Recent HBT-EP Kink Mode Control Results

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for

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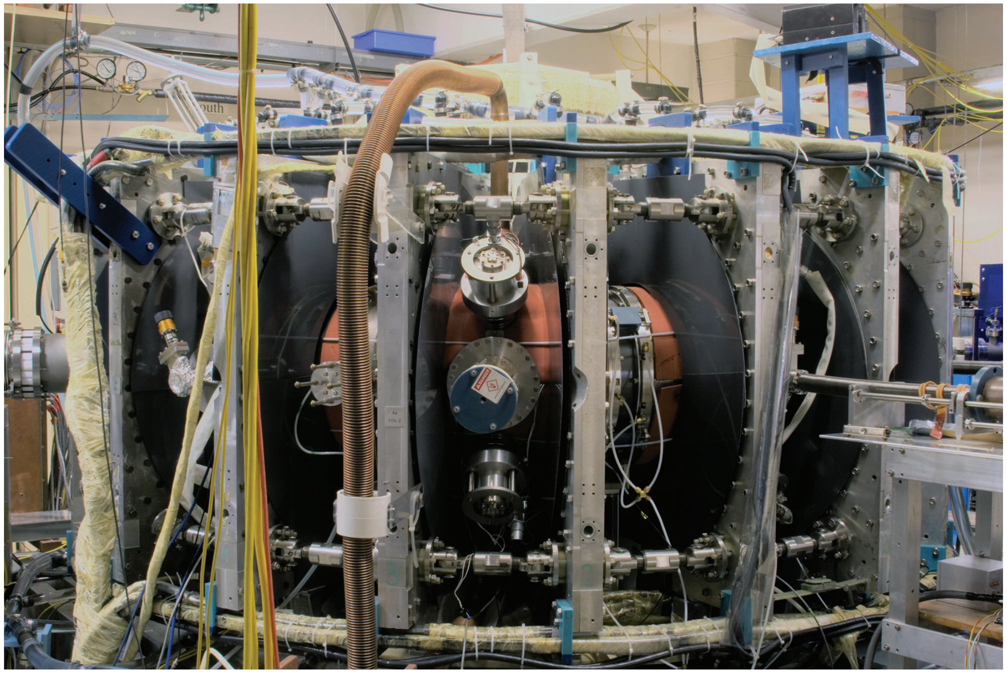
HBT-EP

Previous results from HBT-EP

HBT-EP has been used to study external kink and resistive wall modes for over a decade.



- The effect of a close-fitting conducting wall on external kink stability was investigated as early as 1996 [Ivers, *et. al.*, 1996].
- Next, feedback suppression of the external kink mode was demonstrated using radial sensors and radial control coils [Cates, *et. al.*, 2000].

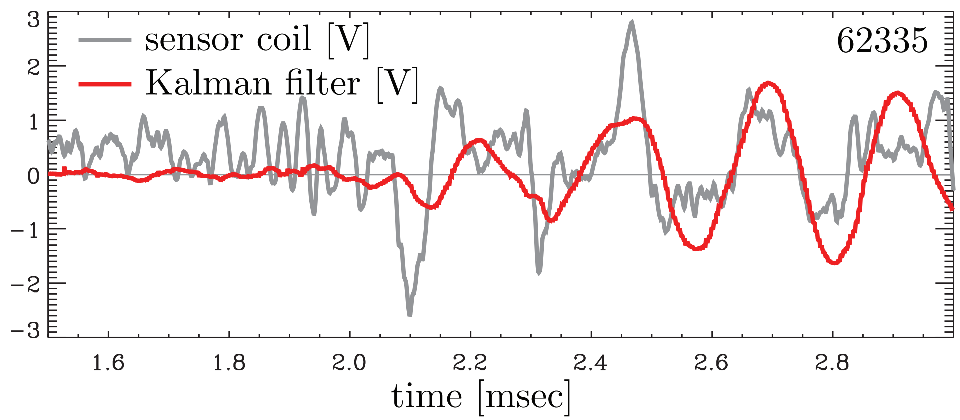
- This coil set was also used to apply resonant perturbations to plasmas near the marginal stability limit. Agreement with the Fitzpatrick-Aydemir model in the high-dissipation limit was found [Shilov, *et. al.*, 2004].
- Feedback suppression of the external kink *near the ideal wall limit* was achieved using the present network of poloidal sensor coils and radial control coils. [Klein, *et. al.*, 2005].

New results

- A **Kalman filter** algorithm has been implemented on a set of low-latency digital feedback controllers, and used to **excite** and **suppress** the $(m, n) = (3, 1)$ external kink instability on HBT-EP.
- The Kalman filter uses a simple, internal model for a growing, rotating mode.
- **Excellent performance** was achieved with initial guesses for filter model parameters; performance was a bit better when the parameters were optimized.
- Kalman filter feedback **works** under **noisy conditions** that disrupt feedback with conventional algorithms.

Motivation: tokamaks are noisy

- We are interested in using **magnetic feedback** to suppress the tokamak **external kink instability**.
- The external kink is a **rotating, helical perturbation** to the plasma's surface and magnetic field. As it rotates by a magnetic pickup coil, oscillations are observed.
- However, there's a lot of noise in measurements due to other magnetic activity: **tokamaks are noisy!**
- **Noise can impair feedback**, but an advanced filtering algorithm called the Kalman filter can come to the rescue.



- Future burning plasma experiments will likely need advanced filtering to optimize feedback in the presence of edge-localized modes and other MHD noise.

Outline

Simulating external kink mode feedback

The Kalman filter

HBT-EP diagnostics and control hardware

Experimental results

Conclusions and future work

Simulating external kink mode feedback

The reduced Fitzpatrick-Aydemir model is used to simulate feedback

- The reduced Fitzpatrick-Aydemir equations[a,b] are used to simulate the $(m, n) = (3, 1)$ external kink mode.[c]
- The Fitzpatrick-Aydemir model has been shown to accurately characterize experimental observations on HBT-EP.[d]
- Growing, rotating, $n = 1$ plasma and wall modes are produced.
- The stabilizing effects of viscous damping and plasma rotation are accounted for.
- Coupling physics for the wall and feedback coils is included from a `VALEN`[e] model for HBT-EP.
- Noise can be added to measurements of the mode, in the form of a random amplitude and phase.
- It's interesting to see how noise affects feedback!

[a] R. Fitzpatrick and A. Y. Aydemir, *Nucl. Fusion* **36**, 11 (1996).
[b] R. Fitzpatrick, *Phys. Plasmas* **9**, 3459 (2002).
[c] M. E. Mauel, *et. al.*, *Nucl. Fusion* **45**, 285 (2005).
[d] M. Shilov, *et. al.*, *Phys. Plasmas* **11**, 2573 (2004).
[e] J. Bialek, *et. al.*, *Phys. Plasmas* **8**, 2170 (2001).

The reduced Fitzpatrick-Aydemir model is used to simulate feedback

- The reduced Fitzpatrick-Aydemir equations[a,b] are used to simulate the $(m, n) = (3, 1)$ external kink mode.[c]
- The Fitzpatrick-Aydemir model has been shown to accurately characterize experimental observations of the RWM on HBT-EP.[d]
- Growing, rotating, $n = 1$ plasma and wall modes are produced.
- The fluxes at the plasma and the wall are given as a function of plasma parameters, coupling parameters and a control flux.

$$\frac{\mathrm{d}\vec{y}}{\mathrm{d}t} = A\vec{y} + \vec{R}\psi_c,$$

where

$$\vec{y} = \begin{pmatrix} \psi_a \\ \psi_w \end{pmatrix}, \quad A = \begin{pmatrix} (1 - \bar{s} - i\bar{\alpha})\frac{\gamma_{mhd}^2}{\nu_d} & -\frac{\gamma_{mhd}^2}{\nu_d\sqrt{c}} \\ \frac{\gamma_w\sqrt{c}}{1-c} & -\frac{\gamma_w}{1-c} \end{pmatrix}, \quad \text{and} \quad \vec{R} = \begin{pmatrix} \frac{-c_f\gamma_{mhd}^2}{\nu_d} \\ \frac{\gamma_w(1-cc_f)}{1-c} \end{pmatrix}$$

With the normalized stability parameter $\bar{s} = 1.0$ and the rotation-dissipation parameter $\bar{\alpha} = -\nu_d\Omega/\gamma_{mhd}^2 = -1.41$ (corresponding to a rotation rate of 5 kHz).

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[a] R. Fitzpatrick and A. Y. Aydemir, *Nucl. Fusion* **36**, 11 (1996).
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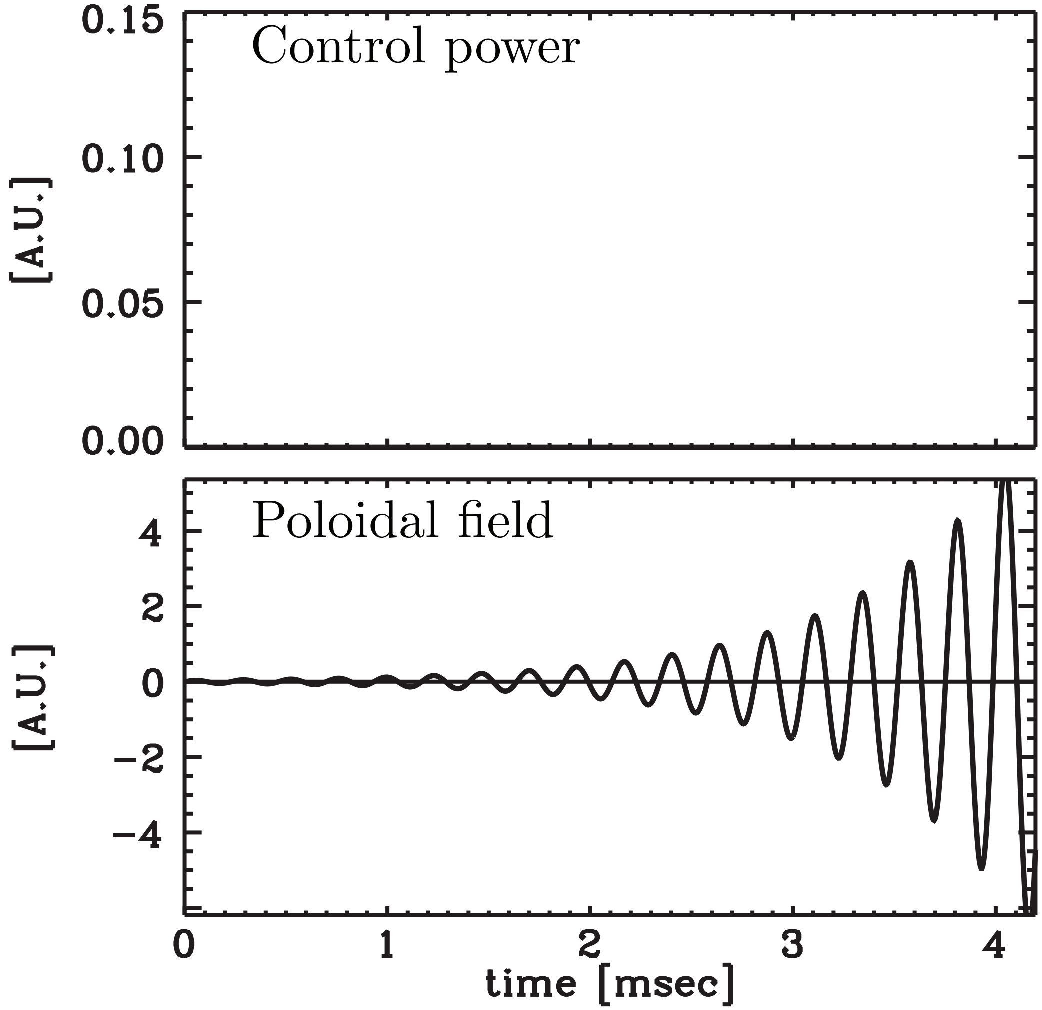
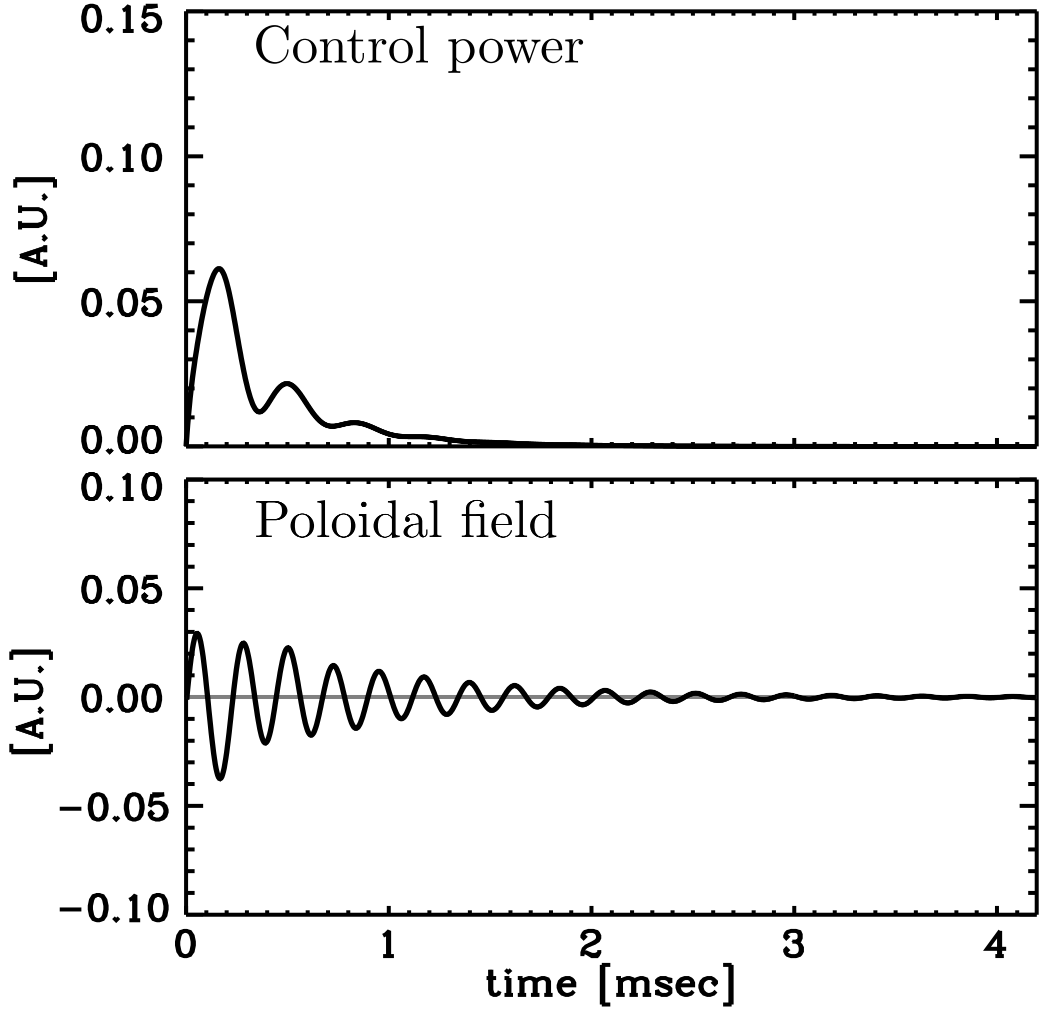
Instability grows exponentially without feedback



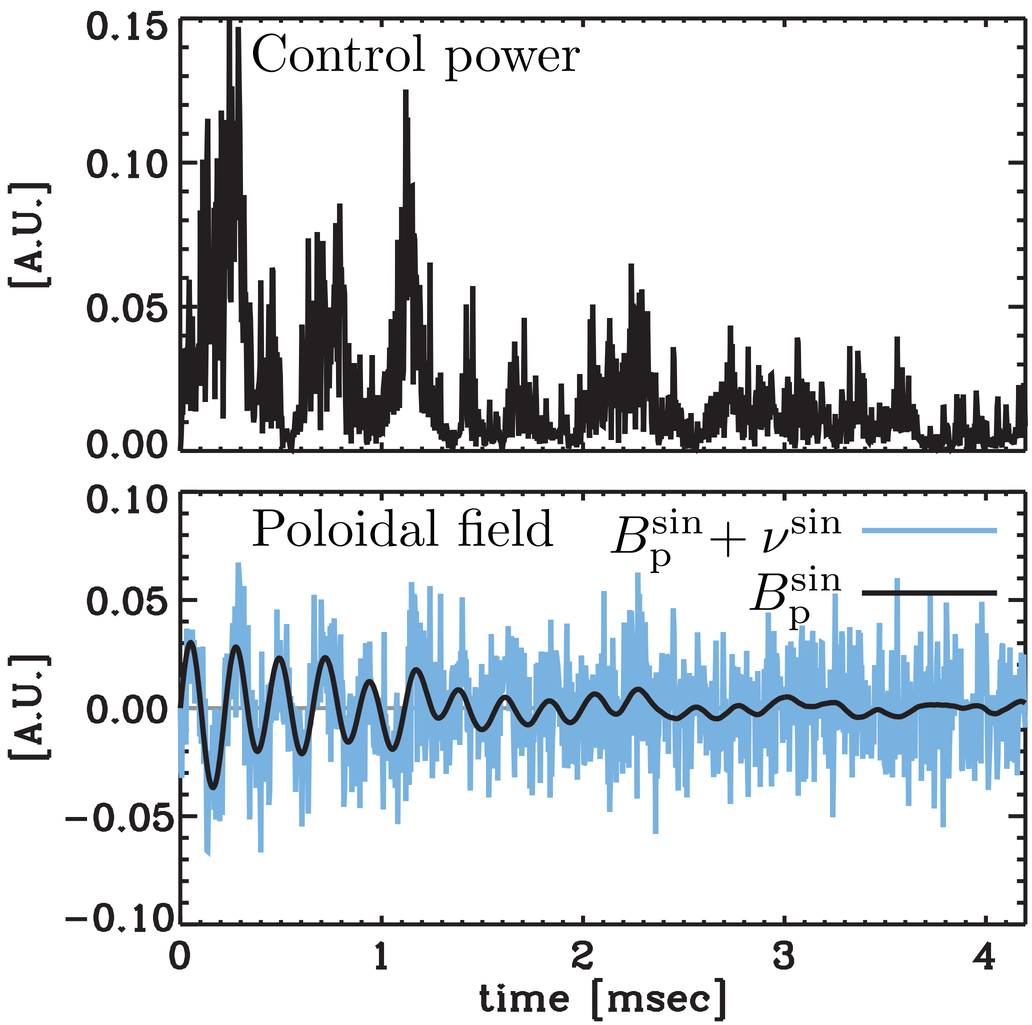
With feedback turned off, we see a growing, rotating mode.

Proportional gain feedback can stabilize the mode



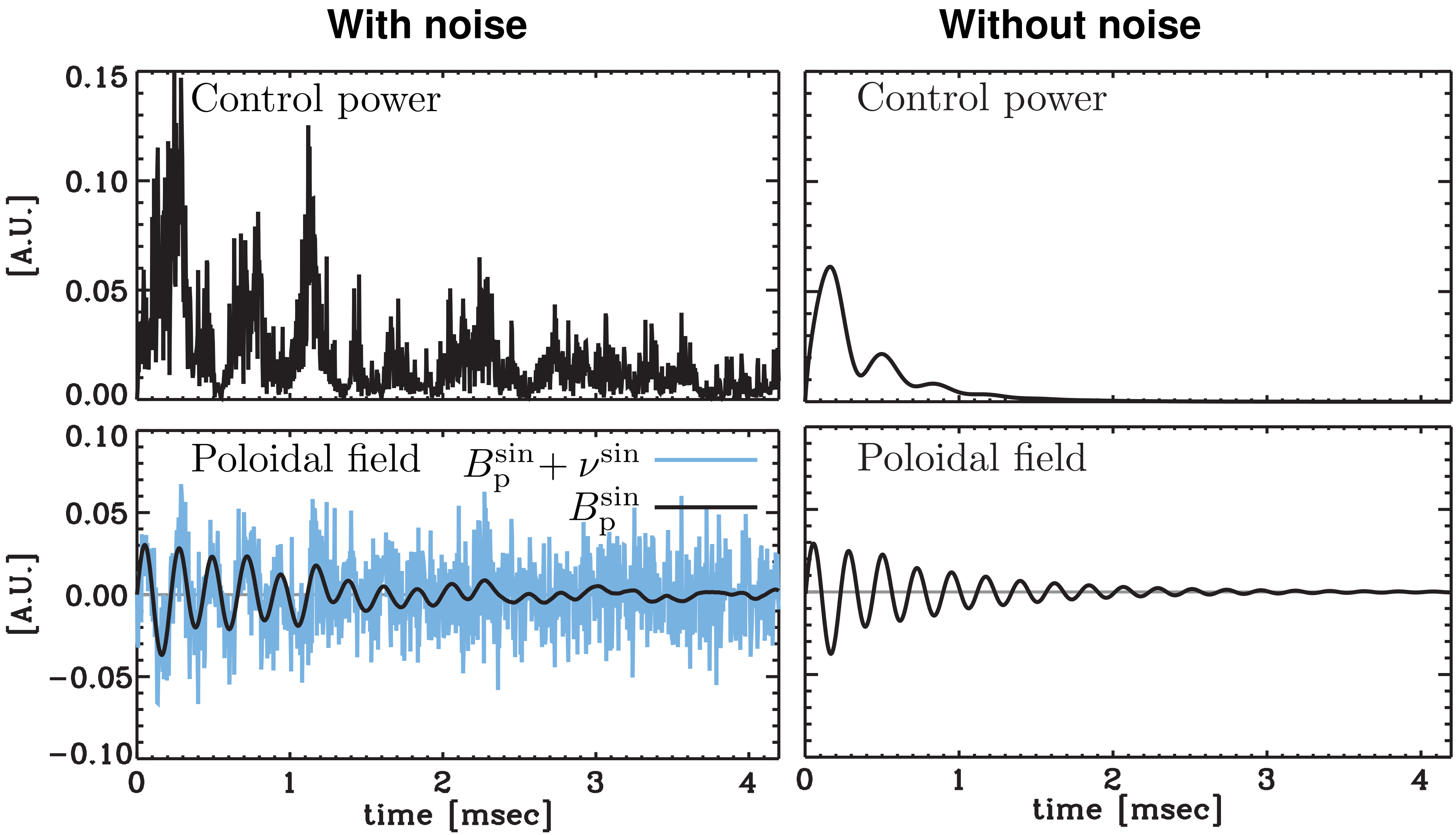
- Proportional gain feedback efficiently stabilizes the mode.
- What about when we add noise?

Stabilization is possible with noise, but power requirement is high



- Feedback still works, but quite a bit of noise makes it into the controller's output.
- Power consumption is high, even after the mode is stabilized.
- Because the simulation is *linear*, feedback works with arbitrary amounts of noise.
- In experiments, *non-linear* effects such as latency and saturation can make feedback performance much more sensitive to noise.

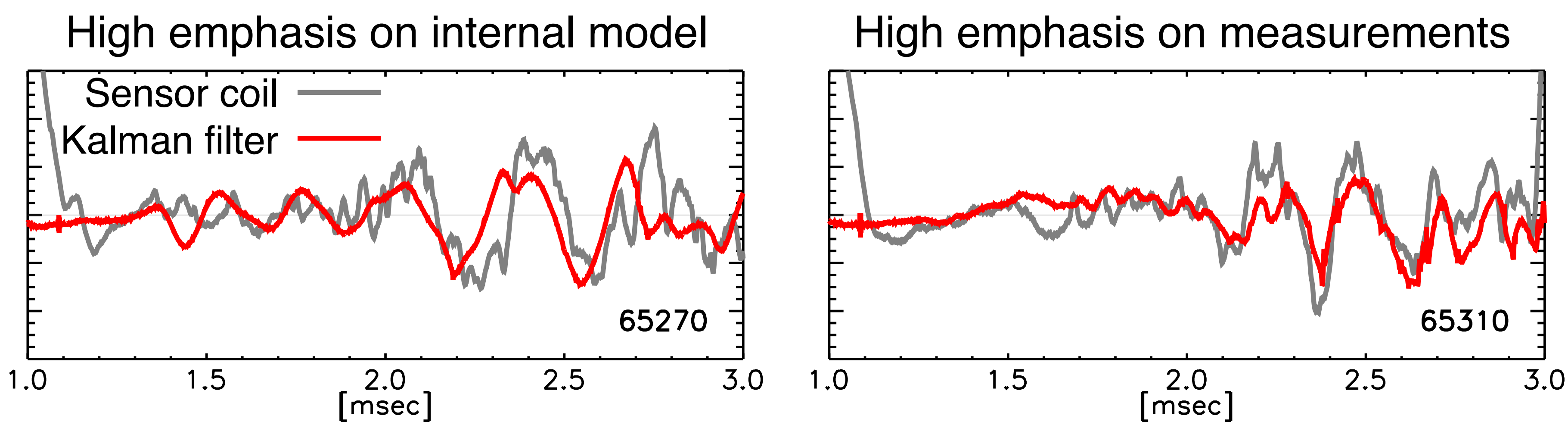
Stabilization is possible with noise, but power requirement is high



The Kalman filter

What is a Kalman filter?

- The Kalman filter[a] combines an internal, linear *model* for a system of interest with *measurements* to produce a real time estimate for the system's state.
- The internal model might be slightly inaccurate or incomplete.
- Likewise, measurements could be incomplete or noisy.
- With the Kalman filter, reliance on the model vs. the measurements can be adjusted.



- In simulation and experiments, we use a Kalman filter with a moderate amount of emphasis on the internal model.

[a] R. E. Kalman, *Transactions of the ASME–Journal of Basic Engineering* **82**, (Series D), 35 (1960).

The Kalman filter is a simple matrix equation

$$\vec{x}_i = \Phi_i \vec{x}_{i-1} + K_i \vec{z}_i,$$

where $\vec{x}$ is the optimal estimate of the system state and $\vec{z}$ is a vector of measurements.

Here,

$$\Phi_i = (I - K_i H)(\boldsymbol{A} + BGH), \quad \text{and}$$
$$K_i = (\boldsymbol{A} P_i \boldsymbol{A}' + \boldsymbol{Q})H'(H(\boldsymbol{A} P_i \boldsymbol{A}' + \boldsymbol{Q})H' + \boldsymbol{R})^{-1}.$$

$\boldsymbol{A}$	A model for the system dynamics
B	System response to a control input
G	Gain
H	Model for measurement dynamics
P_i	Error covariance of estimate
$\boldsymbol{Q}, \boldsymbol{R}$	System and measurement noise covariances

- Increasing terms in $\boldsymbol{Q}$ means the model is trusted less. Increasing terms in $\boldsymbol{R}$ means measurements are trusted less.
- In experiments, we use a filter with a constant Kalman gain, $K_i \to K$.

The internal model depends only on the mode's growth and rotation rates

- The state-vector contains the $\cos\varphi$ and $\sin\varphi$ Fourier components of an $n = 1$ mode.

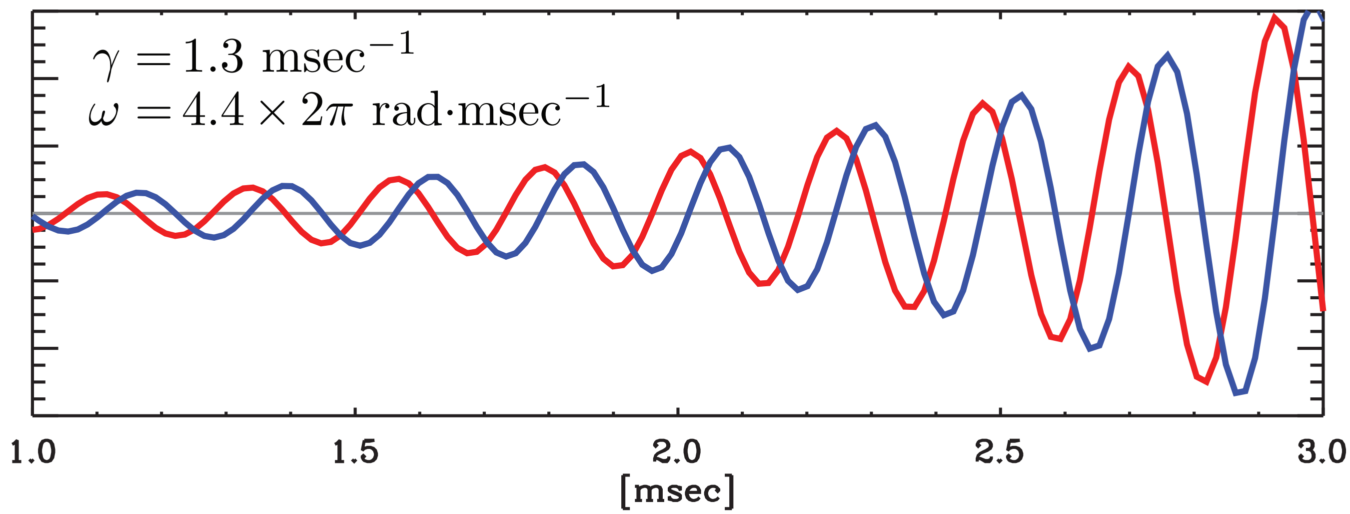
$$\vec{x} = [B_\mathrm{p}^{\cos}, B_\mathrm{p}^{\sin}]$$

- The internal model advances the mode at a prescribed growth and rotation rate.

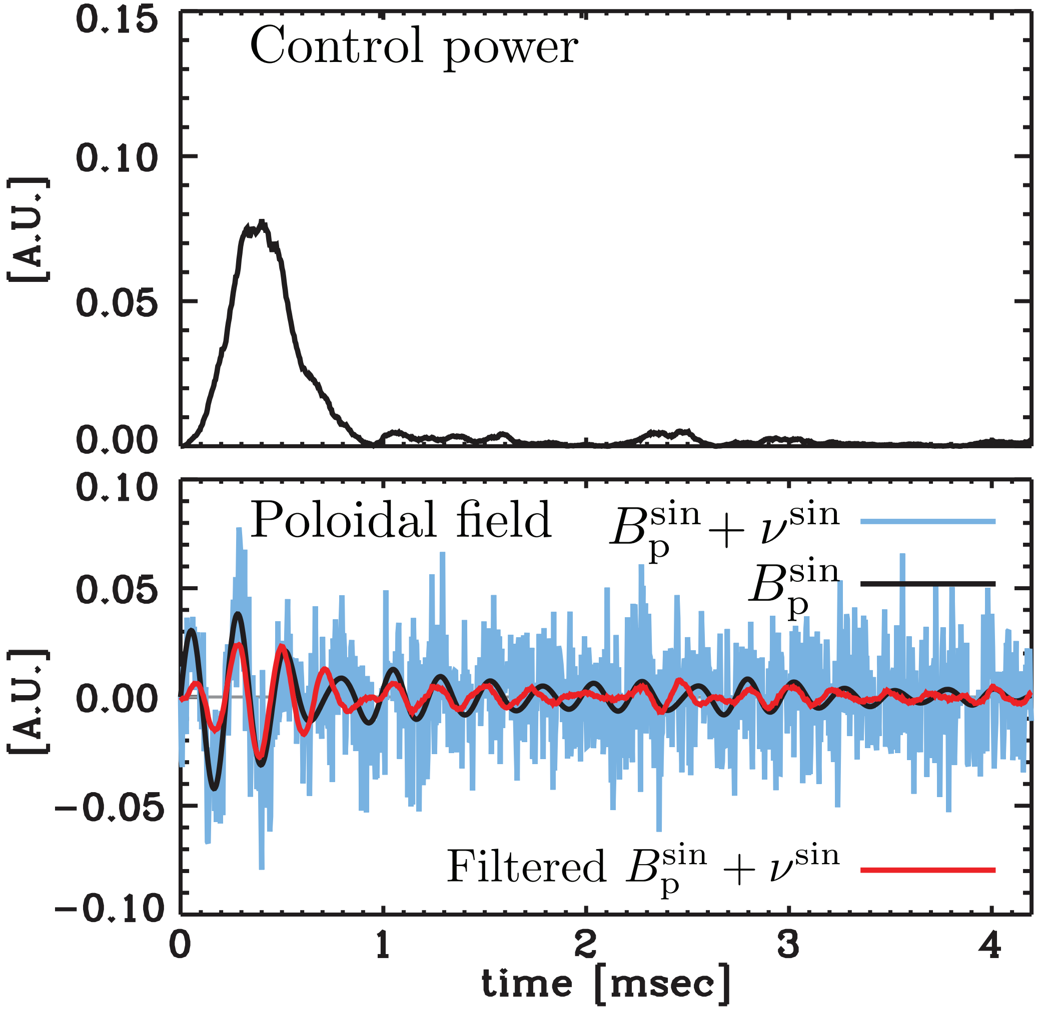
$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = \begin{pmatrix} \gamma & -\omega \\ \omega & \gamma \end{pmatrix} \vec{x}$$

- The solution is

$$\vec{B}_\mathrm{p}(t) = \exp(\gamma\, t)\mathfrak{R}(\omega\, t)\vec{B}_\mathrm{p}(0).$$

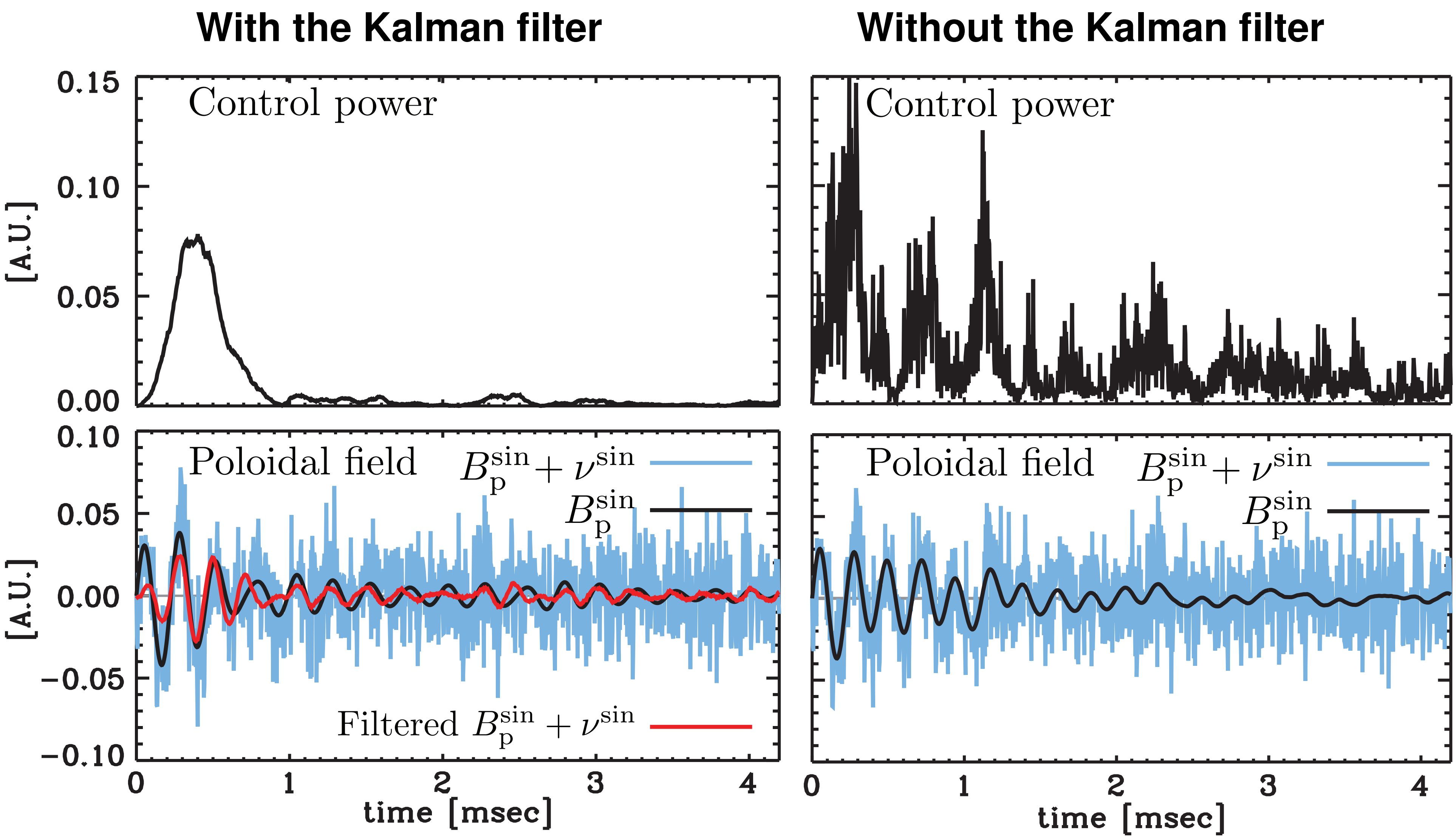


The Kalman filter stabilizes the mode quickly and efficiently



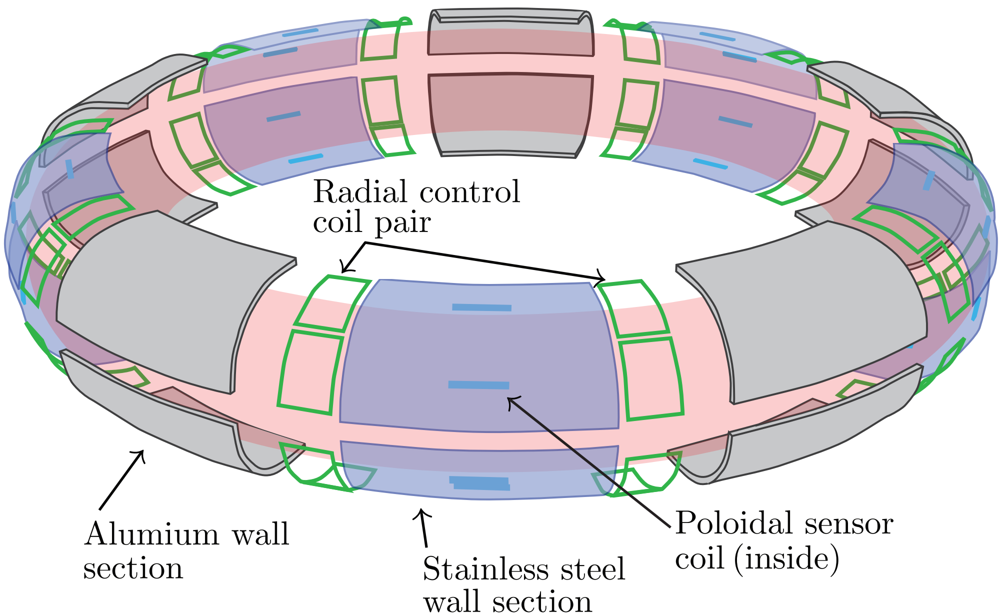
- With the Kalman filter, less control power is used, especially after the mode is stabilized.
- The mode is stabilized more quickly.
- The Kalman filter neatly removes the noise from the feedback signal.

The Kalman filter stabilizes the mode quickly and efficiently



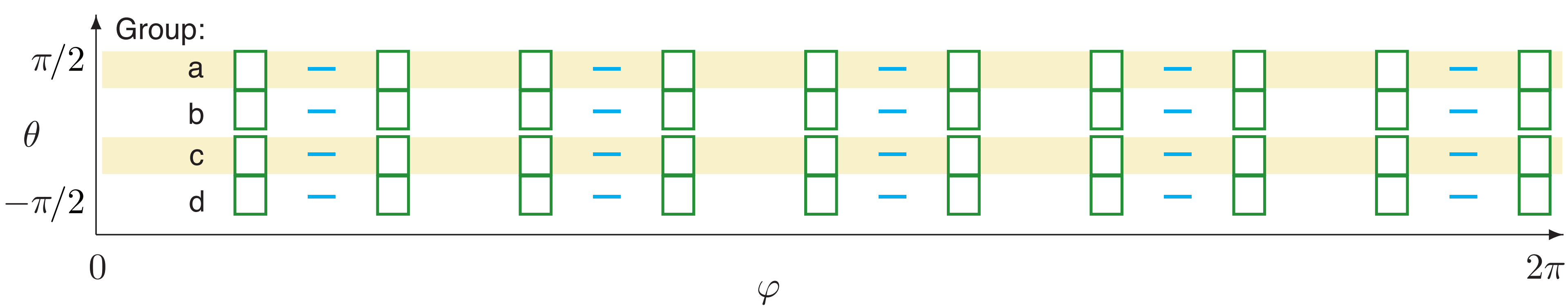
HBT-EP diagnostics and control hardware

HBT-EP has passive and active stabilization hardware

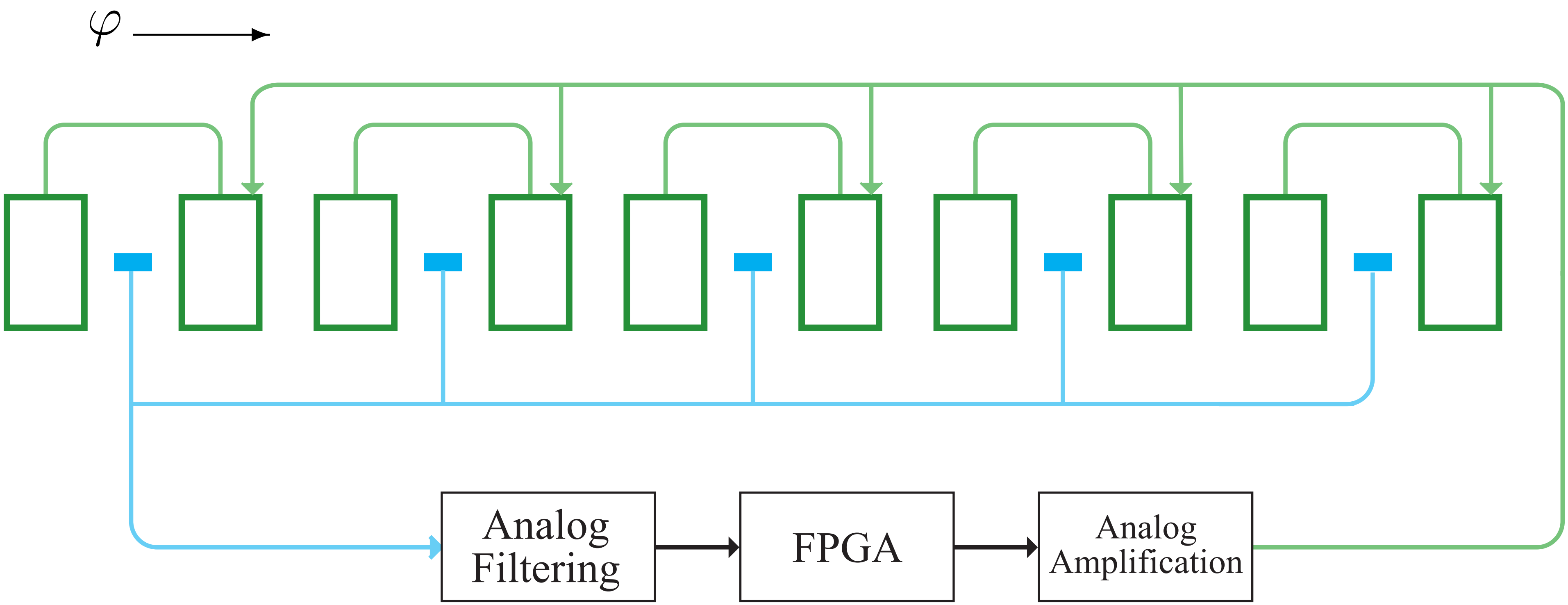


- There are 10 *thick aluminum shells* with a long wall time.
- There are 10 *thin stainless steel* shells with a short wall time.
- For feedback experiments, Al shells are pulled back 4 cm from the plasma surface, for $1/\gamma_w \approx 300$ μsec.

- We have 20 *poloidal sensor coils* and 20 pairs of *radial control coils*.
- The control coils are *small* and *localized*: they only cover 15% of the plasma surface.
- The coils are divided into four independent, parallel feedback loops.

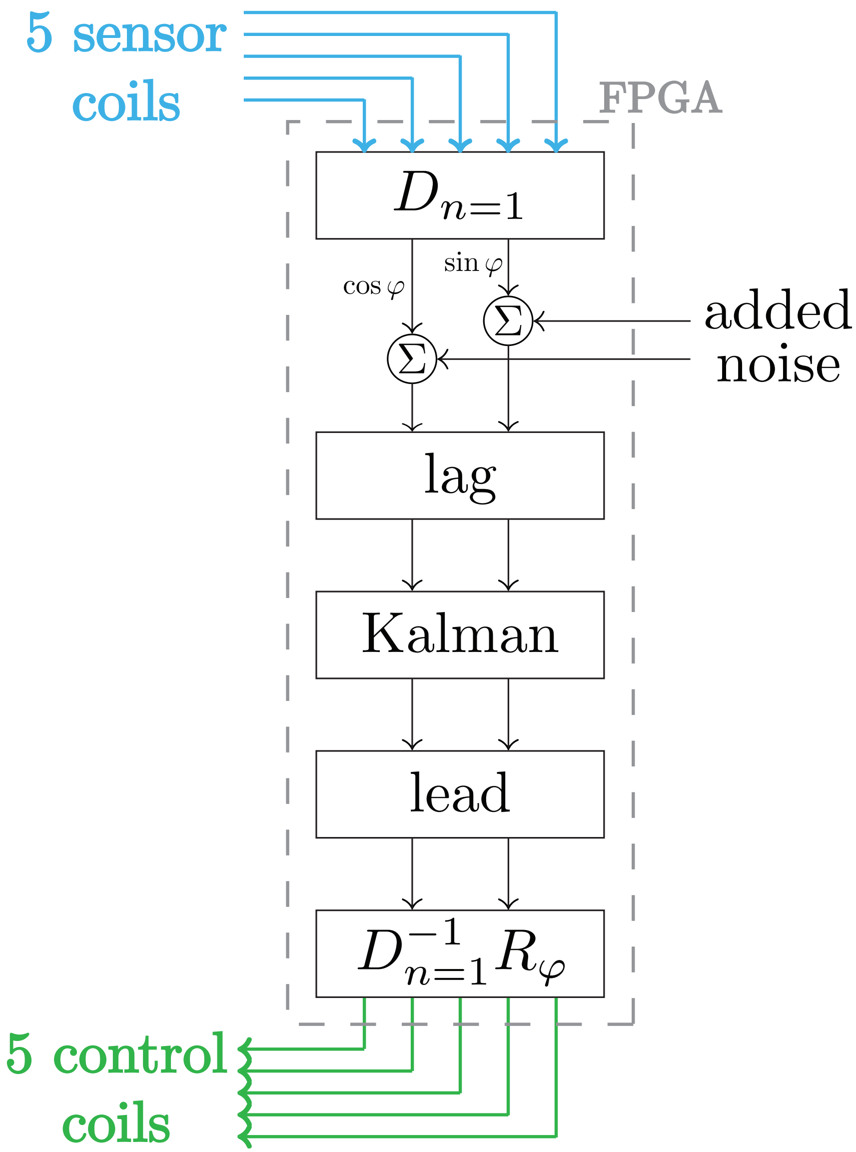


Each feedback loop consists of 5 sensor coils and control coil pairs



- Each feedback loop has its own field programmable gate array (FPGA) controller.
- With 5 evenly spaced sensors, $n = 1$ instabilities can be detected with Fourier analysis.

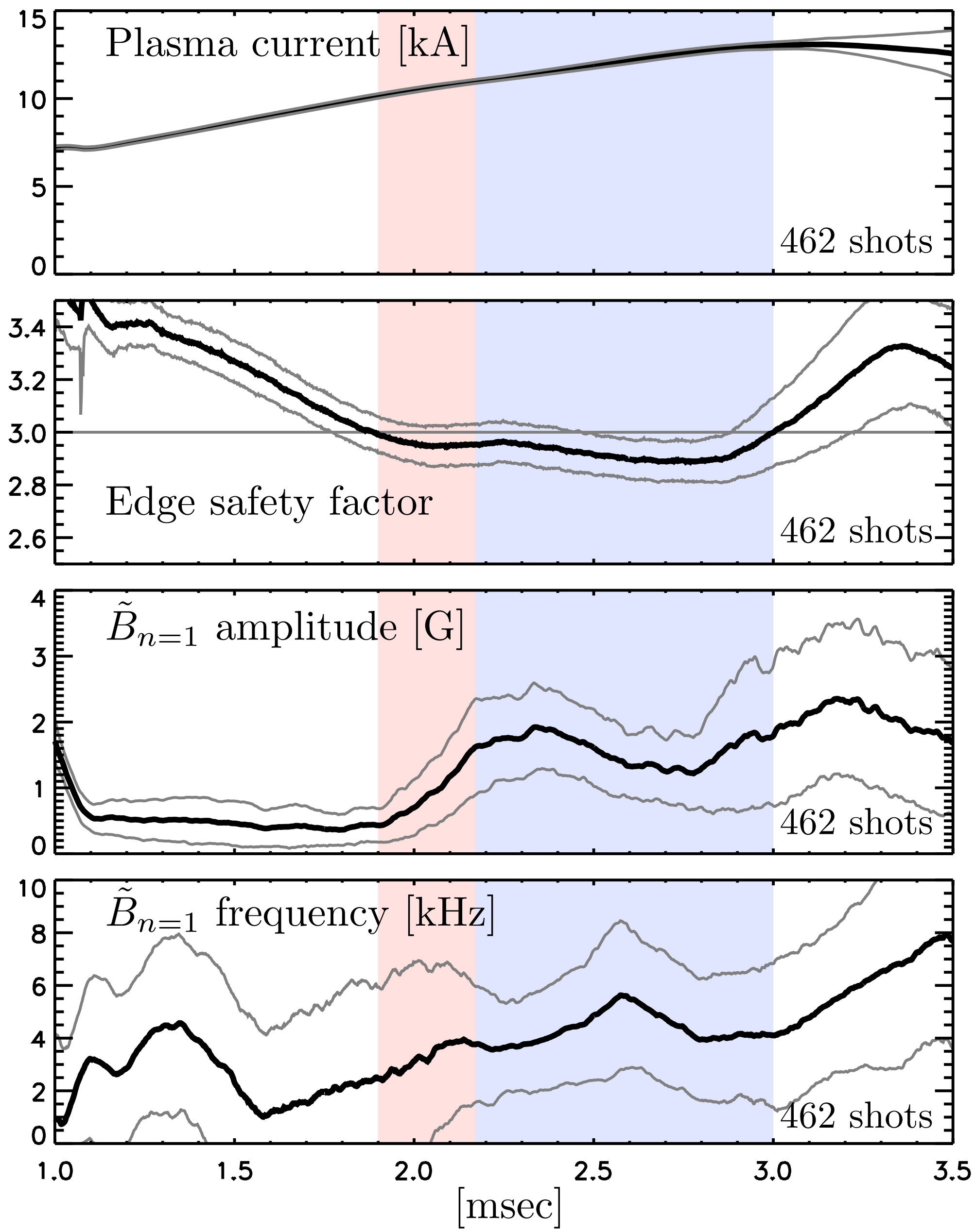
The feedback algorithm is implemented on low-latency field programmable gate array controllers



- A spatial discrete Fourier transform (DFT) is used to select the $n = 1$ mode.
- Phase lag and lead compensators correct for hardware transfer functions.
- Noise can be added after the DFT to test the robustness of control algorithms.
- The Kalman filter can be bypassed to get a conventional algorithm.
- The toroidal phase of the output is adjustable.
- Total latency is $\sim$10 μsec, external kink instabilities grow and rotate on a 100–500 μsec timescale.

Experimental results

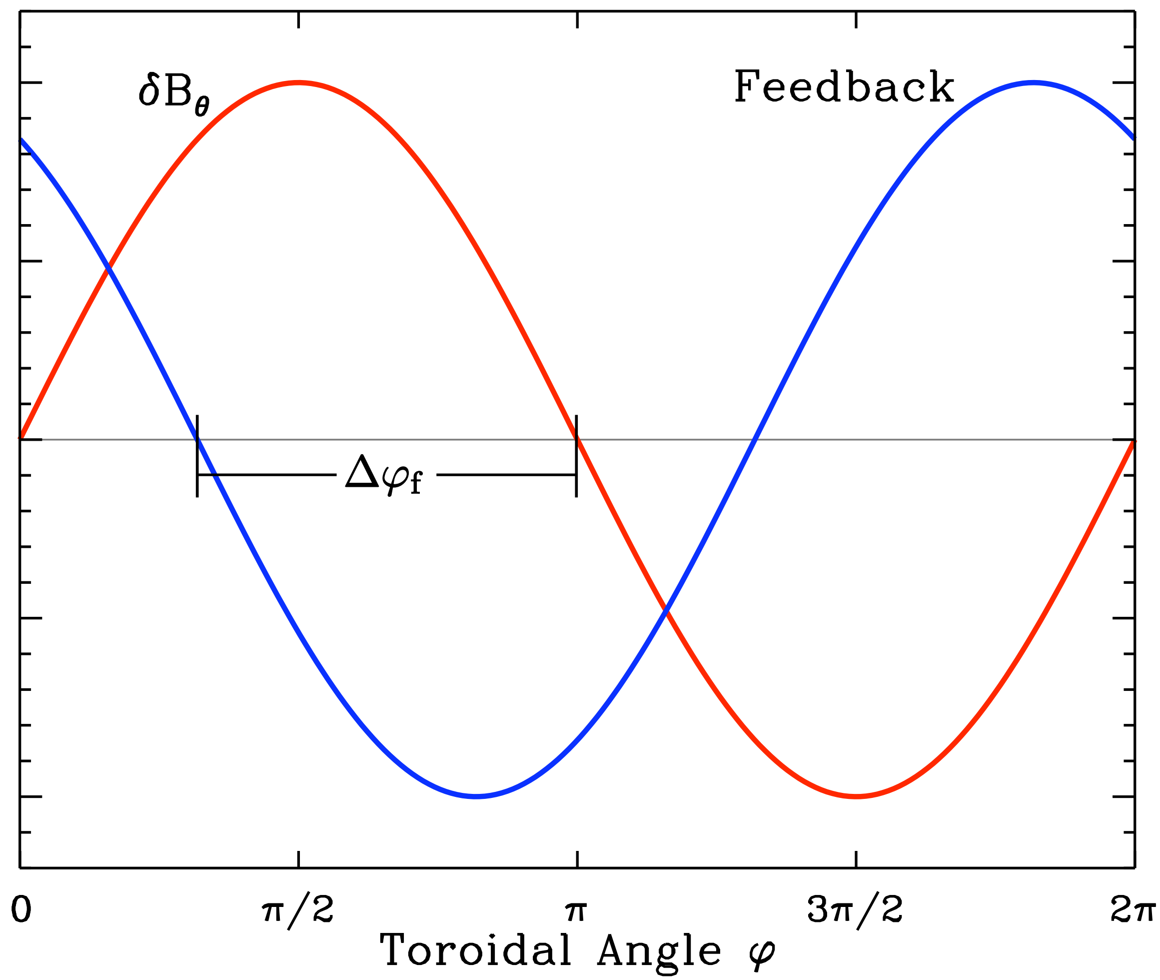
A current ramp is used to create external kink modes



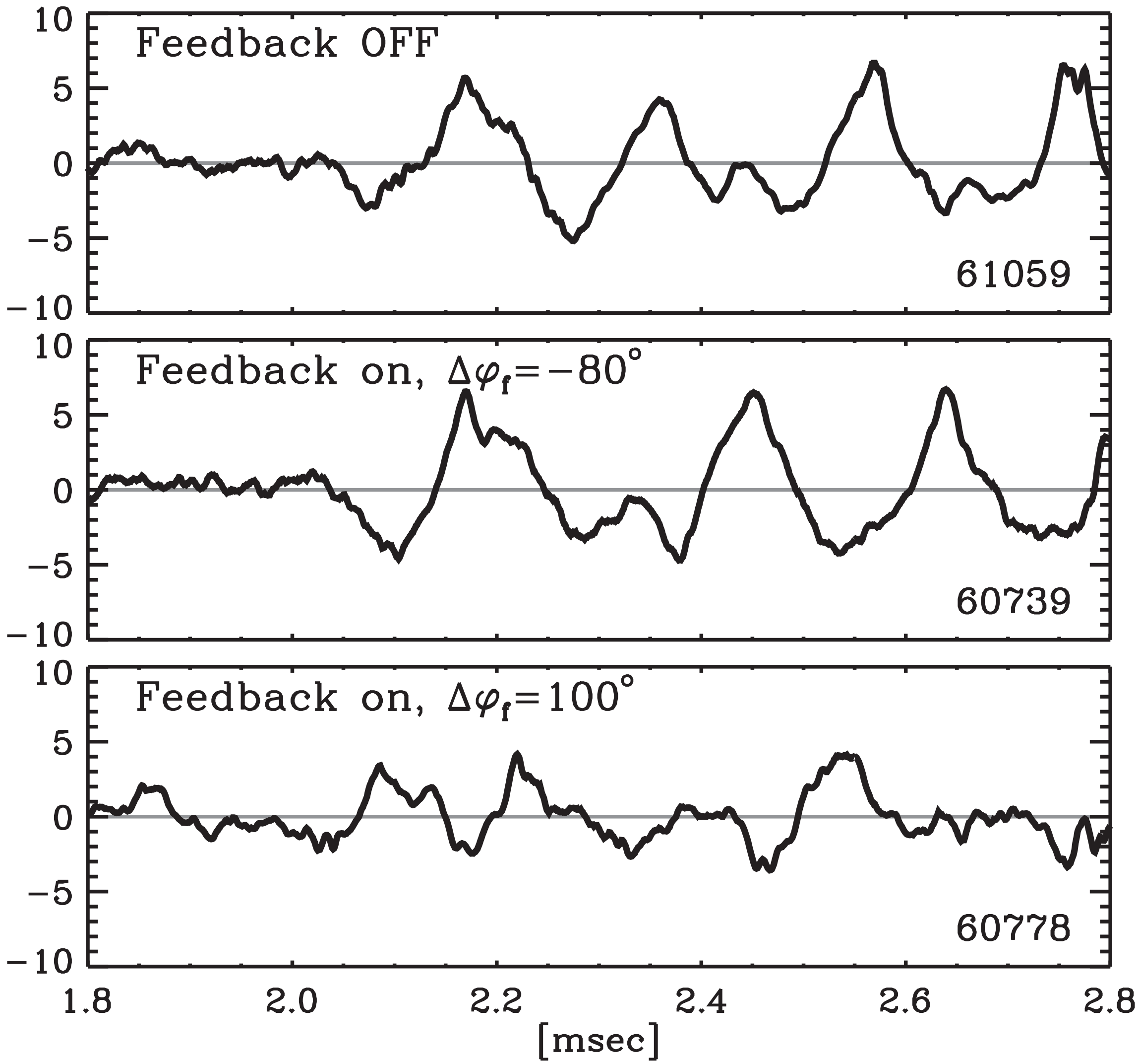
- The plasma current is ramped at ~3 MA/sec to create a current density gradient at the edge of the plasma.
- As the $q = 3$ surface goes external, a rotating $n = 1$ mode is observed in the magnetics.
- The mode grows exponentially, then saturates.
- Average growth rate is 5 msec^{-1}, but it varies shot to shot.
- Rotation frequency is near 3–5 kHz, and it often sweeps in time.

The toroidal phase of feedback is an adjustable control parameter

- Sensors measure the *poloidal* field, but feedback coils are *radial*. So a phase-shift is needed for negative feedback.
- Phase shifts also appear due to controller latency and imperfect optimization of the system transfer function.

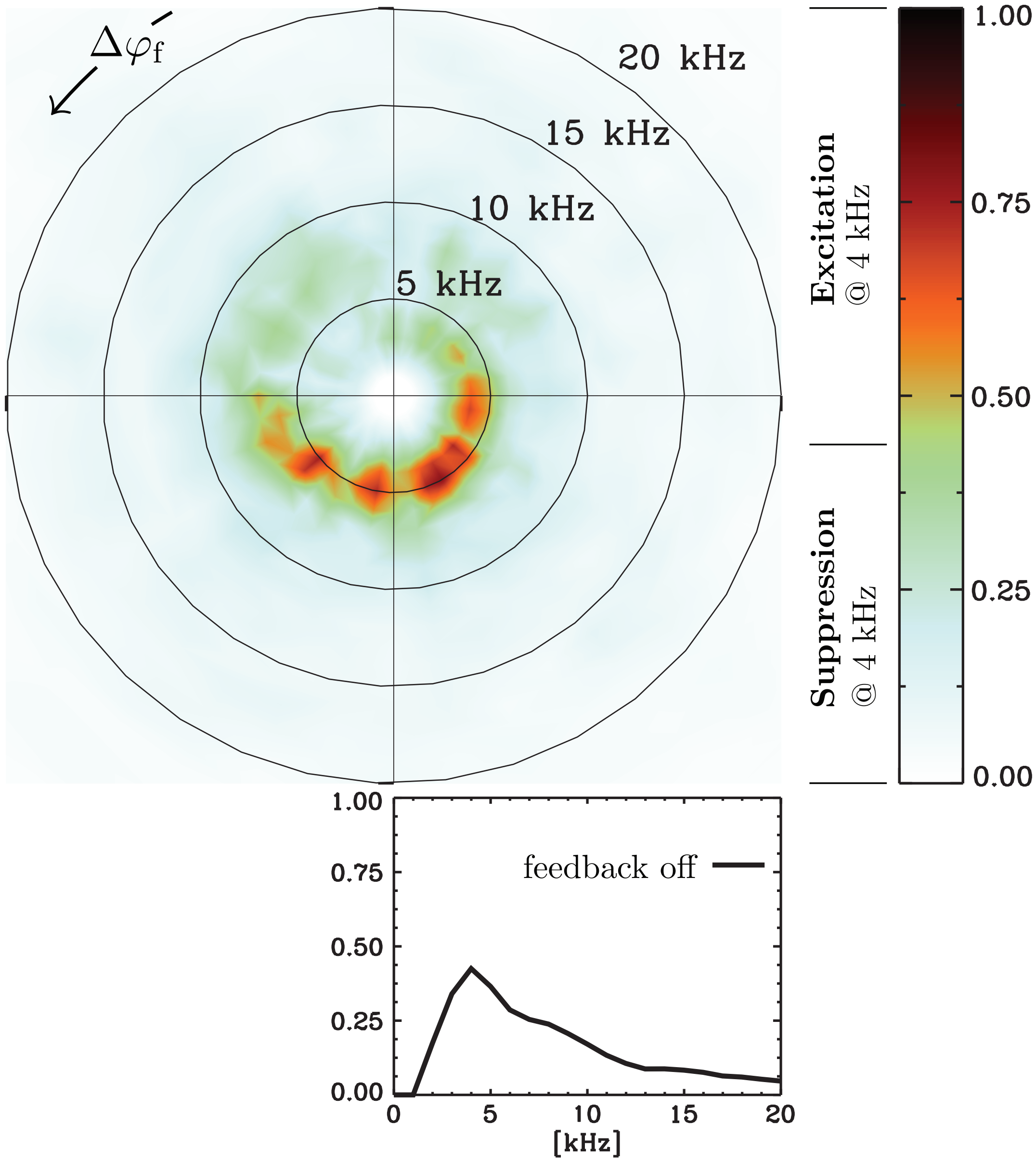


Feedback can be phased to either suppress or excite the mode



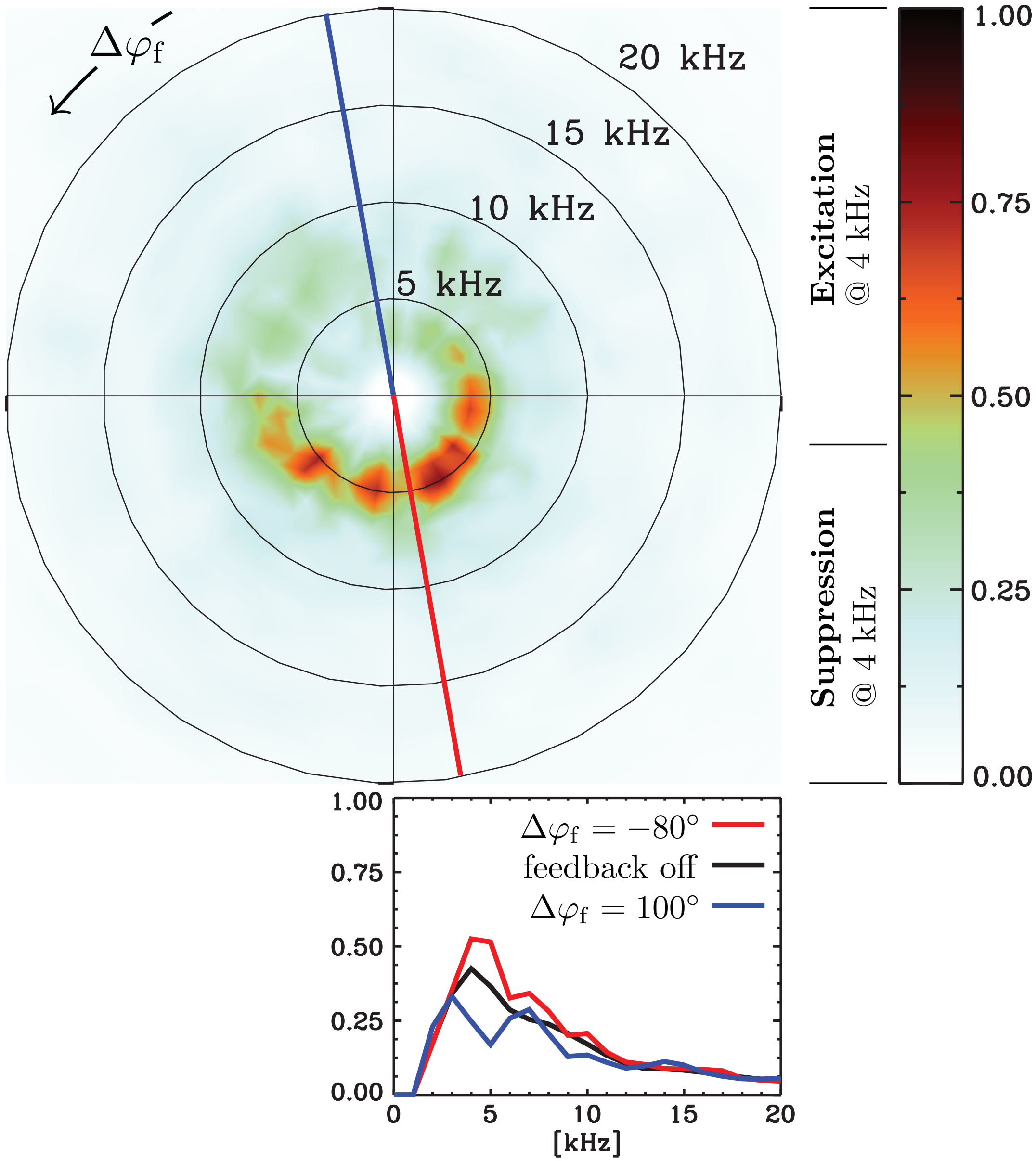
- With feedback off, fluctuations near 4 kHz are observed on a poloidal sensor coil.
- Positive feedback can be used to excite the mode.
- Negative feedback decreases mode's amplitude.

Feedback phase angle scan shows clear regions of excitation and suppression



- As the feedback phase angle is scanned through 360°, the frequency spectrum of poloidal field fluctuations is observed.
- In the upper-left quadrant, suppression is observed relative to feedback off.
- Peak feedback excitation is about 180° away.

Feedback phase angle scan shows clear regions of excitation and suppression



- As the feedback phase angle is scanned through 360°, the frequency spectrum of poloidal field fluctuations is observed.
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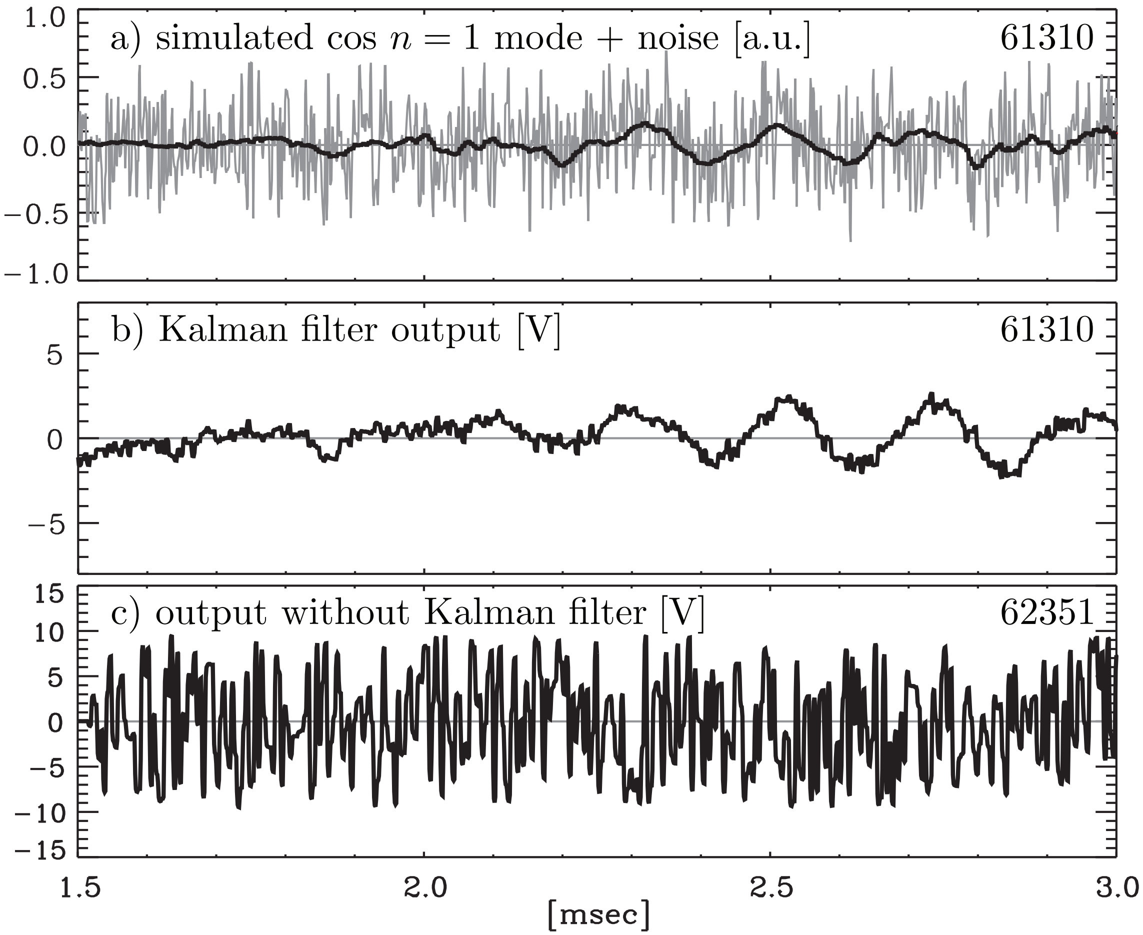
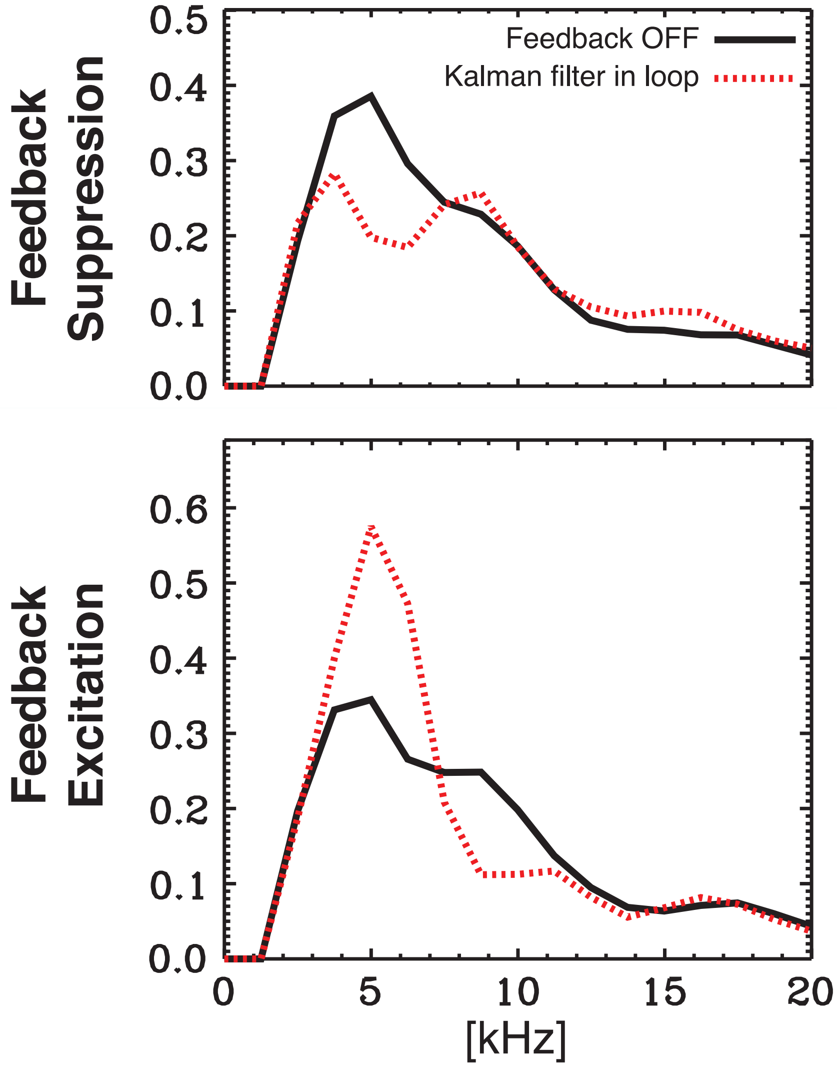
- Peak feedback excitation is about 180° away.

The Kalman filter removes added noise very effectively



- White frequency spectrum, Gaussian noise was added to the algorithm after the DFT stage.
- This effectively added a random amplitude and phase to the $n = 1$ mode.
- The Kalman filter was able to track the mode well, even with large amplitude noise.

Kalman filter feedback remains robust when additional noise is added to the system



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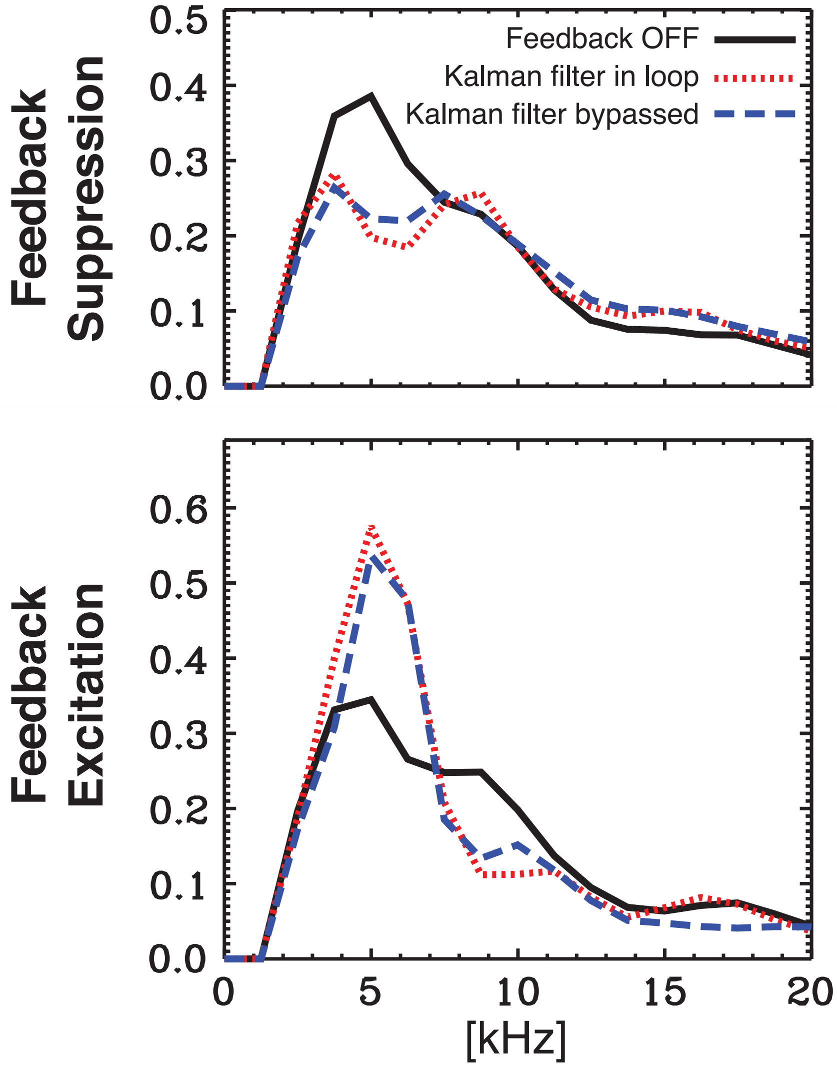

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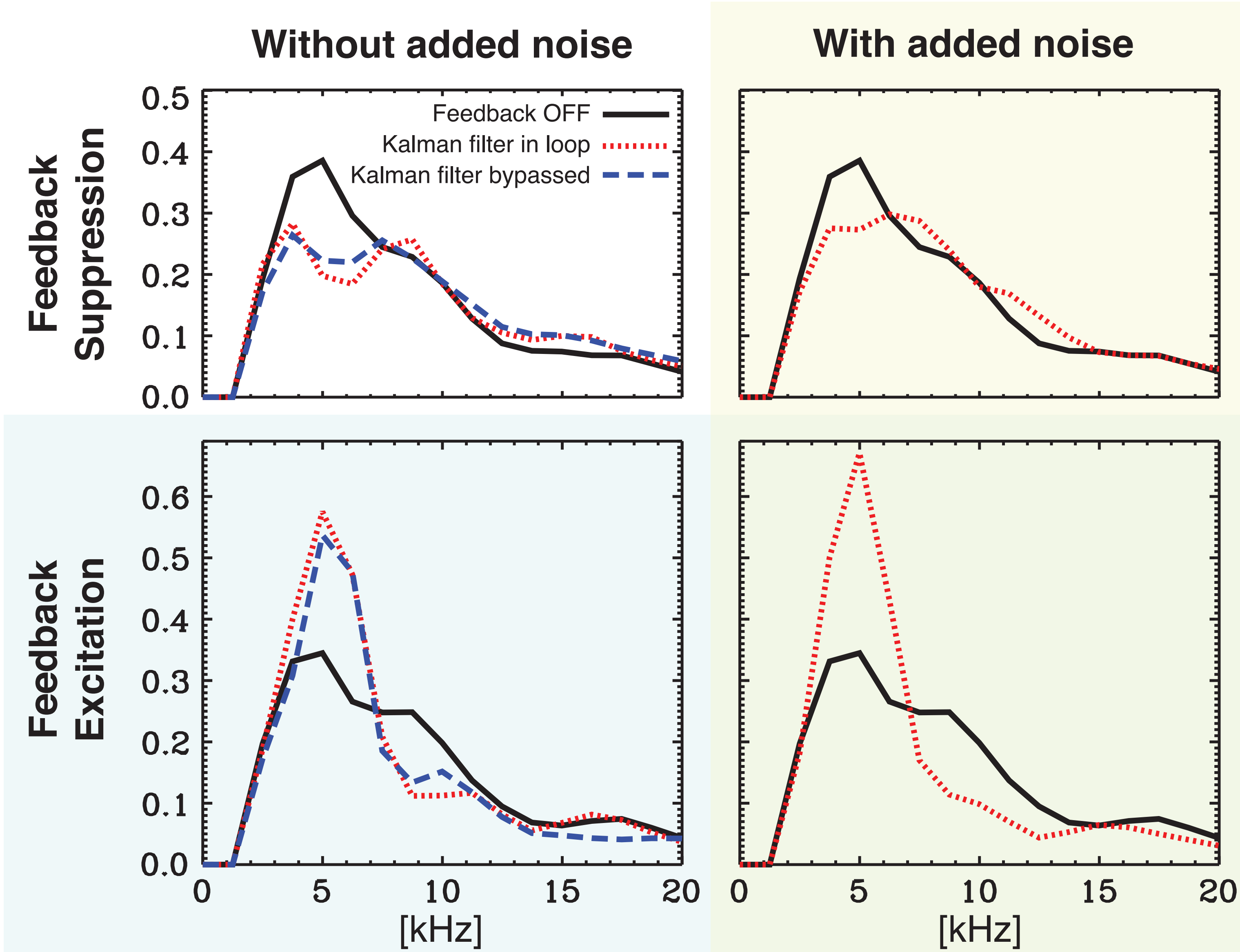

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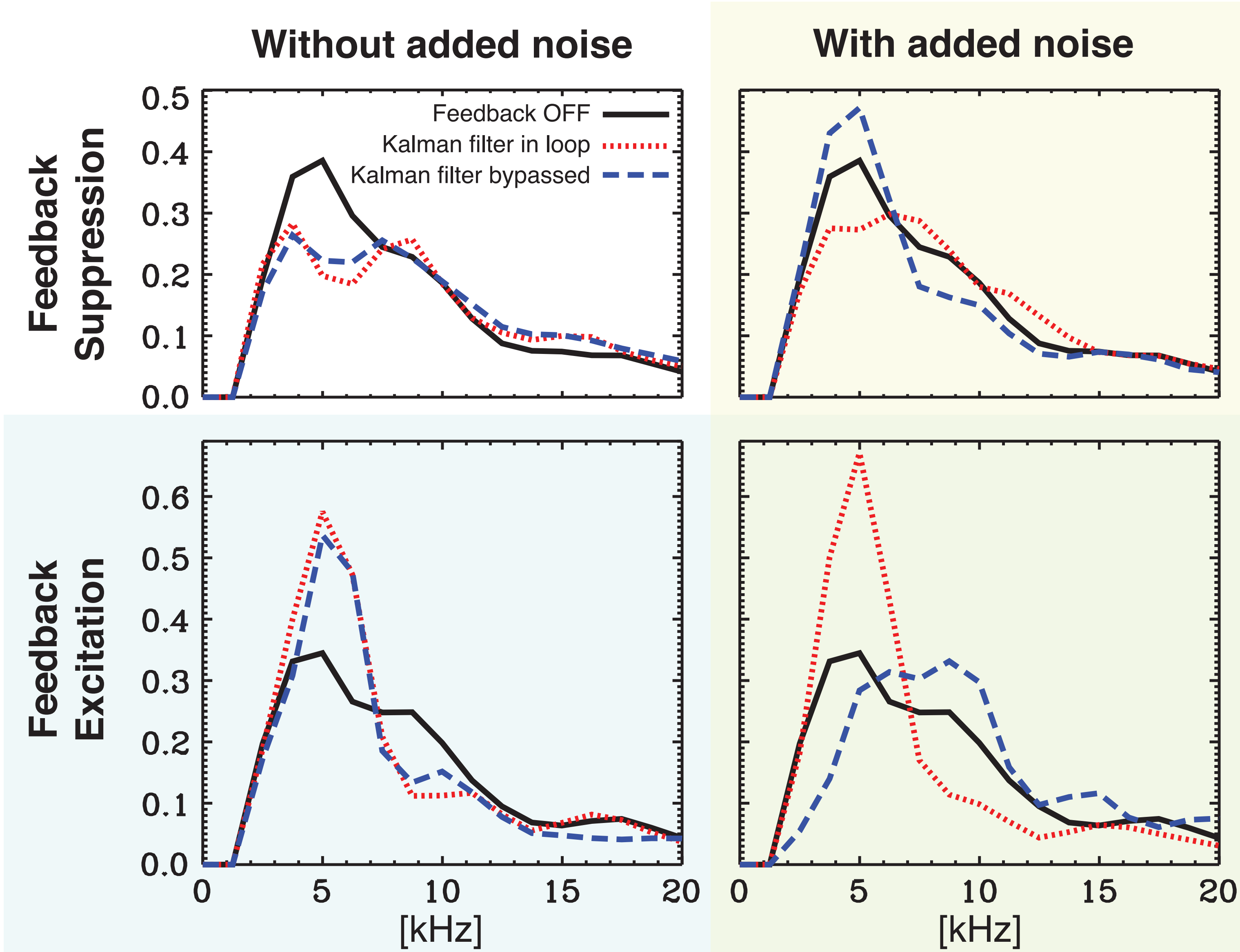


Kalman filter parameter scans are used to determine best settings

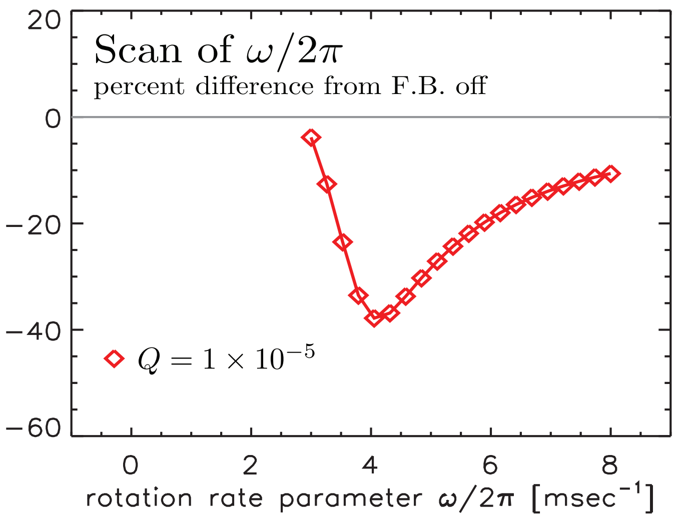
- We are interested in how the following parameters in the Kalman filter's system model affect feedback:

γ	Estimate of the mode's growth rate
ω	Estimate of the mode's rotation rate
Q	Uncertainty in the system model

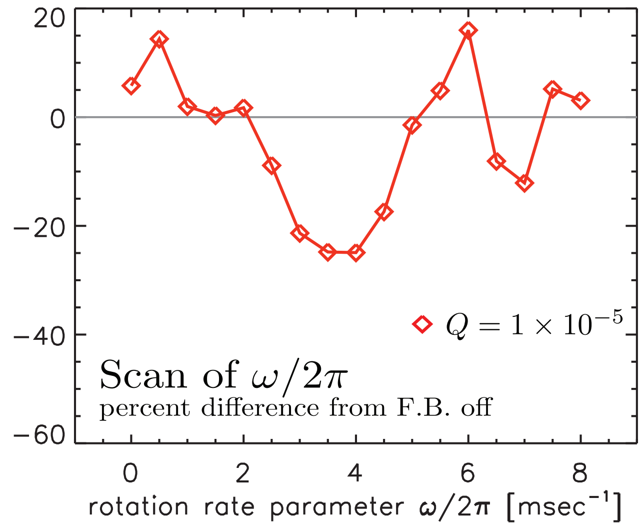
- The impact of these settings can be investigated in both the simulation and experiment by scanning one parameter at a time.
- For each scan point, the RMS average of the poloidal field is subtracted from feedback off average and normalized to a percentage.
- Percent differences greater than zero imply feedback excitation, percent differences less than zero imply suppression.
- This makes it straightforward to compare simulation results with those from the experiment.

Kalman filter parameter scans point to best settings

Simulation results



Experimental results



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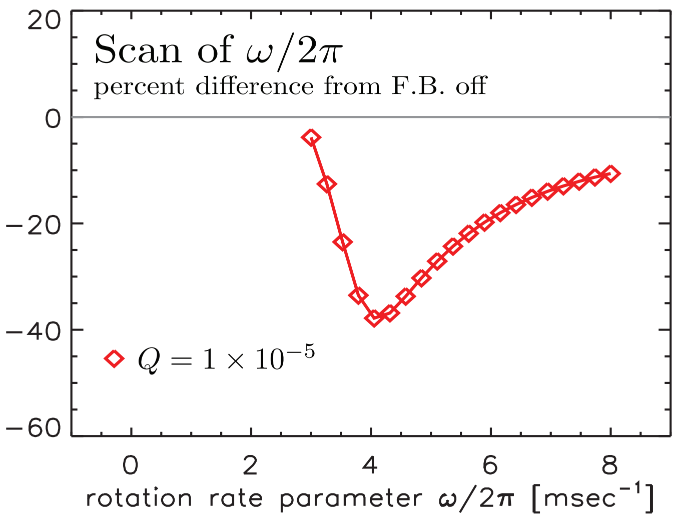

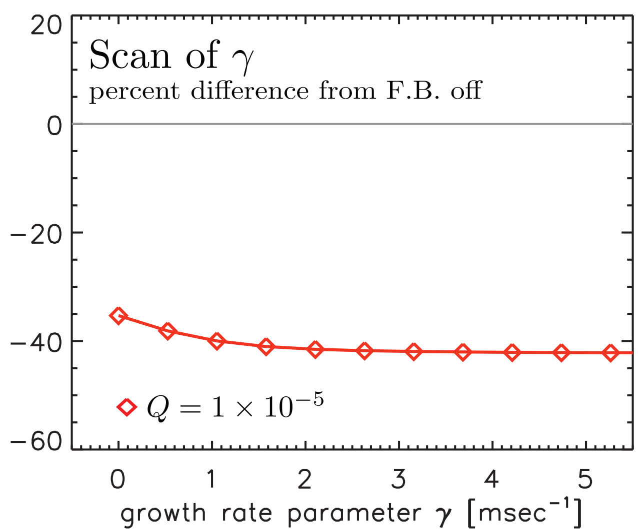

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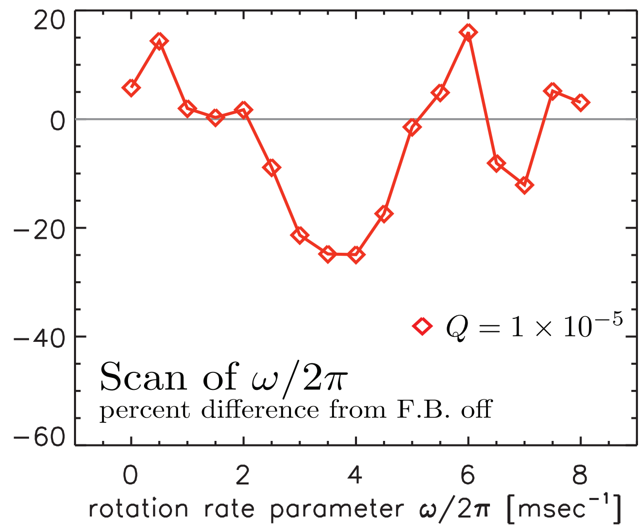

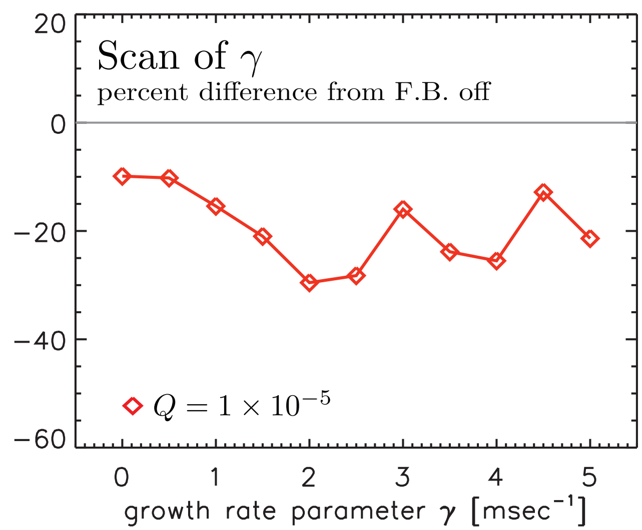

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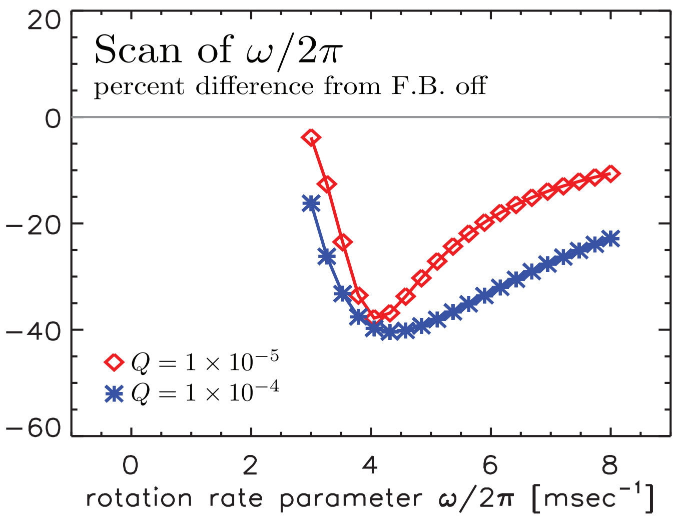

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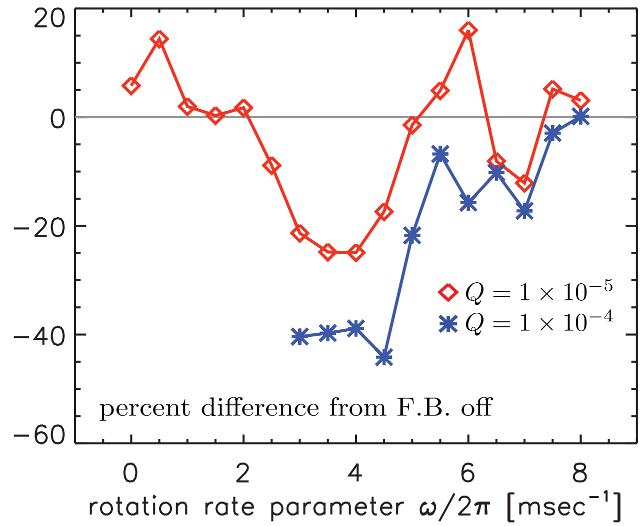
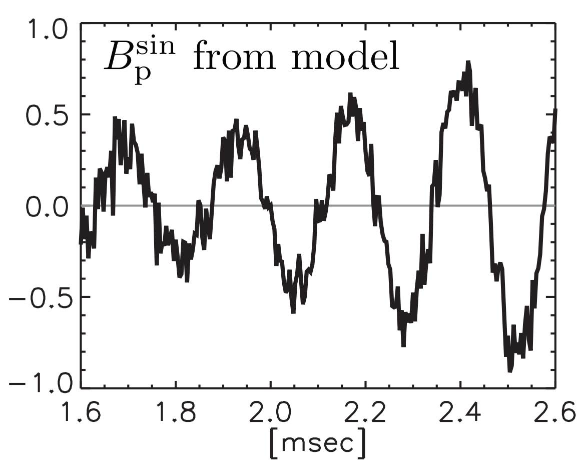
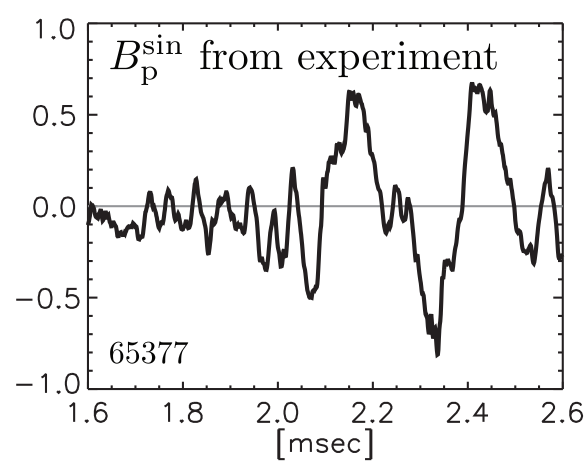


- Scans of ω were done at two settings of Q
- Remember, Q is the amount of uncertainty in the Kalman filter's internal model.
- The larger value of Q leads more flexibility in the setting of ω and better suppression of the mode.

Key differences between simulation and experiment

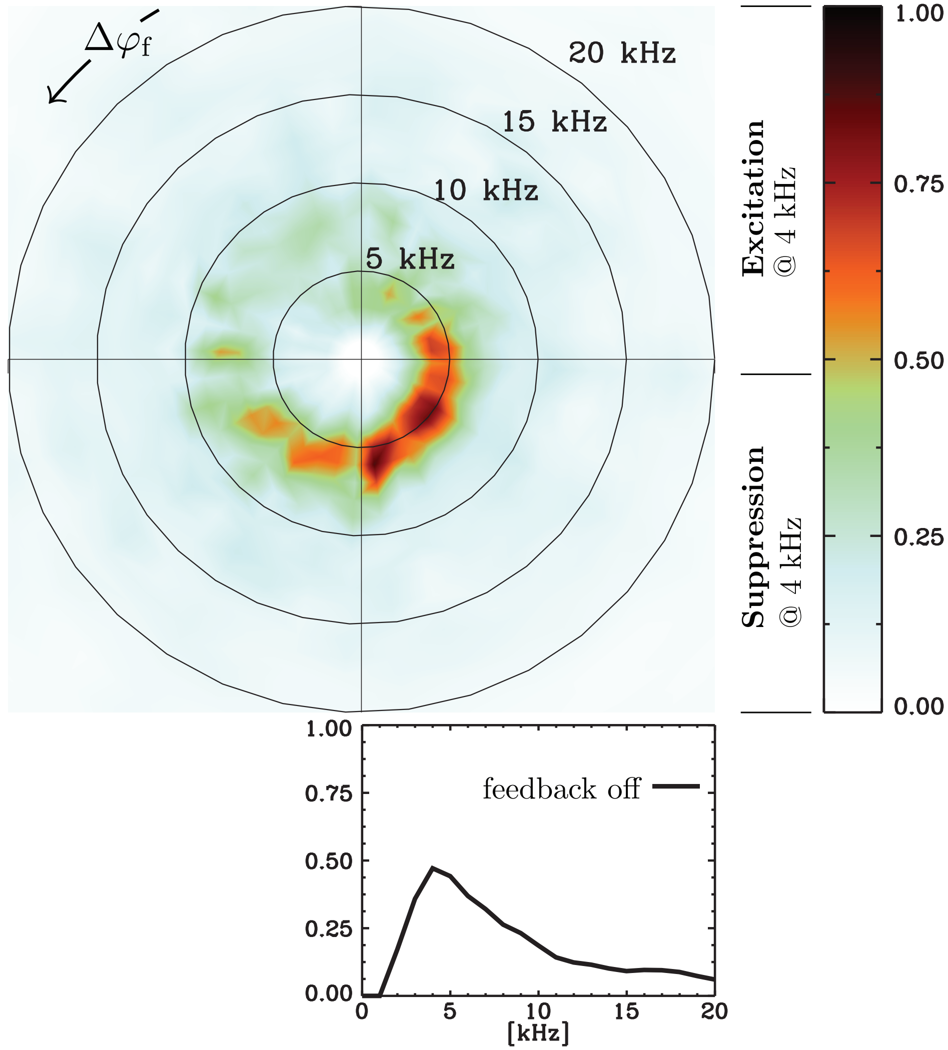
Simulation	HBT-EP
Without feedback, instabilities grow infinitely large.	Instabilities saturate at finite amplitude.
Mode lasts as long as we like.	Can only maintain edge current gradient for $\sim$1 msec.
Unique, fixed growth and rotation rates.	A range of growth and rotation rates are observed.
Mode rotation is smooth, fluctuations are sinusoidal.	Fluctuations are not always sinusoidal, maybe due to interaction with a complex conducting boundary.





Choosing a more optimal set of filter parameters leads to improved performance

- The original filter worked quite well.
- With the new settings, feedback suppression and excitation are stronger.



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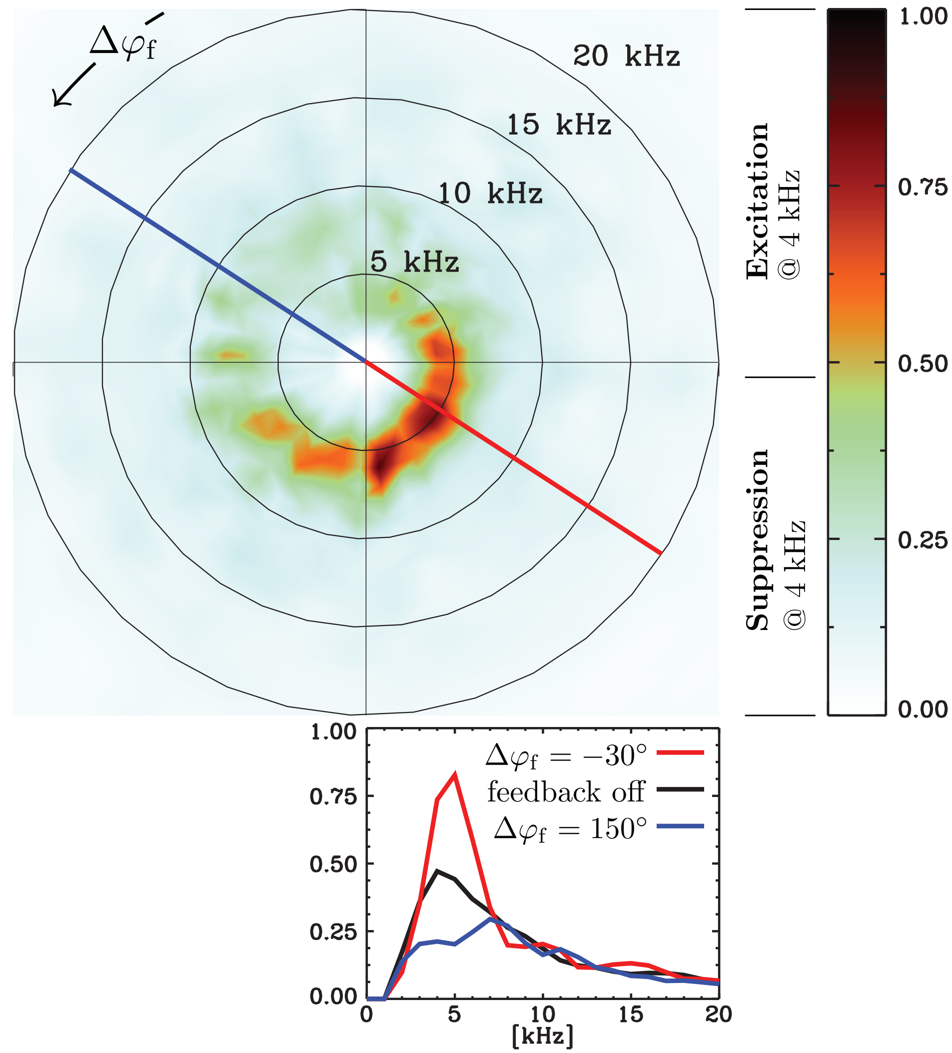

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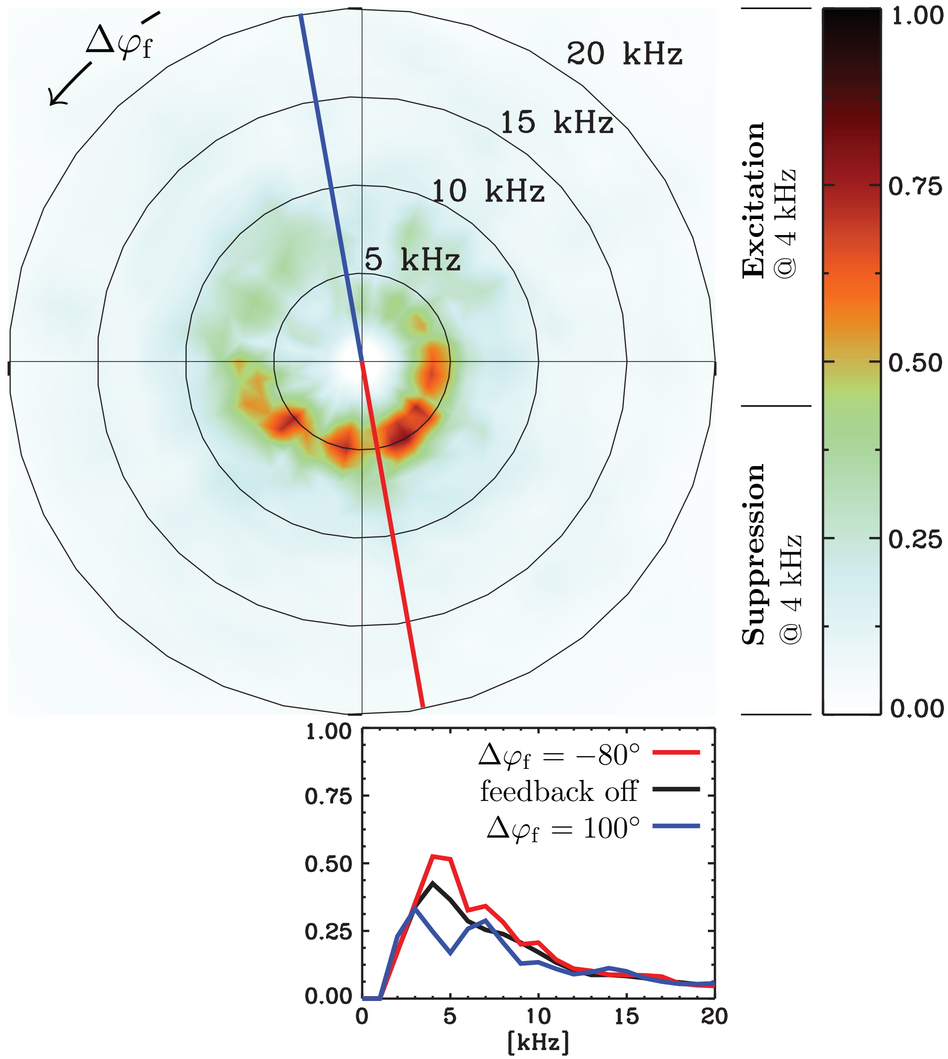
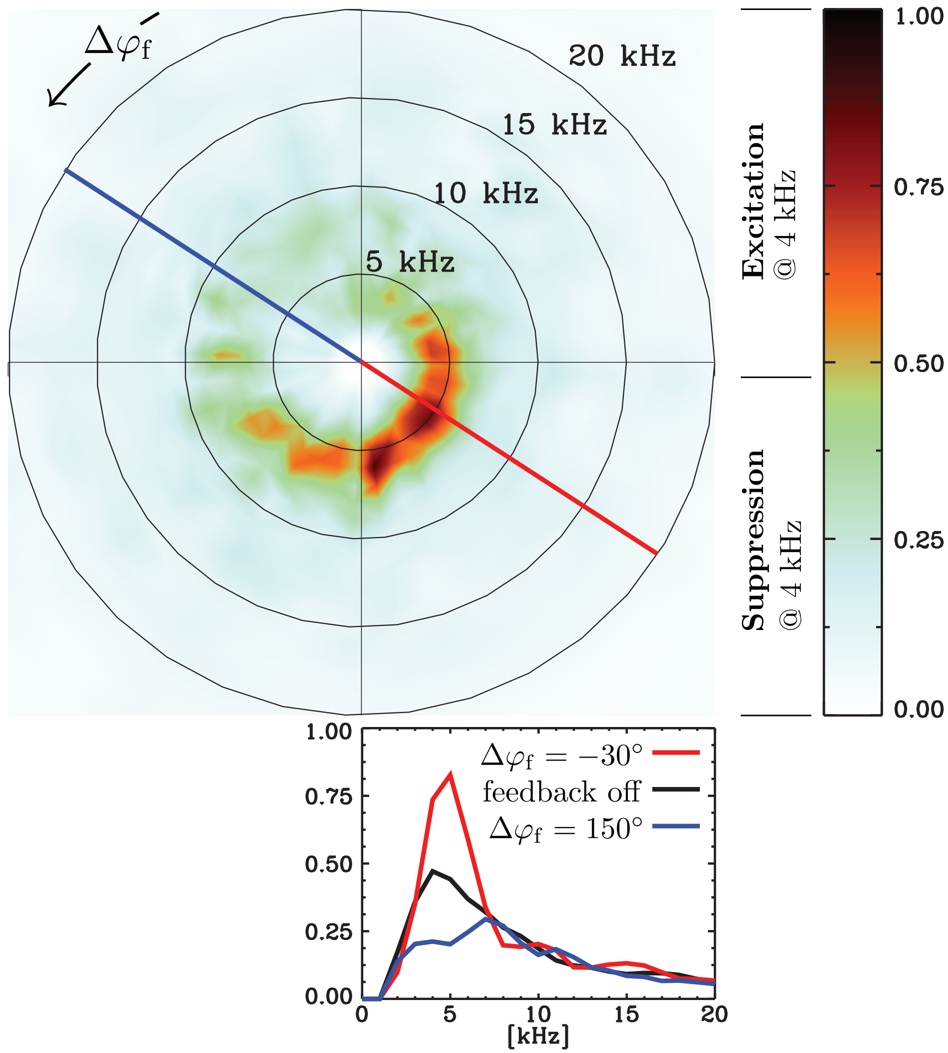
Before optimization



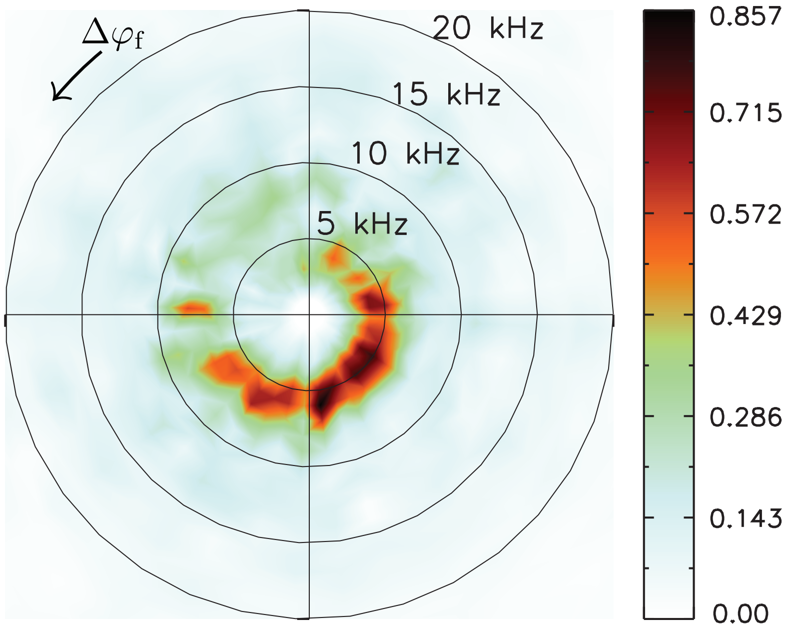
After optimization



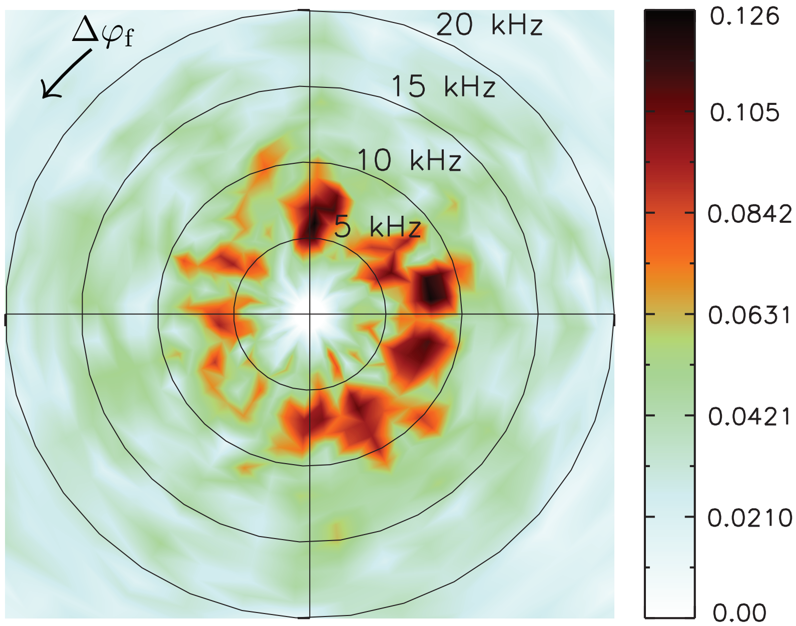
There's evidence of multiple simultaneous modes

- HBT-EP has a high-density poloidal array of pickup loops that can resolve high m-number fluctuations.
- A combined analysis of these and the sensor coils turned up evidence of a $(m, n) = (6, 2)$ mode.

$(m, n) = (3, 1)$ fluctuations



$(m, n) = (6, 2)$ fluctuations



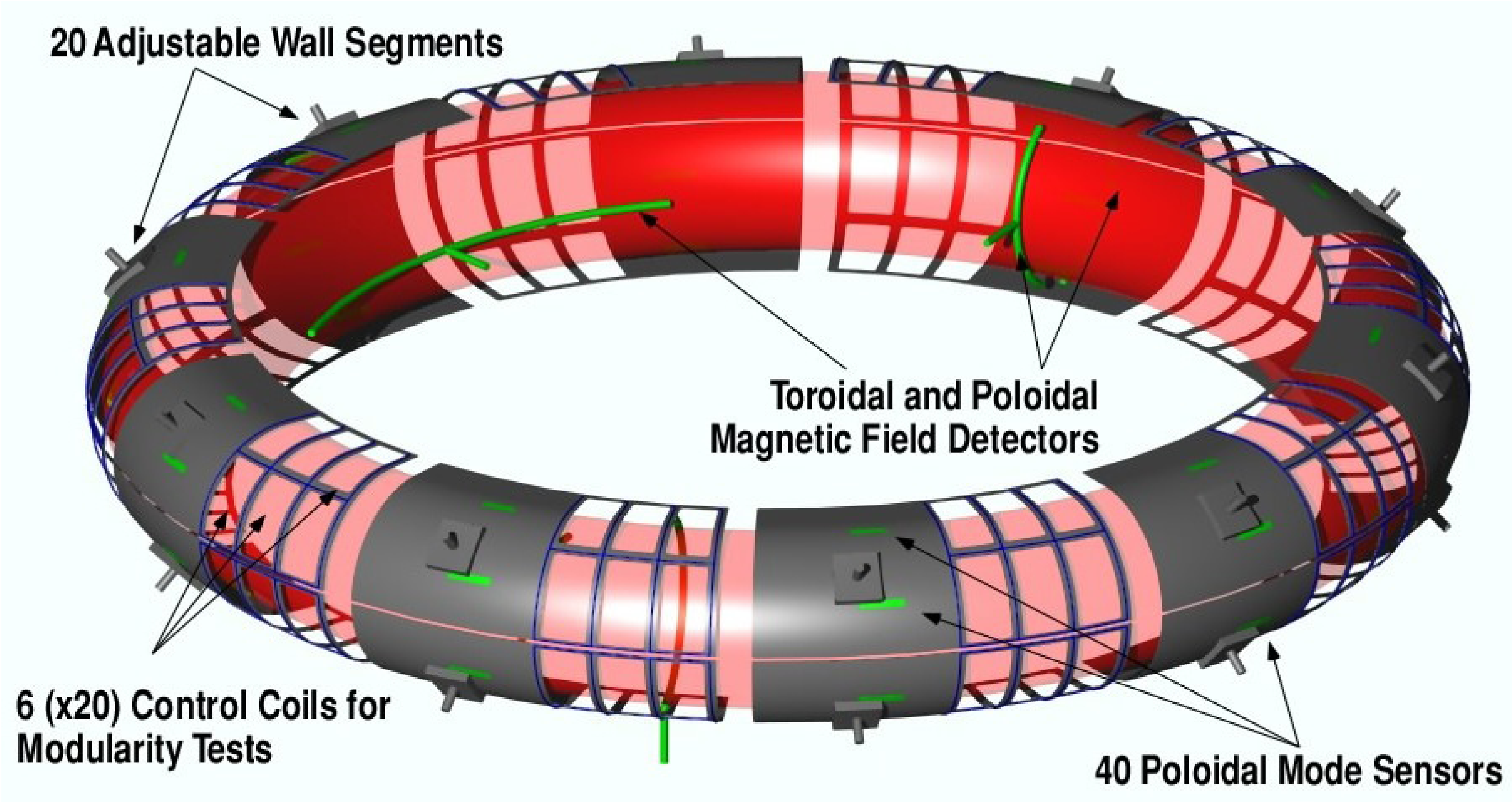
- The $(6, 2)$ mode is a lot smaller than the $(3, 1)$, and higher in frequency.
- It also seems to be mostly unaffected by feedback.

Conclusions and future work

Conclusions

- A **Kalman filter** algorithm has been implemented on a set of low-latency digital feedback controllers, and used to **excite** and **suppress** the $(m, n) = (3, 1)$ external kink instability on HBT-EP.
- The Kalman filter uses a simple, internal model for a growing, rotating mode.
- **Excellent performance** was achieved with initial guesses for filter model parameters; performance was a bit better when the parameters were optimized.
- Kalman filter feedback **works** under **noisy conditions** that disrupt feedback with conventional algorithms.

Future work



- A new wall will be installed in early 2009.
- There will be an increased number of small control coils for modularity and mode rigidity studies.
- ITER will also have small, modular control coils. We will address whether these types of coils can suppress low-wavelength modes.

HBT-EP Collaborators

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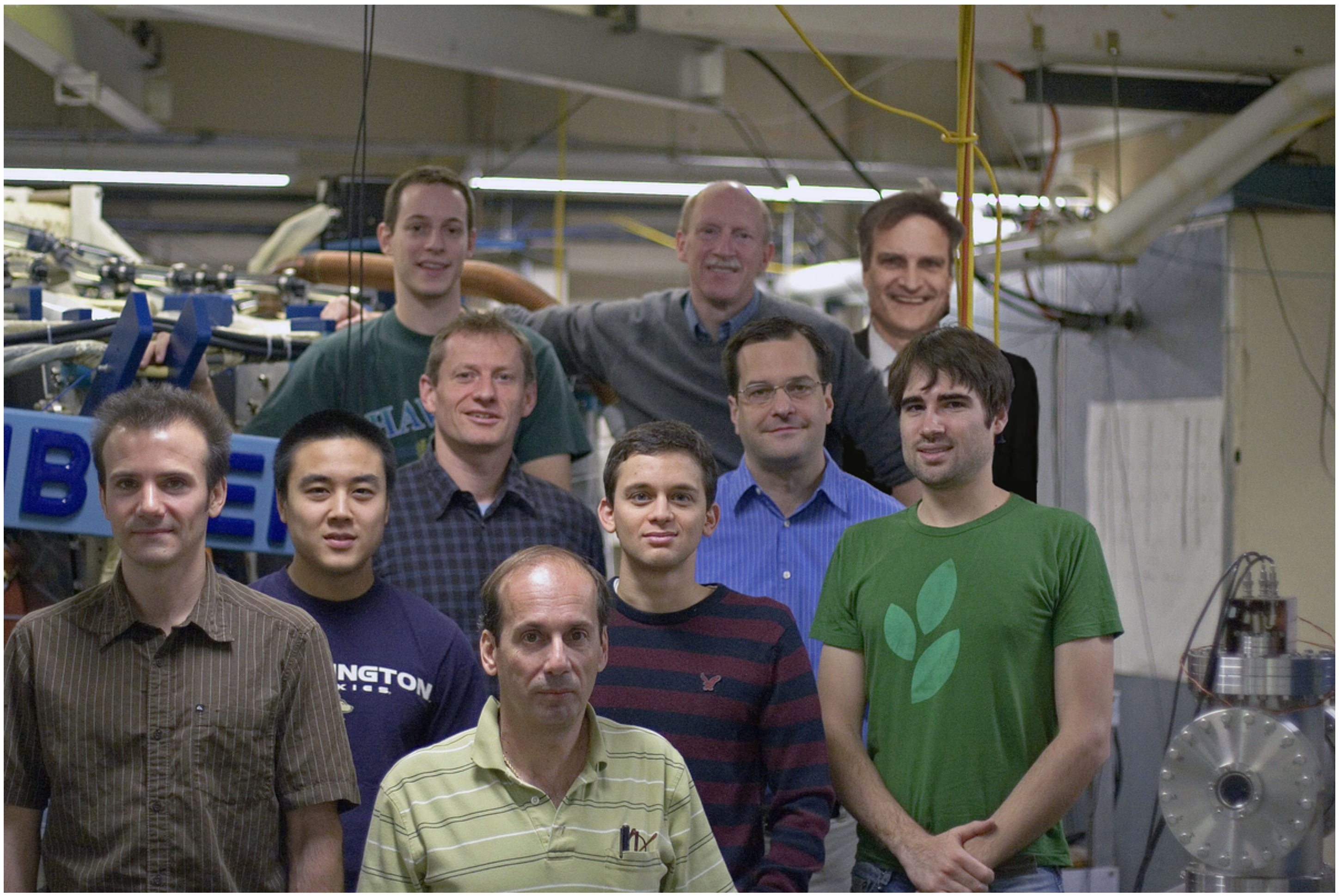

Conclusions

- A **Kalman filter** algorithm has been implemented on a set of low-latency digital feedback controllers, and used to **excite** and **suppress** the $(m, n) = (3, 1)$ external kink instability on HBT-EP.
- The Kalman filter uses a simple, internal model for a growing, rotating mode.
- **Excellent performance** was achieved with initial guesses for filter model parameters; performance was a bit better when the parameters were optimized.
- Kalman filter feedback **works** under **noisy conditions** that disrupt feedback with conventional algorithms.

Extra slides

A Simple Control Coil Model

The control coils are modeled by[a]

$$\frac{\mathrm{d}\psi_c}{\mathrm{d}t} + \frac{R_c}{L_c}\psi_c = \frac{M_c}{L_c}V_c$$

or, equivalently

$$\psi_{c\,n} = \epsilon\psi_{c\,n-1} + \frac{M_c}{R_c}(1-\epsilon)V_{c\,n}$$

with $\epsilon = \exp(-R_c/L_c\,\delta t)$.

[a] M. E. Mauel, *et. al.*, *Bull. Amer. Phys. Soc. Paper* BP1.00007

The Time-varying Kalman Filter uses a simple, internal model

$$\begin{aligned}
\vec{x}_n^* &= \hat{A}\vec{x}_{n-1} + B\vec{u}_n \\
\vec{x}_n &= \vec{x}_n^* + K_n(\vec{z}_n - H\vec{x}_n^*) \\
\hline
P_n^* &= \hat{A}P_{n-1}\hat{A}' + Q \\
K_n &= P_n^* H'(HP_n^* H' + R)^{-1} \\
P_n &= (I - K_n H)P_n^*
\end{aligned}$$

The state vector is

$$\vec{x}(n) = (B_p^{\cos}(n), B_p^{\sin}(n), B_p^{\cos}(n-1), B_p^{\sin}(n-1)).$$

The system model A comes from the dynamics of a growing, rotating mode.

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} B_p^{\cos} \\ B_p^{\sin} \end{pmatrix} = \begin{pmatrix} \mathrm{Re}\gamma_k & -\mathrm{Im}\gamma_k \\ \mathrm{Im}\gamma_k & \mathrm{Re}\gamma_k \end{pmatrix} \begin{pmatrix} B_p^{\cos} \\ B_p^{\sin} \end{pmatrix}.$$

with $\gamma_k = 1.27 + 4.26i$ kHz.

Time-varying Kalman Filter

The $B\vec{u}$ term must give the response of the kink mode to a control flux.

$$B\vec{u} = 2\delta t \left(\frac{3}{1-c}\right) \mathrm{Re}\left[((2\sqrt{c}, -(1+c)) \cdot \vec{\xi}_k)(\Xi^{-1} \cdot \vec{R}) \cdot \hat{e}_k \begin{pmatrix} 1 \\ \mathrm{e}^{-i\pi/2} \end{pmatrix} \psi_c\right]$$

Here, Ξ contains the eigenvectors of the reduced Fitzpatrick-Aydemir system matrix in its columns, and k the index of the unstable eigenvalue.[a]

Note: to solve this equation, we must measure the flux in the control coils.

[a] M. E. Mauel, *et. al.*, *Bull. Amer. Phys. Soc. Paper* BP1.00007

The Steady-State Kalman Filter is easy to implement

The time-varying Kalman Filter is probably too large to implement on HBT-EP's present mode control system.

Take the limit of the time-varying Kalman filter in which $n \to \infty$.

This filter has a simple form – the controller does not need to compute a matrix inverse.

$$\vec{x}_n = \Phi \vec{x}_{n-1} + K \vec{z}_n$$

The matrices can be calculated in advance.

$$\Phi = (I - KH)(\hat{\hat{A}} + BGH)$$
$$K = (\hat{\hat{A}} P \hat{\hat{A}}' + Q)H'(H(\hat{\hat{A}} P \hat{\hat{A}}' + Q)H' + R)^{-1}$$

The control flux ψ_c must be added to the state vector.

$$\vec{x}(n) = (\psi_c^{\cos}(n), \psi_c^{\sin}(n), B_p^{\cos}(n), B_p^{\sin}(n))$$

Steady-State Kalman Filter

There is a subtlety: if the control flux is not measured, it must be computed from the control voltage.

$$G = \frac{M_c}{R_c}(1-\epsilon)\left(\begin{array}{cc|cc} \frac{\epsilon R_c}{M_c(1-\epsilon)} & 0 & g_p & ig_p \\ 0 & \frac{\epsilon R_c}{M_c(1-\epsilon)} & -ig_p & g_p \\ \hline \multicolumn{2}{c|}{0_{22}} & \multicolumn{2}{c}{0_{22}} \end{array}\right)$$

The response of the system to the control flux must be calculated, too.

$$B = \left(\begin{array}{cc|c} \sigma & 0 & \\ 0 & \sigma & 0_{22} \\ \hline \multicolumn{2}{c|}{0_{22}} & 0_{22} \end{array}\right)$$

Here, $\sigma = 2\delta t \frac{3}{1-c}((2\sqrt{c}, -(1+c)) \cdot \vec{\xi_i})(\Xi^{-1} \cdot \vec{R}) \cdot \hat{e}_i$.

When calculating the filter parameters, the real part of the product BG is used.