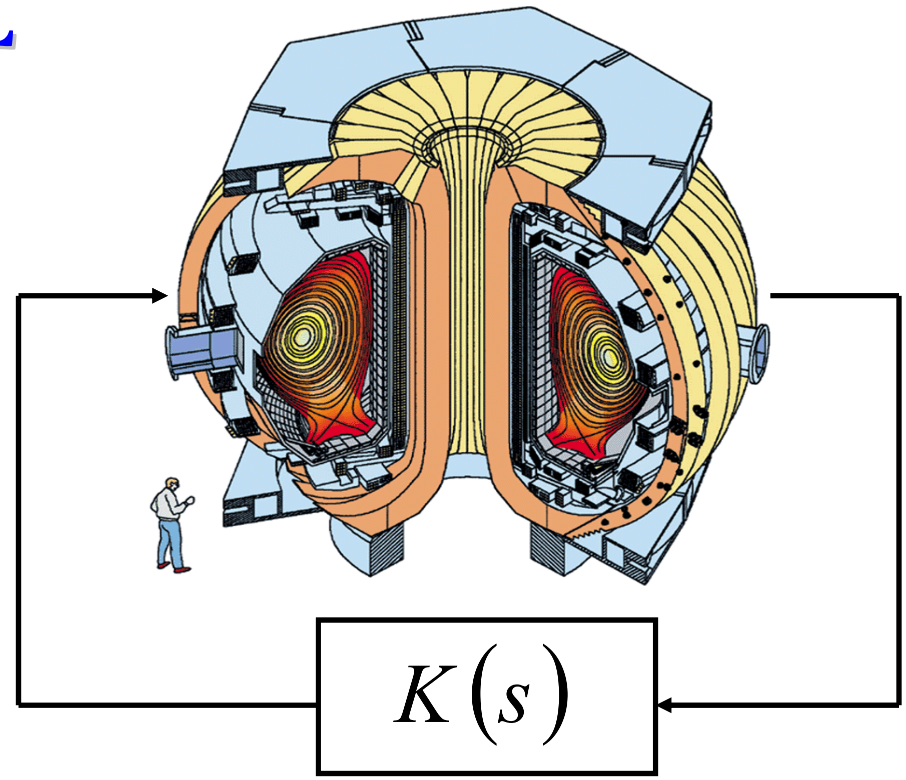
PROGRESS ON MODEL-BASED CURRENT PROFILE CONTROL IN DIII-D



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Current Profile Control: Why?

- A key goal in control of an advanced tokamak discharge is to maintain current and pressure profiles that are compatible with both MHD stability at high plasma pressure and a high fraction of the self-generated bootstrap current → Steady-state operation.

- Simultaneous real-time control of the current and pressure profiles can lead to the steady-state sustainment of an internal transport barrier (ITB) → Confinement improvement .

- Global current profile control, eventually combined with pressure profile control, could be an effective mechanism for neoclassical tearing mode (NTM) control and avoidance.

Current Profile Control: Model

ψ	poloidal magnetic flux
$I(t)$	total plasma current

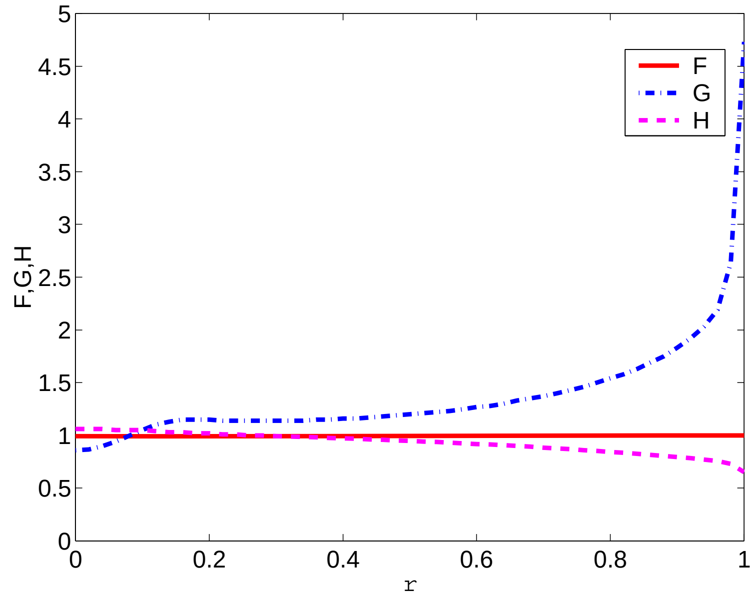
Magnetic diffusion equation:

$$\frac{\partial \psi}{\partial t} = \frac{\eta(T_e)}{\mu_o \rho_b^2 \hat{F}^2} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left(\hat{\rho} \hat{F} \hat{G} \hat{H} \frac{\partial \psi}{\partial \hat{\rho}} \right) - R_o \hat{H} \eta(T_e) \frac{\langle \bar{j}_{NI} \cdot \bar{B} \rangle}{B_{\phi,o}}$$

Boundary conditions:

$$\left. \frac{\partial \psi}{\partial \hat{\rho}} \right|_{\hat{\rho}=0} = 0$$

$$\left. \frac{\partial \psi}{\partial \hat{\rho}} \right|_{\hat{\rho}=1} = \frac{\mu_o}{2\pi} \frac{R_o}{\hat{G}\big|_{\hat{\rho}=1} \hat{H}\big|_{\hat{\rho}=1}} I(t)$$



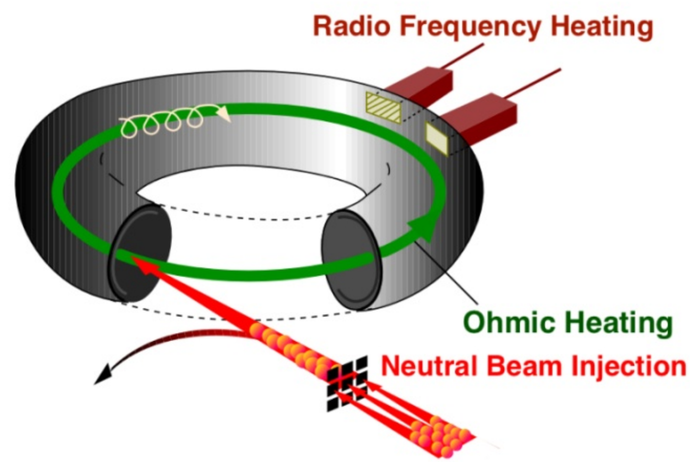
Current Profile Control: Approach

- One unique feature of the problem of current profile control in tokamaks is that it allows for **interior**, **boundary** and **diffusivity control** mechanisms. These three types of control are functions of three physical actuators:

  i. Line-averaged density.

  ii. Non-inductive current drive power. Simplified models for current drives (neutral beams and RF current drives) can be derived from more complex codes.

  iii. Total plasma current.



- Modeling of additional non-inductive current drives such as bootstrap current may be necessary depending on the phase of the discharge.

- Real-time measurement of the current density profile by Motional Stark Effect (MSE) is available for feedback control implementation.

Profile Control: Important Issues

Motivation:

The development of profile controllers is aimed at saving long trial-and-error periods of time currently spent by fusion experimentalists trying to manually adjust the time evolutions of the actuators to achieve a desired profile.

The high dimensionality of the problem and the strong coupling between the different variables describing the plasma profile evolution call for a model-based, multivariable approach to obtain improved closed-loop performance.

Model for Control Design:

The goal is to develop a model based controller to be used toward the achievement of desirable profiles. A necessary prior task is the development of a dynamic model to use for controller design.

Modeling and Control Challenges:

- **Coupling of the controlled variable with kinetic variables.**
  - The controlled-variable PDE equation is accompanied by transport PDE equations for the kinetic variables.
- **High dimensionality of the problem.**
  - Model reduction (PDE $\rightarrow$ ODE) may be necessary. Particularly for closed-loop control.
- **Unknown parameters in transport models.**
  - Model identification may be necessary.

Profile Control: Modeling Approaches

Open-loop Control

1. The controlled-variable PDE equation is accompanied by transport PDE equations for the kinetic variables.

2. Time-scale separation through singular perturbation methods may be possible: PDE system $\rightarrow$ PDE equation (magnetic variables) + algebraic equations (kinetic variables).

3. The magnetic-variable PDE equation is accompanied by simplified, scenario-oriented, models for the kinetic variables (algebraic equations).

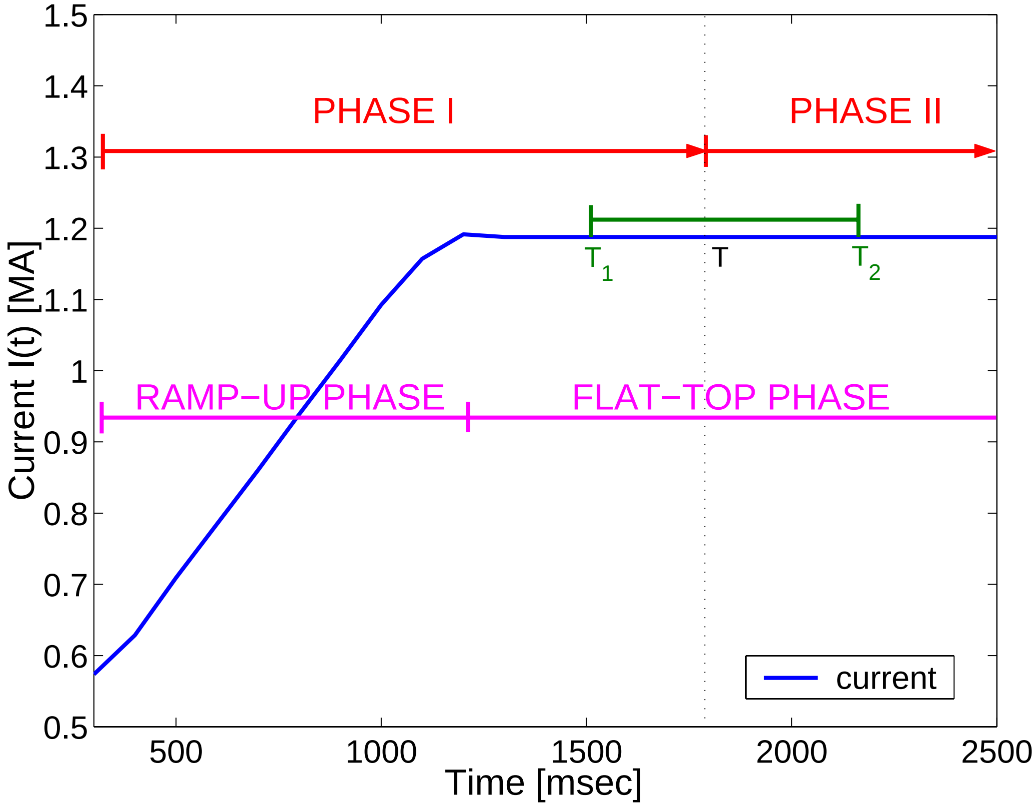
Closed-loop Control $\leftrightarrow$ Model reduction (PDE $\rightarrow$ ODE) may be necessary

4. The controlled-variable PDE equation is evaluated with real-time measurements of the kinetic variables (measurable disturbances).

This approach defines a different control problem: Easier? Better?

Profile Control: Objective

The phases of the discharge define the modeling and control objectives.



During "Phase I" the control goal is to drive the current/rotation profile from any arbitrary initial condition to a prescribed target profile at some time $T \in (T_1, T_2)$ in the flat-top phase of the total current $I(t)$ evolution. The prescribed target profile is not an equilibrium profile during "Phase I."

During "Phase II" the control goal is to regulate the current/rotation profile around a target equilibrium profile.

"Phase I" → Mainly inductive

"Phase II" → Mainly non-inductive (Stronger magnetic/kinetic coupling)

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Current Profile Control: Phase I

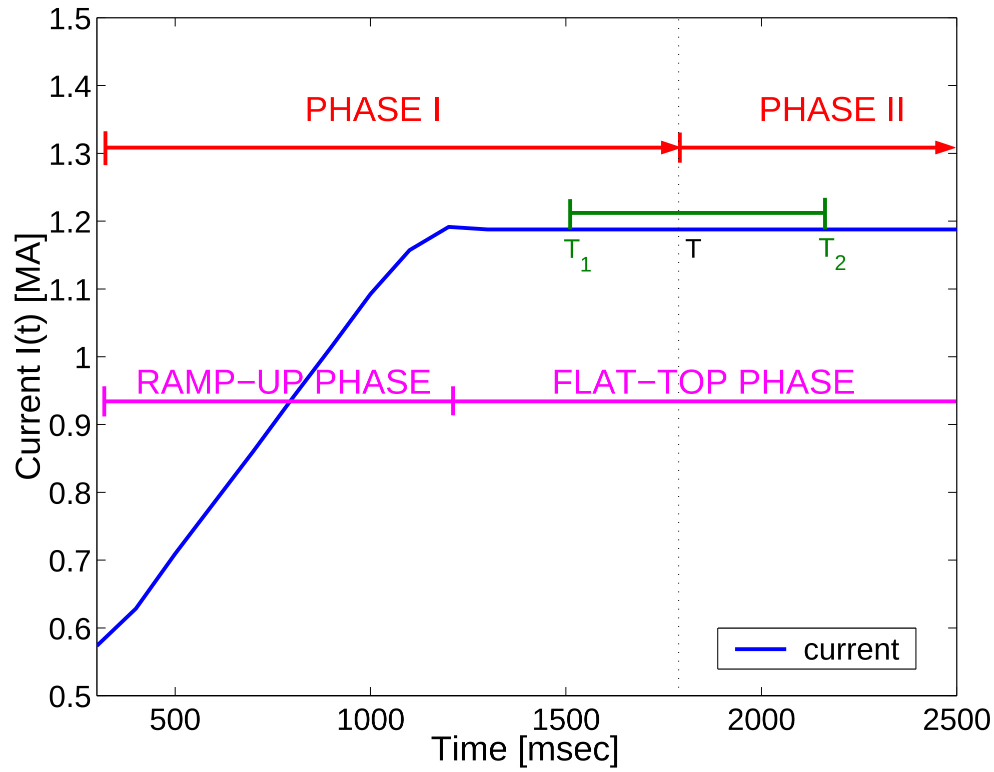
Open-loop Finite-time Optimal Control

GOAL: During "Phase I" an optimal control problem must be solved, where time evolution for three actuators ($I(t)$, $\bar{n}(t)$ $\left(u_n(t)\right)$, $P_{tot}(t)$) are sought to minimize the functional.

$$J_{\min} = \min_{t \in [T_1, T_2]} \left(J(t) \right)$$

where

$$J(t) = \int_0^1 \left(\iota(\hat{\rho}, t) - \iota^{des}(\hat{\rho}) \right)^2 d\hat{\rho}$$



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Current Profile Control: Phase I

The physical ranges for $I(t)$, $\bar{n}(t)$ and $P_{tot}(t)$, are given by

$$0 \leqslant I(t) \leqslant I_{\max} \qquad \left|\frac{dI(t)}{dt}\right| \leqslant dI_{\max}$$

$$I(MA) \leqslant \frac{\bar{n}(t)}{10^{19}} \leqslant 5I(MA) \qquad dn_{\min} \leqslant \frac{d\bar{n}(t)}{dt} \leqslant dn_{\max}$$

$$P_{\min} \leqslant P_{tot}(t) \leqslant P_{\max}$$

To accurately reproduce experimental discharges, we must add constraints for *I(t)* and $\bar{n}(t)$, at the initial time of "Phase I", i.e.,

$$I(t = 0s) = I_0$$

$$\bar{n}(t = 0s) = \bar{n}_0$$

In addition, a value of the total current *I(t)* is prescribed for the flattop phase, i.e.,

$$I(t \geq T_1) = I_{target}$$

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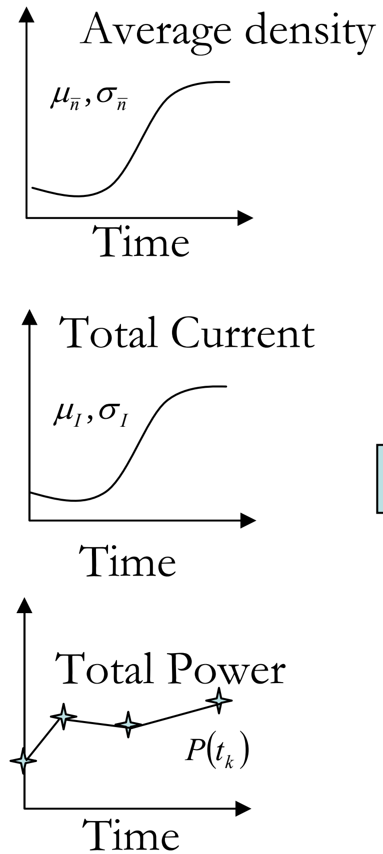
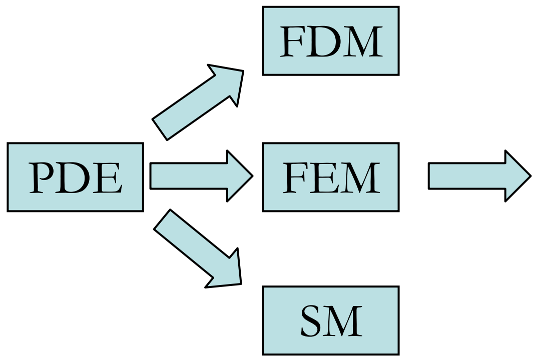
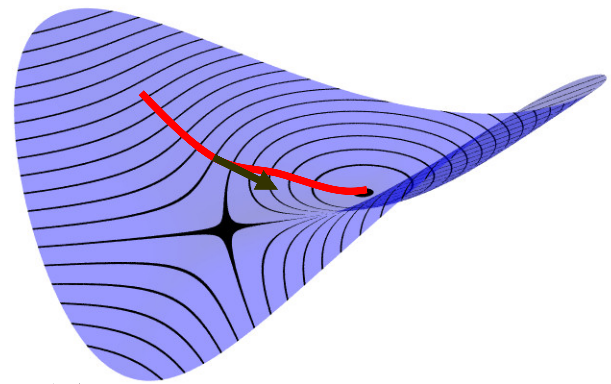
Current Profile Control: Phase I – Open Loop



Parameterization
Repetitive simulation of PDE
Cost functional checking
Parameter modification



Finite difference
Finite element
Spectral/Pseudo



Trajectory in parameter space

$$J_{\min} = \min_{t \in [T_1, T_2]} \int_0^1 \left(\iota(\hat{\rho}, t) - \iota^{des}(\hat{\rho}) \right)^2 d\hat{\rho}$$

Current Profile Control: Phase I – Open-loop

- The vector parameter θ has 10 components

$$\theta = \begin{cases} I(0.4s), I(0.8s), \\ P_{tot}(0s), P_{tot}(0.4s), P_{tot}(0.8s), P_{tot}(1.2s), \\ \bar{n}(0.3s), \bar{n}(0.6s), \bar{n}(0.9s), \bar{n}(1.2s) \end{cases}$$

- By taking into account that $I(0s)=I_0$ and $I(T_1)=I_{target}$, and using curve fitting for the points $I(0s)$, $I(0.4s)$, $I(0.8s)$, $I(T_1=1.2s)$, we can reconstruct the profile $I(t)$ for $t\in[0,T_1]$. In addition, we make $I(t)=I_{target}$ for $t\in[T_1,T_2=2.4s]$.

- Following similar procedure, we can construct the law for $P_{tot}(t)$ and $\bar{n}(t)$.

- The reconstructed control laws are in turn fed into the PDE model. Given initial ψ, the PDE system is integrated to obtain $\psi(\hat{\rho},t)$, and finally $\iota(\hat{\rho},t)$, which are necessary to evaluate the cost function

Current Profile Control: Phase I – Open-loop

The magnetic-variable PDE equation is accompanied by simplified, scenario-oriented, models for the kinetic variables (algebraic equations).

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Magnetic diffusion equation:

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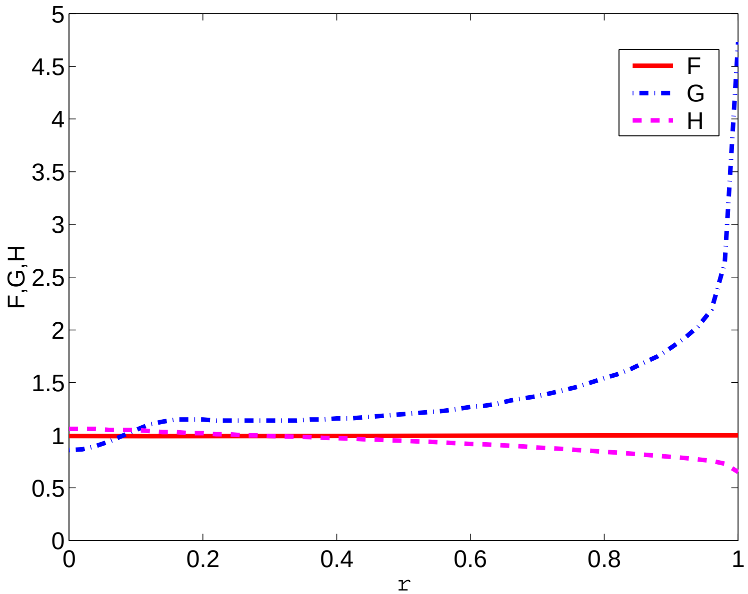

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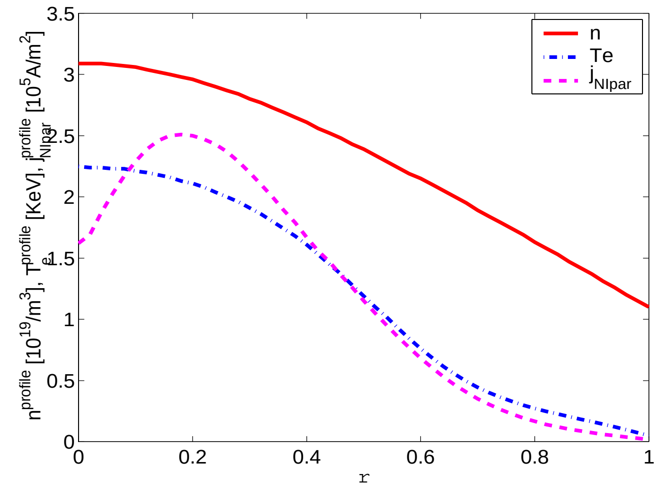
Current Profile Control: Phase I – Open-loop

Highly simplified models for the density and temperature are chosen for the inductive phase (Phase I). The profiles are assumed to remain fixed. The temperature and density responses to the actuators are simply scalar multiples of the reference profiles. These reference profiles are taken from a DIII-D tokamak discharge. Bootstrap current is neglected.



Density:

$$n(\hat{\rho},t) = n^{profile}(\hat{\rho})\,u_n(t)$$

$$\bar{n}(t) = \int_0^1 n(\hat{\rho},t)\,d\hat{\rho}$$

Temperature:

$$T_e(\hat{\rho},t) = k_{T_e} T_e^{profile}(\hat{\rho}) \frac{I(t)\sqrt{P_{tot}(t)}}{\bar{n}(t)}$$

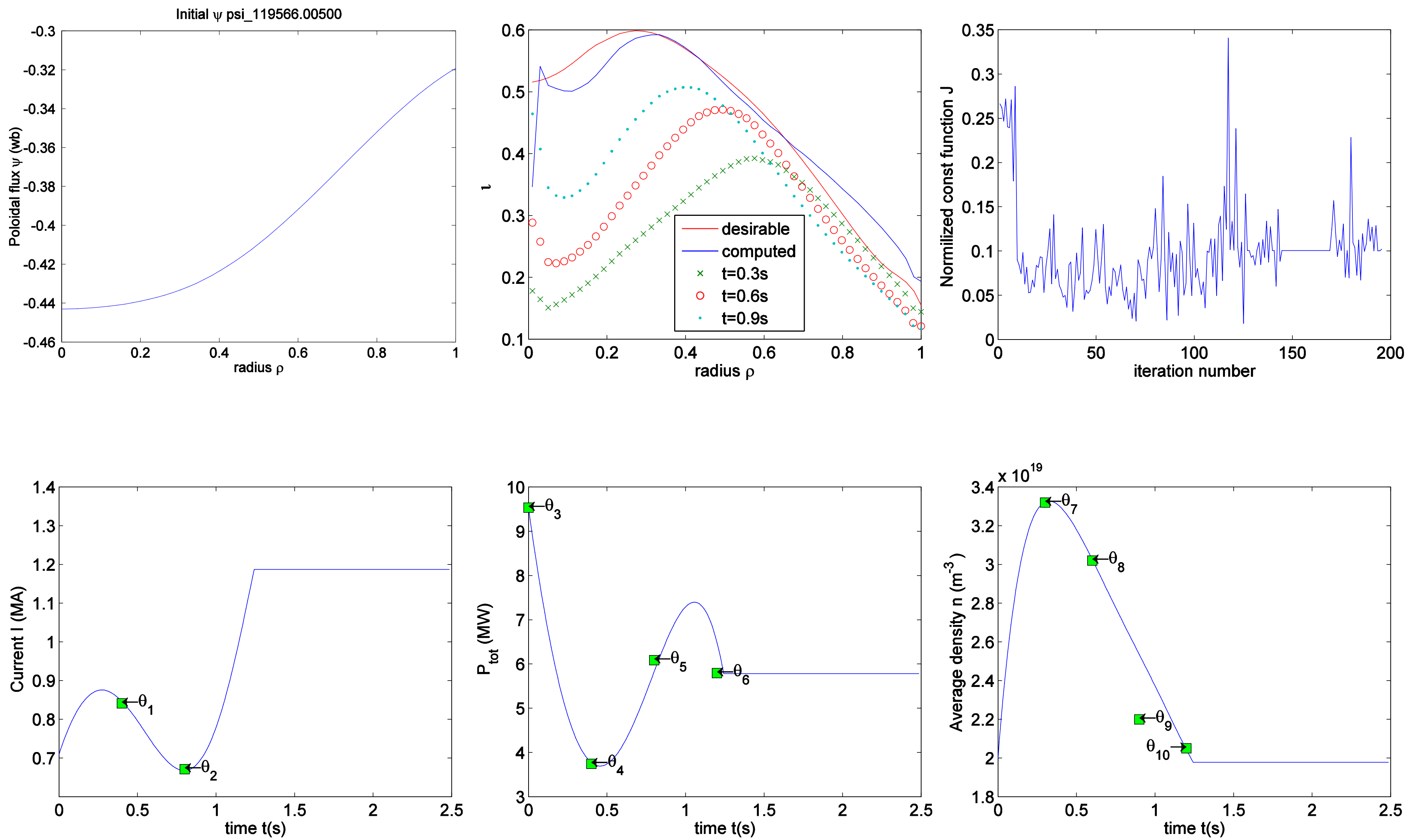
$$\eta(\hat{\rho},t) = \frac{k_{eff} Z_{eff}}{T_e^{3/2}(\hat{\rho},t)}$$

Parallel Current:

$$\frac{\langle \bar{j}_{NI} \cdot \bar{B} \rangle}{B_{\phi,o}} = k_{NI_{par}} j_{NI_{par}}^{profile}(\hat{\rho}) \frac{I(t)^{1/2} P_{tot}(t)^{5/4}}{\bar{n}(t)^{3/2}}$$

We consider $I(t)$, $\bar{n}(t)$ and $P_{tot}(t)$ the physical actuators of the system.

Current Profile Control: Phase I – Open-loop



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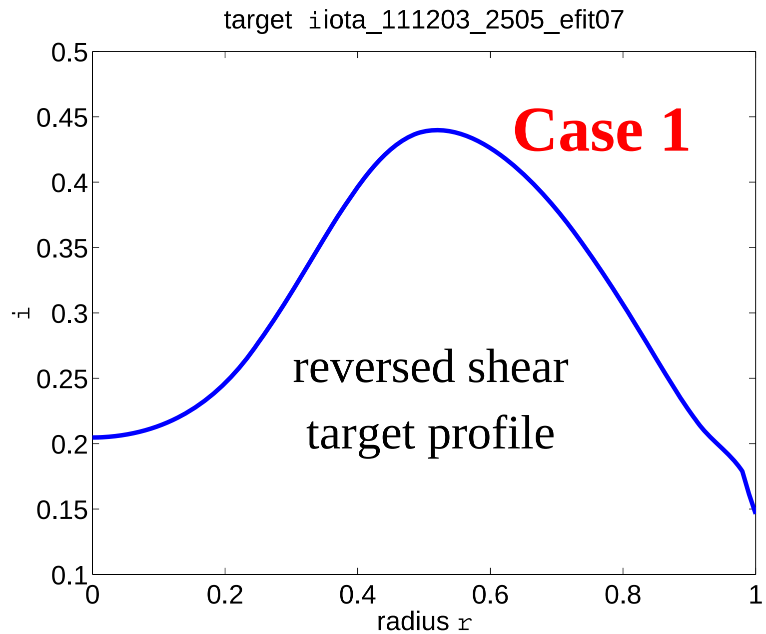
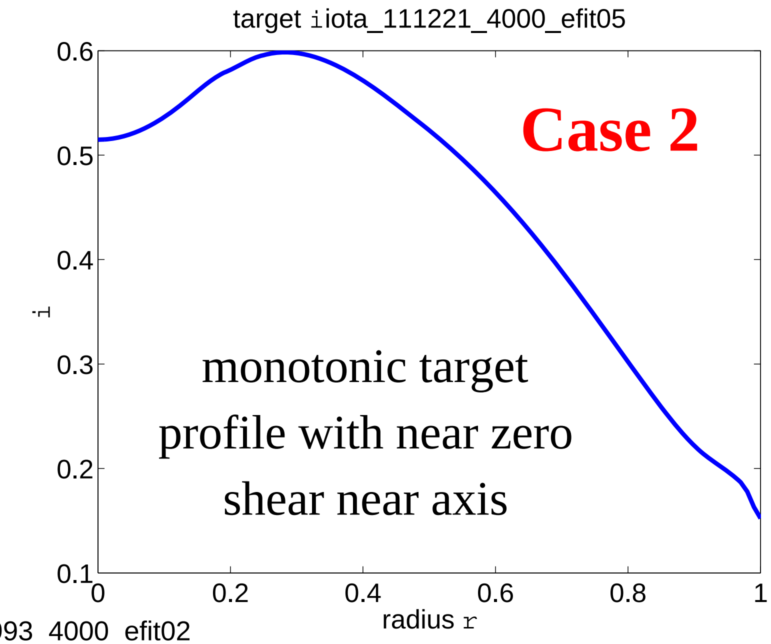
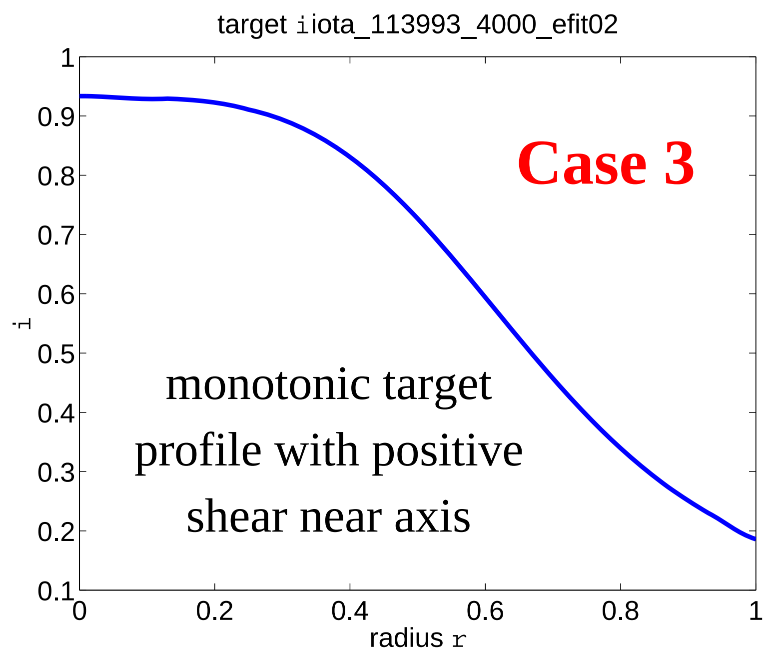
Current Profile Control: Phase I – Open-loop





Desired ι profiles



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Current Profile Control: Phase I – Open-loop

Case 1: Experiment June 23, 2008

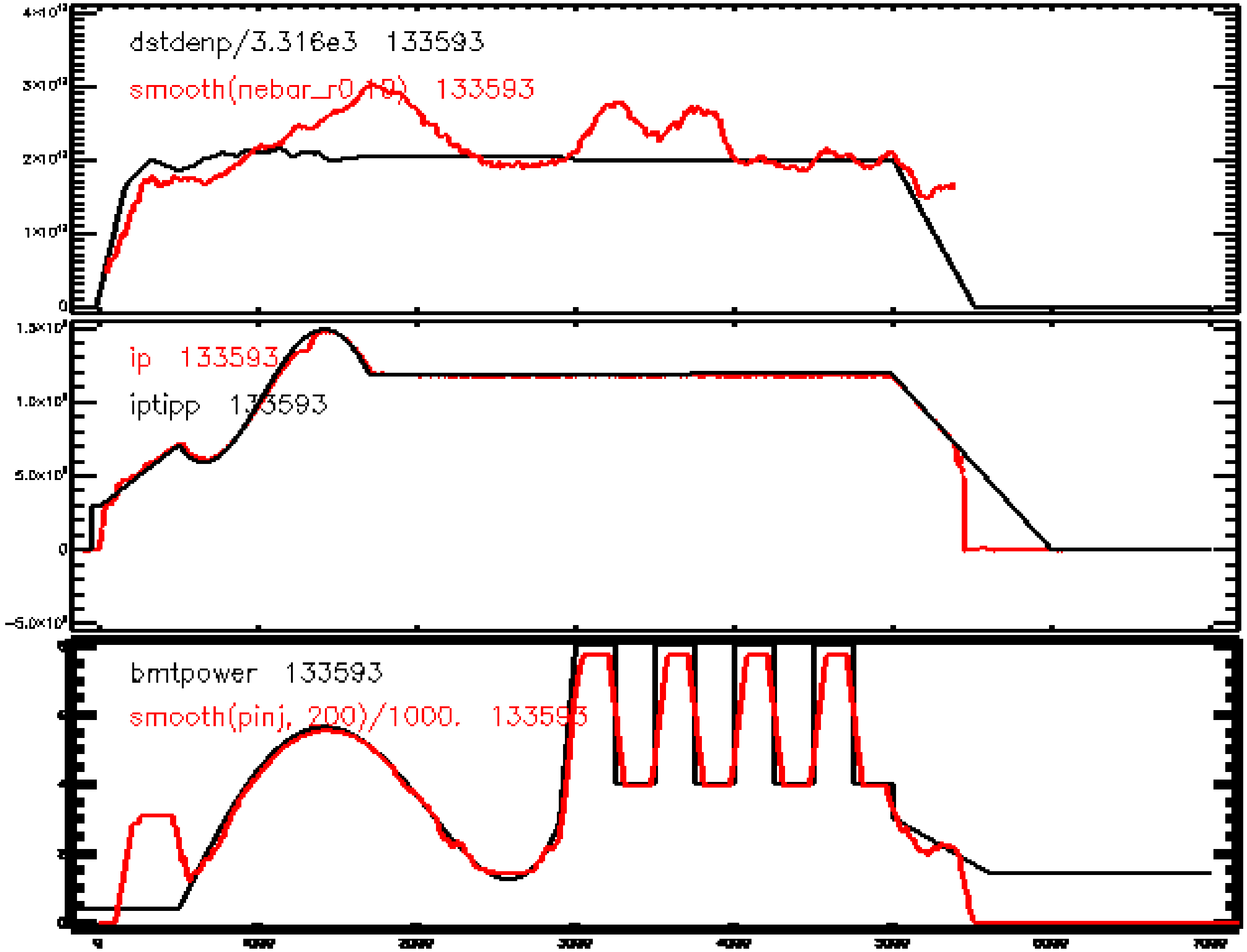

Figure: Comparison of desired and actual actuators for shot 133593 (Case 1).

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Current Profile Control: Phase I – Open-loop

Case 2: Experiment June 23, 2008

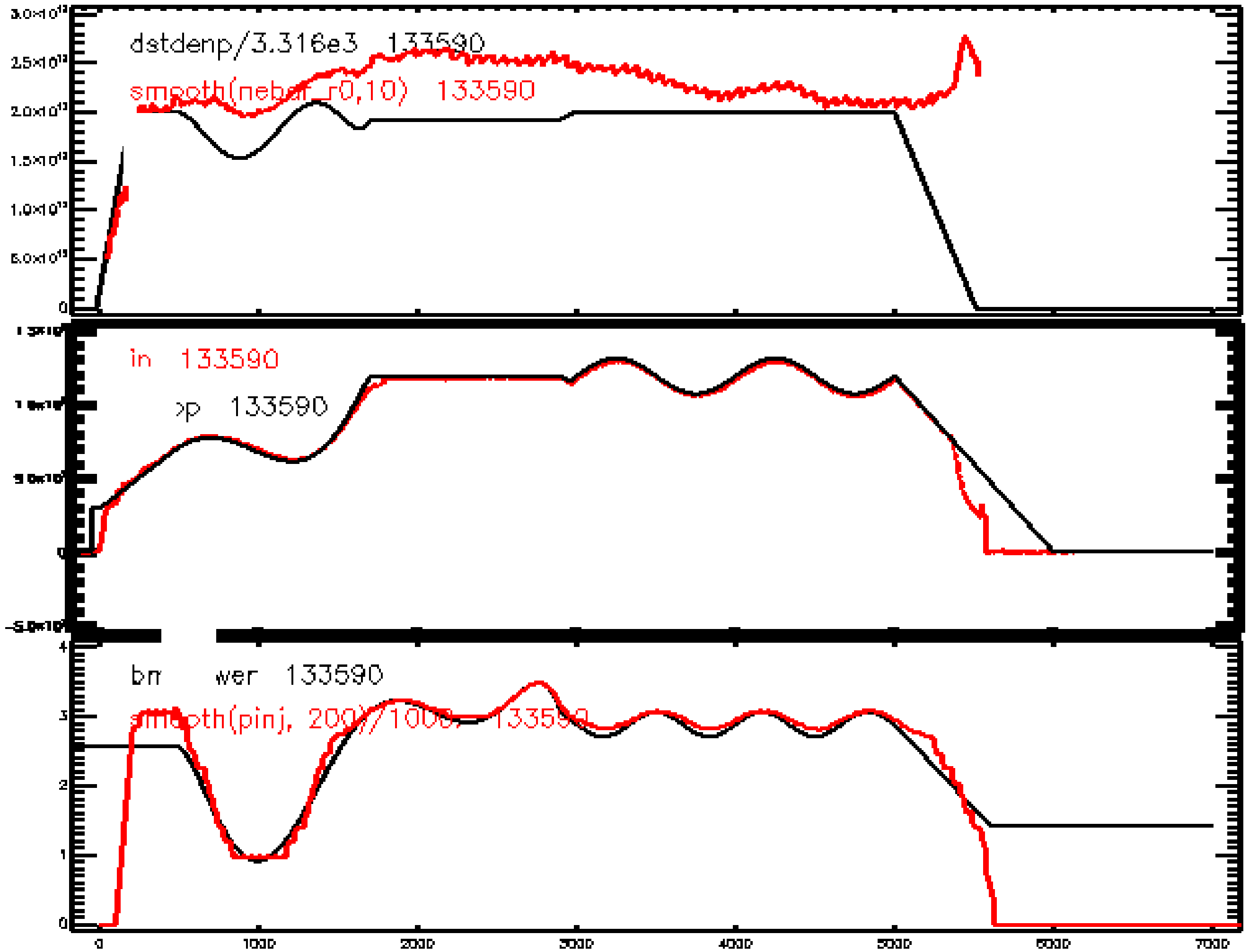

Figure: Comparison of desired and actual actuators for shot 133590 (Case 2).

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Current Profile Control: Phase I – Open-loop

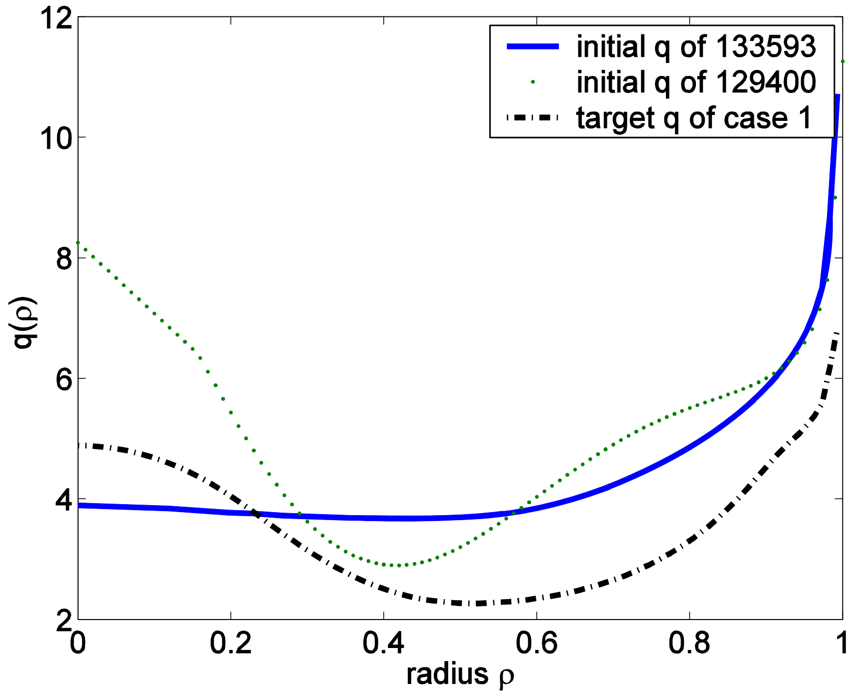


Figure: Initial q profile used in case 1

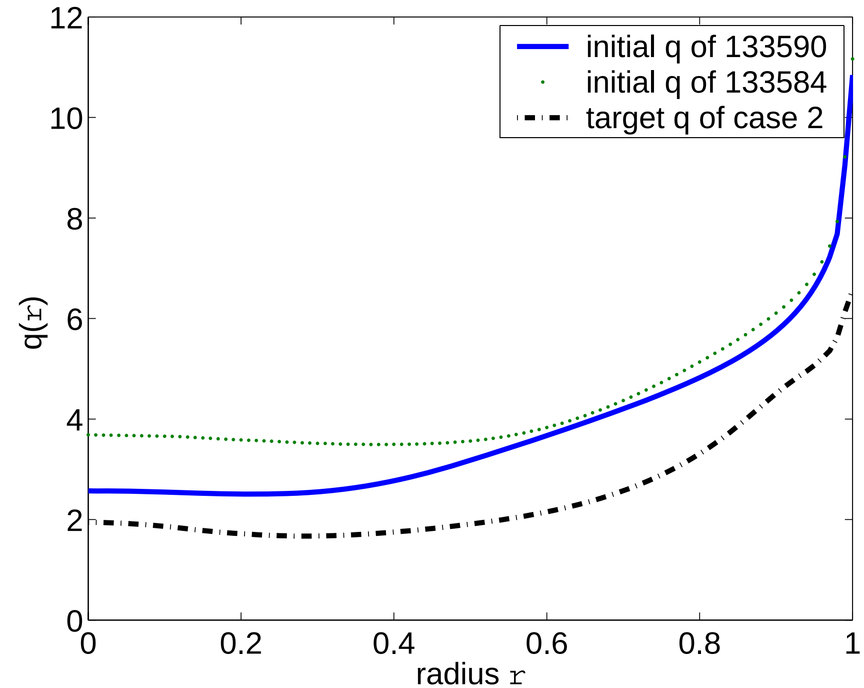
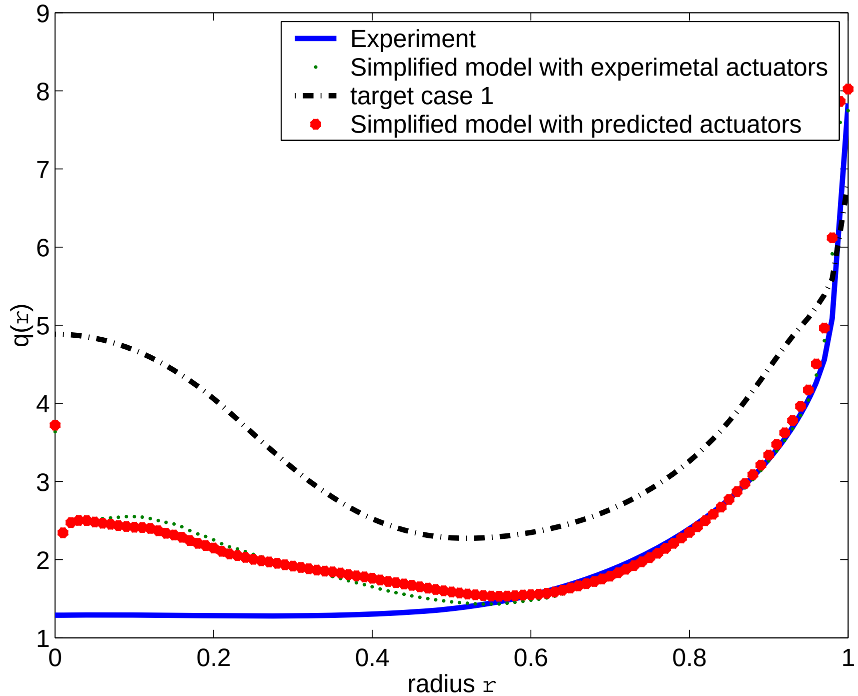
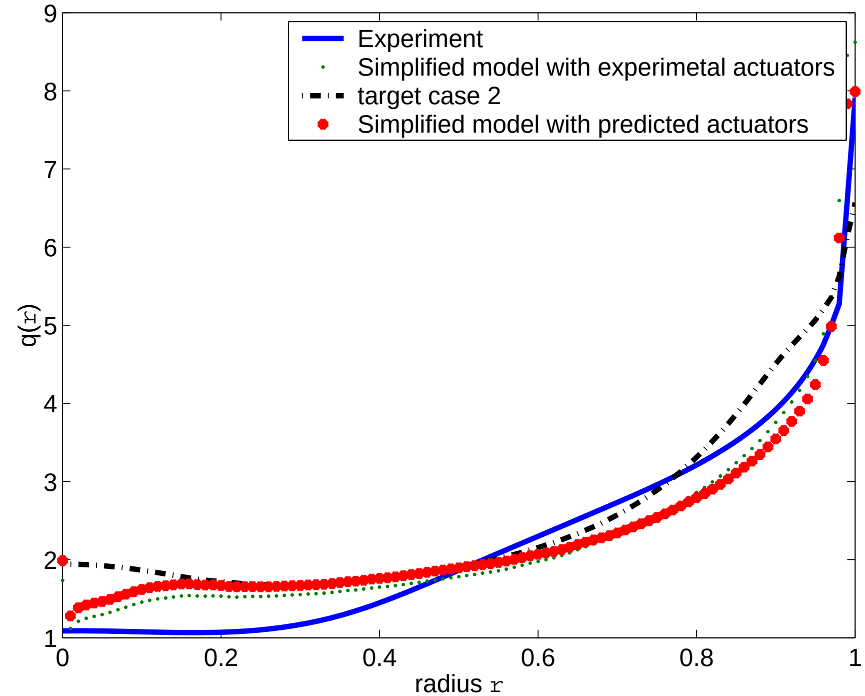


Figure: Initial q profile used in case 2

The figures illustrate initial conditions of safety factor q profiles for shots 133593 (Case 1) and 133590 (Case 2) at 500ms, where the initial profiles of shots 129400 and 133584 at 500 ms (green-dotted curves) were used offline for the synthesis of the ESOC actuator trajectories.

Current Profile Control: Phase I – Open-loop





Comparisons of the q profiles at time T=1,700 ms are illustrated in these figures, where the black curves are the target profiles, the blue curves represent the experimental results, the green curves illustrate the q profile achieved using the simplified model with experimental initial conditions and actuator trajectories, and the red curves depict the q profiles achieved using the simplified model with initial conditions of shot 129400 (case 1) and 133584 (case 2) and actuator trajectories predicted by the ESOC procedure.

Current Profile Control: Phase I – Open-loop

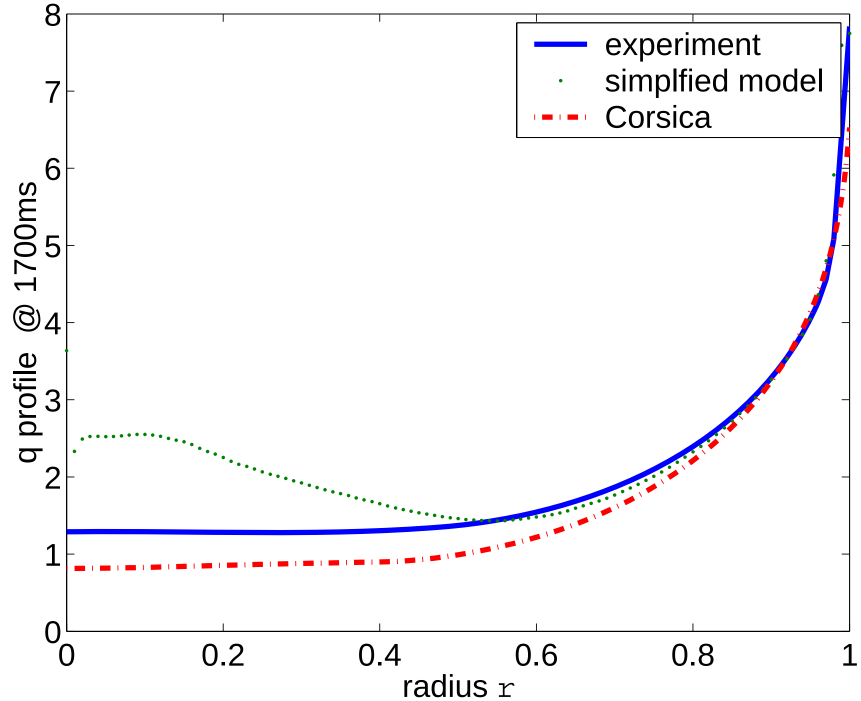


Figure: Comparison of q profile in Case 1

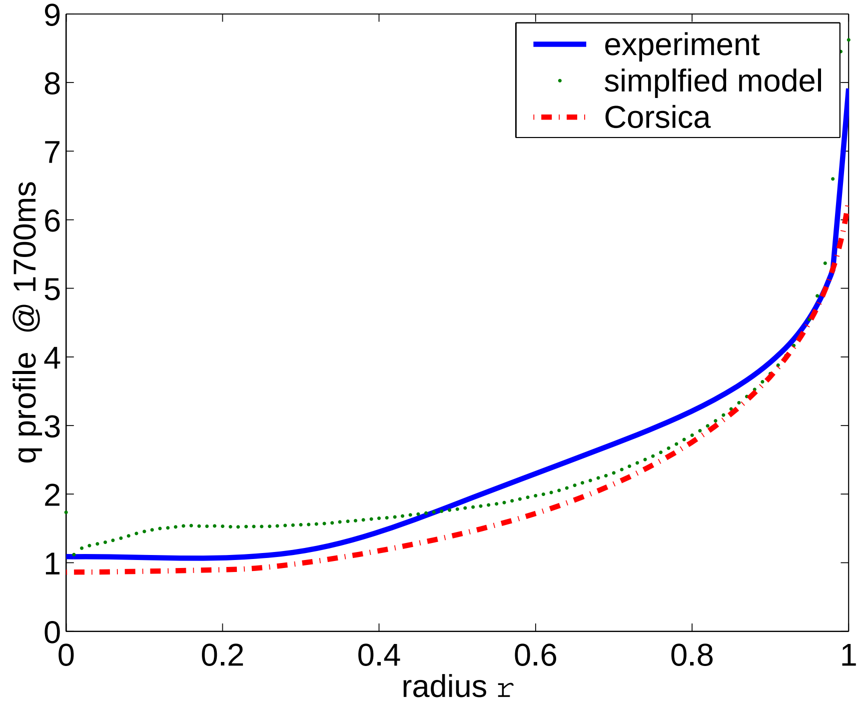
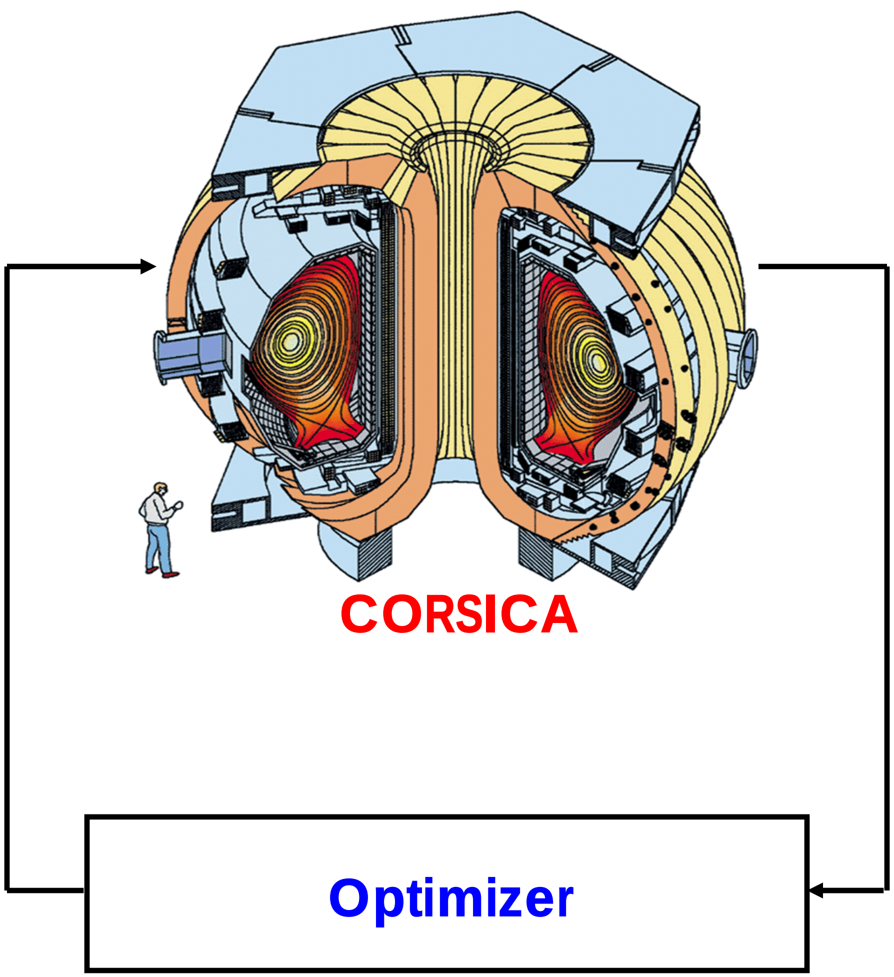


Figure: Comparison of q profiles in Case 2

Current Profile Control: Phase I – Open-loop

The controlled-variable PDE equation is accompanied by transport PDE equations for the kinetic variables.



Open-loop design admits the use of highly complex models

Integration of CORSICA into MATLAB SIMULINK environment

Current Profile Control: Phase I

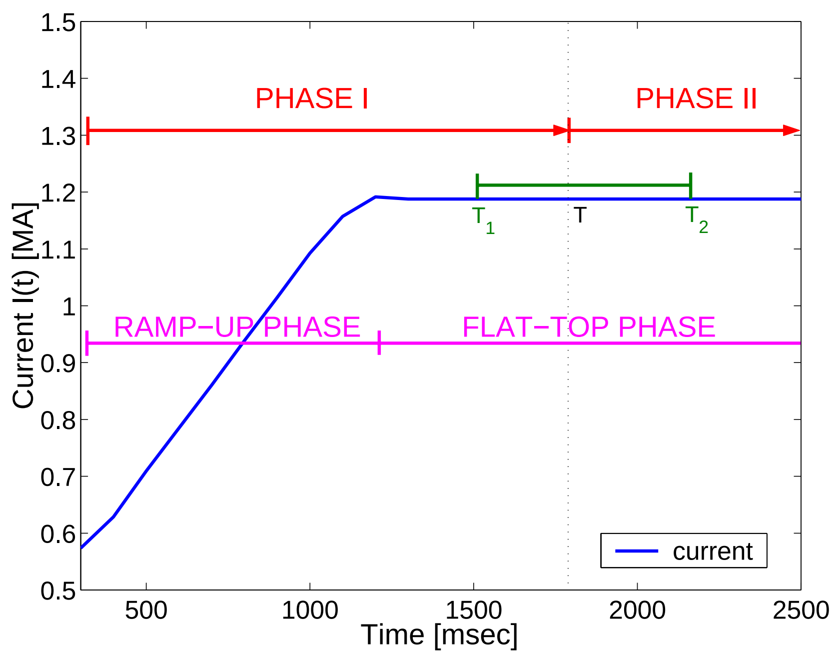
Closed-loop Finite-time Optimal Control

GOAL: Identical to open-loop control. During "Phase I" an optimal control problem must be solved, where time evolution for three actuators ($I(t)$, $\bar{n}(t)$ $(u_n(t))$, $P_{tot}(t)$) are sought to minimize the functional. Closed-loop control is expected to be more effective in dealing with model and IC uncertainties, and measurement noise.

$$J_{min} = \min_{t \in [T_1, T_2]} \left(J(t) \right)$$

where

$$J(t) = \int_0^1 \left(\iota(\hat{\rho}, t) - \iota^{des}(\hat{\rho}) \right)^2 d\hat{\rho}$$



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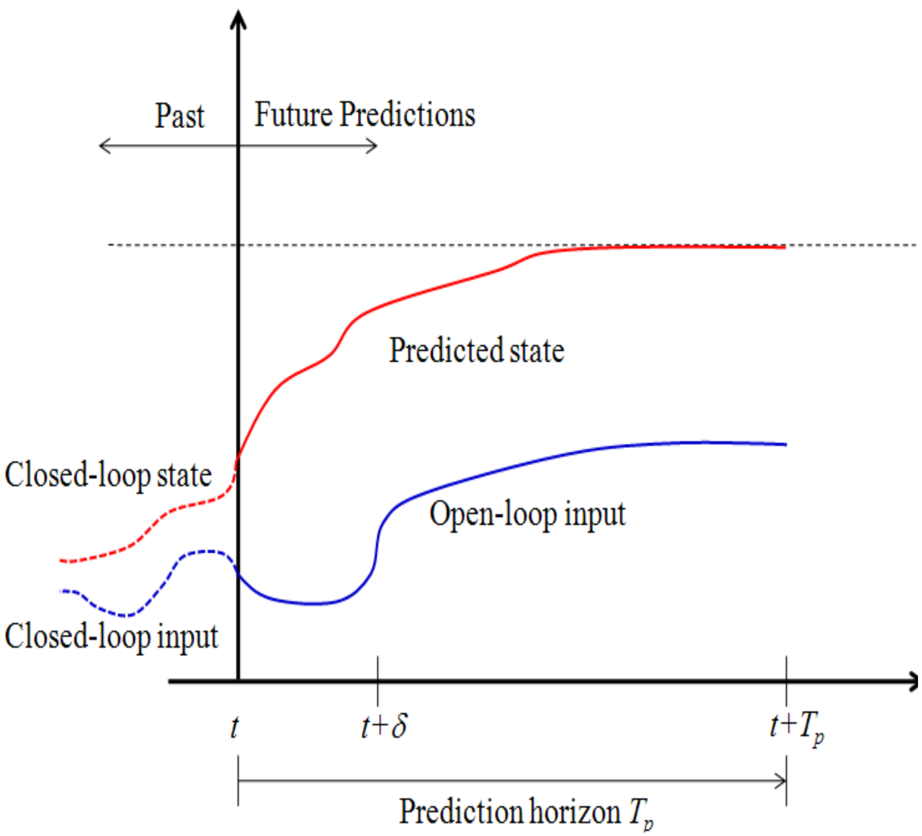
Current Profile Control: Phase I – Closed-loop

Receding-horizon closed-loop control



- Based on measurements obtained at time t, the controller uses the dynamic model to predict the future dynamic behavior of the system over a prediction horizon T_p, and determines the input such that a predetermined open loop performance objective functional is optimized subject to the system dynamics, and input and state costraints.
- In order to incorporate some feedback mechanism to compensate for disturbances and model-plant mismatch, the open-loop manipulated input function obtained is implemented only until the next measurement becomes available. The recalculation/measurement takes place every δ sampling time-units. Using the new measurement at time $t+\delta$, the entire procedure (prediction and optimization) is repeated to find a new input function with the prediction horizon moving forward.

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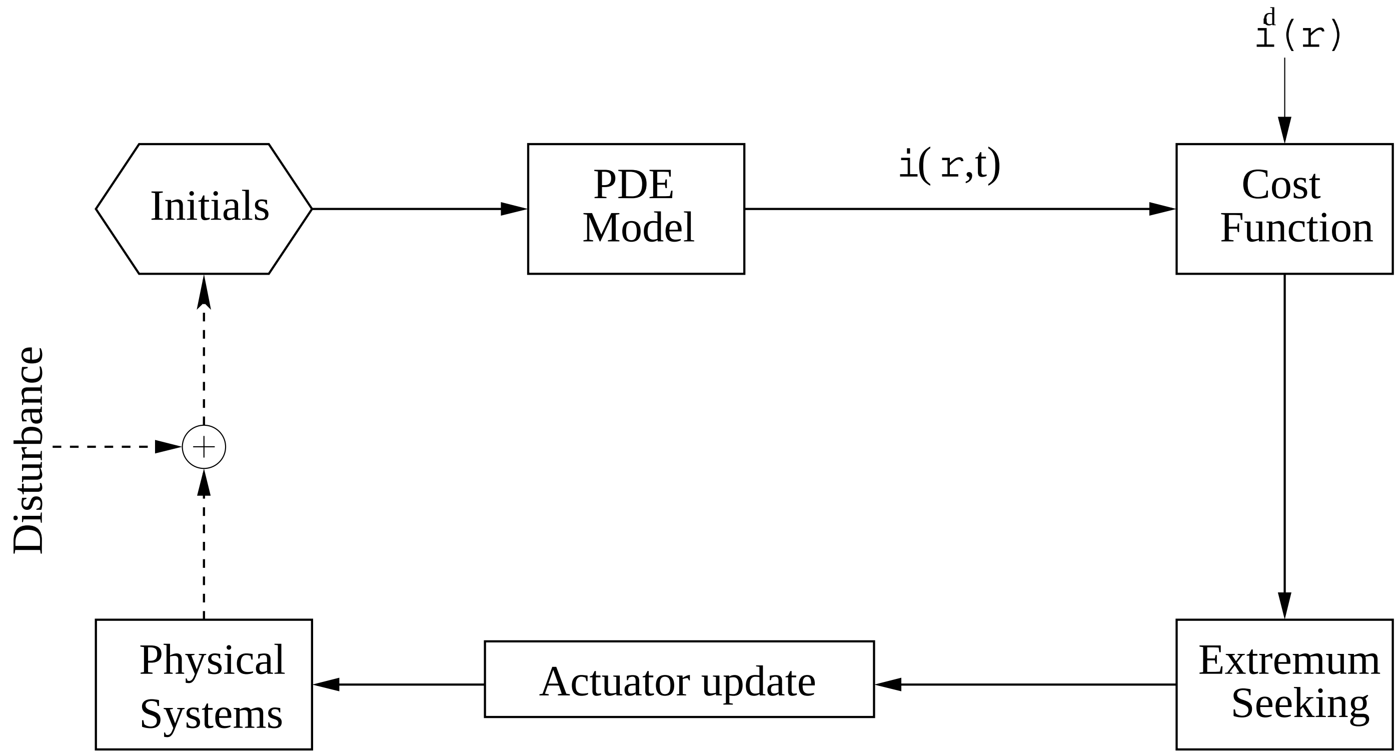
Current Profile Control: Phase I – Closed-loop



Receding-horizon closed-loop control scheme

This figure shows a closed-loop, receding-horizon, optimal controller based on an extremum-seeking optimization framework.

Limitation: Computational demand

Current Profile Control: Phase I – Closed-loop

1. Select the tolerance $\varepsilon > 0$ and the maximum number of iterations for the extremum seeking control algorithm.
2. Define $t_i = t_0$. Provide the off-line actuator trajectories $u(t)$, for $t > t_i = t_0$, and the actual initial poloidal flux profile $\psi(t_0)$ to the PDE model.
3. Compute the predicted $\iota(T)$ (control target) from the output sequence $\psi(t)$, for $t > t_i$, obtained from the PDE model.
4. Calculate the cost function. If it is less than ε, go to step 6.
5. Adjust the parameters θ (or $u(t)$) of the extremum seeking algorithm, until the cost function is less than ε or the maximum number of iteration is reached.
6. Implement the calculated actuator trajectories on the actual system for $[t_i + \Delta t, t_i + 2\Delta t]$.
7. Move the control horizon one sampling interval Δt ahead, measure the output of the actual system $\psi(t_i + \Delta t)$, make $t_i = t_i + \Delta t$, and go to step 3.

Model Reduction

$$\frac{\partial y}{\partial t} + f\left(y, \frac{\partial y}{\partial r}, \frac{\partial^2 y}{\partial r^2} \right) = 0$$

Infinite-dimensional model (weak form) + Galerkin Projection:

$$\int_0^1 \phi_i(r) \frac{\partial y}{\partial t} dr = \int_0^1 \phi_i(r) f\left(y, \frac{\partial y}{\partial r}, \frac{\partial^2 y}{\partial r^2} \right) dr$$

assuming $$y(t,r) = \sum_{j=1}^{l} \alpha_j(t) \phi_j(r)$$

Finite-dimensional model (low dimensional):

$$\frac{dz}{dt} + g(z,t) = 0 \qquad z = [\alpha_1, \cdots, \alpha_l]^T$$

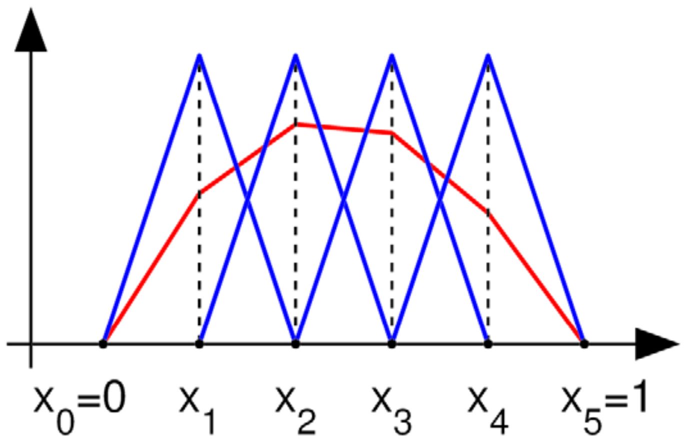
Model Reduction

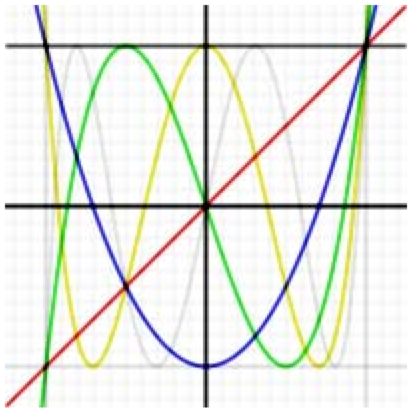
We seek reduced-order solutions

$$y(t,r) = \sum_{j=1}^{l} \alpha_j(t)\phi_j(r)$$

- Finite Element Method uses piecewise linear functions

- Pseudo-spectral Method uses orthogonal polynomials, e.g., Chebyshev





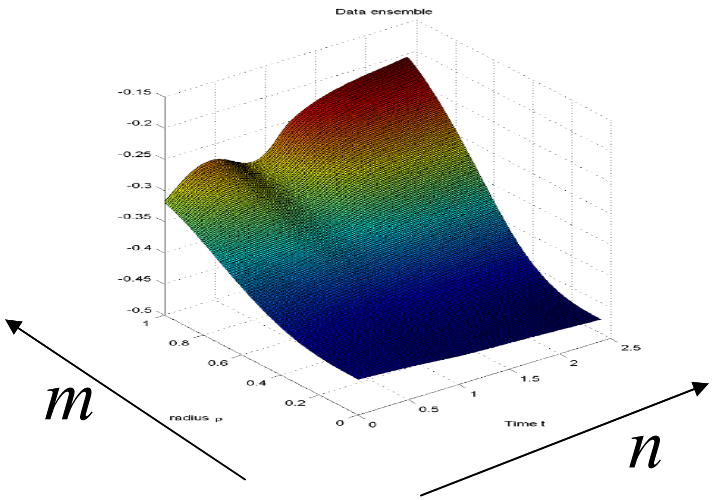
Which are the optimal basis functions?

Model Reduction

Data: $\upsilon = span\{y_1, \ldots, y_n\} \in R^m$

m : points in space

n : points in time



Orthonormal basis:

$$\{\phi_k\}_{k=1}^{d}, \qquad d = \dim \upsilon \leq m \qquad\qquad y_j = \sum_{k=1}^{d} (y_j, \phi_k)\phi_k, \qquad j = 1, \ldots, n.$$

Objective: Choose l out of this d eigenfunctions such that

$$\min_{\{\phi_k\}_{k=1}^{l}} \frac{1}{n} \sum_{j=1}^{n} \left\| y_j - \sum_{k=1}^{l} (y_j, \phi_k)\phi_k \right\|^2$$

$$s.t. \ (\phi_i, \phi_j) = \delta_{ij}, 1 \leq i \leq l, 1 \leq j \leq i.$$

$$where \ \|y\| = \sqrt{y^T y}$$

The solution of this problem is giving by the Proper Orthogonal Decomposition (POD) theory

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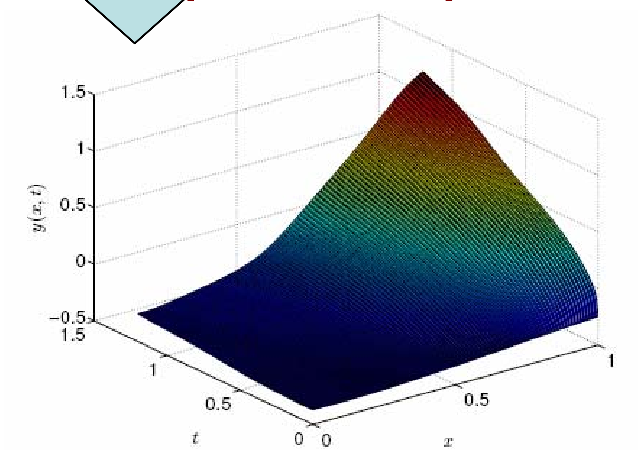
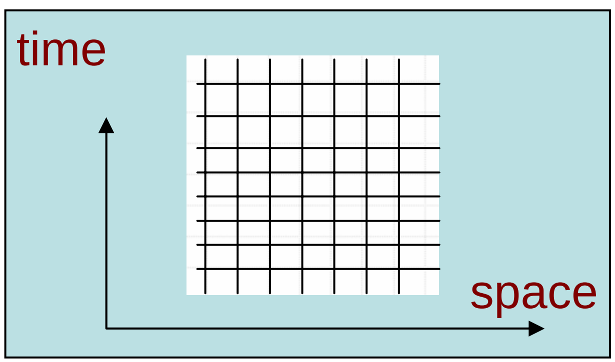
Model Reduction

$$\frac{\partial y}{\partial t} + f\left(y, \frac{\partial y}{\partial r}, \frac{\partial^2 y}{\partial r^2} \right) = 0$$

Numerical methods (FD/FE/SMs)





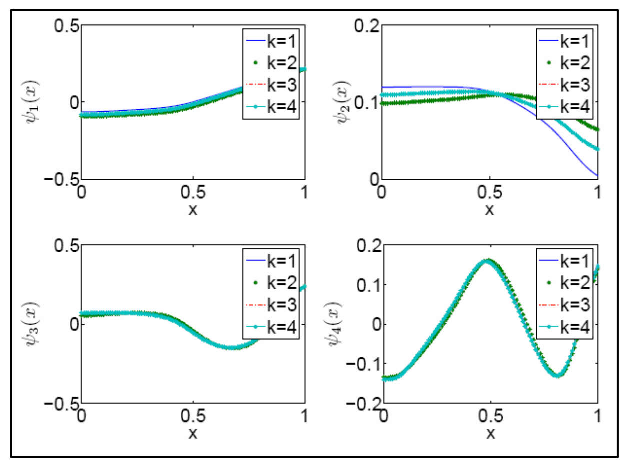
$$Y = \lfloor y(r_i, t_j) \rfloor_{m \times n},$$

$$Y' Y v_k = \lambda_k v_k,$$

$$\varphi_k = \frac{1}{\sqrt{\lambda_k}} Y v_k$$

POD-mode extraction

Data reconstruction



$$y(t,r) = \sum_{j=1}^{l} \alpha_j(t) \varphi_j(r)$$

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Current Profile Control: ROM Control

$$\frac{\partial y}{\partial t} = f_1(r)u_1(t)\frac{1}{r}\frac{\partial}{\partial r}\left(rf_4(r)\frac{\partial y}{\partial r}\right) + f_2(r)u_2(t), \left.\frac{\partial y}{\partial r}\right|_{r=0} = 0, \left.\frac{\partial y}{\partial r}\right|_{r=1} = u_3(t)$$

Infinite-dimensional model (weak form):

$$u_1 = \frac{\bar{n}}{I\sqrt{P}}, u_2 = \frac{\sqrt{P}}{I}, u_3 = I$$

$$\int_0^1 r\psi_i(r)\frac{\partial y}{\partial t}dr = \int_0^1 r\psi_i(r) f_1(r)u_1(t)\frac{1}{r}\frac{\partial}{\partial r}\left(rf_4(r)\frac{\partial y}{\partial r}\right)dr$$

$$+\int_0^1 r\psi_i(r) f_2(r)u_2(r)dr$$

assuming $$y(t,r) = \sum_{j=1}^{l}\alpha_j(t)\psi_j(r)$$

Finite-dimensional model (low dimensional):

$$M\frac{d\alpha}{dt} = A\alpha + A\alpha\hat{u}_1(t) + b\hat{u}_2(t) + c\hat{u}_3(t)$$

$$\hat{u}_1 = u_1, \hat{u}_2 = u_2, \hat{u}_3 = u_1u_3$$

Current Profile Control: ROM Control

Generalization of finite-dimensional model (low dimensional):

$$\dot{y} = Ay + Kyu_D(t) + Fu_I(t)$$

Cost function:

$$\min_{u_I,u_D} J = \frac{1}{2} y^T\left(t_f\right)Sy\left(t_f\right) + \frac{1}{2}\int_{t_0}^{t_f} y^T(t)Qy(t) + \left(r_I u_I^2(t) + r_D u_D^2(t)\right)dt$$

Hamiltonian + Optimality conditions:

$$H\left(y,u_I,u_D,p\right) = \frac{1}{2} y^T Qy + \frac{1}{2} r_I u_I^2 + \frac{1}{2} r_D u_D^2 + p^T(Ay + Kyu_D + Fu_I)$$

$$\frac{\partial H}{\partial u_I} = 0 \Rightarrow u_I = -r_I^{-1}F^T p, \frac{\partial H}{\partial u_D} = 0 \Rightarrow u_D = -r_D^{-1}\left(Ky\right)^T p$$

$$\dot{y} = \frac{\partial H}{\partial p} = Ay - Kyr_D^{-1}\left(Ky\right)^T p - Fr_I^{-1}F^T p, y\left(t_0\right) = y_0$$

$$\dot{p} = \frac{\partial H}{\partial y} = -Qy - A^T p + K^T pr_D^{-1}\left(Ky\right)^T p, p\left(t_f\right) = Sy\left(t_f\right)$$

Current Profile Control: ROM Control

Quasilinearization of optimality condition:

$$\dot{y}^{(k+1)} = Ay^{(k+1)} - Vp^{(k+1)} - G^{(k)}, y^{(k+1)}(t_0) = y_0$$

$$\dot{p}^{(k+1)} = -Qy^{(k+1)} - A^T p^{(k+1)} + H^{(k)}, p^{(k+1)}(t_f) = Sy^{(k+1)}(t_f)$$

$$H^{(k)} = r_D^{-1}\left(Ky^{(k)}\right)^T p^{(k)} K^T p^{(k)}$$

$$V = Fr_I^{-1}F^T = V^T$$

$$G^{(k)} = Ky^{(k)} r_D^{-1}\left(Ky^{(k)}\right)^T p^{(k)}$$

Solution: $p^{(k+1)} = Py^{(k+1)} + q^{(k+1)}, P^T = P$

$$\dot{P} = -PA - A^T P + PVP - Q, P(t_f) = S$$

$$\dot{q}^{(k+1)} = -(A - VP)^T q^{(k+1)} + PG^{(k)} + H^{(k)}, q^{(k+1)}(t_f) = 0$$

Current Profile Control: ROM Control

Closed-loop system:

$$\dot{y}^{(k+1)} = (A - VP)y^{(k+1)} - Vq^{(k+1)} - G^{(k)}, y^{(k+1)}(t_0) = y_0$$

Optimal Control:

$$u_I(t) = -r_I^{-1} F^T (Py + q^*)$$

$$u_D(t) = -r_D^{-1} (Ky)^T (Py + q^*)$$

$$\lim_{k \to \infty} q^{(k)} = q^*$$

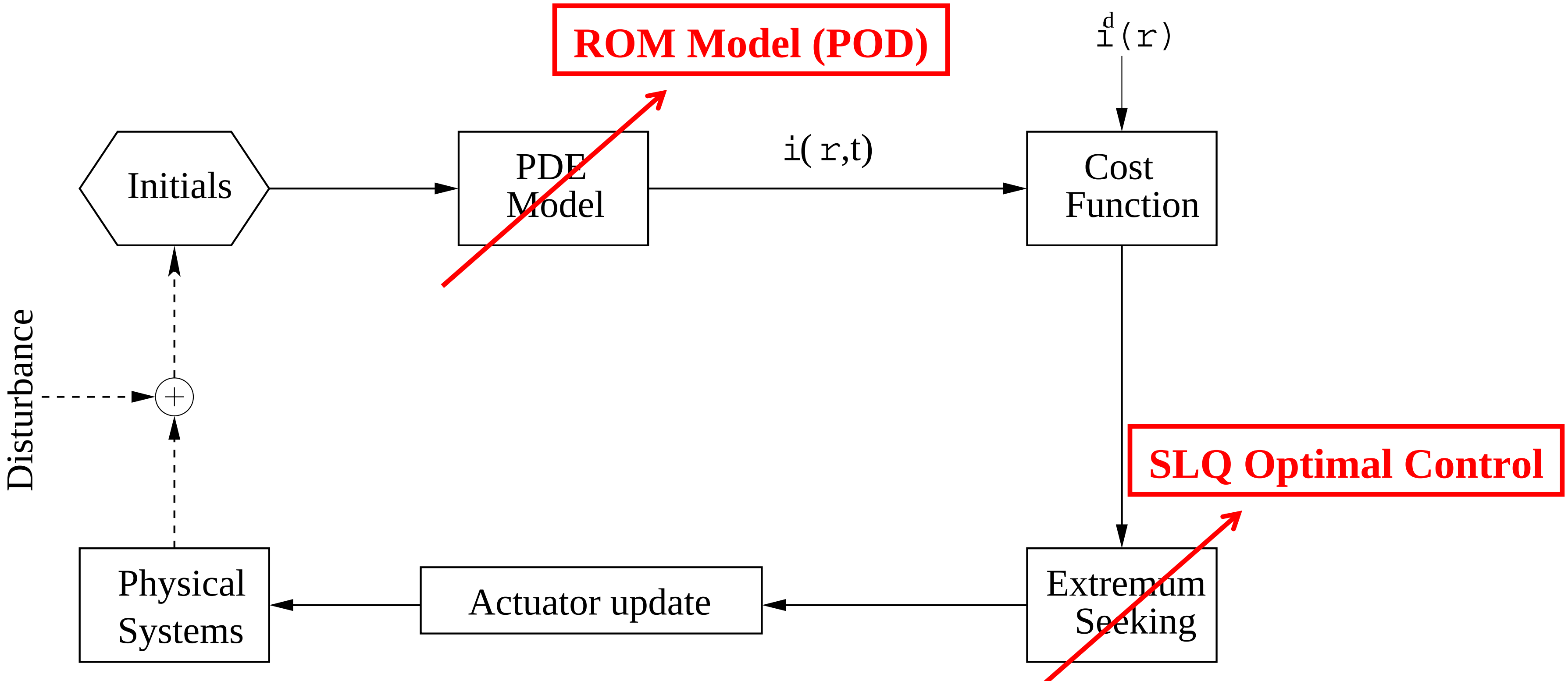
Convergence:

- Construction of a contraction mapping
- Fixed point theorem in Banach space

Caveat:

- The closed-loop control solution still depends on the IC
- A receding horizon control scheme is still necessary

Current Profile Control: CL-POD-SLQ-RH



Receding-horizon closed-loop control scheme

This figure shows a closed-loop, receding-horizon, optimal controller based on a proper-orthogonal-decomposition (POD) sequential-linear-quadratic (SLQ) optimization framework.

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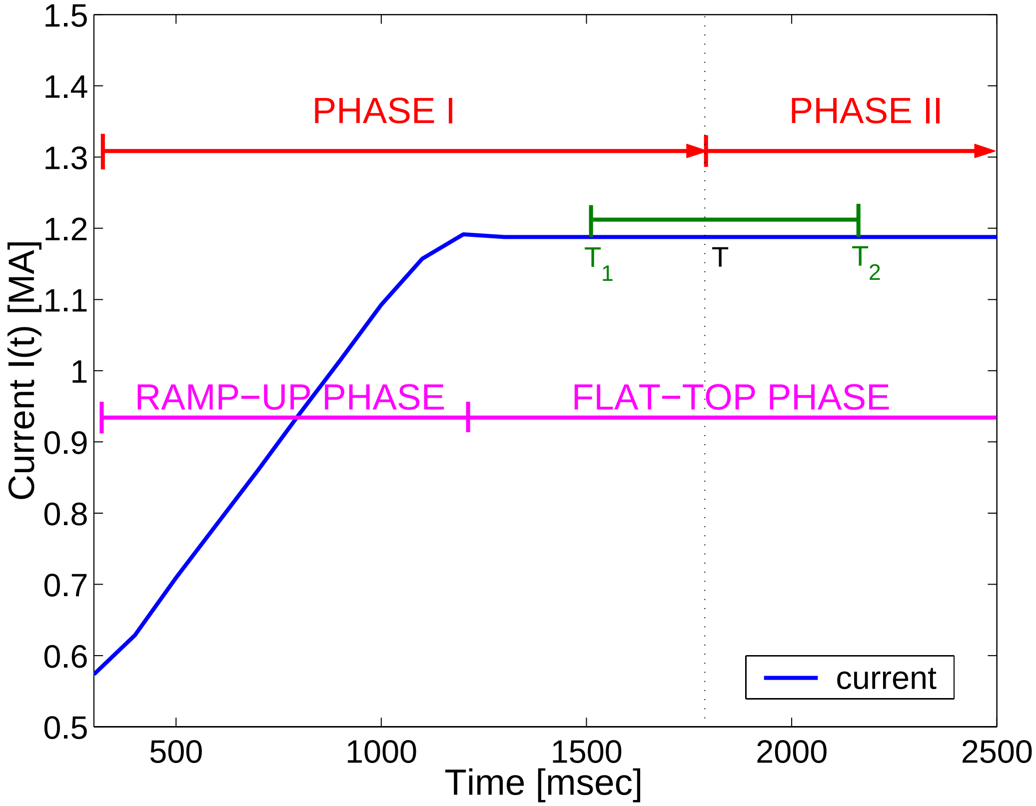
Profile Control: Objective

The phases of the discharge define the modeling and control objectives.



During "Phase I" the control goal is to drive the current/rotation profile from any arbitrary initial condition to a prescribed target profile at some time $T \in (T_1, T_2)$ in the flat-top phase of the total current $I(t)$ evolution. The prescribed target profile is not an equilibrium profile during "Phase I."

During "Phase II" the control goal is to regulate the current/rotation profile around a desired equilibrium profile.

"Phase I" → Mainly inductive

"Phase II" → Mainly non-inductive (Stronger magnetic/kinetic coupling)

Current Profile Control: Phase II

Regulation Control

GOAL: During "Phase II" a regulation control problem must be solved, where time evolution for three actuators ($I(t)$, $\bar{n}(t)$ $(u_n(t))$, $P_{tot}(t)$) are sought to regulate the profile around a desired profile.

First-principles Approach ➡ **Data-based Approach**

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Current Profile Modeling: Identification I

Model: $$\frac{\partial y}{\partial t} = \nabla\big(D(\nabla y, y)\nabla y\big) + S(r,t)$$

We are interested in identifying the unknown coefficient $D(\cdot)$ based on known experimental data $\tilde{y}$ and $\nabla\tilde{y}$

$$\min_{D(\cdot)} J = \|y - \tilde{y}\| + \|\nabla y - \nabla\tilde{y}\|$$

Nonlinear structure of the transport coefficient:

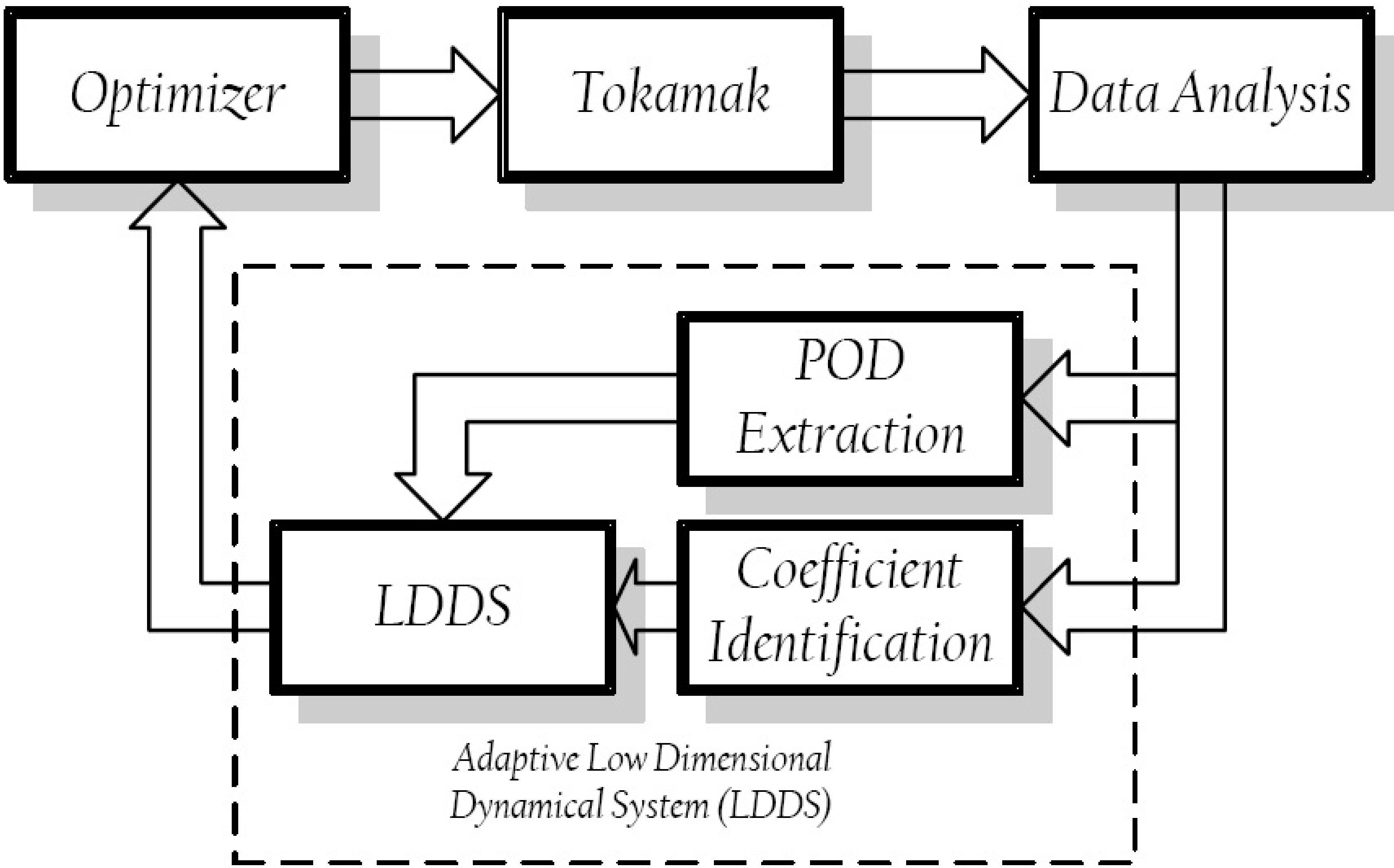
- Known from first principles
- Identified from data → Nonlinear regression

Assume (just an example): $$D(\nabla y, y) = D_0(r) + D_1(r)\nabla y + D_2(r)y$$

POD: $$\frac{\partial y}{\partial t} = \nabla\big(D(\nabla y, y)\nabla y\big) + S(r,t) \Rightarrow \frac{dz}{dt} = K(D)Z + L(D)u$$

D is the finite-dimensional representation of D_0, D_1, D_2 → Standard System Identification problem

Reduced Order Model for PDE Control



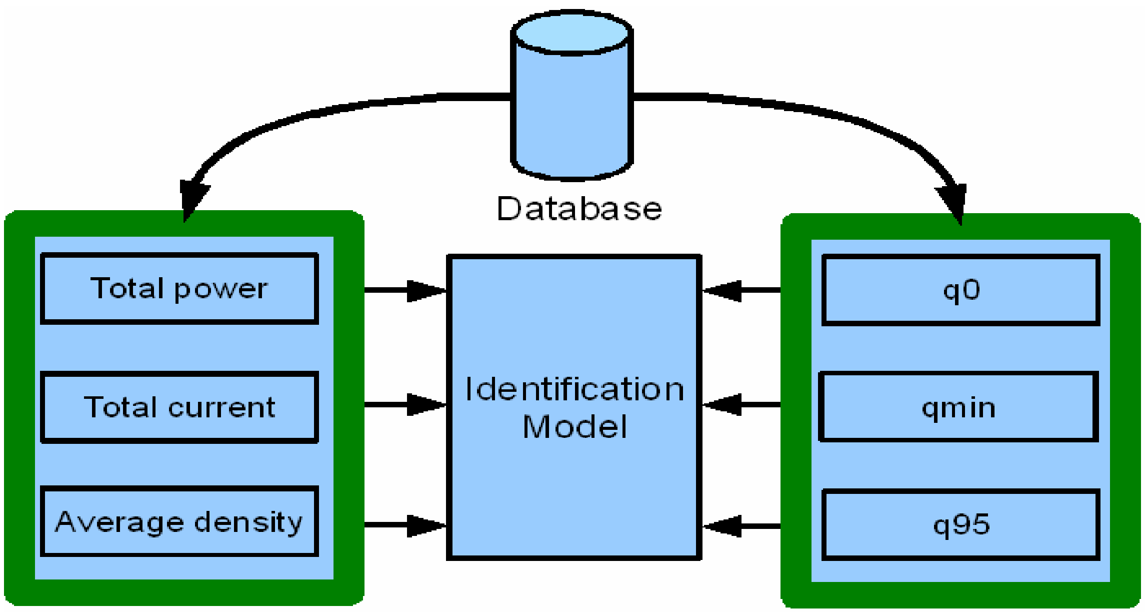
Current Profile Modeling: Identification II

Model: Unknown.

We are interested in identifying the whole realization

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Subspace Identification problem



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Current Profile Modeling: Identification II

- In general, the following linear model with unknown system matrices is used to model the input-output response obtained from diagnostics

$$x(k+1) = Ax(k) + Bu(k), \quad y(k) = Cx(k) + Du(k)$$

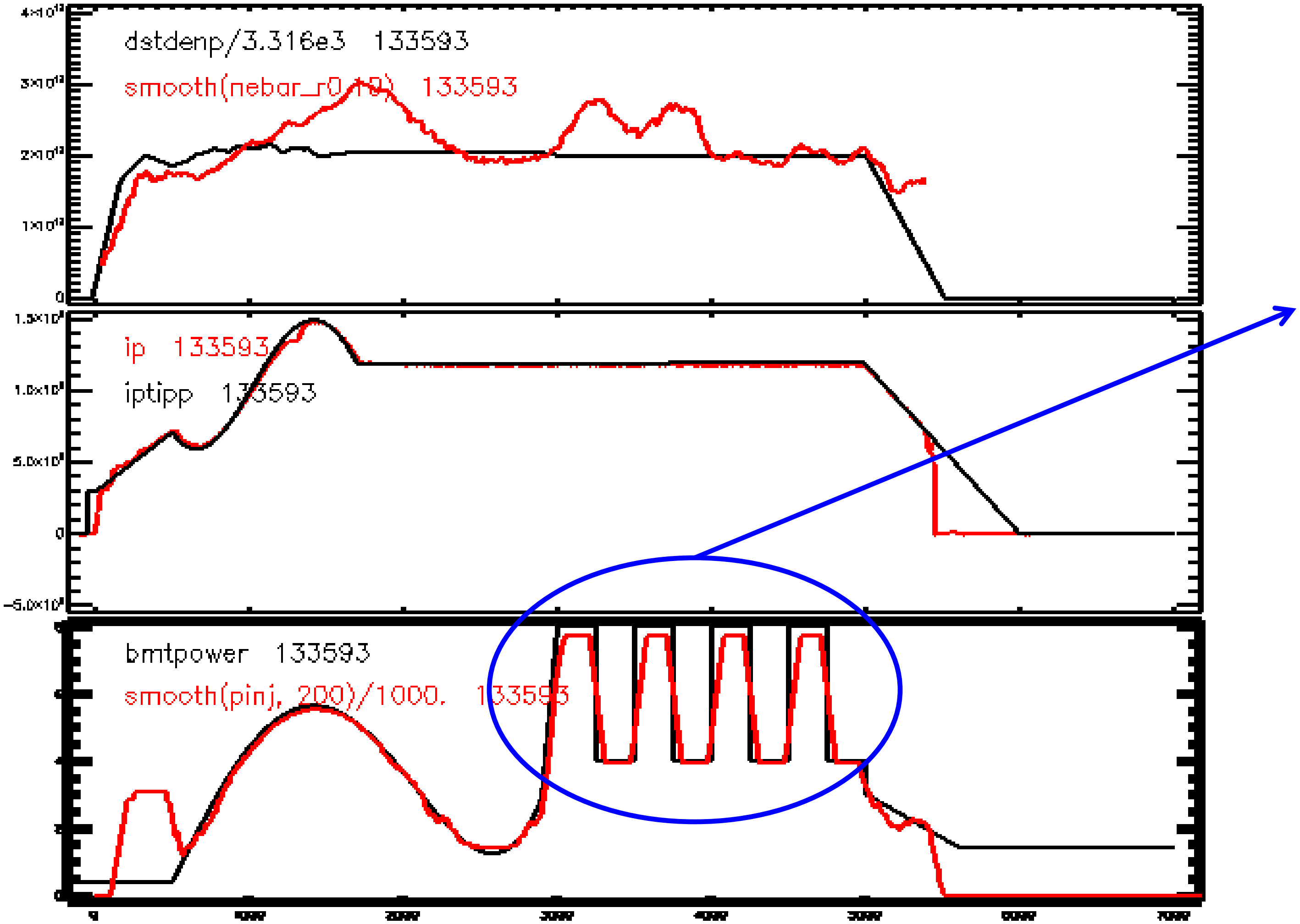
- The solution of the linear model can be obtained by recursive computation of the system equation with zero initial value:

$$x(k) = \sum_{i=1}^{k} A^{i-1} Bu(k-i), \quad y(k) = \sum_{i=1}^{k} CA^{i-1} Bu(k-i) + Du(k)$$

- Impulse response: Let $u(0)=1$ and $u(k)=0$, $k>0$, the response sequence is then given by $y(0) = D$, $y(1) = CB$, $y(2) = CAB$, …, $y(k) = CA^{k-1}B$
- Markov parameters are defined as $Y_0 = y(0) = D$, $Y_1 = y(1) = CB$, $Y_2 = y(2) = CAB$, …, $Y_k = y(k) = CA^{k-1}B$, which characterize the system dynamics of the linear model.
- We note $D = Y_0$, then a realization can be obtained by the computation of a triplet $\{A, B, C\}$ from the Markov parameters Y_1 , …, Y_k (the diagnostics of the impulse excitation experiment).

Current Profile Control: Identification II

Case 1:



Input excitation for system identification experiment

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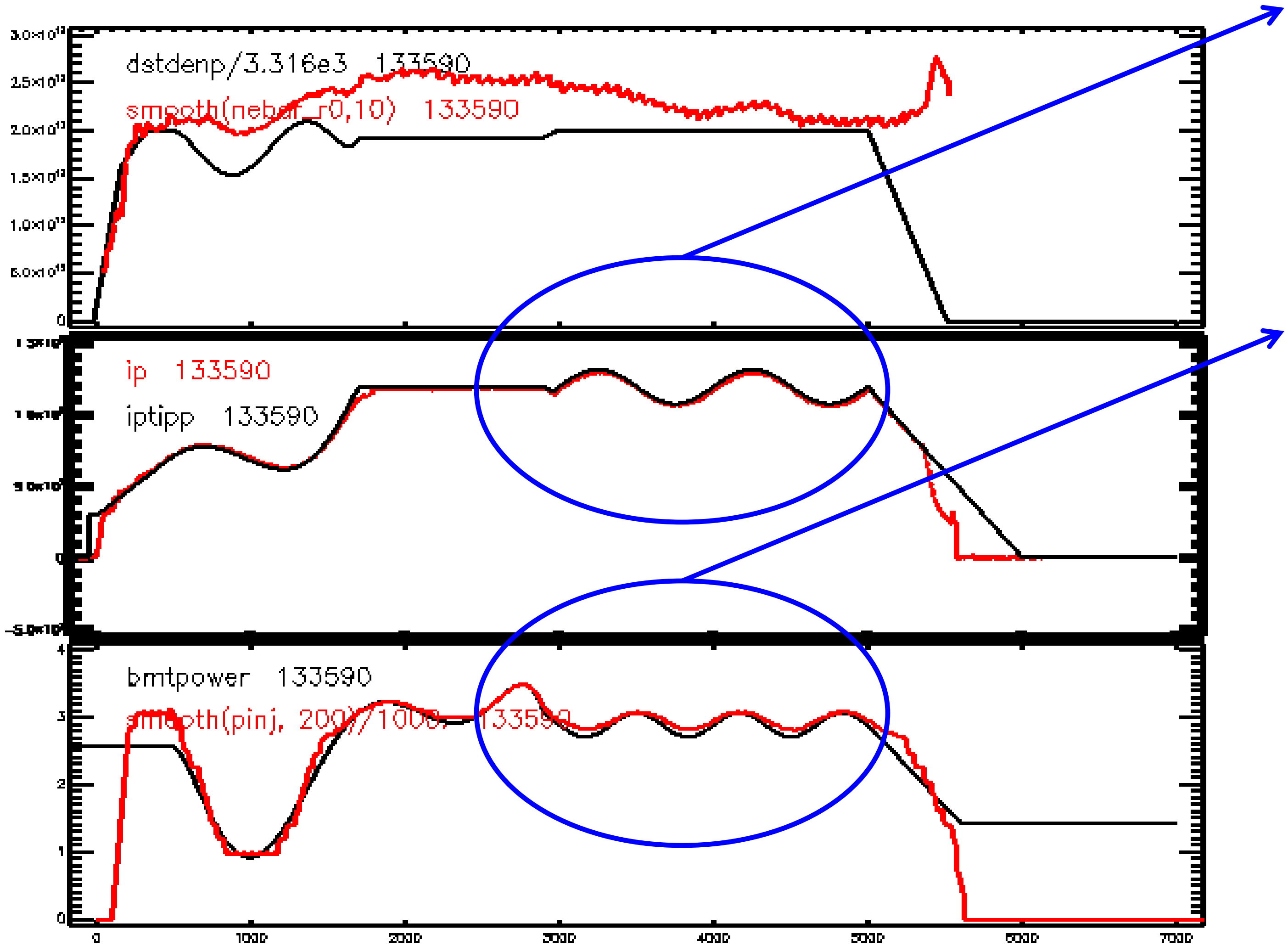
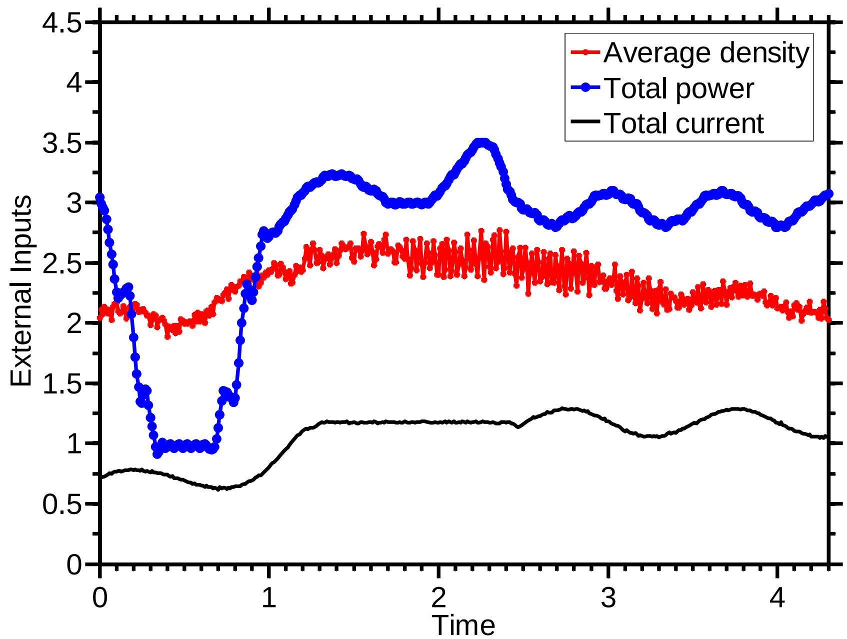
Current Profile Control: Identification II

Case 2:

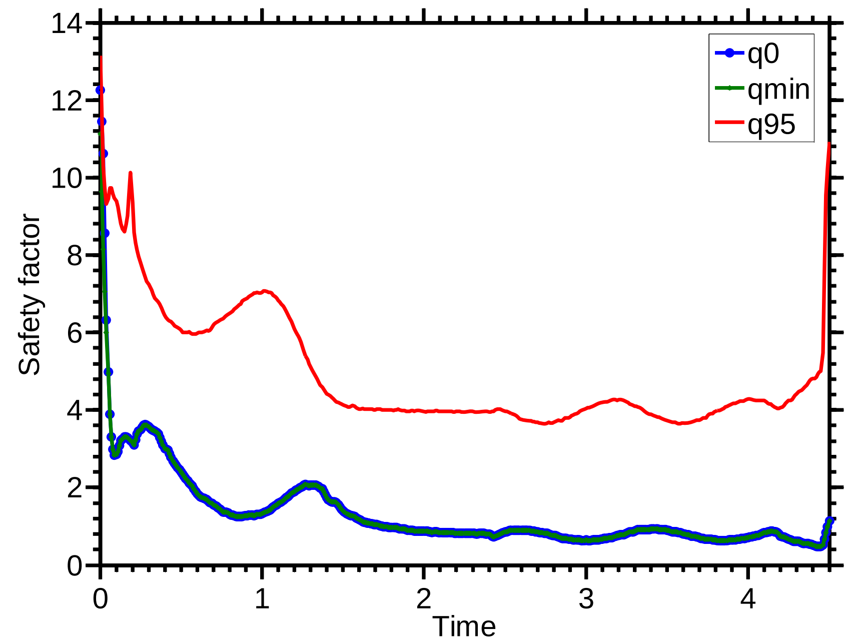


Input excitation for system identification experiment

Current Profile Modeling: Identification II



Actuator trajectories in discharge 133590



Safety factor in discharge 133590 ($q_{\min}$ is similar to q_0)

Inputs

Outputs

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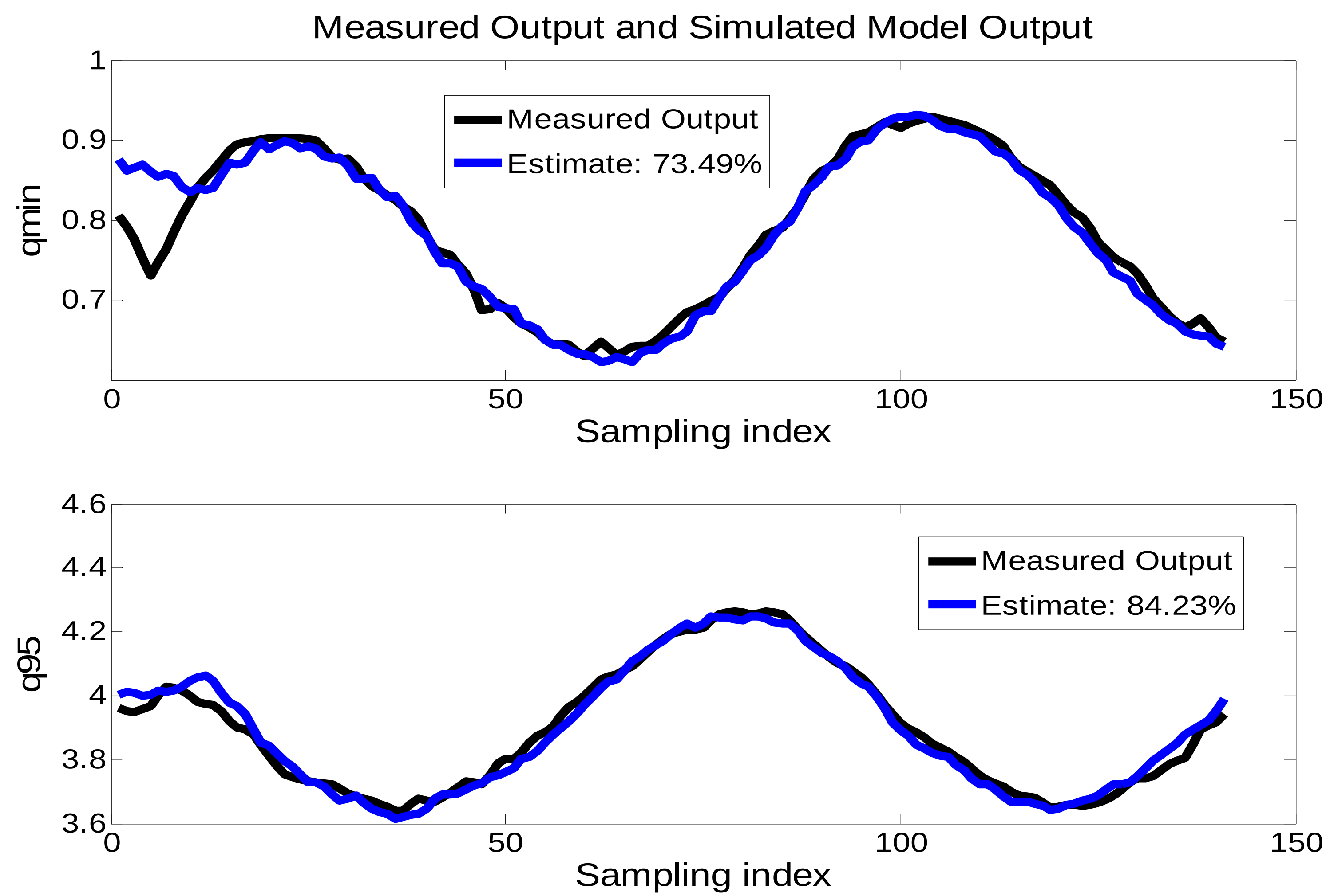
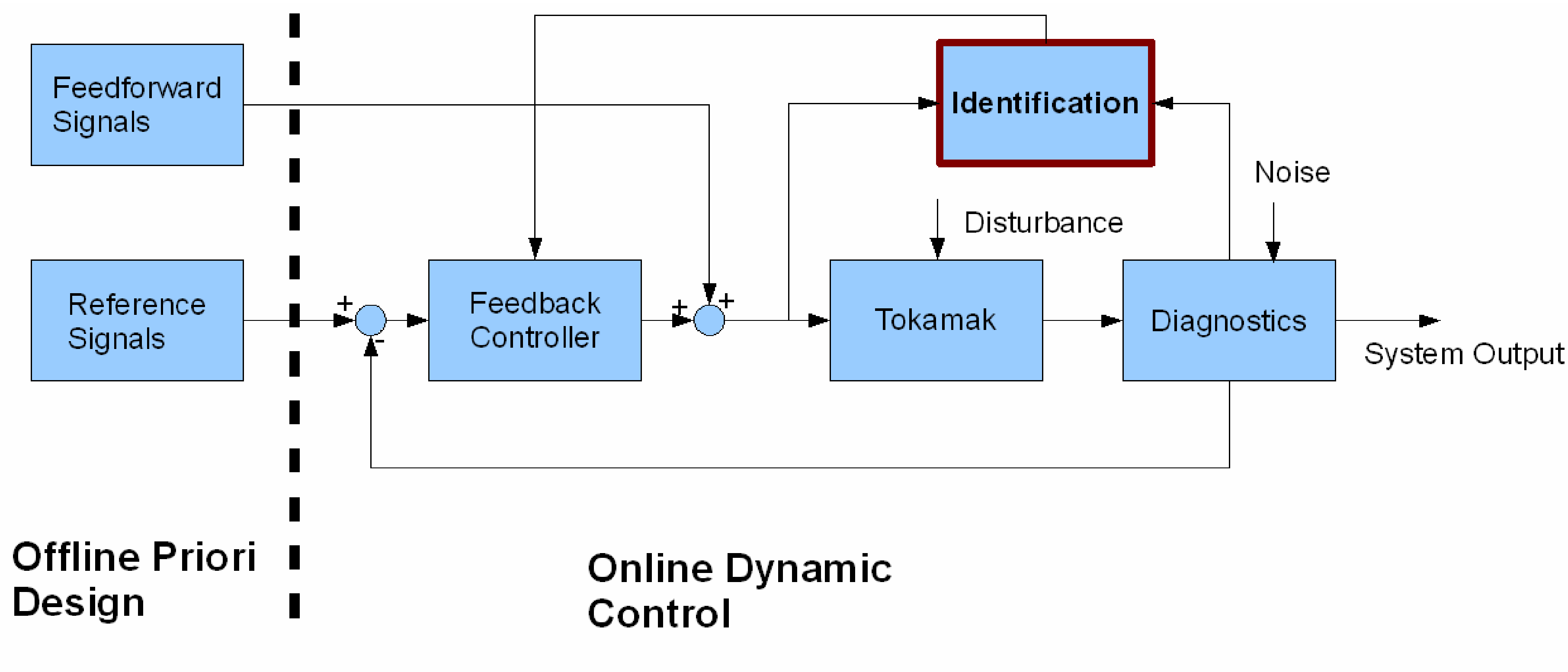
Current Profile Modeling: Identification II



Reduced Order Model for PDE Control



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Current Profile Control: Conclusions

- **Demonstrated existence of model-based solutions to the problem of defining actuator trajectories that achieve desired profiles during ramp-up (inductive) phase.**

- **Overall control combines open-loop + closed-loop optimal controllers.**
    - Open-loop control design admits highly complex models
    - Closed-loop control design requires simplified reduced-ordered models

This strategy is being extended to the non-inductive phase of the discharge.

- **Incorporation of predictive codes (CORSICA) for model/controller design/validation**
    - CORSICA in the loop for open-loop control design (ES, NLP)

- **Simplified scenario-oriented model development and validation**
    - Absolutely necessary for closed-loop control
    - The simpler the model, the faster the convergence for open-loop control

- **Reduced order modeling**
    - Necessary for closed-loop control (Bilinear Opt. Cont., Receding Hor. Cont.)
    - Useful for open-loop control design (ES, NLP)

- **Data-driven reduced-order modeling as an alternative to first-principle**
    - Black-box approach.
    - Gray-box approach.