

Interaction between resonant magnetic perturbations and plasma

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1. RMP (resonant magnetic perturbation) penetration
2. Effect of RMP on plasma rotation
3. Effect of RMP on particle confinement

1. Introduction

* The resonant magnetic perturbations (RMP) exist in tokamaks due to machine error field, actively applied RMP, and plasma instabilities of the same or different helicities.

* Major physics issues of concern:

(1) ELM mitigation

What is the mechanism of ELM mitigation?

Stochastic magnetic field (obtained under vacuum assumption).

(2) Field penetration (mode penetration)

(3) Effect of RMP on plasma rotation and particle confinement.

2. Mode penetration

Experimental results

* La Haye et al, Phys. Fluids B, 1992

DIII-D Br/Bt=1.7×10⁻⁴

ITER Br/Bt=2×10⁻⁵

* R. J. Buttery et al, Nucl. Fusion, 1999, 2000

JET, Compass-D, DIII-D $Br / B_t \sim n_e B_t^\alpha q^\beta$ $\alpha = -1.2 - 2.9$, $\beta = 0.05 - 1.6$

ITER Br/Bt~10⁻⁴ (ohmic phase)

* H. R. Koslowski et al., Nucl. Fusion, 2006

TEXTOR Lowest threshold when $\omega_f = \omega_E + \omega^*_E$

The relative frequency between the field and the mode is important.

Theoretical works

* Most works are based on reduced MHD equation.

R. Fitzpatrick, et al., Nucl. Fusion 1993, PoP 1991, 1998.

T.C. Hender et al., Nucl. Fusion 1992.

* Recent works are extended to two fluids equations and the four field model.

A. Cole and R. Fitzpatrick, PoP 2006,

Y. Kikuchi, PRL 2006, PPCF 2006

* Our works: nonlinear two fluids equations.

Instability + 2D transport (Yu, et al., NF 2008)

Significant progress in numerical methods for high $\chi_{\parallel}/\chi_{\perp}$ and S.

S. Günter et al., J. Comp. Phys, 2005, Yu et al., PoP 2004

Equations

Two fluids equation in periodic cylinder geometry, cold ion assumption

$$\frac{d\Psi}{dt} = E - \eta(j - j_b) + \Omega \nabla_{||} n + \Omega \nabla_{||} T_e$$

$$\rho \left[\frac{d}{dt} + \mathbf{v} \cdot \nabla \right] \nabla^2 \phi = \mathbf{e}_t \cdot (\nabla \Psi \times \nabla j) + \rho \mu \nabla^4 \phi + S_M$$

$$\frac{3}{2} n \frac{dT_e}{dt} = 0.78 d_1 \nabla_{||} j + n \nabla \cdot (\chi_{||e} \nabla_{||} T_e) + n \nabla \cdot (\chi_{\perp e} \nabla_{\perp} T_e) + S_e$$

$$\frac{dn}{dt} = d_1 \nabla_{||} j - \nabla_{||} (n u) + \nabla \cdot (D_{\perp} \nabla_{\perp} n) + S_n$$

$$\frac{du}{dt} = -c_s^2 \frac{\nabla_{||} p}{n} + \mu_{\perp} \nabla_{\perp}^2 u$$

where $\mathbf{B} = B_t \mathbf{e}_t - (kr/m) B_t \mathbf{e}_{\theta} + \nabla \Psi \times \mathbf{e}_t$, $\mathbf{v} = u \mathbf{e}_{||} + \mathbf{v}_{\perp}$, $\mathbf{v}_{\perp} = -\nabla \phi \times \mathbf{e}_t$, $d/dt = \partial/\partial t + \mathbf{v}_{\perp} \cdot \nabla$

$$d_1 = \omega_{ce}/v, \quad \Omega = \beta d_1, \quad c_s = (T/m_i)^{1/2}, \quad \beta = 4\pi n T / B^2, \quad T = T_e + T_i, \quad p = n T$$

The momentum source S_M leading to a poloidal plasma rotation.

Parameters for comparison with TEXTOR results

$B_t=2.25\text{T}$, $a=0.47\text{m}$, $R=1.75\text{m}$,

$T_e= 1800[1-(r/a)^2]+ 300$ (eV)

$n_e= 3.2\times 10^{19}[1-(r/a)^2]+ 3.0\times 10^{18}$ (m^{-3})

These lead to the normalized parameters

$S=1.97\times 10^8$, $\Omega=6.3\times 10^4$, $C_s=1.2\times 10^7$, $d_l=2.5\times 10^7$, and $\chi_{\parallel}=1.1\times 10^{19}$, $\chi_{\perp}=\mu=21$ (a^2/τ_R) (0.5m/s), and $D_{\perp}=\chi_{\perp}/5$.

A monotonic safety q profile is used, and the $q=2$ rational surface locates at $r=0.628a$.

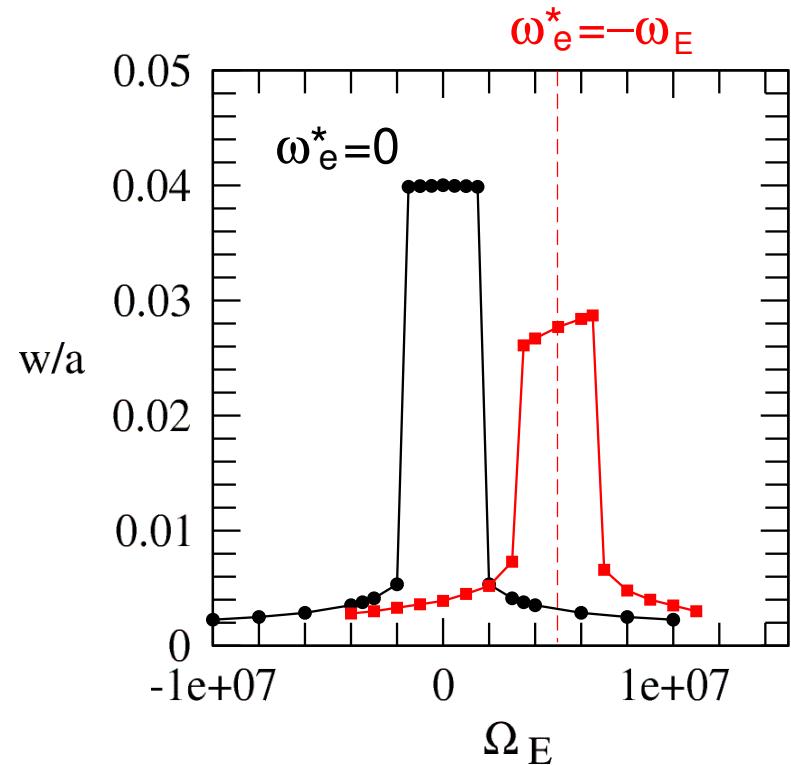
The $m/n=2/1$ is stable without an externally applied helical field.

The effect of the error field is taken into account by the boundary condition

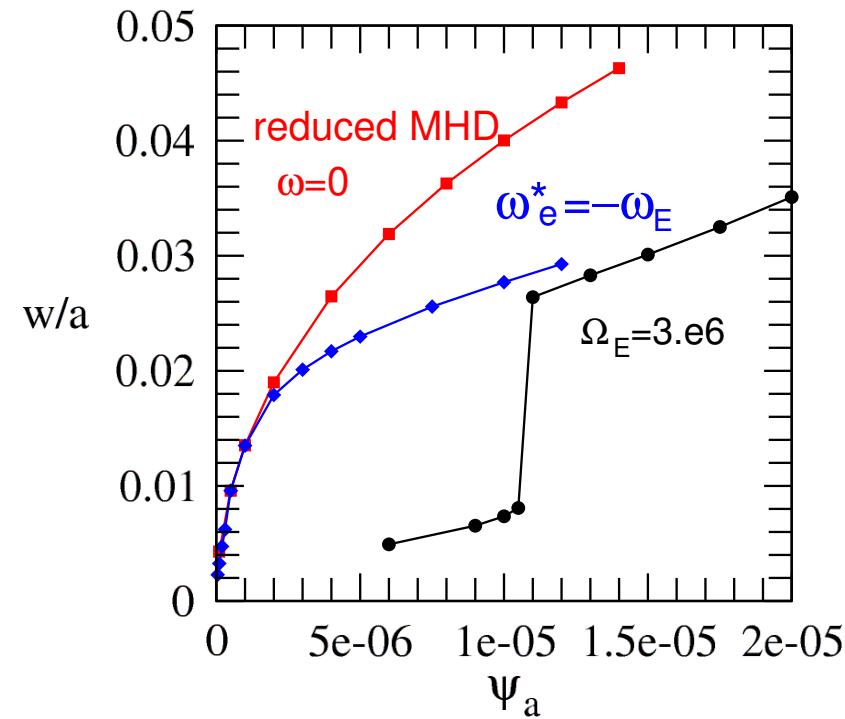
$$\psi_{m,n}(r=a)=\psi_a \cos(\omega ft + m\theta + n\phi)$$

Island width versus plasma rotation frequency and RMP amplitude

static helical field $B_{ra}/B_T = 2.e-5$
 $\Omega = 6.31e4$



When $\omega = \omega_F$, there is no penetration threshold.

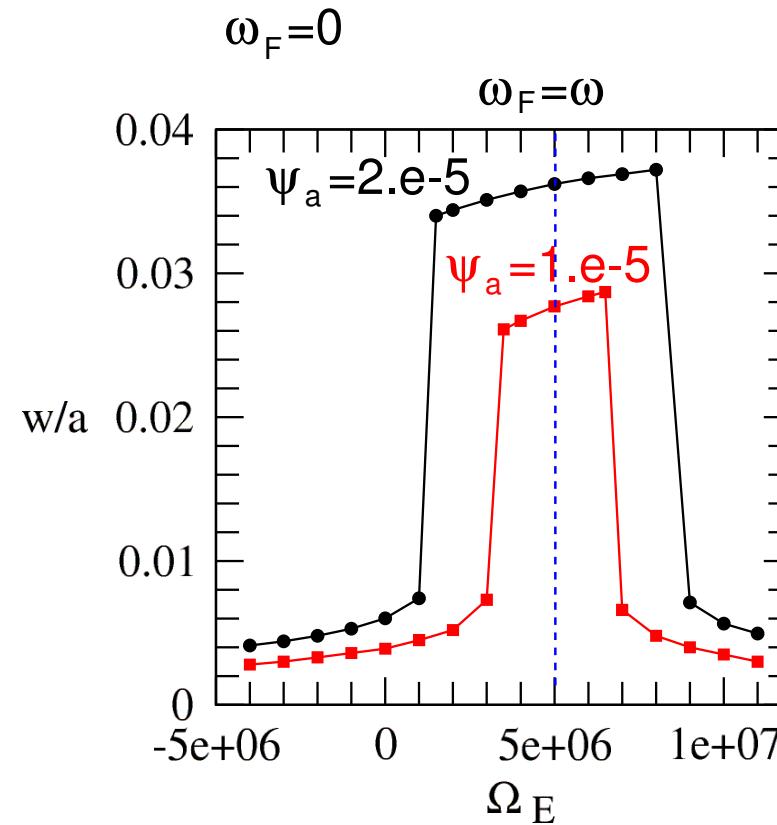
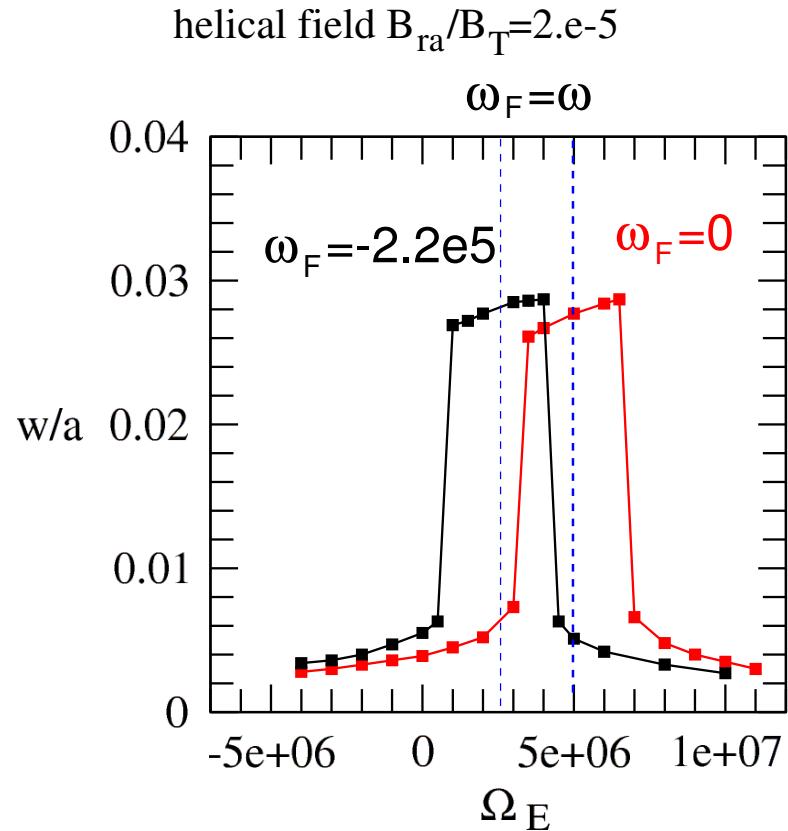


The resonance occurs around $\omega_e^* = -\omega_E$ so that the mode frequency is zero.

There is a finite frequency band width for resonance.

The ion polarization current is stabilizing.

Island width versus helical field rotation frequency

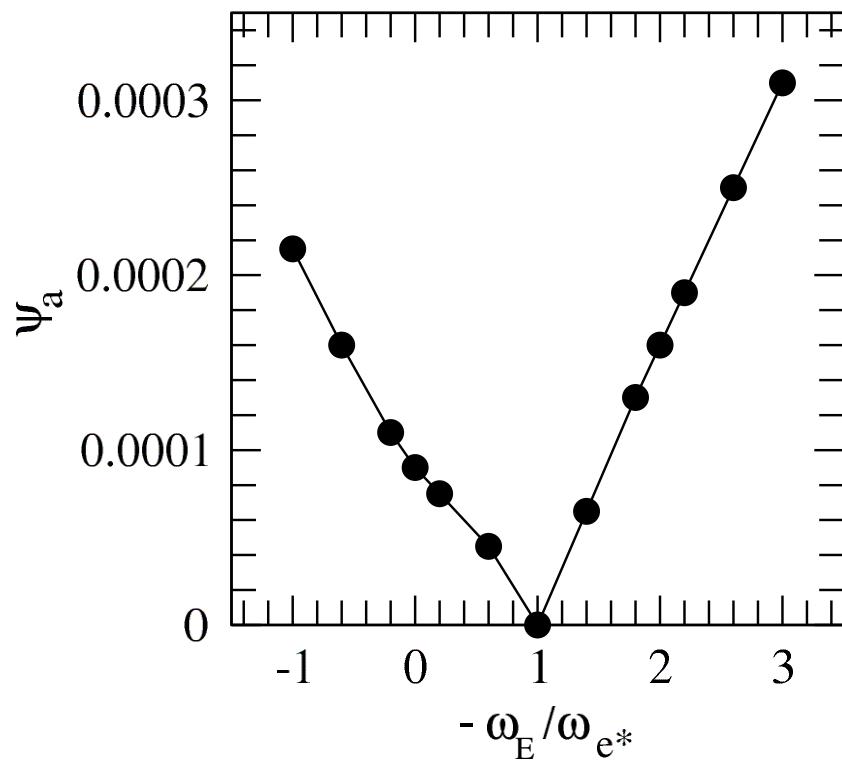


The resonance occurs around $\omega_F = \omega$, the field frequency equals the mode frequency.

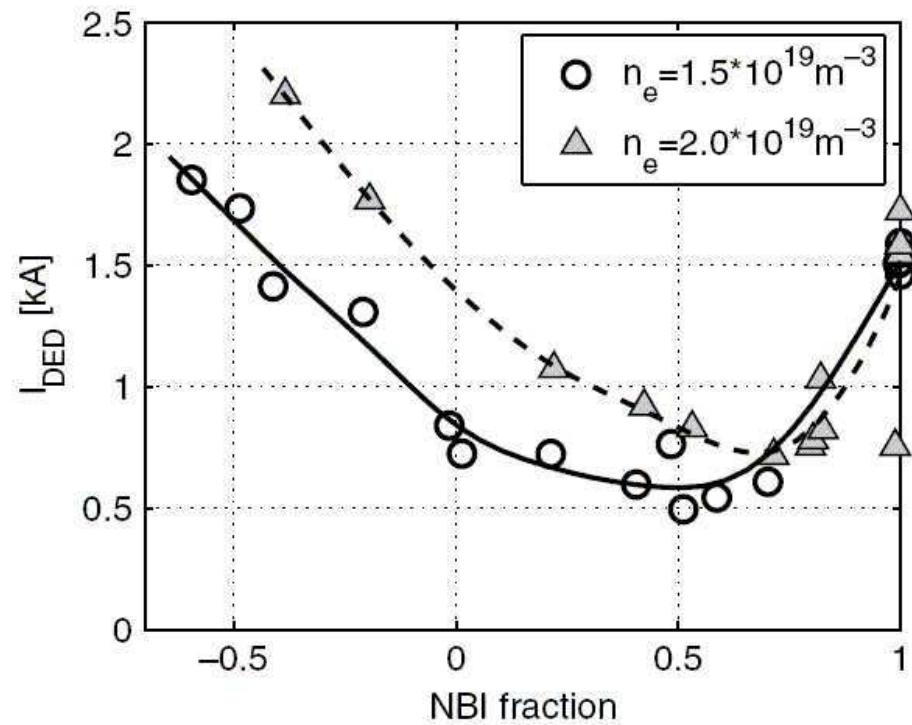
The resonance frequency band width increases with helical field amplitude.

Required field amplitude for mode penetration

numerical results



H.R. Koslowski, et al., Nucl. Fusion 2006



When $\omega \neq \omega_F$, the parallel heat transport is important for mode penetration.

Discussion on mode penetration

- * The penetration threshold has a minimal value when the RMP frequency equals the mode frequency determined by the plasma rotation and the diamagnetic drift, in agreement with TEXTOR experimental results.
- * There is a finite band width in the plasma rotation frequency for the penetration with a given RMP amplitude.
- * The penetrated island width is smaller obtained from two fluids equations than that from single fluid. The ion polarization current (electron diamagnetic drift) is stabilizing.

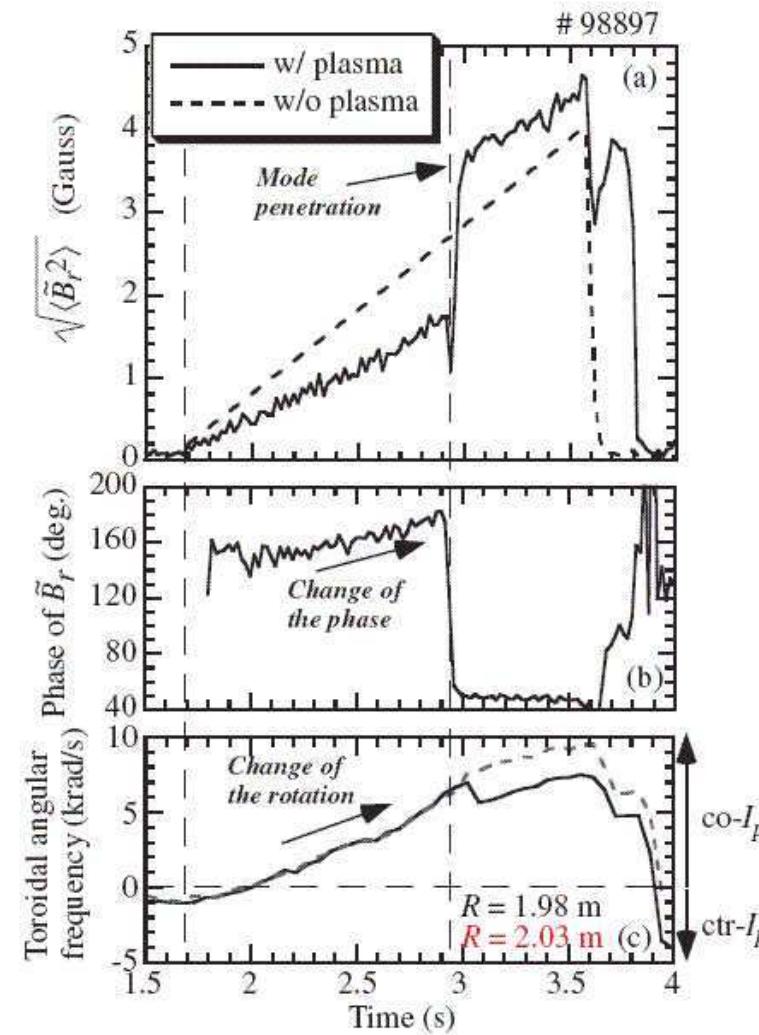
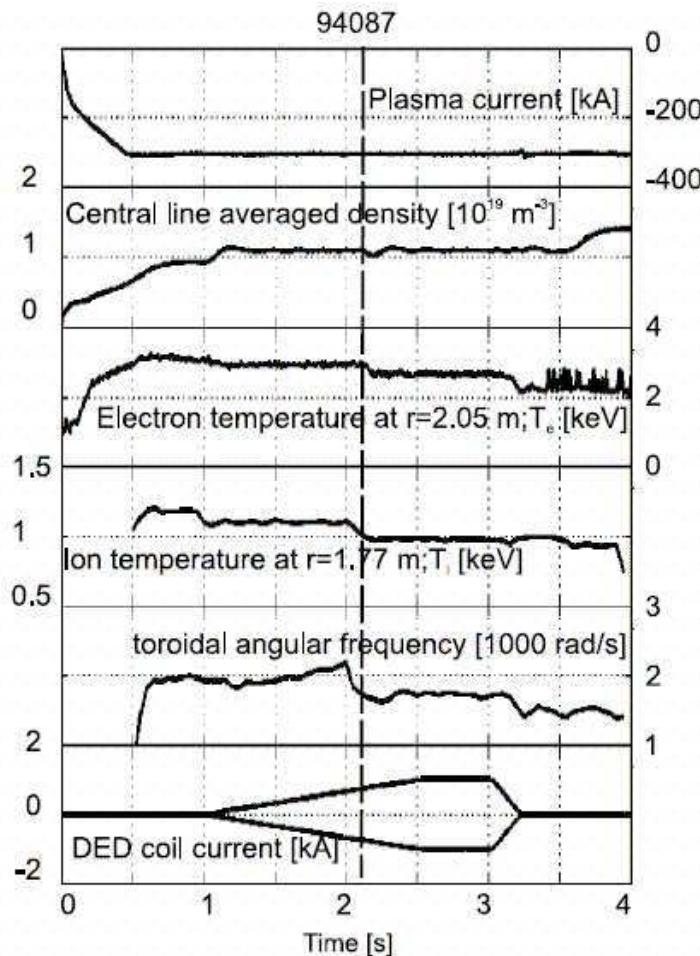
3. Effect of RMP on plasma rotation

- * A sufficiently large RMP leads to mode locking.
- * While a small RMP changes the plasma rotation speed in a more complicated way.

TEXTOR results

Kikuchi et al. PRL 2005

Finken et al. PRL 2005

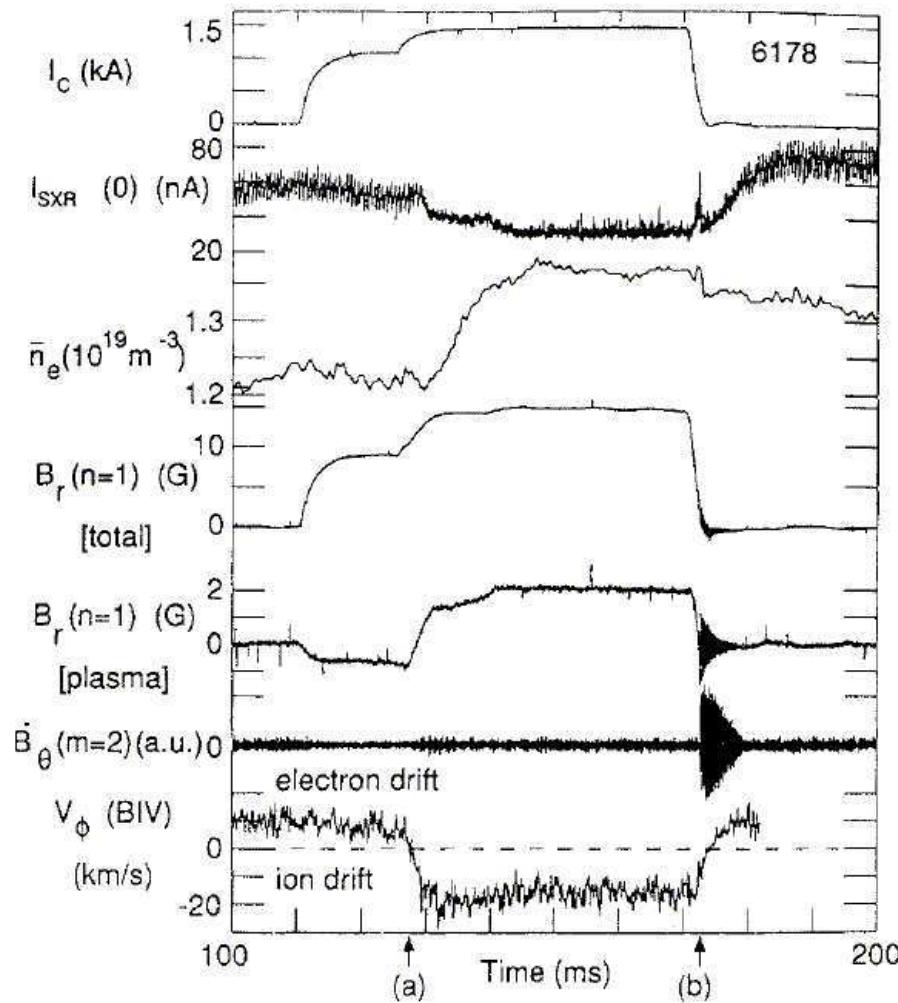


Static RMP speeds up plasma rotation before RMP penetration.

Compass-C results

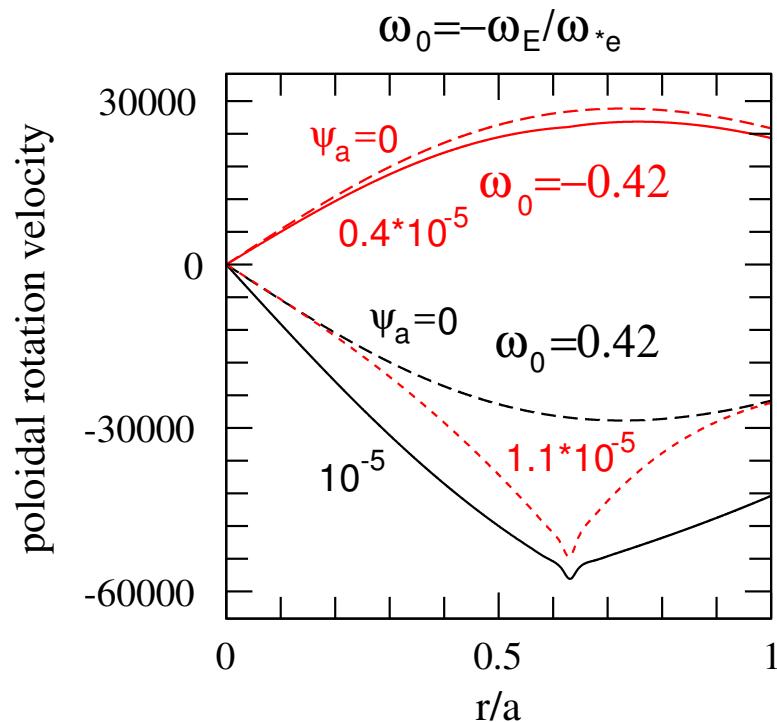
Hender et al. NF 1992

Static RMP changes plasma rotation from electron diamagnetic drift direction to the ion's one after RMP penetration.



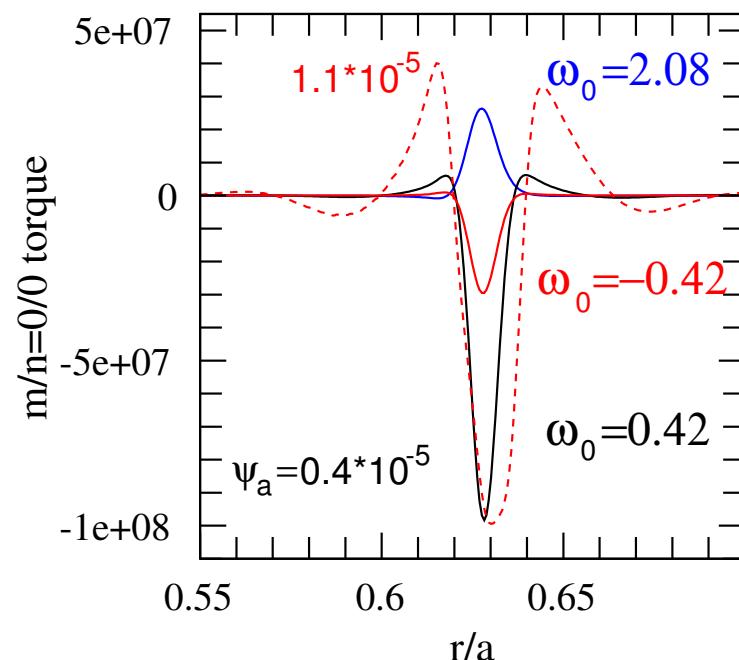
Effect of RMP on plasma rotation

Radial profile of the poloidal plasma rotation velocity



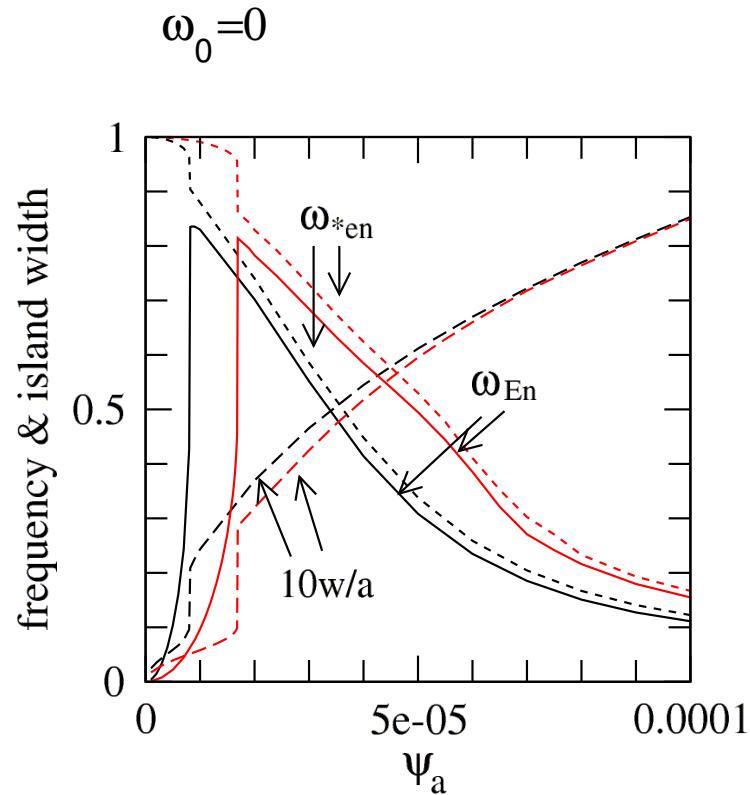
RMP can either increase or decrease rotation speed

Radial profile of the JxB torque in the poloidal direction

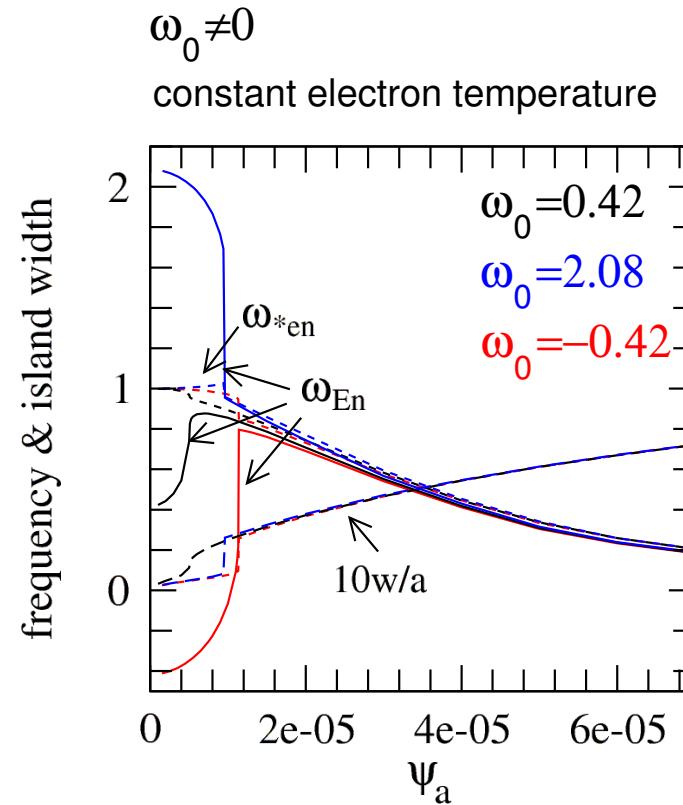


the JxB torque can be either in or against the rotation direction

Effect of RMP amplitude on plasma rotation frequency

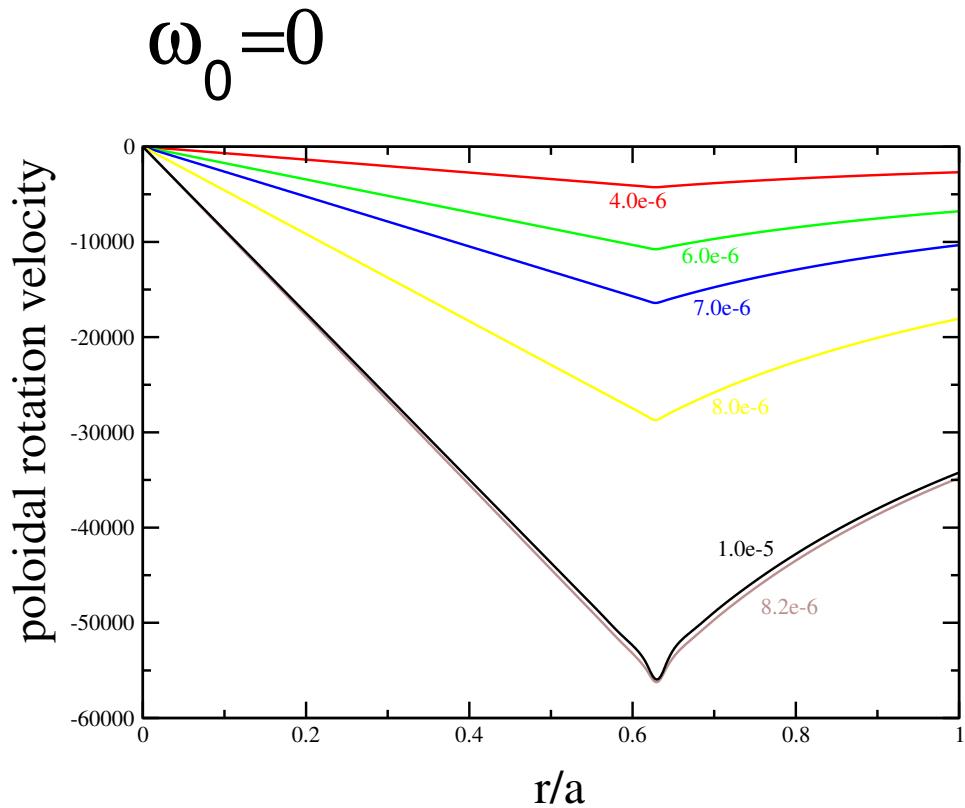


The RMP rotates a static plasma in the ion drift direction.

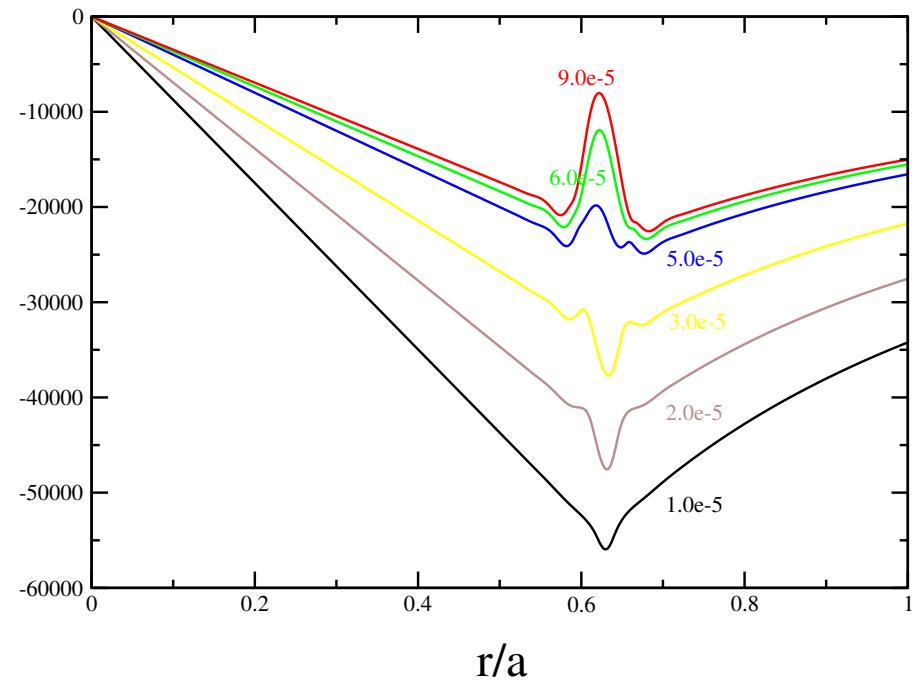


The RMP drives the plasma rotation towards the ion drift direction.

Rotation velocity profiles versus RMP amplitude



The plasma rotation speed Increases with increasing RMP amplitude in the ion drift direction.



The plasma rotation speed decreases for large RMP.

Discussion: effect of RMP on plasma rotation

- * Depending on the original plasma rotation direction and frequency, a small amplitude RMP can either increases or decreases plasma rotation frequency.
- * A large amplitude RMP rotates the plasma in the ion diamagnetic drift direction at the local electron diamagnetic drift frequency.
- * Spontaneous plasma rotations in co-current direction are observed in tokamak H-mode plasmas.
Does the error field play a role in it?
- * For a fusion reactor, the plasma rotation velocity is expected to be very low. While the rotation and its shear is important for plasma stability and transport. How to speed up plasma rotation in a reactor?
 - Using NBI? But NBI is not needed for a burning plasma.
 - Using a helical field? The RMP rotating in the co-current is better.
 - The diamagnetic drift frequency is large in the pedestal of H-mode plasmas.

4. Effect of RMP on particle confinement

* RMPs sometimes improves the tokamak particle confinement

(Finken et al., PRL 2007, Hender, et al., NF 1992)

* Explanations:

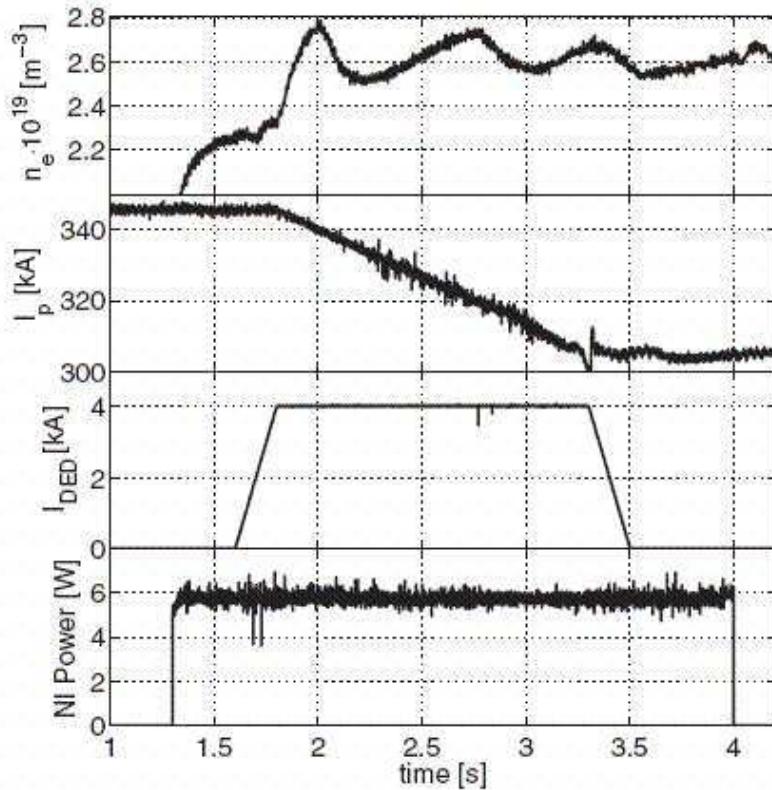
Difference in wall conditioning. (Hender, et al., NF 1992)

Stochastic field. (Finken et al., PRL 2007)

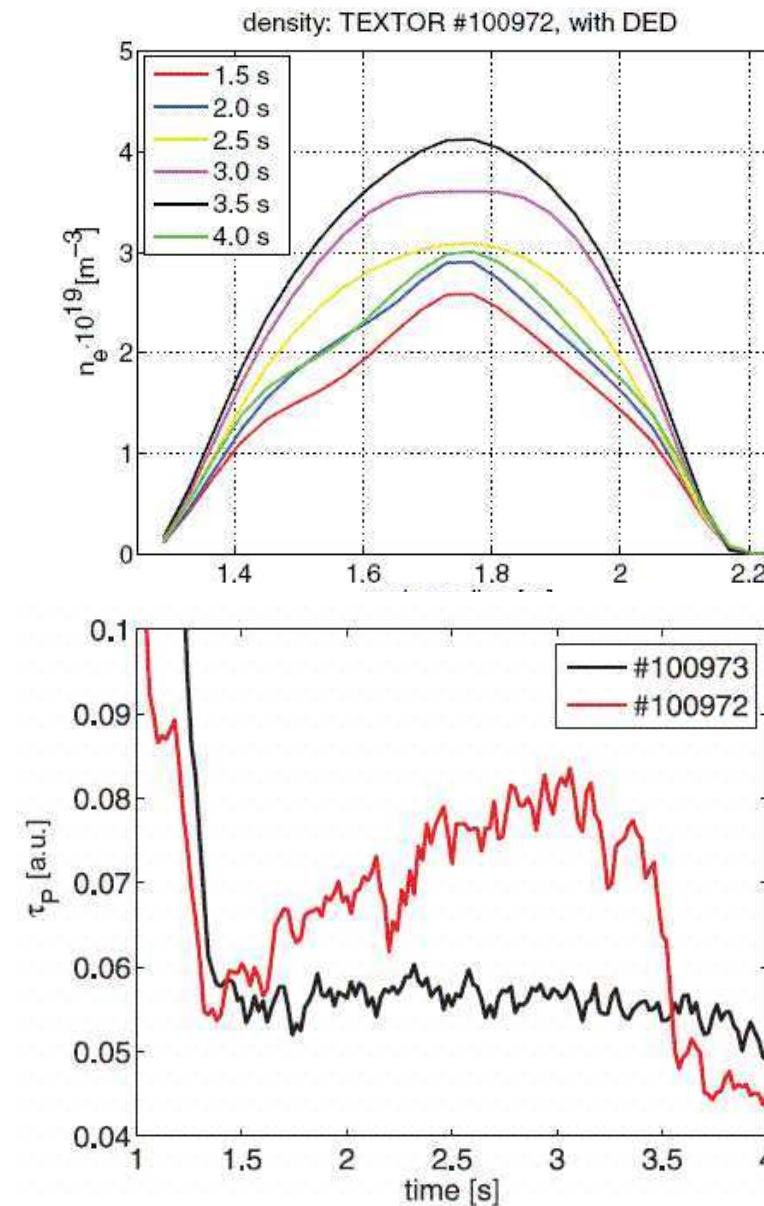
* On TEXTOR the improved particle confinement occurs in the low density, low collisionality and well heated plasma with co-NBI.

TEXTOR results

Finken et al. PRL 2007



Static RMP increases plasma density.



Analysis

* Mass conservation equation

$$\partial n_e / \partial t + \mathbf{v}_\perp \cdot \nabla n_e = d_\parallel \nabla_\parallel j - \nabla_\parallel (n_e v_{i\parallel}) + \nabla_\perp (D_\perp \nabla_\perp n_e) + S_n. \quad (1)$$

* For a small RMP and in the steady state,

$$d_\parallel \nabla_\parallel j + \nabla_\perp (D_\perp \nabla_\perp n_e) + S_n = 0. \quad (2)$$

Away from the rational surface at $r=r_s$, $\nabla_\parallel j=0$.

In the inner region around r_s ,

$$(\Delta n_e)' = (n_{e,0/0} - n_{e0})' = -0.5(d_\parallel/D_\perp)(b_{1r}j_1^* + b_{1r}^*j_1), \quad (3)$$

* For a static RMP $\omega=0$ and at $r=r_s$,

$$(\Delta n_e)' / n_e = \chi_{e\parallel} (1 - \omega_0) |b_{1r}/B_{0\parallel}|^2 / (L_{pe} D_\perp) \quad (4)$$

* When $\omega_0 = -\omega_{E0}/\omega_{*e0} > 1$ (plasma rotation in the ion drift direction with $\omega_{E0} > \omega_{*e0}$), $(\Delta n_e)' < 0$.

* When $\omega_0 < 1$, $(\Delta n_e)' > 0$.

* As $(\Delta n_e)'/n_e \sim \chi_{\text{ell}}$, being larger for a higher T_e and a lower n_e .

* For edge plasma with $T_e=300\text{eV}$, $n_e=2\times 10^{19}\text{m}^{-3}$, $\omega_0=3$, $L_{pe}=a$, $D_\perp=0.1\text{m}^2/\text{s}$, and $|b_{1r}/B_{0t}|=2\times 10^{-5}$ at $r=r_s$,

$$(\Delta n_e)'/n_e = -2.2/a.$$

* On the other hand, the RMP drives the plasma rotation frequency ω_E to $\omega_E \approx -\omega_{*e}$.

$$\Delta\omega_n = [\Delta\omega/\omega_{E0}]/[(\Delta n_e)'/(n_e/r)]$$

$$\approx (D_\perp/\mu)(\omega_{ci}/\omega_{E0})m[w/(qR)]^2, \quad (5)$$

* Two regimes

(a) $\Delta\omega_n \gg 1$, the major change is the plasma rotation frequency.

(b) $\Delta\omega_n \ll 1$, the major change is the electron density gradient (ω_{*e}).

* For a deuterium plasma with $B_{0t}=2.5\text{T}$, $\omega_{E0}=10^4\text{rad/s}$, $w=0.01a$, $q=3$, $R/a=3$, $m=6$, $\mu=5D_\perp$,

$$\Delta\omega_n = 0.018.$$

Numerical results

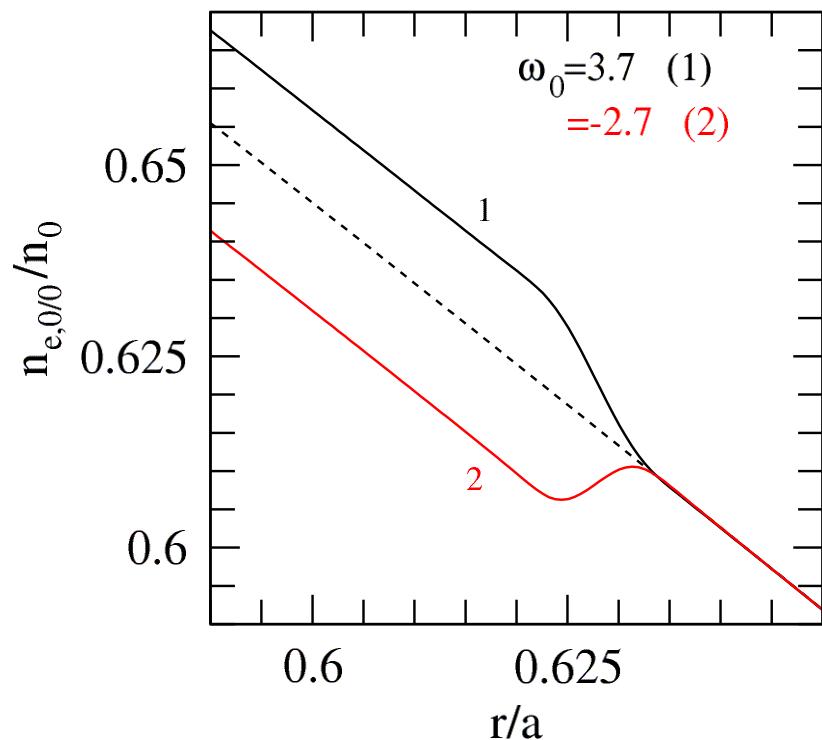
A single helicity RMP with $m/n=2/1$.

$S=1.97\times10^8$, $\Omega=6.3\times10^4$, $c_s=1.2\times10^7(a/\tau_R)$, $D_\perp=0.1\text{m}^2/\text{s}$ ($4.2a^2/\tau_R$),

$d_1=2.5\times10^8$, $\mu=2.1\times10^3(a^2/\tau_R)$, if not mention elsewhere.

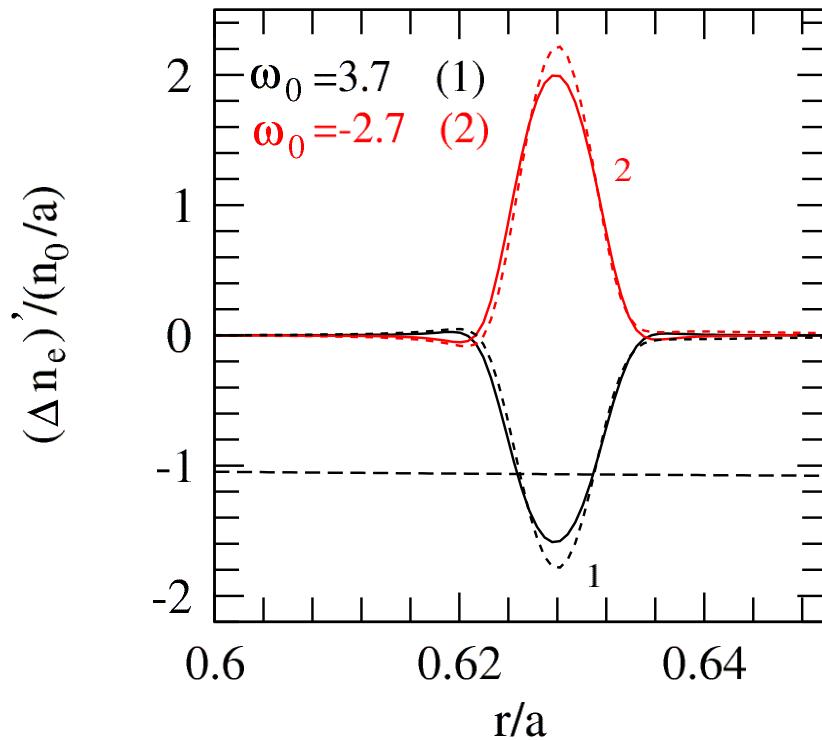
Effect of RMP on plasma density

Radial profile of the electron density



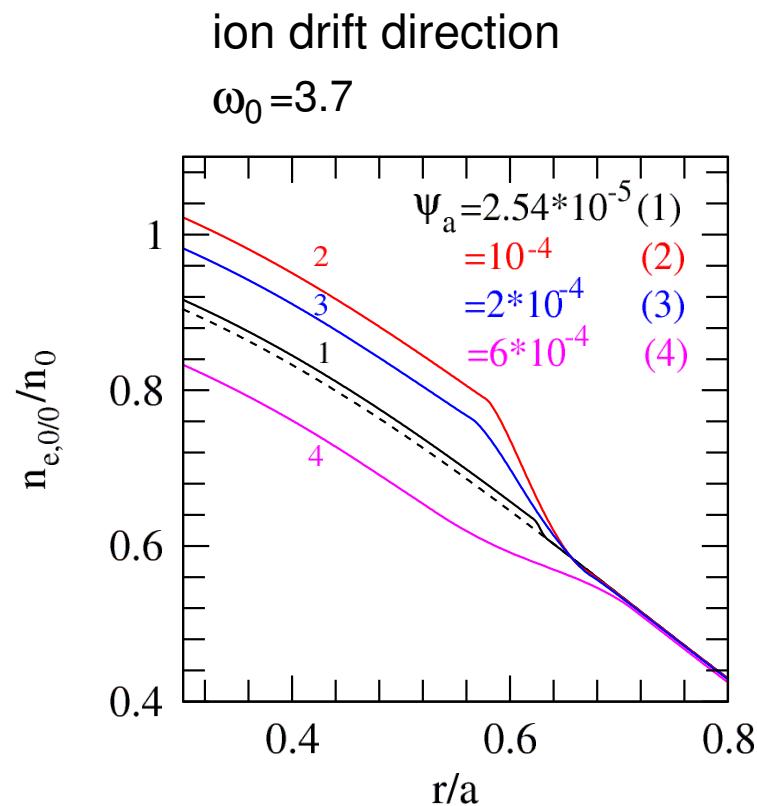
RMP can either increases or decreases local plasma density.

Radial profile of density gradient

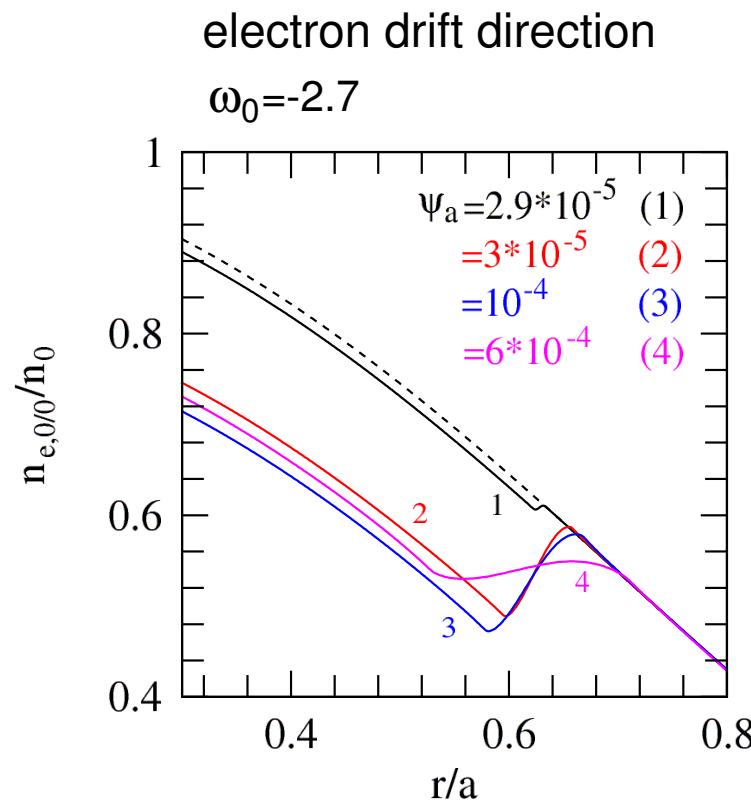


The density gradient is decreased for $\omega_0 > 0$ but increased for $\omega_0 < 0$

Effect of RMP amplitude

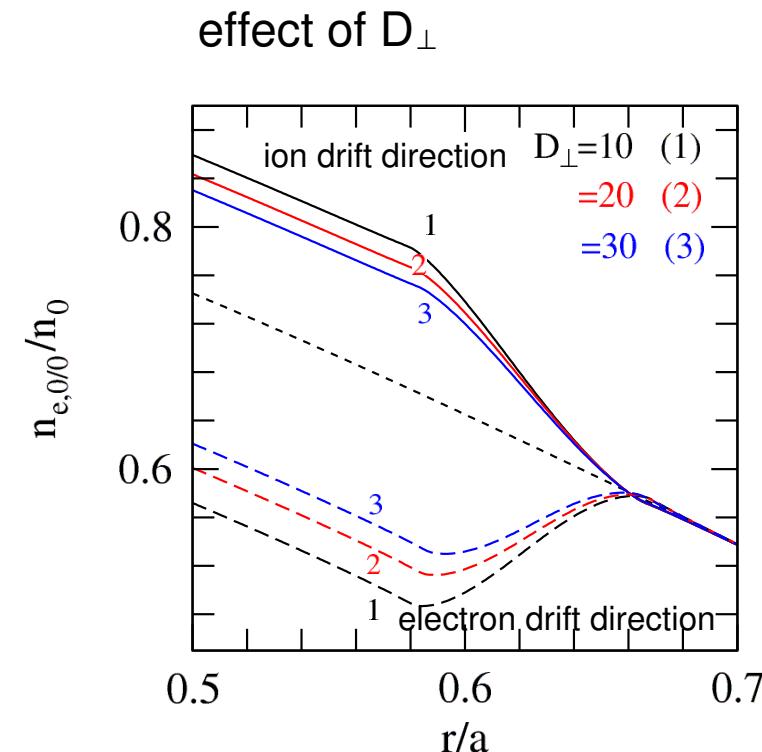
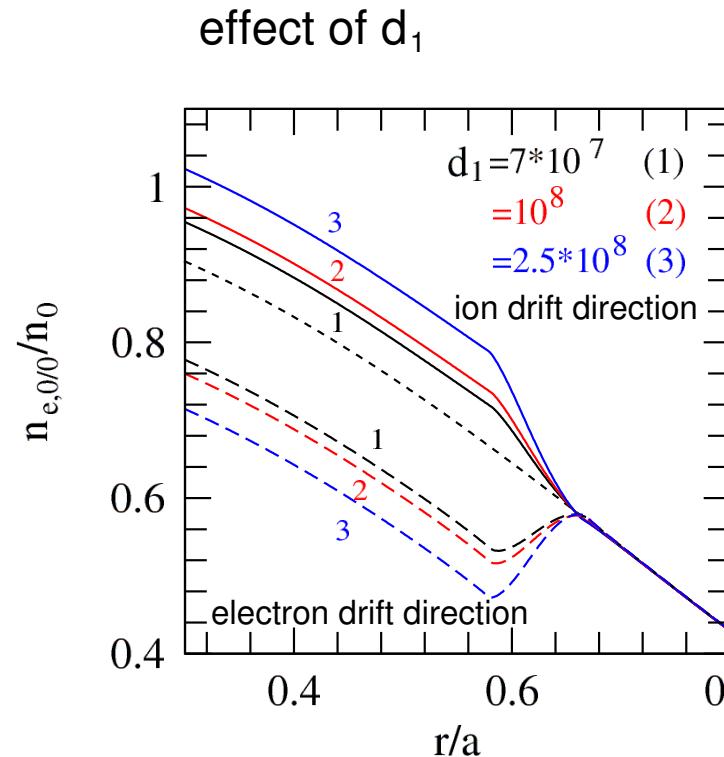


RMP first increases and then decreases local plasma density.



The density gradient becomes positive for a small magnetic island.

Effect of the parameters d_1 and D_{\perp}



The effect of the RMP on plasma density gradient is larger for a large value of d_1/D_{\perp} , as predicted by the analytical theory.

Discussion: effect of RMP on particle confinement

- * The electron density is increased by a small RMP if the plasma rotates in the plasma current direction with a frequency being larger than the electron drift frequency. In the opposite limit the electron density is decreased, and its local gradient can become positive.
- * The highest improved confinement is observed on TEXTOR in a low density and high temperature plasma with co-NBI, being consistent with our theoretical results. Also, the island width is small with high mode numbers ($m \sim 6$) of the applied RMPs.
- * Since the required RMP amplitude for changing the density gradient is quite small,
Is the particle confinement affected by RMPs from intrinsic machine error field?
- * Our results suggests that an applied RMP of an appropriate frequency and amplitude can be utilized to either increase or decrease the local electron density gradient.

5. Summary

*When the applied RMP frequency equals the mode frequency determined by both the background plasma rotation and the diamagnetic drift, the penetration threshold reaches the minimal value.

* For a small or moderate RMP amplitude, there are two regimes:

- (1) Changing local electron density gradient in a low density and high temperature plasma
- (2) Changing plasma rotation frequency in the opposite limit.
- (3) The increases or decreases in plasma density and rotation frequency depends on the original plasma rotation direction and frequency.

* A sufficiently large RMP rotates the plasma in the ion diamagnetic drift direction at the local electron diamagnetic drift frequency, or stop the rotation if $\omega_{*e} \sim 0$ (single fluid limit).

*** Experiments are needed for a better understanding**