Closed-loop direct parametric identification of magnetohydrodynamic spectrum in EXTRAP-T2R reversed-field pinch

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Overview

CLID

Closed-loop identification

A research field related to signal processing and automatic control for construction of models from measured data is *system identification*. The particular application to (partly) unknown systems in a feedback loop is *closed-loop identification*.

CLID

Closed-loop identification ...

Identification methods that only use plant input and output are known as *direct*. There are also *indirect* and *joint* approaches, that explicitly incorporate information on the feedback circuit.

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. . . for EXTRAP-T2R

A *direct parametric* method has been considered. Closed-loop operation stabilizes T2R but reduces observability: *dithering* applied.

CLID

Motivation

Determining MHD mode response to externally applied magnetic perturbations is beneficially performed in the closed-loop:

1. longer batches of data
2. transients sorted out
3. plasma stabilized; maintained equilibrium

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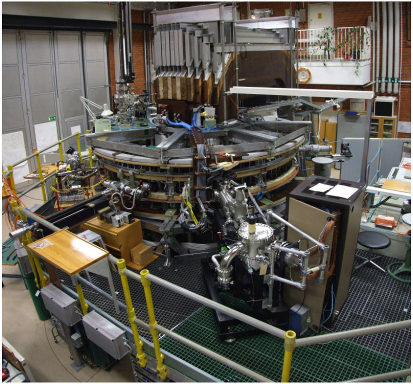
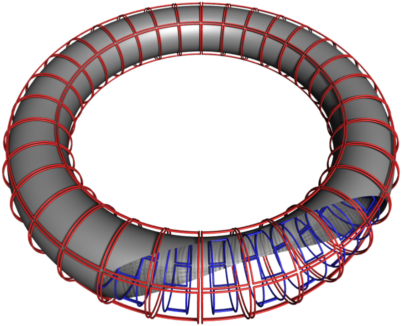
NB: Special care needed, some analysis methods *not applicable* in CLID due to feedback-induced correlation between output and input (via unmeasurable noise).

T2R recap





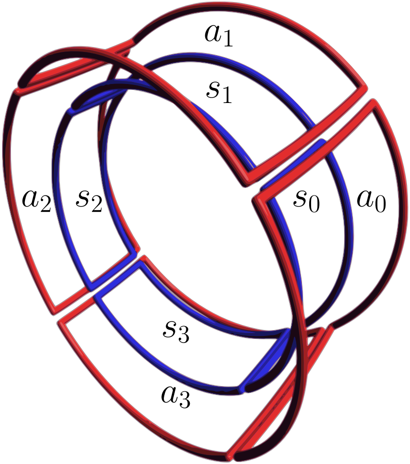
4 × 32 sensor coils & 4 × 32 active coils

T2R actuator & sensor array

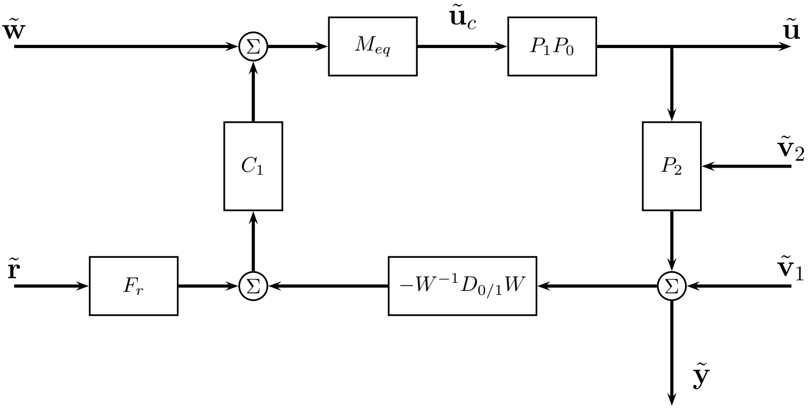
$$\begin{cases} \tilde{y}_i = \frac{1}{2}\left(s_{0,i} - s_{2,i}\right) \\ \tilde{y}_{i+32} = \frac{1}{2}\left(s_{1,i} - s_{3,i}\right) \end{cases}$$

$$\begin{cases} a_{0,i} = \frac{1}{2}\tilde{u}_i,\ a_{2,i} = -\frac{1}{2}\tilde{u}_i \\ a_{1,i} = \frac{1}{2}\tilde{u}_{i+32},\ a_{3,i} = -\frac{1}{2}\tilde{u}_{i+32} \end{cases} \quad (1)$$

for section $i = 1 \ldots 32$



T2R closed-loop stabilization



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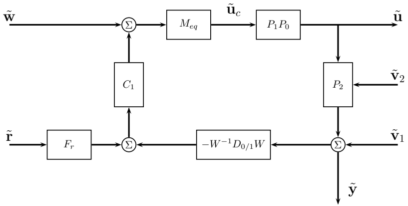

Basic direct CLID

Put $\tilde{\mathbf{r}} = \mathbf{0}$, randomly perturb vector $\tilde{\mathbf{w}}$ and forget (2) altogether, store $(\tilde{\mathbf{u}}, \tilde{\mathbf{y}})$.

Stabilizing feedback

$$\tilde{\mathbf{u}}_c = M_{eq} \left(C_1(s)(F_r(s)\tilde{\mathbf{r}} - W^{-1} D_{0/1} W \tilde{\mathbf{y}}) + \tilde{\mathbf{w}} \right) \tag{2}$$

Generic spatial aliasing

For $M_b \times N_b$ actuator and $M_c \times N_c$ sensor arrays

$$\tau_{m,n}\dot{x}_{m,n} = \hat{\gamma}_{m,n}x_{m,n} + b_{m,n}u_{p,q} + w_{m,n}, \text{ for } m = p + kM_b, n = q + lN_b$$
$$y_{r,s} = \sum_{k,l} c_{r+kM_c,s+lN_c}x_{r+kM_c,s+lN_c} + v_{r,s} \tag{3}$$

where (T2R) $\Delta\theta = \delta\theta = \frac{2\pi}{4}$, $\Delta\phi = \frac{2\pi}{32}$, $\delta\phi = \frac{2\pi}{64}$

$$b_{m,n} \sim \frac{n}{m}\sin(m\Delta\theta/2)\sin(n\Delta\phi/2) \times K'_m(\frac{|n|r_c}{R})I'_m(\frac{|n|r_w}{R}) \tag{4}$$
$$c_{m,n} \sim \frac{1}{mn}\sin(m\delta\theta/2)\sin(n\delta\phi/2) \tag{5}$$

and $u_{p,q}$, $y_{r,s}$ are DFT transformations of $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{y}}$.

A minimalistic truncation for T2R

Assemble the discrete-time linear system with state $\mathbf{x} \in \mathbb{R}^2$

$$\begin{aligned} \mathbf{x}(k+1) &= A(\bar{\theta}_n)\mathbf{x}(k) + B(\bar{\theta}_n)u(k) + \mathbf{w}(k) \\ y(k) &= C(\bar{\theta}_n)\mathbf{x}(k) + v(k) \end{aligned} \tag{6}$$

having parameterization $A(\bar{\theta}_n) = \begin{pmatrix} e^{\hat{\gamma}_n T_s/\tau_n} & 0 \\ 0 & e^{-\zeta_n T_s/\tau_n} \end{pmatrix}$,

$B(\bar{\theta}_n) = \begin{pmatrix} \frac{\alpha_n}{\hat{\gamma}_n}\left(e^{\hat{\gamma}_n T_s/\tau_n} - 1\right) \\ \frac{\alpha_n}{\eta_n}\left(1 - e^{-\zeta_n T_s/\tau_n}\right) \end{pmatrix}$, $C(\bar{\theta}_n) = \begin{pmatrix} 1 & 1/\xi_n \end{pmatrix}$ where

$\eta_n = b_{1,n}/b_{3,n}$, $\zeta_n = c_{1,n}/c_{3,n}$, $\xi_n = \tau_{1,n}/\tau_{3,n}$, $\bar{\theta}_n = \{\alpha_n, \tau_n, \hat{\gamma}_n\}$

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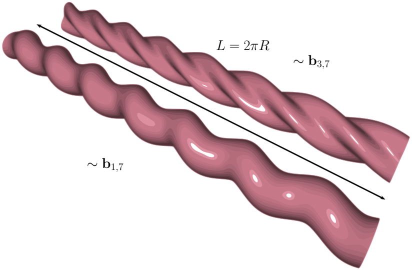
Key truncation & approximation

i) only include $m = 1$ and $m = 3$ for arbitrary $n = -16 \ldots +15$ and *ii)* prescribe $\hat{\gamma}_{m=3,n} = -1$, which theoretically is close to true.

A minimalistic truncation for T2R, continued



Phenomenological representation

A potentially unstable mode ($m = 1$) possibly camouflaged behind a fast stable mode ($m = 3$).

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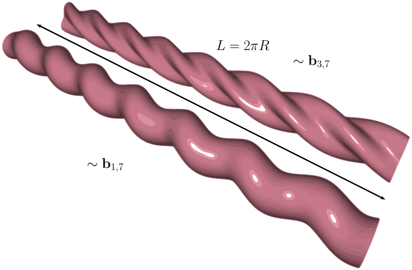

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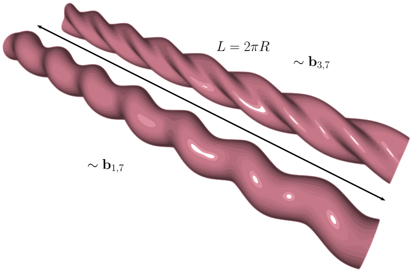


Phenomenological representation

A potentially unstable mode ($m = 1$) possibly camouflaged behind a fast stable mode ($m = 3$). Model (6) is a compact packaging of this, using three parameters to be experimentally identified, of which *only one* is attributed to the plasma.

Pseudorandom binary sequence (PRBS)

Employ a computationally efficient identification vector input generator

$$z_j(k) = \mathrm{rem}\left(\sum_{l=1}^{n} a_l q^l z_j(k), 2\right) \tag{7}$$

$$\tilde{w}_j^{(0)}(k) = \kappa\left\{2\mathrm{rem}\left(z_j(k), 2\right) - 1\right\},\ j = 1 \ldots 64$$

$$\tilde{w}_j(k) = \begin{cases} \tilde{w}_j^{(0)}(k) - \frac{1}{32}\sum_{i=1}^{32} \tilde{w}_i^{(0)}(k),\ j = 1 \ldots 32 \\ \tilde{w}_j^{(0)}(k) - \frac{1}{32}\sum_{i=33}^{64} \tilde{w}_i^{(0)}(k),\ j = 33 \ldots 64 \end{cases} \tag{8}$$

The identification program

Prediction-error minimization program

$$\bar{\theta}^{\star} = \arg\min_{\bar{\theta}} V(\bar{\theta}) \quad (9)$$

where $V(\bar{\theta}) \equiv \frac{1}{N}\sum_{k=1}^{N} e^2(k,\bar{\theta})$ and the *prediction errors* $e(k,\bar{\theta}) \equiv y(k) - \hat{y}(k,\bar{\theta})$ are produced by a *predictor* $\hat{y}(k,\bar{\theta}) = f\left(\bar{\theta}, \{y(l), u(l)\}_{l=0,\ldots,k-1}\right)$

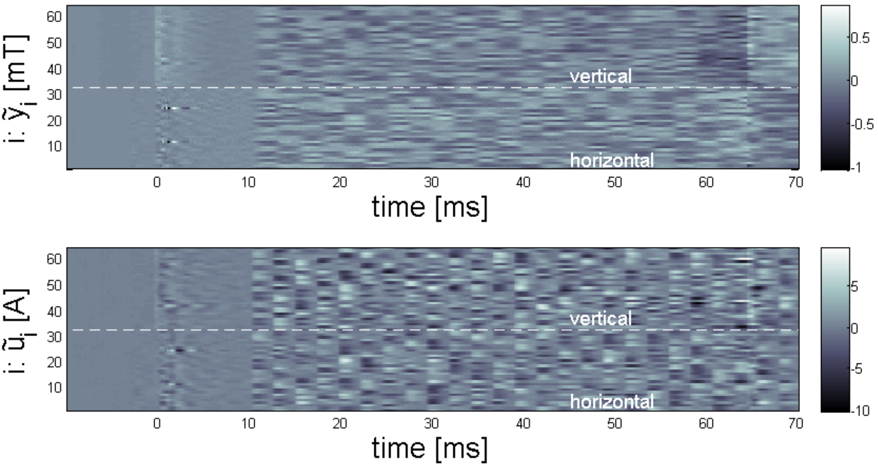
A kalman predictor

$$\begin{cases} \hat{\mathbf{x}}^-(k) = A\hat{\mathbf{x}}^+(k-1) + Bu(k-1) \\ P^-(k) = AP^+(k-1)A^T + Q \end{cases}$$

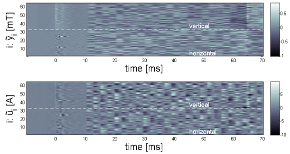
$$\begin{cases} e^-(k) = y(k) - C\hat{\mathbf{x}}(k)^- \\ K(k) = P^-(k)C^T\left(CP^-(k)C^T + R\right)^{-1} \\ \hat{\mathbf{x}}^+(k) = \hat{\mathbf{x}}^-(k) + K(k)e^-(k) \\ P^+(k) = (I - K(k)C)\,P^-(k) \end{cases} \quad (10)$$

takes input data $u(k)$, $y(k)$ and outputs $e^-(k) = e(k,\bar{\theta})$

Dithering shot #21716, visually



Spatial DFT → SISO PEM programs



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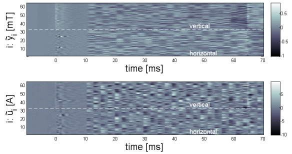

Sort-out normal modes

Recorded data interpreted assuming *independent* Fourier modes.

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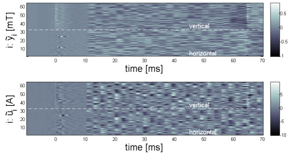

Sort-out normal modes

Recorded data interpreted assuming *independent* Fourier modes.

With $\iota^2 = -1$ and for $n = 0 \ldots 31$ set

$$Y_n(t) = \sum_{k=0}^{31} e^{-2\pi\iota\frac{nk}{32}} \left(\tilde{y}_{k+1}(t) + \iota\tilde{y}_{k+33}(t)\right)$$

$$y_{n+1}(t) = \mathrm{Re}\; Y_n(t),\; y_{n+33}(t) = \mathrm{Im}\; Y_n(t)$$

implicitly defining a DFT matrix $W \in \mathbb{R}^{64\times 64}$ with the properties

$$\mathbf{y} = W\tilde{\mathbf{y}},\; \tilde{\mathbf{u}} = W^{-1}\mathbf{u} \qquad (11)$$

Spatial DFT → SISO PEM programs

$\mathbf{y} = W\tilde{\mathbf{y}},\ \tilde{\mathbf{u}} = W^{-1}\mathbf{u}$

Analyze according to (9)-(10) data $(y_i, u_i) \forall i = 1 \ldots 64$ independently as SISO systems (sine/cosine components of the Fourier modes $n = -16 \ldots +15$)

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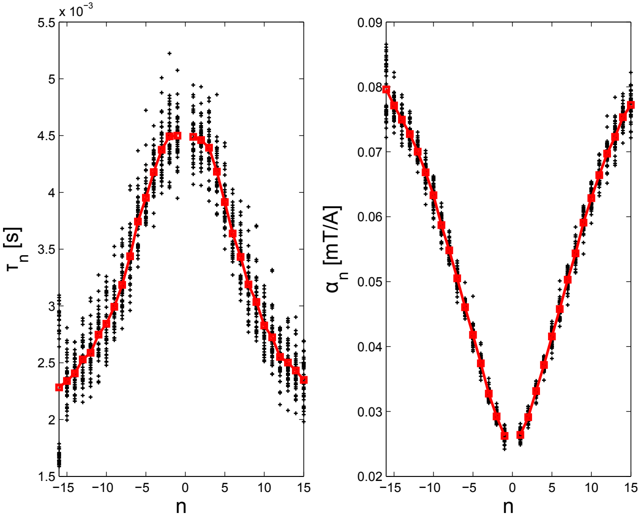
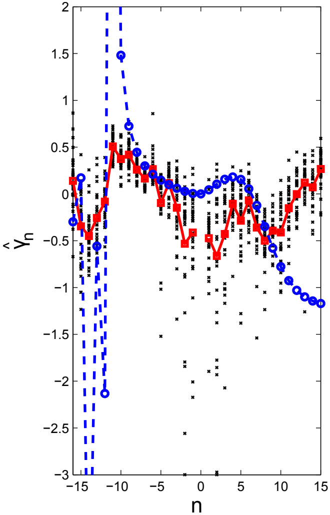
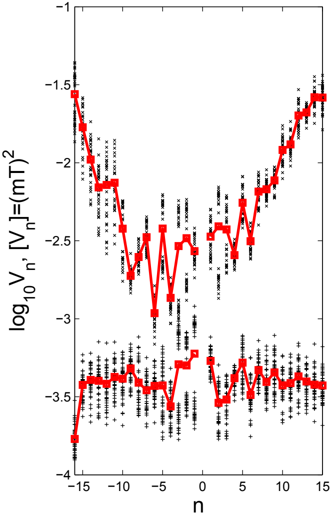
1. Calibrate (vacuum batches): $\alpha_n^\star$, $\tau_n^\star$
2. Spectrum (plasma batches): given $\alpha_n^\star$, $\tau_n^\star$, minimize w.r.t. $\hat{\gamma}$, i.e. find $\hat{\gamma}_n^\star$

Dry plant (vacuum) calibration parameters

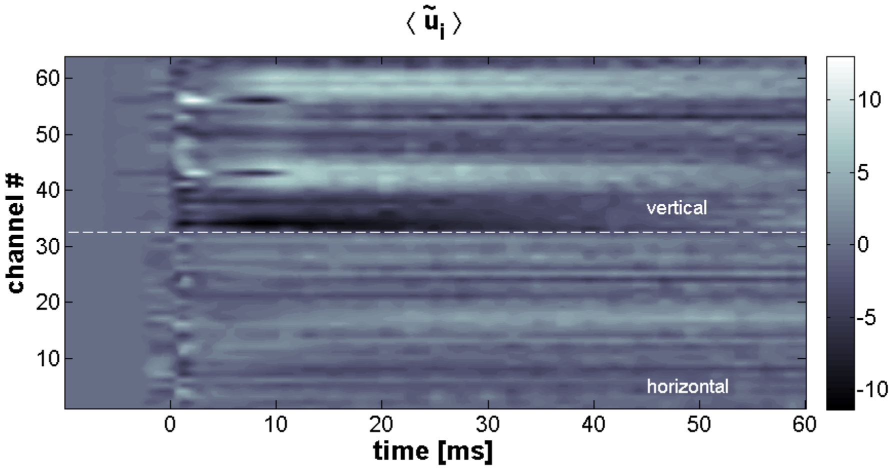


T2R spectrum & prediction-errors





Reproducible machine error-field



Some notes on first results

overall : instability and RFP spectrum shape do ‘emerge’ from ‘obfuscated’ data!

error-field : time-resolved ‘negative’ as a by-product of signal preprocessing

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higher $\|n\|$	: model prediction-error *larger*
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TODO

- Truncate model at a higher order and/or in a more clever way
- Sophistication of error-field separation

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- Data-driven modeling of RWM dynamics assessed
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- Promising and useful for further development of control system, among other things

Thank you