Two-Dimensional Magnetohydrodynamic Simulations of Time-Dependent Poloidal Flow.

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Motivation

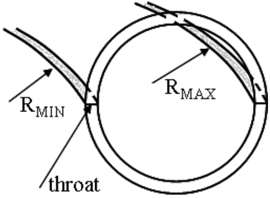
- Poloidal flows in tokamaks are receiving an increasing attention, as newer and better flow measurements keep increasing the amount of available experimental information.
- MHD theory predicts that when the poloidal velocity is transonic with respect to the poloidal sound speed ($C_{sp} \equiv C_s B_p / B$), a discontinuity/pedestal will form at the transonic surface[1]. This discontinuity is NOT a shock.
- We want to verify the theoretical predictions with time-dependent simulations, and to determine how much flow is required to form such a pedestal at the plasma edge.

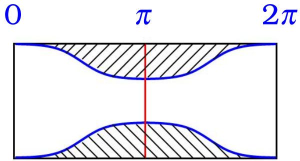
[1] R. Betti and J. P. Freidberg, Phys. Plasmas **7**, 2439 (2000)

Outline

The Magnetic Field creates a de Laval Nozzle for the Poloidal Flow

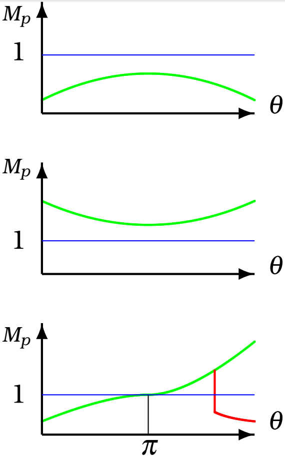
- In ideal MHD, the *frozen in* condition holds: plasma cannot flow across magnetic surfaces.
- Due to toroidal geometry, in a tokamak the cross section between any two nested magnetic surfaces varies with the poloidal angle.
- For the poloidal flow, nested magnetic surfaces act as a de Laval nozzle.





Due to the Geometry, Shocks (time-dependent) or Discontinuities/Pedestals (steady-state) form in the plasma

- The relevant velocity is the poloidal sound speed $C_{sp} = C_s B_p / B$.
- Due to periodicity:
  - If the flow becomes supersonic at the nozzle throat, a shock will form.
  - At steady state shocks are not allowed, and the flow can be sonic only at the nozzle throat.
  - At steady state, contact discontinuities will remain between the subsonic and supersonic region.



The standard MHD model is used...

We solve the standard ideal-MHD model time-dependent equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{V}) = 0, \qquad \text{(Continuity)}$$

$$\frac{\partial \rho \underline{V}}{\partial t} + \nabla \cdot \left(\rho \underline{V}\,\underline{V} - \underline{B}\,\underline{B} + P\underline{\underline{I}} + \underline{\underline{\Pi}}\right) = 0, \qquad \text{(Momentum)}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{V} \times \underline{B}), \qquad \text{(Faraday's Law)}$$

$$\frac{\partial \mathscr{E}}{\partial t} + \nabla \cdot [(\mathscr{E} + P)\,\underline{V} - \underline{B}(\underline{V} \cdot \underline{B})] = 0. \qquad \text{(Energy)}$$

The equations are written in conservative form to ensure conservation of physical quantities.

...with the usual definitions

The standard definitions are used:

$$P \equiv p + \frac{B^2}{2} \qquad \text{(Total Pressure)}$$

$$\mathscr{E} = \frac{p}{\gamma - 1} + \rho\frac{V^2}{2} + \frac{B^2}{2}, \qquad \text{(Total Energy)}$$

where γ is the adiabatic index, p the pressure, ρ the density, $\underline{V}$ the total plasma velocity, $\underline{B}$ the magnetic field. In this presentation, $\mu_0 = 1$ is used for convenience. The pressure tensor $\underline{\underline{\Pi}}$ represents numerical viscosity.
Axisymmetry is assumed.

Simulations use a predictor-corrector scheme in Cartesian coordinates

- A standard predictor-corrector method in Cartesian coordinates gives the best performance.
- Artificial numerical dissipation is needed to prevent spurious overshoots.
  - In particular, numerical diffusion is needed at (rather, behind) the shock front.
- A projection scheme[2] is used to enforce the $\nabla \cdot \underline{B} = 0$ constraint.

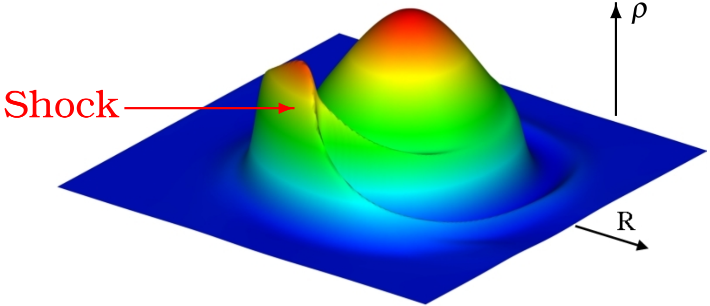
[2] J. U. Brackbill and D. C. Barnes, J. Comput. Phys., **35**, 426 (1980)

Initial and Boundary Conditions

- The initial conditions are assigned using a *static* equilibrium; equilibrium calculations are performed with FLOW[3].
- Velocity is added with an “arbitrary” initial profile, or a momentum source is turned on at $t = 0$.
- Reflective boundary conditions are used at the plasma boundary (superconductive wall).
- A gridless method was implemented for the corner points.

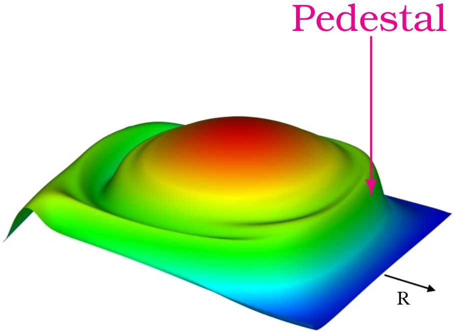
[3] L. Guazzotto, R. Betti, J. Manickam and S. Kaye, Phys. of Plasmas, **11**, 604 (2004)

In ideal MHD, Shocks form due to Poloidal Flow



- A shock is observed at the transonic surface.
- The shock travels in the poloidal direction from the outboard to the inboard part of the plasma.

Simulations show that Density Pedestals form due to Poloidal Flow

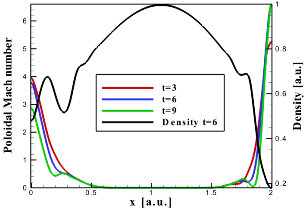


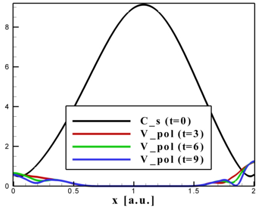
Density Pedestal (flat ρ input)

Density Pedestal

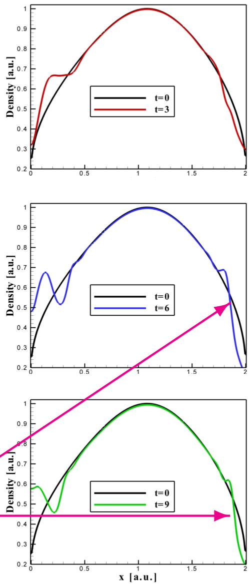
Transonic Discontinuities develop even with small velocities.

- A system with initial fairly flat density profile is evolved.
- Momentum is inserted with a source $\sim r$ for $r/a > 0.5$.
- Time is in arbitrary units; at the transonic surface $\tau = 2\pi r_{trans}/C_{sp} \simeq 6.7$ (time for a sound wave poloidal revolution).
- Discontinuities form with $V_\theta \ll C_s|_{r=0}$.





- The initial smooth density profile forms a discontinuity that lasts for many revolutions.
- Similarly, velocity, Mach number and pressure are discontinuous at the transonic surface.
- The source is turned off at $t = 1.5$.



Pedestal

Discussion [1]

- Time-dependent simulations show the formation of shocks when the poloidal flow become supersonic ($V_\theta > C_{sp}$).
- Radial contact discontinuities (pedestals) are present at steady-state.
- Results are in agreement with the predictions of theory.

Discussion [2]

- This behavior was verified with simulations in several different conditions (only a few selected cases are presented).
- In particular, discontinuities form when the flow becomes supersonic, either due to a time-dependent source, or a transonic initial condition.
- Interestingly, if the flow is driven by a source, a case-dependent threshold in the source power is observed: If the source is weaker than the threshold, no discontinuities form, even if the source is left on for many poloidal revolutions.

Conclusions

- Transonic poloidal flow is studied in axisymmetric configurations.
- Transonic flow creates discontinuities in plasma pressure and density, separating a subsonic (core) and a supersonic (edge) region.
- Discontinuities form even for poloidal velocities much smaller than the sound speed in the plasma core ($\sim$ 10s km/s).