Advanced RFP control at **EXTRAP T2R** closed-loop system identification, experiment design & mode control

K.E.J. Olofsson, P.R. Brunsell, J.R. Drake and L. Frassinetti

Fusion Plasma Physics (Association EURATOM-VR)
School of Electrical Engineering (EES)
Royal Institute of Technology (KTH) Stockholm, Sweden

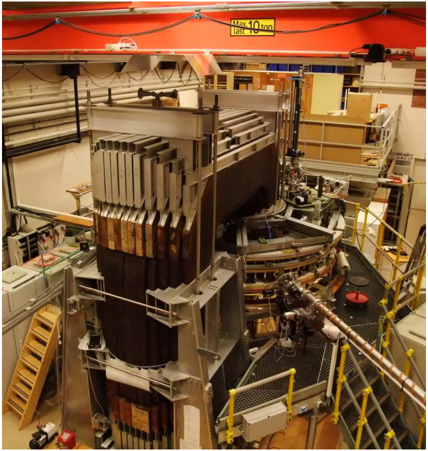
MHD Control Workshop
Princeton Plasma Physics Laboratory (PPPL) Nov. 9-11 2009

Overview

Overview

1. Overview & message
   - T2R recap
   - MHD control activities
   - Message of this talk
2. Applied MHD control topics
   - Closed-loop system identification
   - Applied experiment design
   - Controller retooling from CLID data
   - Reassessment of CLID data in MIMO sense
3. Closing words

Machine specs

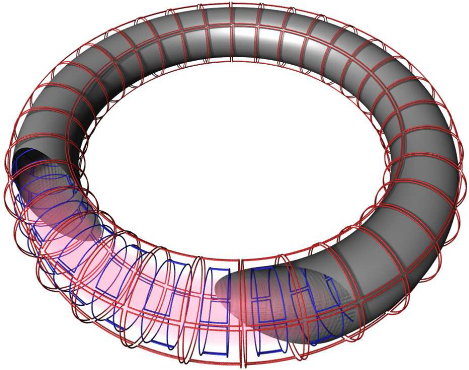


- major radius R=1.24 m
- plasma minor radius a=18.3 cm
- shell norm minor radius $r/a = 1.08$
- shell time constant τ_{shell}=6.3 ms
- plasma current I_p=80-160 kA
- electron temperature T_e=200-400 eV
- pulse length $\tau_{pulse} \leq 90$ ms

MHD unstable!

Without stabilization plasma 'terminates' after $\sim$ 10-15 ms.

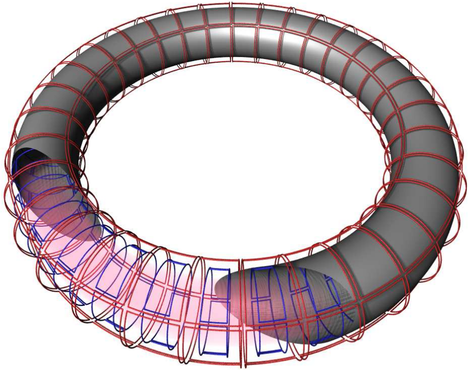
T2R saddle coil arrays; radial magnetic field



4 $\times$ 32 sensor coils
4 $\times$ 32 active coils

Available gear for stabilization of external MHD modes: actuator saddle coils able to produce a maximal radial field of $\sim$ 3% of poloidal field at wall.

T2R saddle coil arrays; radial magnetic field



4×32 sensor coils
4×32 active coils

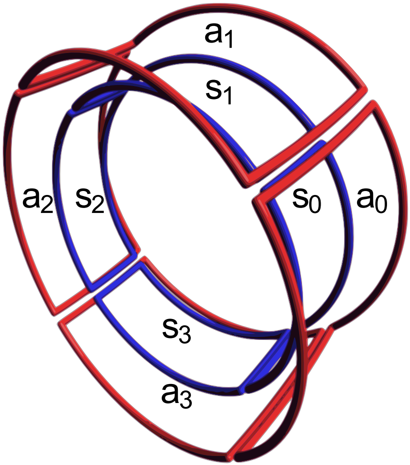
Available gear for stabilization of external MHD modes: actuator saddle coils able to produce a maximal radial field of $\sim 3\%$ of poloidal field at wall. (audio amplifiers!)

T2R actuator & sensor array

$$\begin{cases} \tilde{y}_i = \frac{1}{2}\left(s_{0,i} - s_{2,i}\right) \\ \tilde{y}_{i+32} = \frac{1}{2}\left(s_{1,i} - s_{3,i}\right) \end{cases}$$

$$\begin{cases} a_{0,i} = \frac{1}{2}\tilde{u}_i,\ a_{2,i} = -\frac{1}{2}\tilde{u}_i \\ a_{1,i} = \frac{1}{2}\tilde{u}_{i+32},\ a_{3,i} = -\frac{1}{2}\tilde{u}_{i+32} \end{cases} \quad (1)$$

for section $i = 1 \ldots 32$



A few T2R milestones

1991 Originally built and operated at General Atomic, San Diego (USA) with the name OHTE, the reconstruction of the device now name EXTRAP T2 started at the Alfvén Laboratory in a specially-build experiment hall.

1999 Rebuild started of the front-end of the device; vacuum vessel, copper shell, and toroidal field coil. The new features of the machine included a longer shell time constant, an all metal first wall (molybdenum limiters), reduction of field errors.

2003 The first comprehensive study of resistive wall modes (RWM) in an RFP. Results showed quantitative agreement between experimental growth rates and linear MHD theory predictions.

2005 First demonstration in an RFP experiment of RWM feedback stabilization of the full unstable spectrum of about 15 independent eigenmodes. The RWM suppression allowed a doubling of the plasma pulse length.

http://www.alfvenlab.kth.se/fusion

T2R control-oriented projects

Topics of this talk

system identification of external MHD modes by dithering the closed-loop stabilized plasma; single-input single-output (SISO) *and* multiple (MIMO)

experiment design for the above closed-loop identification

identification-based controller retuning ; one-pass retooling guided by experimental numbers

Central message

Central message pt. I

- Experimental “robustness” and reliability of RWM stabilization
- Successful & precise experimental sustainment of general nonaxisymmetric edge radial field perturbations (output-tracking): a useful MHD-research instrument
- Overlayed random “probing perturbations” (aka. *dithering*) in stabilized operation offer an efficient and practical recipe to statistical experimental linearization of external plasma response dynamics.
- Theories on how to generate/design this dithering are useful (aka. experiment design)

Central message

Central message pt. II

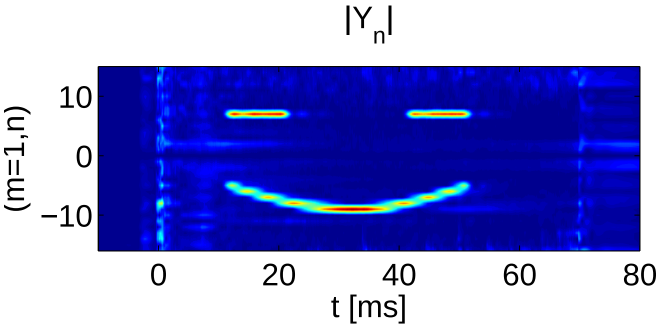
- Closed-loop identification data typically used for subsequent control-system refinement: example will be presented.
- Closed-loop identification data re-analysed in general MIMO-sense: SISO-version seems to hold well, with slight modification.
- Progress in the direction of process control, monitoring, long-pulses, etc.

A nonzero time-varying output-tracking experiment

First message

A nonzero time-varying output-tracking experiment

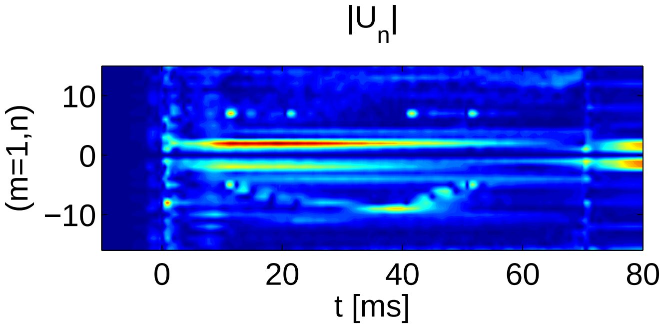
First message



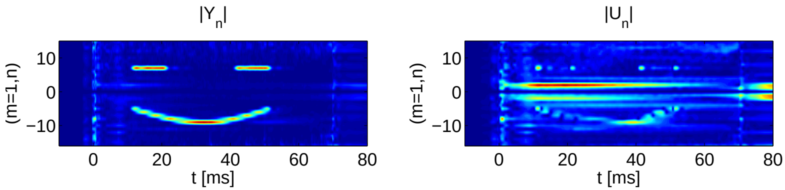
A playful demonstration of output-tracking (shot #21676)

A nonzero time-varying output-tracking experiment

Requires 'nonintuitive' actuator excitation!



A nonzero time-varying output-tracking experiment



RWM control in practice

First message

For EXTRAP T2R RFP

- RWMs routinely stabilized
- Programmable nonaxisymmetric radial field perturbations possible
- Good accuracy

RWM control in practice

First message

For EXTRAP T2R RFP

- RWMs routinely stabilized
- Programmable nonaxisymmetric radial field perturbations possible
- Good accuracy

The rest of this talk goes on from these observations

Overview

1. Overview & message
   - T2R recap
   - MHD control activities
   - Message of this talk
2. Applied MHD control topics
   - Closed-loop system identification
   - Applied experiment design
   - Controller retooling from CLID data
   - Reassessment of CLID data in MIMO sense
3. Closing words

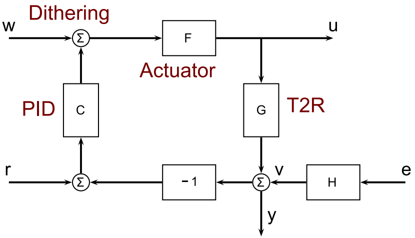
Closed-loop system identification

Motivation

Determining MHD mode response to externally applied magnetic perturbations is beneficially performed in the closed-loop:

1. longer batches of data
2. transients sorted out
3. plasma stabilized, maintained equilibrium
4. joint system: plasma + external feedback (identification for control)
5. process control, refinement, optimization

Basic T2R closed-loop stabilization with dithering



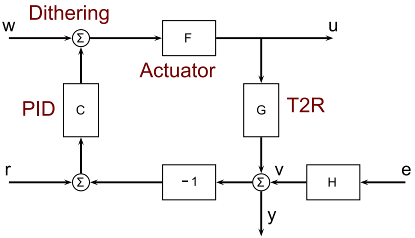
Basic direct CLID

Put $\tilde{\mathbf{r}} = \mathbf{0}$, randomly perturb vector $\tilde{\mathbf{w}}$ and forget (2) altogether, store $(\tilde{\mathbf{u}}, \tilde{\mathbf{y}})$.

Stabilizing feedback

$$\tilde{\mathbf{u}}_c = M_{eq}\left(C(s)(\tilde{\mathbf{r}} - W^{-1}D_{0/1}W\tilde{\mathbf{y}}) + \tilde{\mathbf{w}}\right) \tag{2}$$

Basic T2R closed-loop stabilization with dithering



Basic direct CLID

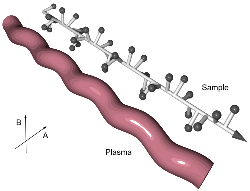
Put $\tilde{\mathbf{r}} = \mathbf{0}$, randomly perturb vector $\tilde{\mathbf{w}}$ and forget (2) altogether, store $(\tilde{\mathbf{u}}, \tilde{\mathbf{y}})$.

Stabilizing feedback

$$\tilde{\mathbf{u}}_c = M_{eq}\left(C(s)(\tilde{\mathbf{r}} - W^{-1}D_{0/1}W\tilde{\mathbf{y}}) + \tilde{\mathbf{w}}\right) \tag{2}$$

Q: RWM signal model?

Aliased subset linear RWM model



$$y_i = G_i(q, \theta_i) u_i$$

Discrete-time formulation

$$G_i(q, \theta_i) = \sum_{(m,n) \in \mathcal{K}_{n(i)}} G_{m,n}(q, \theta_i) \qquad (3)$$

where

$$G_{m,n}(q, \theta) = \frac{\alpha \hat{c}_{m,n} \hat{b}_{m,n} \frac{1}{\hat{\gamma}_{m,n}} \left(\hat{d}_{m,n} - 1 \right) q^{-1}}{1 - \hat{d}_{m,n} q^{-1}} \qquad (4)$$

using $\hat{d}_{m,n} = e^{\frac{\hat{\gamma}_{m,n} T_S}{\hat{\tau}_{m,n} \tau}}$, $\theta_i \equiv (\alpha_i \, \tau_i \, \hat{\gamma}_i)^T$, q^{-1} the one-sample delay operator, T_S sample interval.

The identification program

Prediction-error minimization (PEM) program

$$\bar{\theta}^\star = \arg\min_{\bar{\theta}} V(\bar{\theta}) \quad (5)$$

where $V(\bar{\theta}) \equiv \frac{1}{N}\sum_{k=1}^{N} e^2(k,\bar{\theta})$ and the *prediction errors* $e(k,\bar{\theta}) \equiv y(k) - \hat{y}(k,\bar{\theta})$ are produced by a *predictor* $\hat{y}(k,\bar{\theta}) = f\left(\bar{\theta}, \{y(l), u(l)\}_{l=0,\ldots,k-1}\right)$

A kalman predictor

$$\begin{cases} \hat{\mathbf{x}}^-(k) = A\hat{\mathbf{x}}^+(k-1) + Bu(k-1) \\ P^-(k) = AP^+(k-1)A^T + Q \end{cases}$$

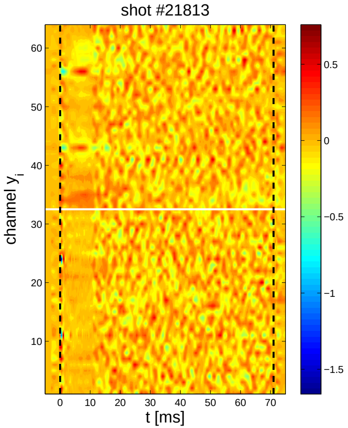
$$\begin{cases} e^-(k) = y(k) - C\hat{\mathbf{x}}^-(k) \\ K(k) = P^-(k)C^T\left(CP^-(k)C^T + R\right)^{-1} \\ \hat{\mathbf{x}}^+(k) = \hat{\mathbf{x}}^-(k) + K(k)e^-(k) \\ P^+(k) = (I - K(k)C)P^-(k) \end{cases} \quad (6)$$

takes input data $u(k)$, $y(k)$ and outputs $e^-(k) = e(k,\bar{\theta})$

The identification program, summarized.

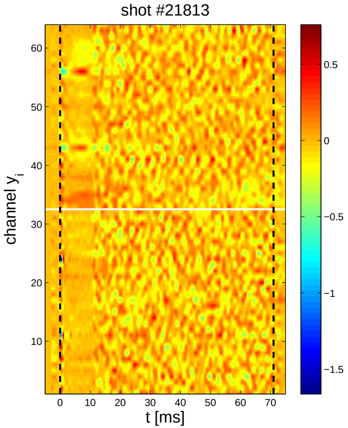
1. identify (PEM) vacuum parameters from dithered system without plasma.
2. identify (PEM) plasma parameter; $m = 1$ growth-rate from dithered plasma experiment (using fixed vacuum parameters from the previous step)

Example instance of dithering



MHD spectroscopy, “plasma massage”, probing, dithering, excitation

Example instance of dithering



MHD spectroscopy, “plasma massage”, probing, dithering, excitation

Nondithered feedback practically renders system dynamics unobservable.
Reintroduce observability, but keep control.
Textbook example of system identification.

Are there any preferred dither signals?

Convex programming FIR design

It turns out that the dithering spectrum is related to the variance of the system parameter estimates and that this variance (or trace of covariance matrix) can be minimized by formulating a convex program for finite-impulse-response (FIR) filter coefficients.

Input design example

$$\min_{Z,Q,\{r_j\}_{j=0}^{M-1}} \operatorname{tr} Z$$

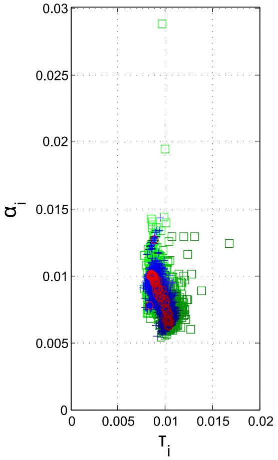
$$\text{s.t.} \begin{cases} \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_w(\omega) d\omega = r_0 \leq \beta_{i,OL} \\ \begin{pmatrix} Z & V \\ V^* & P^{-1} \end{pmatrix} \geq 0 \\ \begin{pmatrix} Q - A_w^T Q A_w & C_w^T - A_w^T Q B_w \\ C_w - B_w^T Q A_w & D_w + D_w^T - B_w^T Q B_w \end{pmatrix} \geq 0 \end{cases} \tag{7}$$

where

$$P^{-1}(\theta_i^{(0)}) = \frac{1}{2\pi\lambda_0}\int_{-\pi}^{\pi} \frac{\partial G}{\partial \theta}\frac{\partial G}{\partial \theta}^* \Big|_{\theta=\theta_i^{(0)}} |F(e^{\iota\omega})|^2 \Phi_w(\omega) d\omega \tag{8}$$

and $\theta_i^{(0)} = \theta_{i,dry}^{(0)}$

Input design result



green : assorted PRBS dithering

blue : optimized dithering

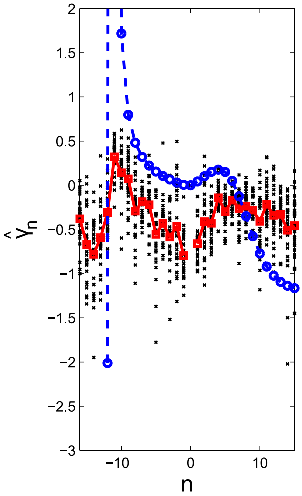
red : averages of the optimized estimates

Controller retooling

Fixed-order control design; one-pass

- Assume independent fourier modes.
- Use experimentally identified growth-rates and time-constants.
- Synthesize an FFT-decoupled fixed-order control system for this data.
- Deploy it.

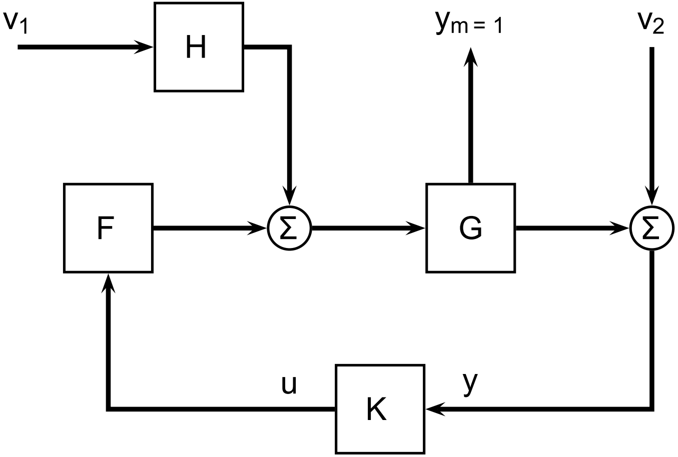
Fixed-order FFT-decoupled implementation



Rebranded FFT-controller solely tuned on identification dataset.

- implementability: fixed-order controller
- optimized: quadratic cost on model-states (or output)
- cost weight: reproduces either IS or CMC or some trade-off combination.

Main idea



Modeling closed-loop operation

Given a discrete-time system F, G, H

$$\begin{aligned} \mathbf{x}(k+1) &= A\mathbf{x}(k) + Bu(k) + Nv_1(k) \\ y(k) &= C\mathbf{x}(k) + v_2(k) \\ y_{m=1}(k) &= M\mathbf{x}(k) \end{aligned} \tag{9}$$

we wan't to find a "optimal" controller K

$$\begin{aligned} \mathbf{x}_K(k+1) &= A_K\mathbf{x}_K(k) + B_K y(k) \\ u(k) &= C_K\mathbf{x}_K(k) + D_K y(k) \end{aligned} \tag{10}$$

Modeling closed-loop operation; output variance

K should minimize the variance of y or $y_{m=1}$. How to achieve this?

Modeling closed-loop operation; output variance

K should minimize the variance of y or $y_{m=1}$. How to achieve this?

$$\min_K \operatorname{tr} P_z V_z \tag{11}$$

$$P_x = \mathcal{A} P_x \mathcal{A}^T + \mathcal{B} P_w \mathcal{B}^T, \; P_w = \mathrm{E}\mathbf{w}(k)\mathbf{w}(k)^T \tag{12}$$

$$P_z = \mathrm{E}\mathbf{z}(k)\mathbf{z}(k)^T = \mathcal{C} P_x \mathcal{C}^T + \mathcal{D} P_w \mathcal{D}^T \tag{13}$$

E is the expectation operator; $\mathbf{z} = \begin{pmatrix} u & y & y_{m=1} \end{pmatrix}^T$; $\mathbf{w} = \begin{pmatrix} v_1 & v_2 \end{pmatrix}^T$.

Modeling closed-loop operation; details

$$\mathcal{A} = \begin{pmatrix} A + BD_K C & BC_K \\ B_K C & A_K \end{pmatrix}, \ \mathcal{B} = \begin{pmatrix} N & BD_K \\ 0 & B_K \end{pmatrix} \tag{14}$$

$$\mathcal{C} = \begin{pmatrix} D_K C & C_K \\ C & 0 \\ M & 0 \end{pmatrix}, \ \mathcal{D} = \begin{pmatrix} 0 & D_K \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{15}$$

Modeling closed-loop operation; trace weights

Repeating (11):

$$\min_K \operatorname{tr} P_z V_z$$

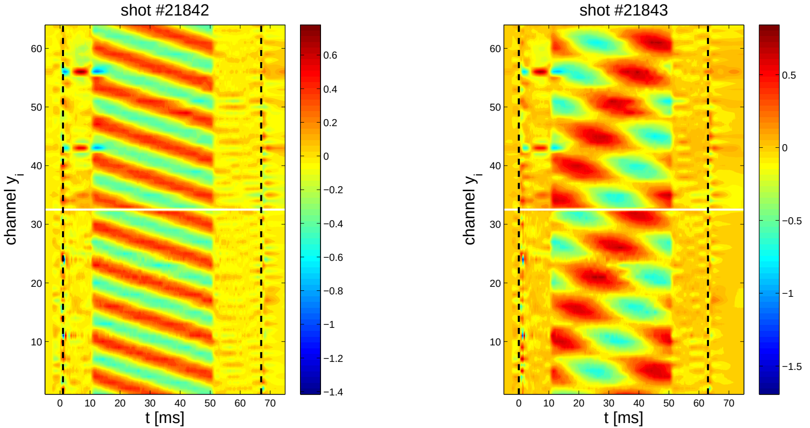
with weights: (i) $\Leftrightarrow$ **IS**, (ii) $\Leftrightarrow$ **CMC**

$$V_z^{(i)} = \begin{pmatrix} q & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{16}$$

$$V_z^{(ii)} = \begin{pmatrix} q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{17}$$

and $q > 0$.

Example shots (output-tracking) FFT-decoupled control

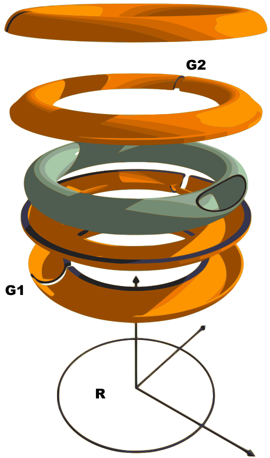


Supposedly procrustean property of FFT

A clean fourier mode on the actuator array possibly does not look very fourier on the other side of the shell.

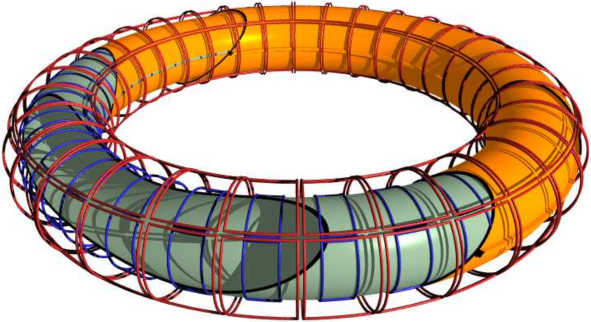
Basic problem of real-world engineering

Anatomy of external conducting structures.



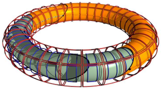
Assembled T2R plausibly distorts actuator input

Incentivizes **full MIMO** identification. Try to



reanalyze data from dithered experiments already made!

MIMO plant from experimental data



For simplicity: assume a state-space of dimension equal to the sensor array. Then

$$\mathbf{y}(k+1) = A\mathbf{y}(k) + B\mathbf{u}(k) + \mathbf{e}(k) \tag{18}$$

with $\mathbf{y}, \mathbf{u}, \mathbf{e} \in \mathbb{R}^{64\times 1}$, $A, B \in \mathbb{R}^{64\times 64}$, and A, B *fully dense and completely free*

Matrices (8192 scalars) to be determined solely from experimental data. *No imposed structure*.

MIMO plant from experimental data

Bundle unknown numbers in the predictor $G \in \mathbb{R}^{64\times 128}$

$$\begin{aligned}\hat{\mathbf{y}}(k+1) &= A\mathbf{y}(k) + B\mathbf{u}(k) = \begin{pmatrix} A & B \end{pmatrix}\begin{pmatrix} \mathbf{y}(k) \\ \mathbf{u}(k) \end{pmatrix} \\ &= G\mathbf{z}(k) \end{aligned} \tag{19}$$

and minimize a norm of the error **e**:

$$\hat{\mathbf{e}}(k+1) = \mathbf{y}(k+1) - \hat{\mathbf{y}}(k+1) = \mathbf{y}(k+1) - G\mathbf{z}(k) \tag{20}$$

Note that (20) linearly maps measured data to measured data (at consecutive time k).

MIMO plant from experimental data; cont'd

Introduce covariance matrices:

$$R^{(\mathbf{ee})} = R^{(\mathbf{y}_1\mathbf{y}_1)} - R^{(\mathbf{y}_1\mathbf{z}_0)}G^T - G{R^{(\mathbf{y}_1\mathbf{z}_0)}}^T + GR^{(\mathbf{z}_0\mathbf{z}_0)}G^T \quad (21)$$

$$R^{(\mathbf{y}_1\mathbf{y}_1)} = \mathrm{E}\mathbf{y}(k)\mathbf{y}^T(k) \in \mathbb{R}^{n\times n} \quad (22a)$$

$$R^{(\mathbf{y}_1\mathbf{z}_0)} = \mathrm{E}\mathbf{y}(k+1)\mathbf{z}^T(k) \in \mathbb{R}^{n\times 2n} \quad (22b)$$

$$R^{(\mathbf{z}_0\mathbf{z}_0)} = \mathrm{E}\mathbf{z}(k)\mathbf{z}^T(k) \in \mathbb{R}^{2n\times 2n} \quad (22c)$$

$n = 64$.

MIMO plant from experimental data; cont'd

Form objective

$$\hat{G} = \arg\min_{G} V(G) \quad (23)$$

where

$$V(G) = \mathrm{tr}\left(R^{(\mathbf{ee})}\right) = \sum_{i=1}^{n} r_{i,i}^{(\mathbf{ee})} \quad (24)$$

MIMO plant from experimental data; cont'd

The solution of (23) must satisfy the *normal* equations

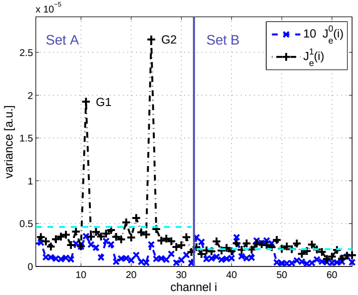
$$\forall p,q:\ 0 = \frac{\partial}{\partial g_{p,q}} V(G) = \ldots$$
$$= -2r_{p,q}^{(\mathbf{y}_1\mathbf{z}_0)} + 2\sum_{j=1}^{2n} g_{p,j} r_{j,q}^{(\mathbf{z}_0\mathbf{z}_0)} \qquad (25)$$

which, written out in full glory, are $2n^2 = 8192$ linear scalar equations in the same number of unknowns. The system is sparse.

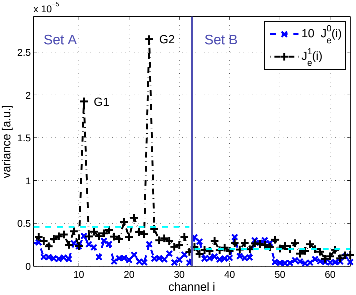
MIMO plant from experimental data; results

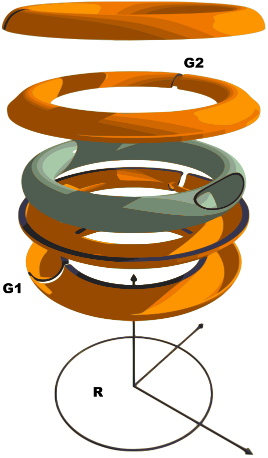
Dithered data appears enough “informative” to actually pose non-ill-conditioned linear equations.

Minimum-trace $R^{(ee)}$-diagonal



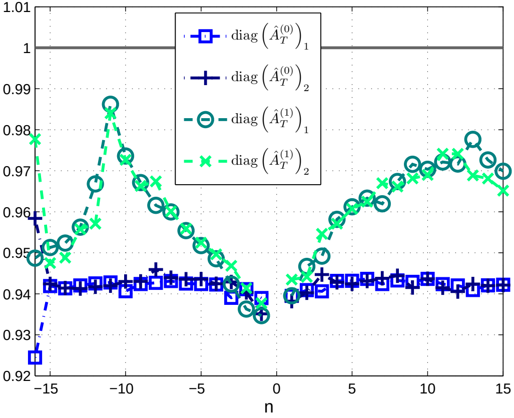
Minimum-trace $R^{(ee)}$-diagonal



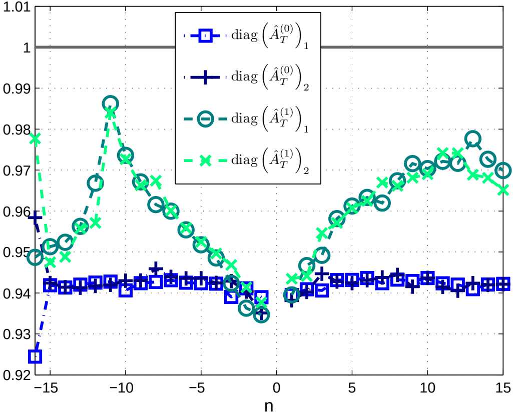


Serious leakage of noise at gap positions $G_{1,2}$.

$\hat{A}$-diagonal in DFT-space

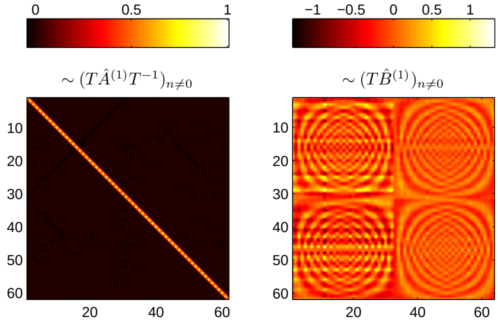


$\hat{A}$-diagonal in DFT-space



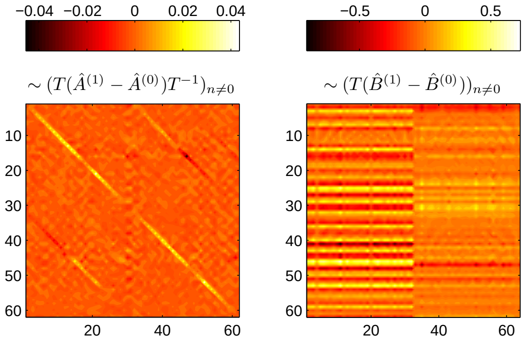
RWM spectrum resemblance. Actually it is almost fully compatible with previous SISO studies.

$\hat{A}$ and $\hat{B}$ in technicolour



Extremely useful information in $\hat{B}$-matrix! In principle it captures the external structures' field distortion.

$\Delta\hat{A}$ and $\Delta\hat{B}$ in technicolour



Weak structural difference in $\hat{A}$ with respect to vacuum. Strong matrix element difference for $\hat{B}$.

Overview

1. Overview & message
   - T2R recap
   - MHD control activities
   - Message of this talk
2. Applied MHD control topics
   - Closed-loop system identification
   - Applied experiment design
   - Controller retooling from CLID data
   - Reassessment of CLID data in MIMO sense
3. Closing words

Message repeat

To remember

- Experimental “robustness” and reliability of RWM stabilization
- Successful & precise experimental sustainment of general nonaxisymmetric edge radial field perturbations (output-tracking): a useful MHD-research instrument
- Overlayed random “probing perturbations” (aka. *dithering*) in stabilized operation offer an efficient and practical recipe to statistical experimental linearization of external plasma response dynamics.
- Theories on how to generate/design this dithering are useful (aka. experiment design)

Recap

More to remember

1. Appealing method for assessment of external-modes MHD growth-rates
2. Applied **experimentally** with encouraging results
3. Spectra resemble RFP-theory qualitatively
4. Modal FFT-decomposition most probably **not** optimal

todo-listing

MIMO work further on methods for automated identification and bespoke control for MCF plants

tokamak the above topic supposedly general $\Rightarrow$ applicability to tokamaks, process control and monitoring of reactor operation

MHD proceed & develop MHD research using the control system as an instrument

Suggested reading

- C. Bishop, "An intelligent shell for the toroidal pinch" *Plasma Physics and Controlled Fusion*, vol. 31, no. 7, pp. 1179–1189, 1989. `http://stacks.iop.org/0741-3335/31/1179`
- P. Brunsell et al., "Initial results from the rebuilt EXTRAP T2R RFP device", *Plasma Physics and Controlled Fusion*, vol. 43, no. 11, p. 1457-1470, 2001
- P. Brunsell et al., "Feedback stabilization of multiple resistive wall modes", *Physical Review Letters*, vol. 93, no. 22, p. 225001, 2004. `http://link.aps.org/abstract/PRL/v93/e225001`
- P. Zanca, L. Marrelli, G. Manduchi, and G. Marchiori, "Beyond the intelligent shell concept: the clean-mode-control" *Nuclear Fusion*, vol. 47, no. 11, pp. 1425–1436, 2007. `http://stacks.iop.org/0029-5515/47/1425`
- E. Olofsson and P. Brunsell, "Controlled magnetohydrodynamic mode sustainment in the reversed-field pinch: theory, design and experiments", *Fusion Engineering and design*, Volume 84, Issues 7-11, June 2009, Pages 1455-1459. `http://dx.doi.org/10.1016/j.fusengdes.2008.11.052`
- E. Olofsson, P. Brunsell and J. Drake, "Closed-loop direct parametric identification of magnetohydrodynamic normal modes spectra in EXTRAP-T2R reversed-field pinch", *Proceedings of the 3rd IEEE Multi-conference on Systems and Control*, July 2009
- E. Olofsson, H. Hjalmarsson, C. Rojas, P. Brunsell, and J. Drake, "Vector dither experiment design and direct parametric identification of reversed-field pinch normal modes", *Proceedings of the 48th IEEE Conference on Decision and Control*, to appear, December 2009
- E. Olofsson and P. Brunsell, "Direct linear multiple RWM system identification for toroidal plasmas", *submitted*, October 2009