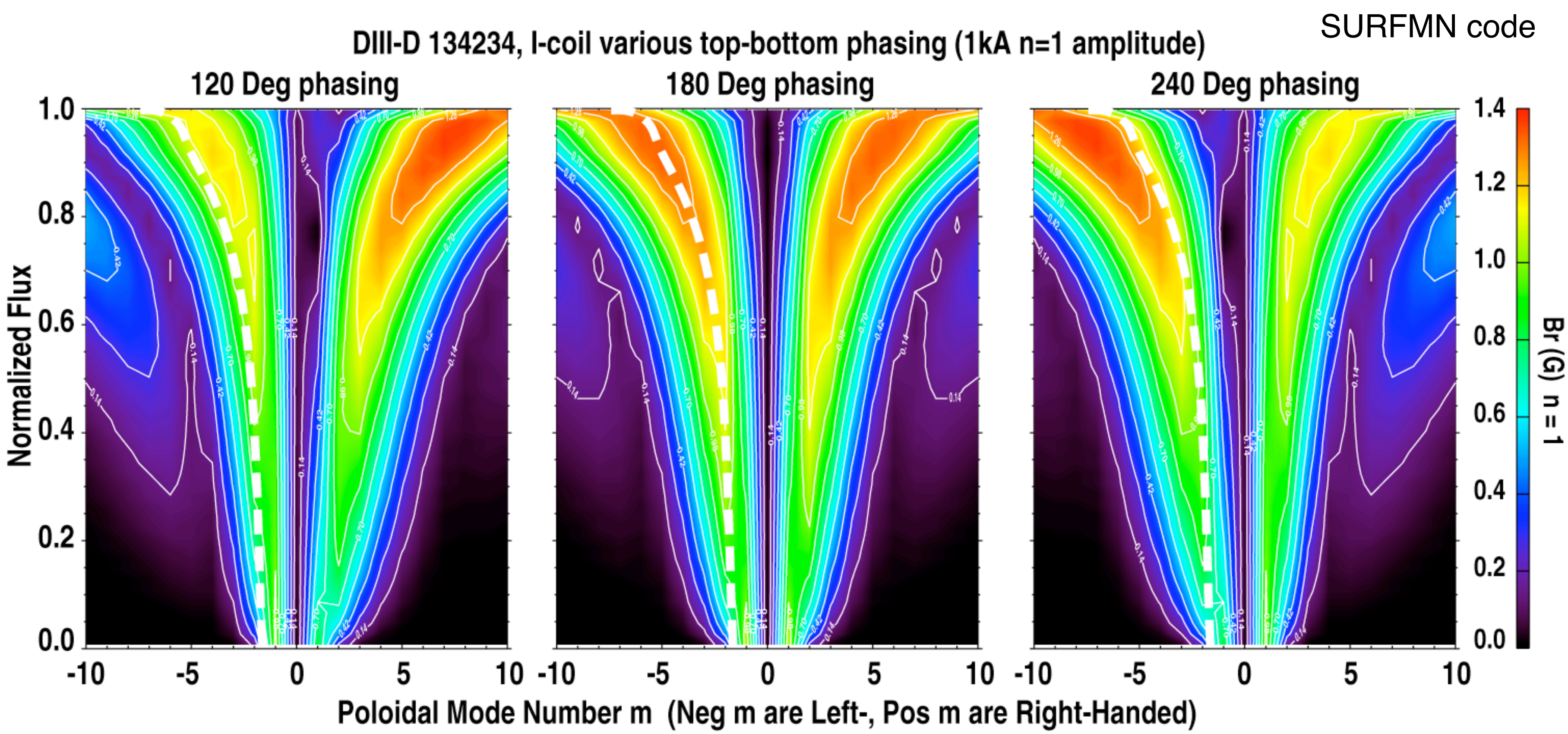
Resonant braking and hence error field tolerance is determined by the external field that is kink-mode resonant - not pitch-resonant at the *q*=2 surface

  - Pointed out by Park, et al. for low density locked modes
    [Park, et al, *Phys. Rev. Lett.* (2007)]

➔ Details in talk by M. Lanctot, "Measurement and modeling of 3D tokamak equilibria" on We



At the LFS, plasma is most sensitive to an external field with a lower pitch angle than the equilibrium field



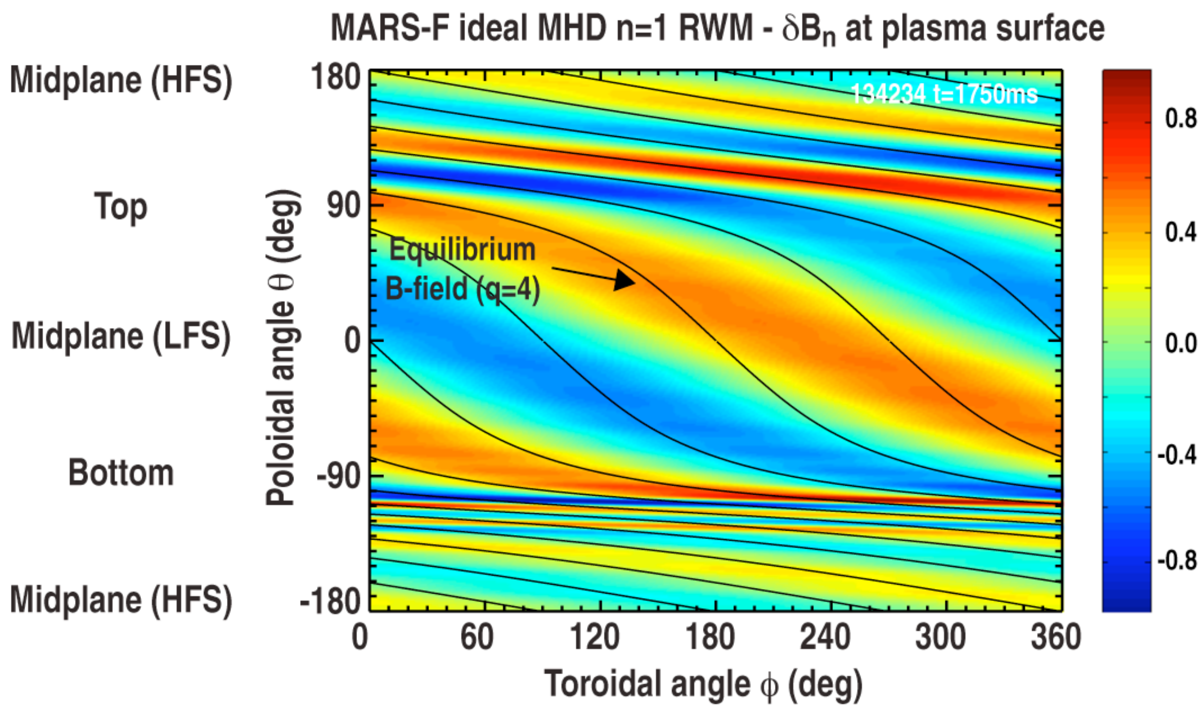
➔ $\delta B_p^{plas} = 1.2G$

➔ $\delta B_p^{plas} = 3.3G$

➔ $\delta B_p^{plas} = 6.3G$

Largest response when *m>n*q* components large

Kink mode (and the most sensitive external field) does not always follow the equilibrium field



- **Fourier harmonics of the external field that the plasma is most sensitive to depend on the geometry of the external coil (e.g. LFS vs. HFS coils)**
  - Exact characterization requires several harmonics + their phases

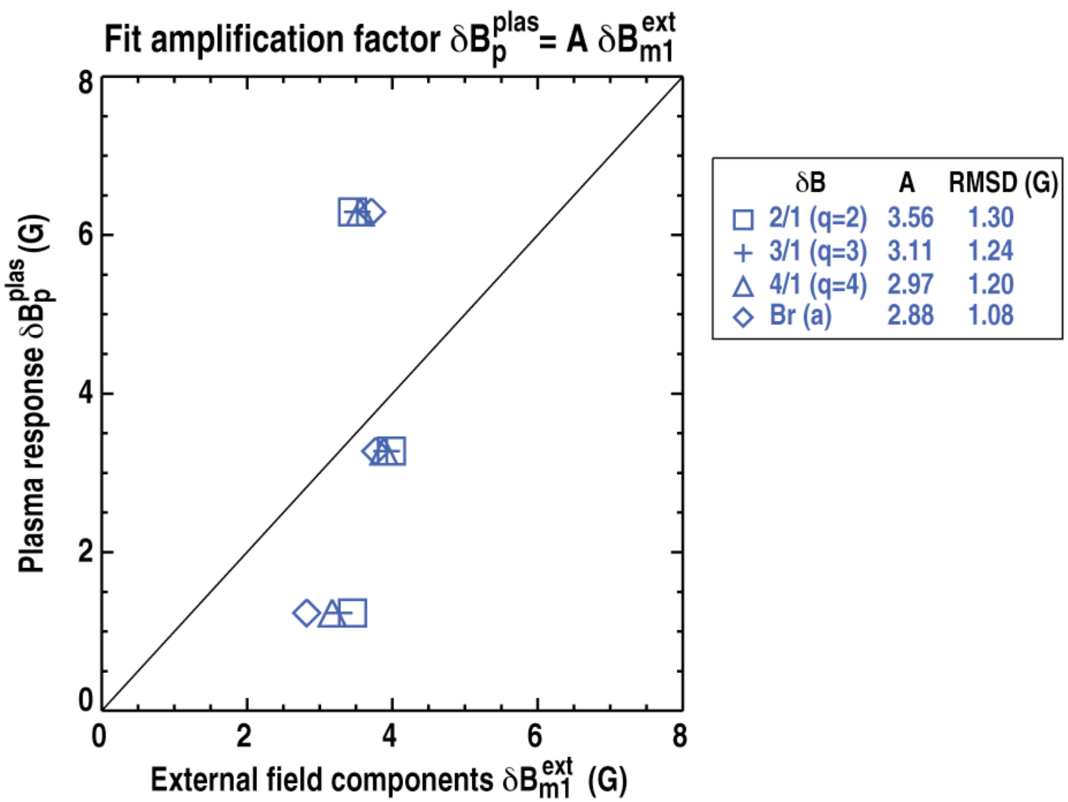
→Fourier harmonics of δB^{ext} in straight field line coordinates are NOT an efficient description of the error field tolerance

Observed amplification yields an estimate of the most sensitive external field component (LFS coils)

- **Correlate various Fourier harmonics of the external field (SURFMN) with the measured plasma response**
  - Fit amplification factor *A*

$$\delta B_{\mathrm{p}}^{\mathrm{plas}} = A\ \delta B_{\mathrm{mn}}^{\mathrm{ext}}(q)$$

- **Resonant components match poorly**



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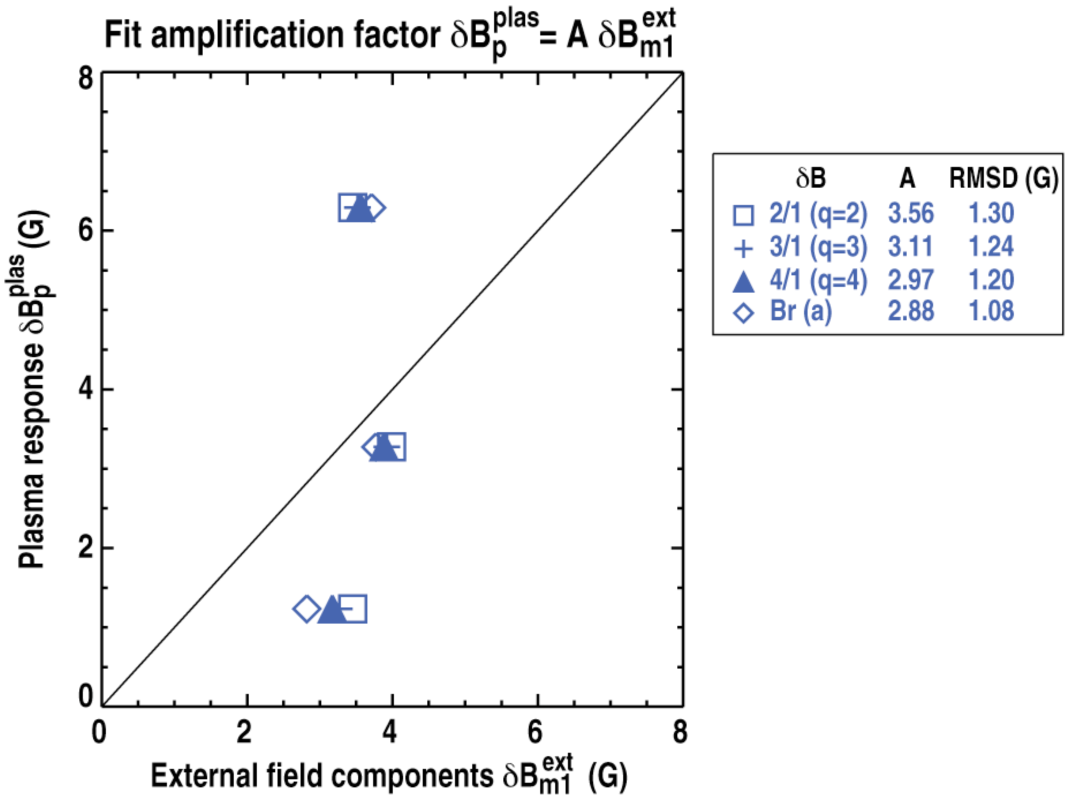

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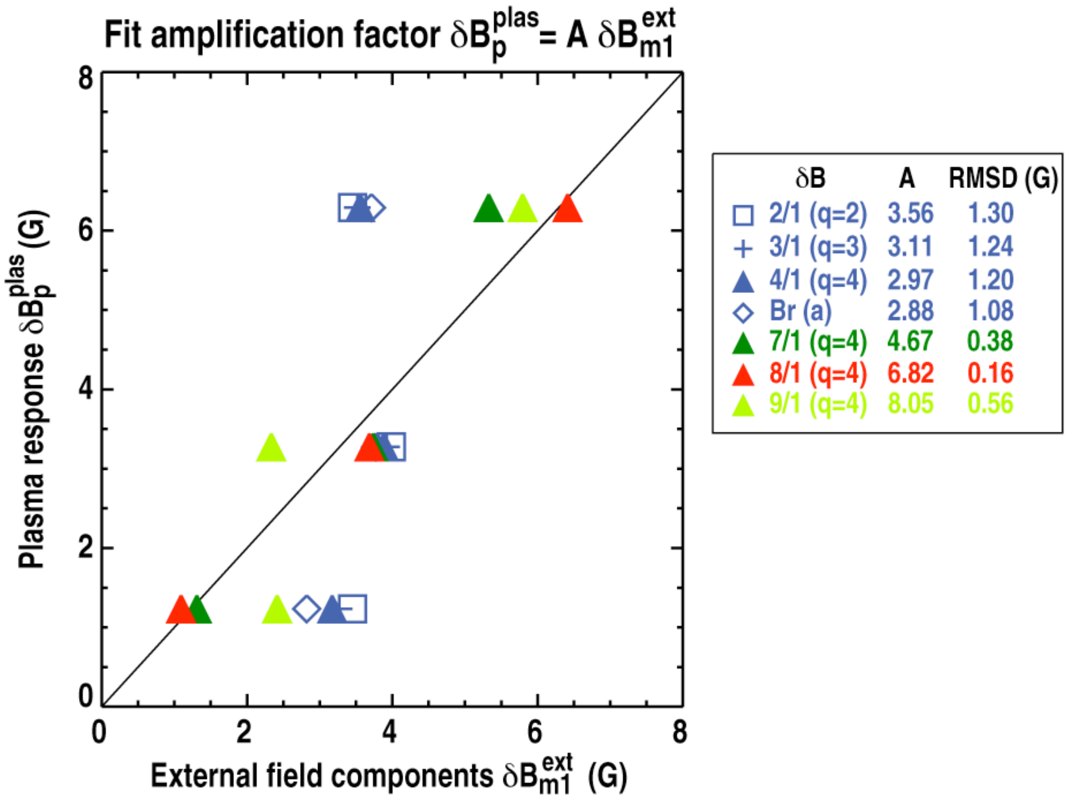

- **Resonant components match poorly**
- **Most sensitive field δB_{mn}**

$$m = 2nq$$

(consistent with M. Schaffer's rule of thumb)

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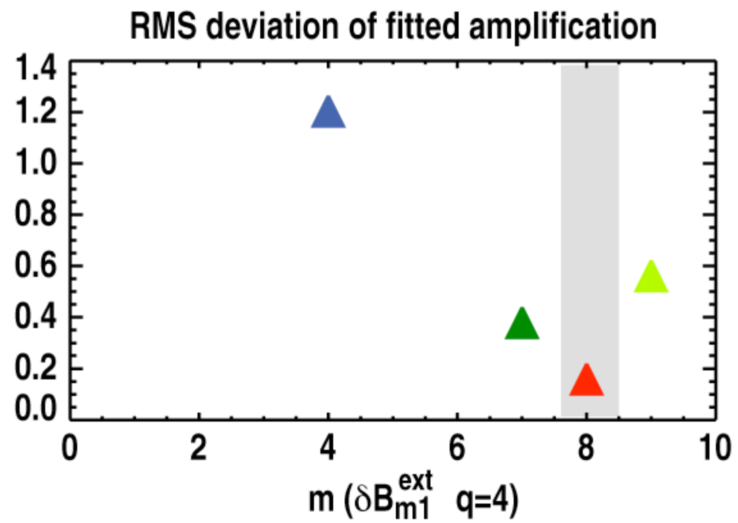

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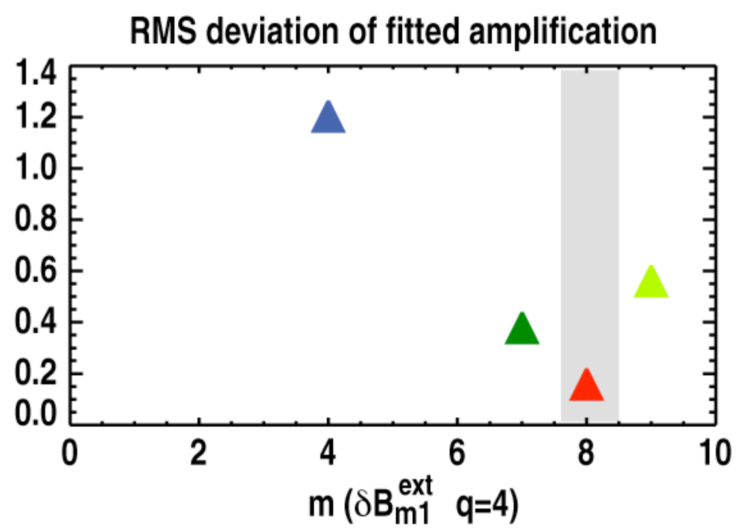


➔ **Use measured plasma response (∝ total perturbed field) for further studies**

Observed dependence of error field tolerance on torque input weaker than prediction for resonant braking

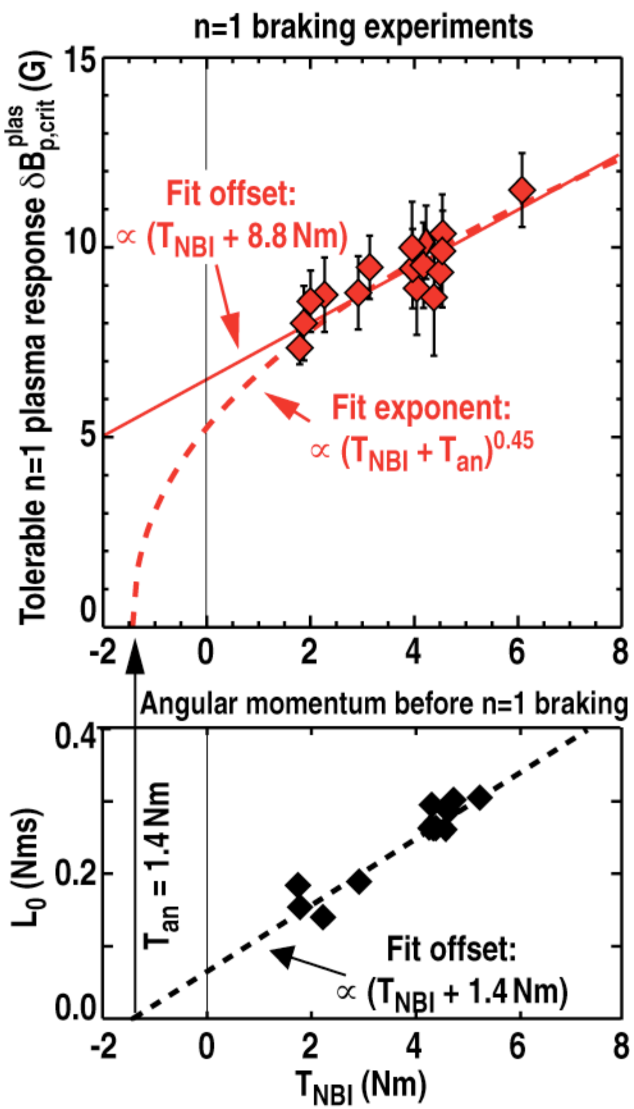
- **Assuming a linear T_{NBI} dependence as predicted by the resonant braking model**

$$\delta B_{R,crit} = T_{in} \sqrt{\frac{\tau_{L,0}}{4IK_R}}$$

leads to an offset that is too large to be explained by the anomalous torque (T_{an}=T_{in}-T_{NBI})

- **A fit based on an estimate of T_{an} from the angular momentum before *n*=1 braking yields**

$$\delta B^{plas}_{p,crit} \propto T_{in}^{0.45}$$



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- **A fit based on an estimate of T_{an} from the angular momentum before n=1 braking yields**

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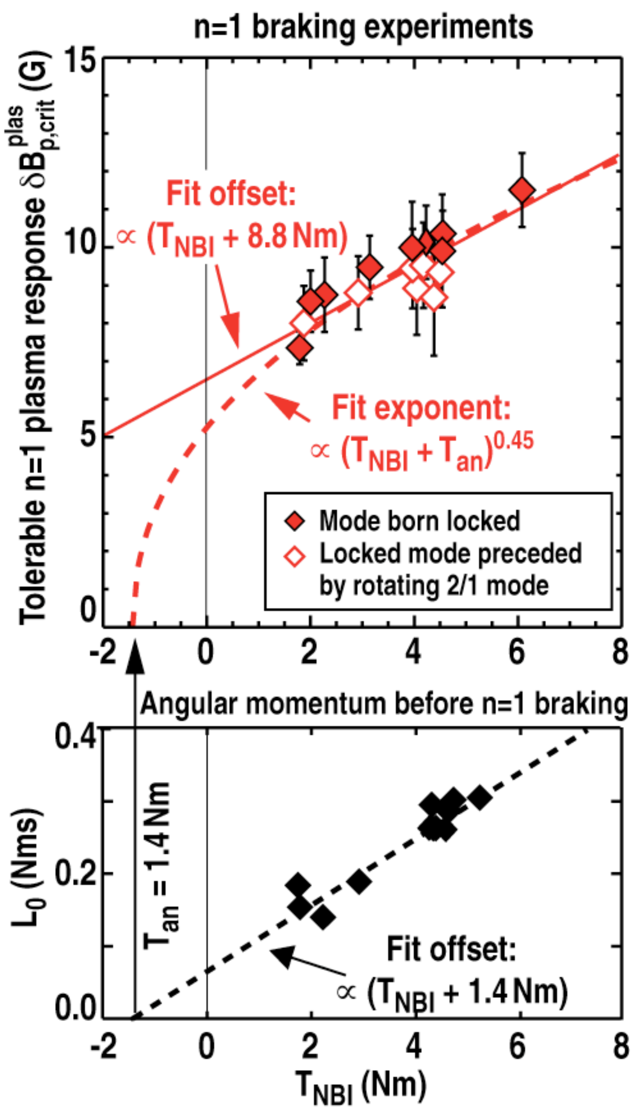


Input torque dependence of error field tolerance translates into a similar rotation dependence

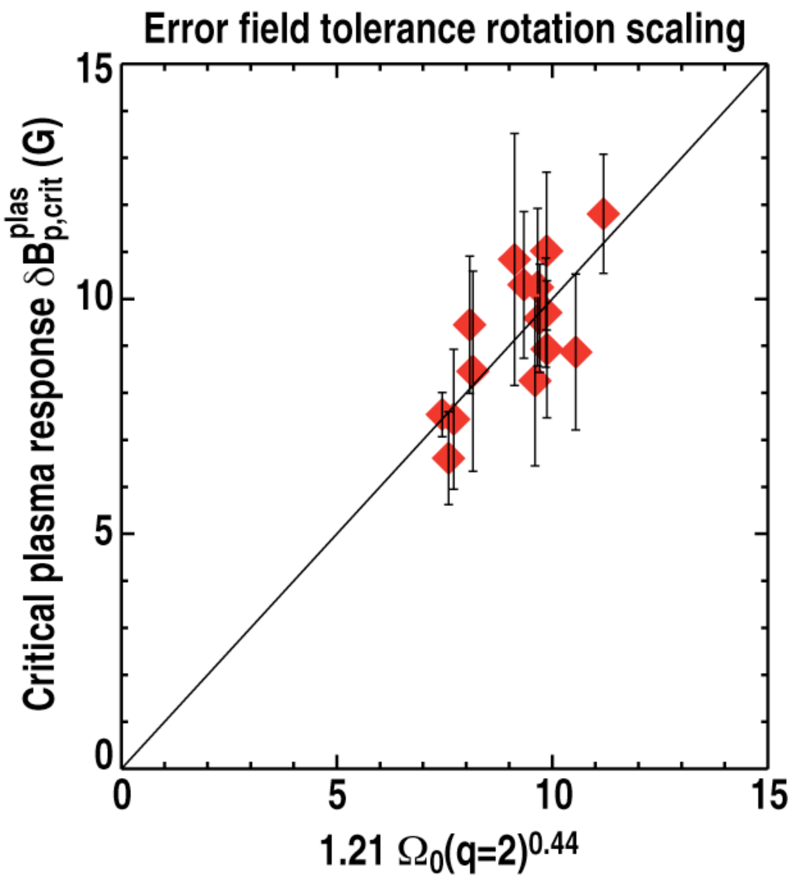
- **A fit of the dependence of the critical plasma response on the unperturbed rotation Ω_0 yields**

$$\delta B_{\mathrm{p,crit}}^{\mathrm{plas}} \propto \Omega_0^{0.44}$$

  - Similar exponent for T_{in} and Ω_0 expected since

$$\Omega_0 = \frac{T_{in}\tau_{L,0}}{I}$$

  - Consistent with previous low β scalings [e.g. E. Lazzaro, et al., *Phys. Plasmas* (2002)]



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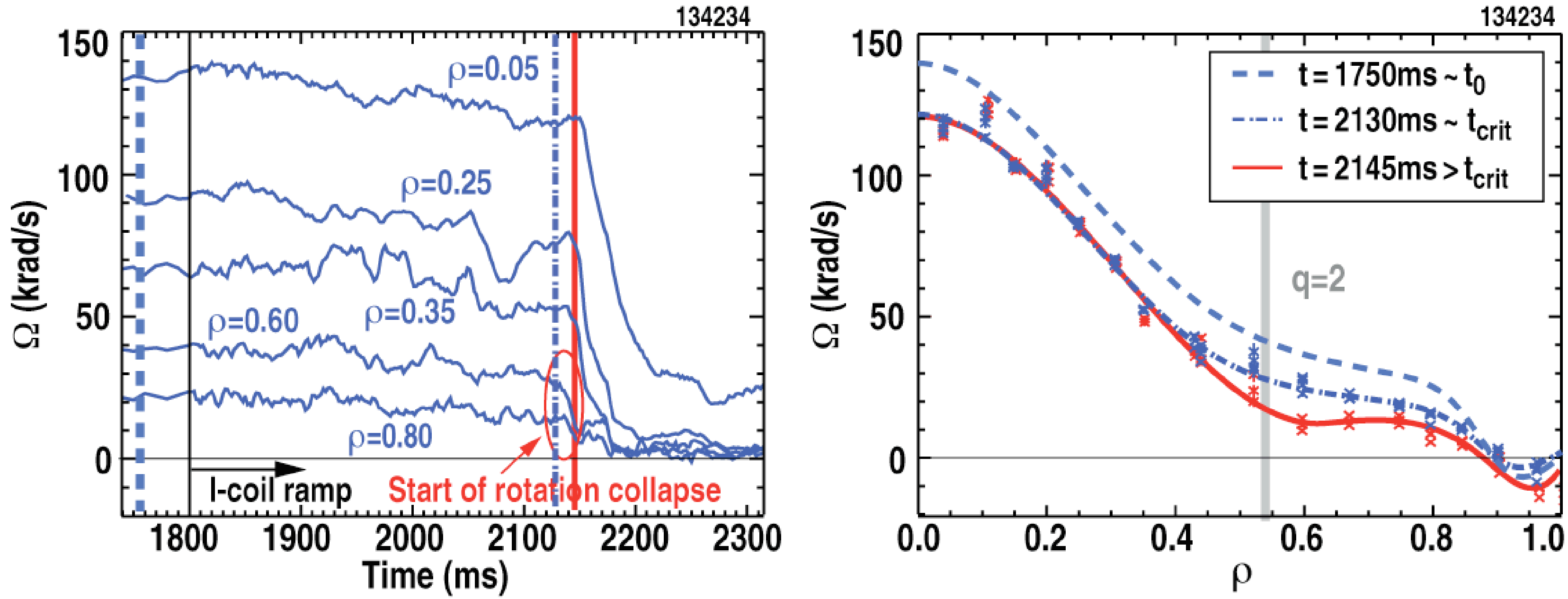
Rotation damping prior to the rotation collapse shows no evidence of a localized braking torque

Toroidal rotation evolution during n=1 magnetic braking



- **Rotation decreases across the entire profile - indicating a non-resonant braking mechanism**
  - Resolution of a localized braking torque limited by uncertainty of Ω' and Ω''
- **Rotation collapse starts at the outer half of the profile**

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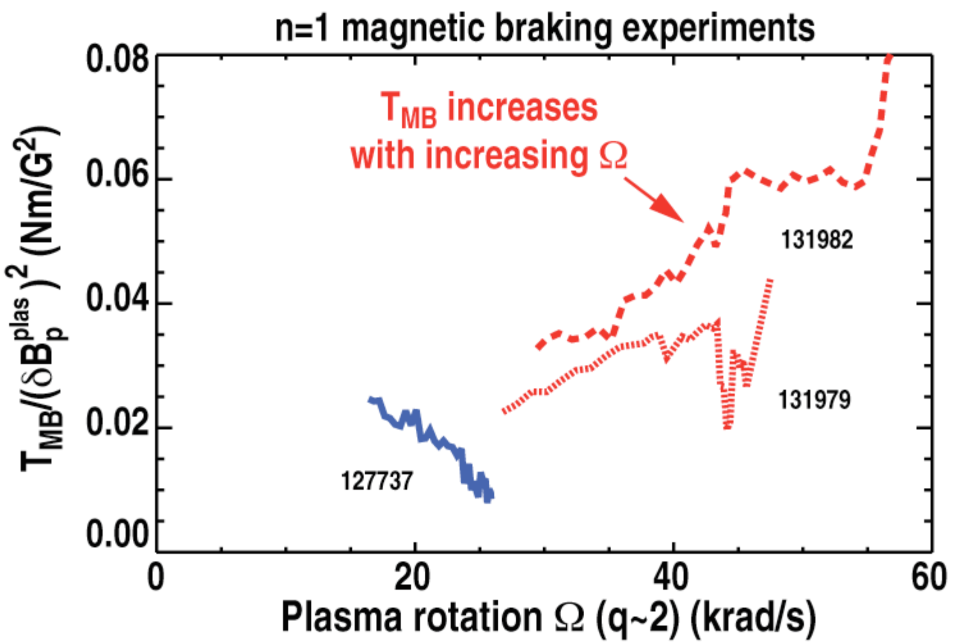
Measured rotation dependence of the braking torque reveals further evidence of a non-resonant component

- **Measured angular momentum evolution yields magnetic braking torque** T_{MB}

$$T_{MB} = T_{NBI} - \frac{L}{\tau_{L,0}} - \frac{dL}{dt}$$

  - Assume $T_{MB} \propto (\delta B^{plas})^2$ to reveal rotation dependence



- **At low rotation T_{MB} increases with decreasing Ω consistent with a resonant torque** [R. Fitzpatrick, *Nucl. Fusion* (1993)]
- **At high rotation T_{MB} increases with Ω → typical for a non-resonant torque** [K.C. Shaing, *Phys. Plasmas* (2003)]

Additional non-resonant component reduces the resonant threshold and weakens the torque dependence

- **Adding a non-resonant component $f_{NR}\delta B$ of the perturbed field δB ($\propto \delta B^{plas}$) to the magnetic braking torque in the 0D torque balance***

$$I\frac{d\Omega}{dt} = T_{in} - \frac{I\Omega}{\tau_{L,0}} \underbrace{- K_R\left(f_R\delta B\right)^2\Omega^{-1}}_{\text{Resonant braking torque}} \underbrace{- K_{NR}\left(f_{NR}\delta B\right)^2\Omega}_{\text{Non-resonant braking torque}}$$

yields

for $f_{NR}=0$:	$\delta B_{crit} = b_R\, T_{in}$
for $f_{NR}\neq 0$:	$\delta B_{crit} = \sqrt{2}\, b_R T_e\left(\left(1+\left(T_{in}/T_e\right)^2\right)^{0.5} - 1\right)^{0.5}$

with $b_R = \frac{1}{2f_R}\left(\frac{\tau_{L,0}}{IK_R}\right)^{0.5}$ **and** $T_e = \underbrace{\frac{I}{\tau_{L,0}}\frac{f_R}{f_{NR}}\left(\frac{K_R}{K_{NR}}\right)^{0.5}}$

Viscous torque at Ω where resonant and non-resonant torque are equally important

***Neglect offset rotation in counter-Ip direction** [A. Cole, et al., *Phys. Rev. Lett.* (2007), A.M. Garofalo, et al., *Phys. Rev. Lett.* (2009)]

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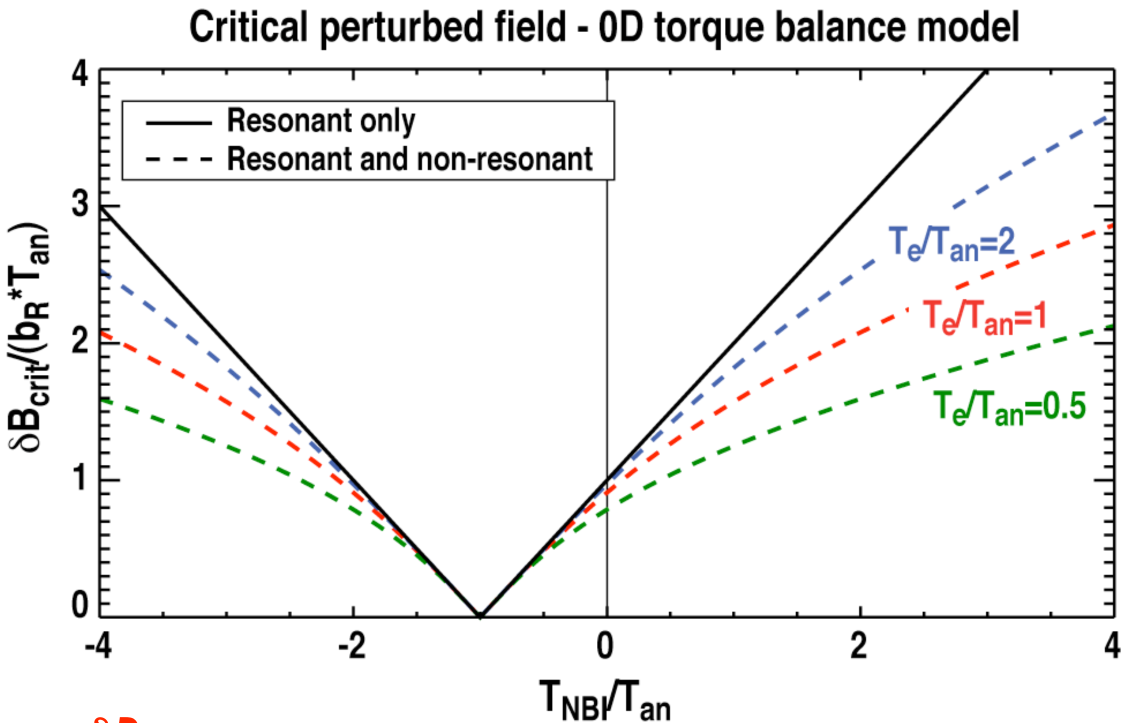
A non-resonant braking component of the perturbed field lowers the tolerance for resonant magnetic field errors

- **Increasing the non-resonant component f_{NR} decreases** $T_e = \frac{I}{\tau_{L,0}} \frac{f_R}{f_{NR}} \left(\frac{K_R}{K_{NR}} \right)^{0.5}$



➔ **Decreases δB_{crit}**

➔ **Weakens the T_{NBI} dependence of δB_{crit} (possible explanation for the negligible T_{NBI} dependence in NSTX** [Park, et al., APS (2009)]**?)**

Observed $\delta B_{crit}^{plas} \propto T_{NBI}^{0.45}$ in DIII-D consistent with a strong non-resonant component of the plasma response

- **Adding a non-resonant component $f_{NR}\delta B$ of the perturbed field δB to the magnetic braking torque in the 0D torque balance**

$$I\frac{d\Omega}{dt} = T_{in} - \frac{I\Omega}{\tau_{L,0}} - K_R\left(f_R\delta B\right)^2\Omega^{-1} - K_{NR}\left(f_{NR}\delta B\right)^2\Omega$$

yields

for $f_{NR}=0$:	$\delta B_{crit} = b_R\, T_{in}$
for $f_{NR}\neq 0$:	$\delta B_{crit} = \sqrt{2}\, b_R T_e\left(\left(1+\left(T_{in}/T_e\right)^2\right)^{0.5} - 1\right)^{0.5}$

with $b_R = \frac{1}{2f_R}\left(\frac{\tau_{L,0}}{IK_R}\right)^{0.5}$ **and** $T_e = \frac{I}{\tau_{L,0}}\frac{f_R}{f_{NR}}\left(\frac{K_R}{K_{NR}}\right)^{0.5}$

- **Assuming a strong non-resonant component/large torque input $T_e \ll T_{in}$ yields**

$$\delta B_{crit} = \sqrt{2}\, b_R\left(T_e T_{in}\right)^{0.5}$$

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A non-resonant component in addition to the *n*=1 plasma response reduces the error field tolerance further

- **Adding a non-resonant torque independent of the *n*=1 plasma response in the 0D torque balance (e.g. *n*=3, TBM, TF ripple)**

$$I\frac{d\Omega}{dt} = T_{\mathrm{in}} - \frac{I\Omega}{\tau_{\mathrm{L,0}}} - K_{\mathrm{R}}\left(f_R\delta B\right)^2\Omega^{-1} - K_{\mathrm{NR}}\left(f_{NR}\delta B\right)^2\Omega - K^*_{\mathrm{NR}}\left(\delta B^*_{\mathrm{NR}}\right)^2\Omega$$

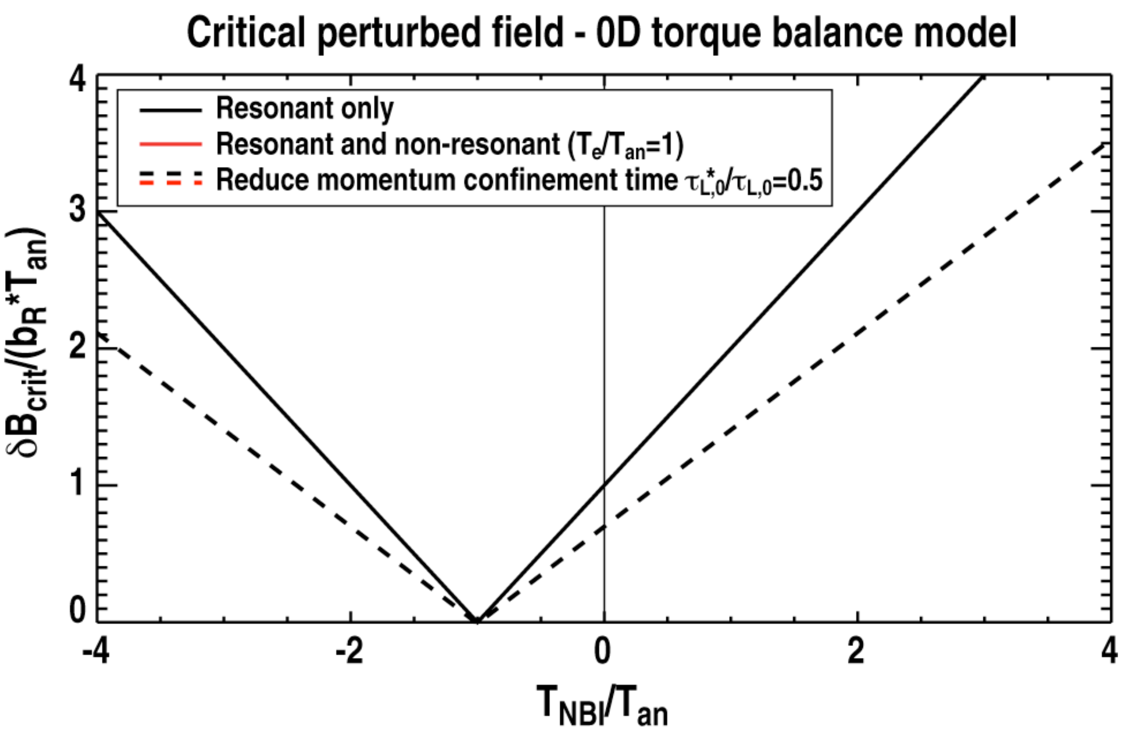
can be expressed by an effective momentum confinement time

$$\tau^*_{\mathrm{L,0}} = \left(\left(\tau_{\mathrm{L,0}}\right)^{-1} + \frac{K^*_{\mathrm{NR}}}{I}\left(\delta B^*_{\mathrm{NR}}\right)^2\right)^{-1}$$

- **For zero and small f_{NR}**

$$\delta B_{\mathrm{crit}} \propto \left(\tau^*_{\mathrm{L,0}}\right)^{0.5}$$

- **For large T_{NR} the error field tolerance becomes independent of $\tau_{\mathrm{L,0}}$***



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can be expressed by an effective momentum confinement time

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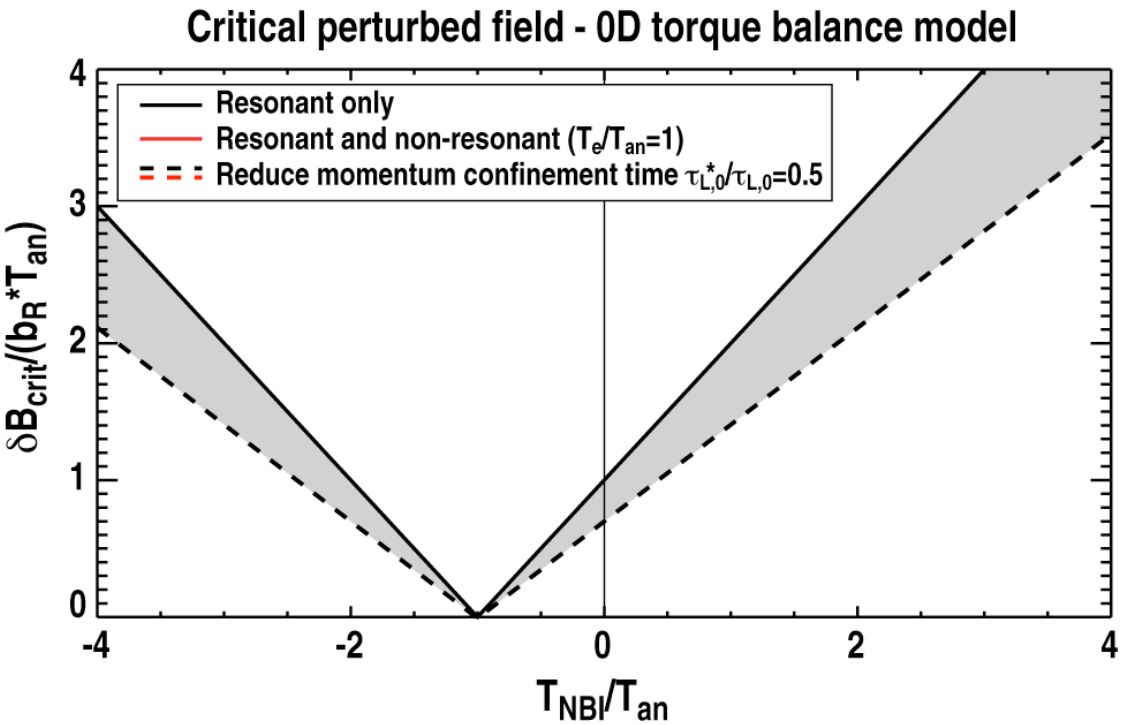

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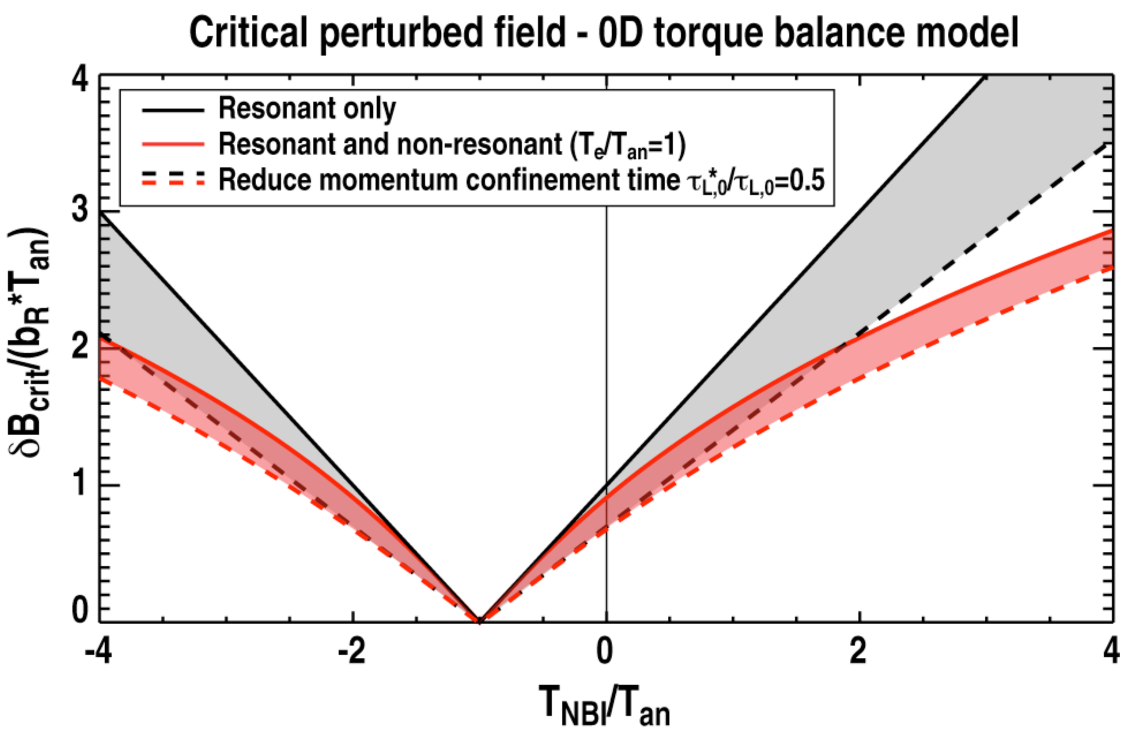

- **For zero and small f_{NR}**

$$\delta B_{\mathrm{crit}} \propto \left(\tau^*_{\mathrm{L},0}\right)^{0.5}$$

- **For large T_{NR} the error field tolerance becomes independent of $\tau_{L,0}$***

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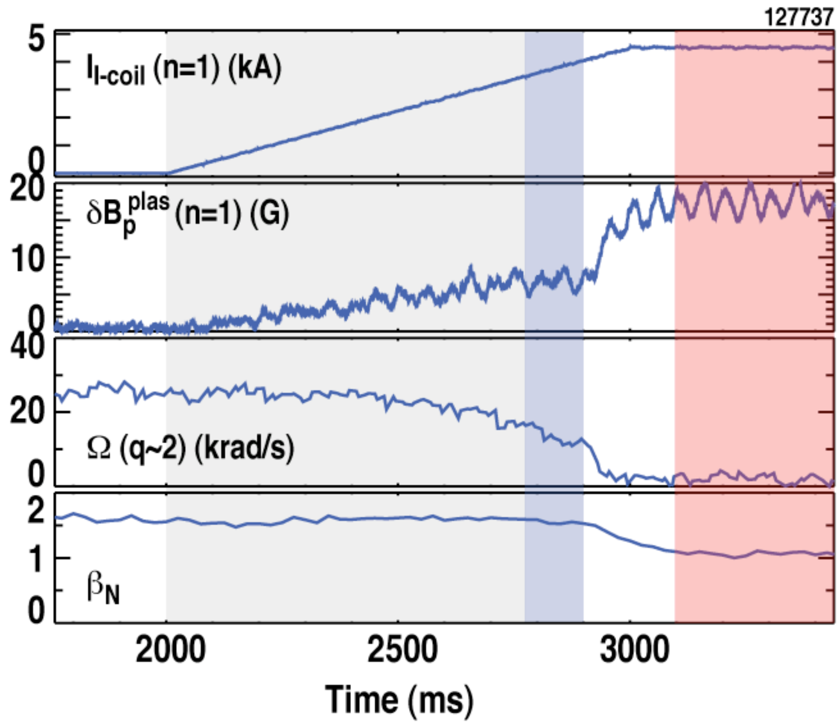
Summary/Main results

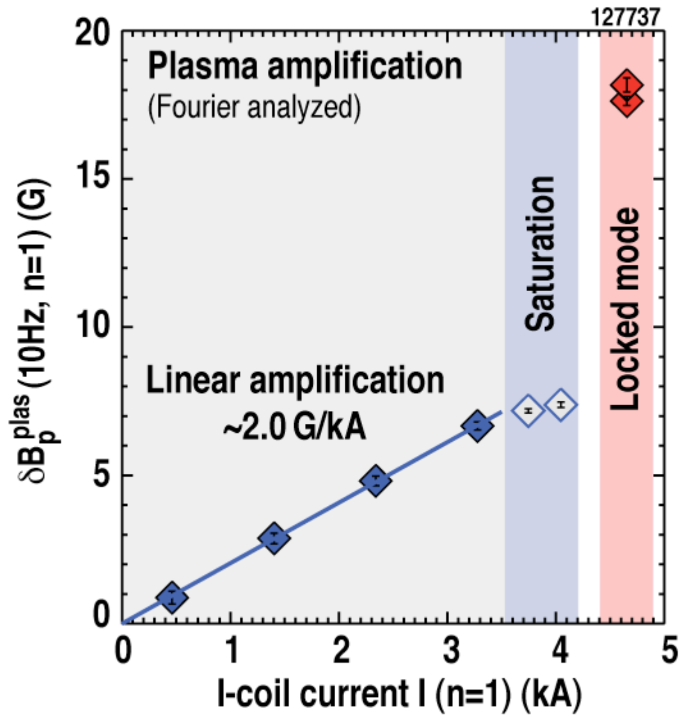
- **Resonant braking leading to a loss of torque balance determines the "ultimate" *n*=1 error field threshold in NBI heated H-modes**
- **Decrease of *n*=1 error field tolerance with increasing β is caused by an increasing plasma amplification**
- **Plasma is sensitive to kink-resonant rather than pitch-resonant external fields (more in M. Lanctot's talk on We)**
  - Fourier harmonics of the external field in straight field line coordinates are not an efficient description of the error field tolerance
  - For *n*=1 errors from the LFS, the plasma is most sensitive to the resulting *m*=2*nq* components
- **Plasma response leads to a resonant and non-resonant magnetic torque**
  - Non-resonant braking component reduces the benefit of additional torque input
  - Non-resonant braking component weakens the torque dependence consistent with experimental observations

Additional slides

Plasma response to an external *n*=1 field is linear as long as the plasma is rotating sufficiently fast

- <u>Linear amplification</u> at β_N=const. despite a rotation reduction by factor 2
- Plasma response saturates prior to the rotation collapse
- Amplification large once the rotation has collapsed, the error field penetrated and a locked mode formed (despite reduction in β_N)

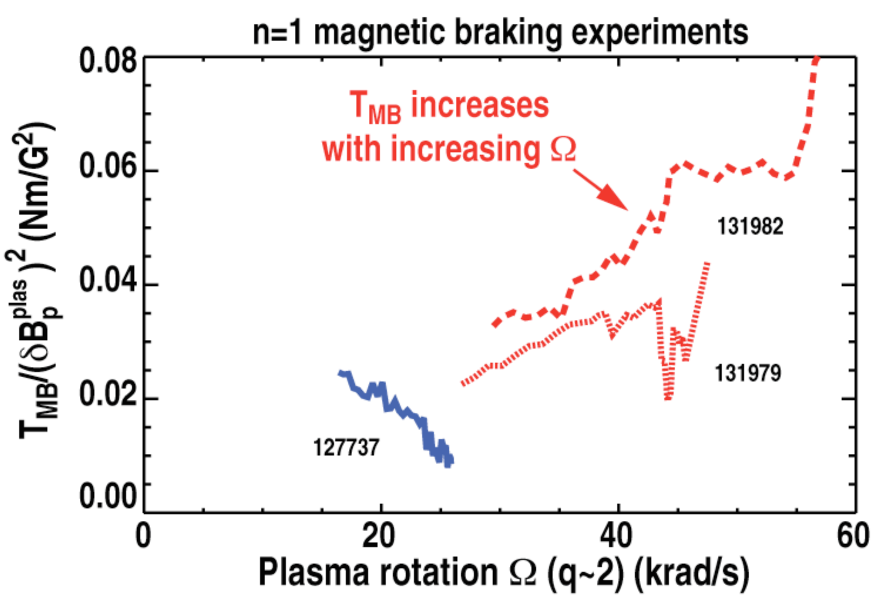


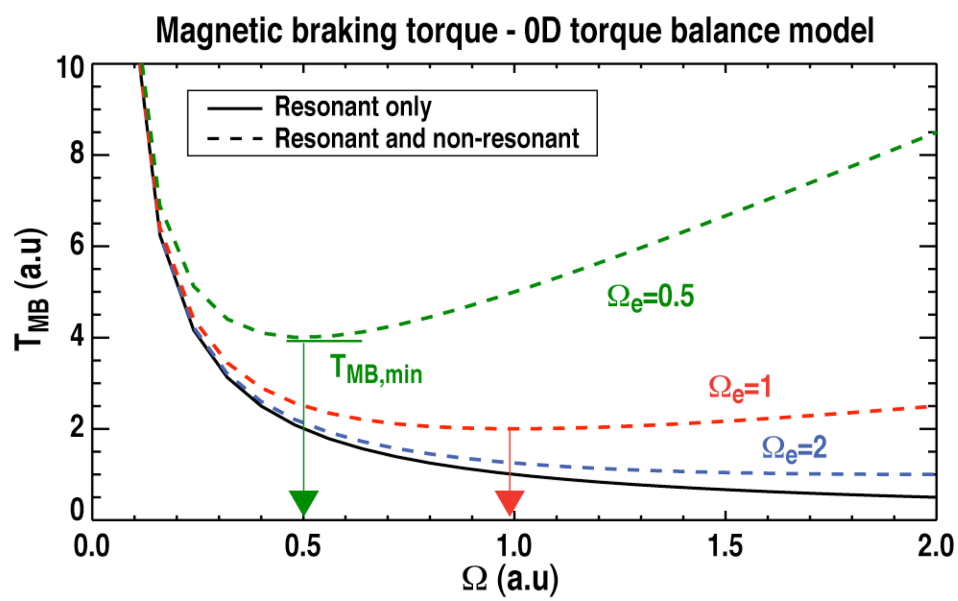


Effect of simultaneous resonant and non-resonant braking

- **Perturbed field has a resonant and a non-resonant component**

$$T_{MB} = K_R (f_R \delta B)^2 \Omega^{-1} + K_{NR} (f_{NR} \delta B)^2 \Omega$$





- T_{MB} **has a minimum for** $\Omega_e = \dfrac{f_R}{f_{NR}}\sqrt{\dfrac{K_R}{K_{NR}}}$ **, where resonant and non-resonant braking torques are equal**

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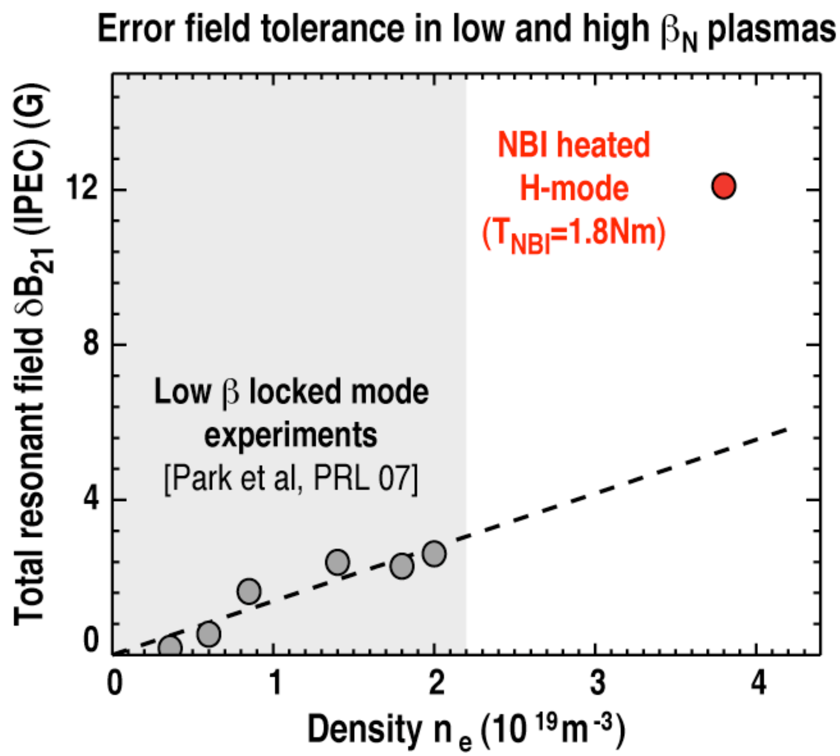
Plasma response and beneficial effect of NBI torque connect L- and H-mode error field threshold

- **Ideal MHD plasma response (IPEC) has been successfully used to restore density dependence of locked mode threshold in L-modes** [J.-K. Park, et al., *Phys. Rev. Lett.* **99** (2007) 195003]
  - Calculate total resonant field at q=2 surface δB_{21}, if shielding currents were absent



- **In H-mode, the error field threshold exceeds the extrapolation of L-mode experiments by a factor of 2**

➤ **~40% increase expected based on extrapolation of measured T_{NBI} dependence towards zero**