Advanced RWM Control Methods on EXTRAP T2R

J.R. Drake , K.E.J. Olofsson, L. Frassinetti, P.R. Brunsell,
Association Euratom-VR, Alfvén Laboratory, Royal Inst. of Technology KTH, Stockholm

e-mail contact: *james.drake@ee.kth.se*

Theme

Going Beyond simply stabilising RWMs.

- *Successful stabilisation of RWMs in RFPs is well known*

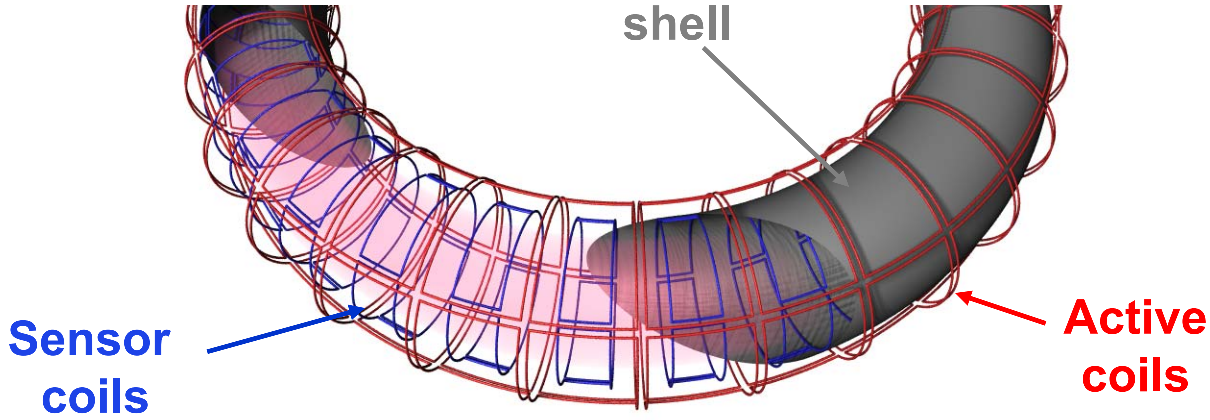
Important to optimize the RWM control systems for future implementation.

- *Develop a multiple input-multiple output (MIMO) controller using state-space model.*
- *Level equalise the transfer functions to account for non-axisymmetric features in the wall* τ_{mn}.
- *Develop control to track an "arbitrary" reference state.*
- *Robust controller stability and acceptable power requirements.*

Use the controller for generic MHD studies.

- *RMP effect on a rotating tearing mode.*

EXTRAP T2R experiment



- R/a=1.24m/0.18m
- $I_p \approx 100kA$
- τ_{wall}=6ms
- τ_{pulse}= up to 90ms
- 4 poloidal x 32 toroidal
  active and sensor saddle coils (m=1 connected)

Generic mode control strategy

RWM control system (the Plant):

- *Plasma, walls, vessel, etc.*
- *Arrays of magnetic sensor coils.*
- *Arrays of active saddle coils (actuators).*
- *Controller hardware plus software.*

Feedback is implemented by a real-time controller which uses, eventually in a well-defined optimal way, the admissible actuators for MHD control in the form of currents in the active coils to constrain the MHD mode evolution at a specified reference spectrum by responding to measured sensor voltages.

The digital controller was originally developed by Consorzio RFX and implemented in a collaborative effort on both the EXTRAP T2R and RFX-mod experiments.

New software incorporating advanced control theory has been developed for EXTRAP T2R and then installed and tested.

RWM Basics

A RWM radial field perturbation characterised by (m,n) measured at the wall

$$b_{m,n} \propto \exp\left(im\vartheta + in\phi\right)$$

has a growth rate, $\gamma_{m,n}$

and the dynamics is described by ,

$$\tau_{m,n}\dot{b}_{m,n} - \tau_{m,n}\gamma_{m,n}b_{m,n} = b_{m,n}^{ext}$$

where $\tau_{m,n}$ *is the wall penetration time for the mode and*

$$b_{m,n}^{ext}$$

is that part of the resonant perturbation measured at the wall by the sensor coils that includes the field produced by the active coils, field errors, MHD noise,...).

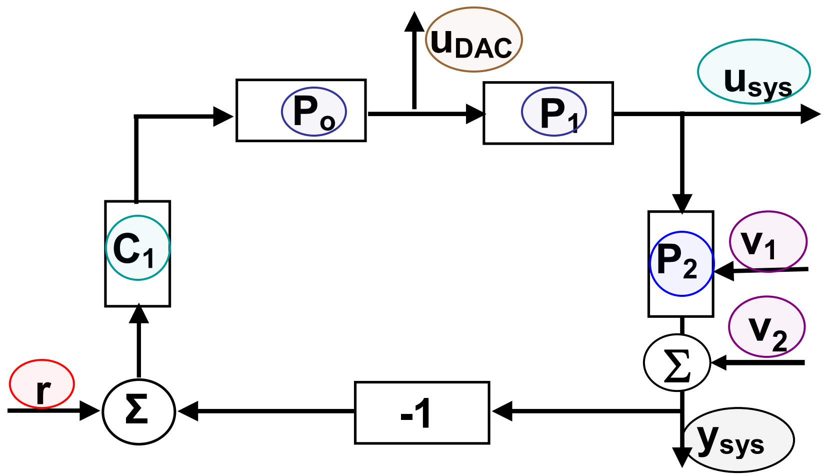
Typical routing of the signals in the closed control loop

$u_{DAC}(t)$ represents the actuator channels' control voltage.

P_o, P_1 represent a time delay and a composite active coil and power amplifier.

C_1 is the feedback control function.

$r(t)$ is a reference.



$u_{sys}(t)$ represents the active coil currents.

P_2 represents the front-end RWM wall/plasma dynamics.

$v_1(t)$ is an exogenous signal representing the field errors and MHD noise.

$v_2(t)$ is white noise.

$y_{sys}(t)$ represent the time-integrated sensor voltages.

ROYAL INSTITUTE OF TECHNOLOGY

Multiple-input multiple-output

The system dynamics for the MIMO model are expressed in state-space form;

$$\frac{dx}{dt} = Ax + Bu_{sys} + Nv_1$$
$$z = Mx$$
$$y_{sys} = Cx + v_2$$

where:

- **x** is a vector of MHD modes (i.e. the *state* of the MHD fluid)
- $\mathbf{u_{sys}}$ (coil currents) and $\mathbf{y_{sys}}$ (sensor signals) are defined in the previous slide.
- *A, B* and *C* are system matrices defined by the parameters and geometry of the wall, actuators and sensors.
- $N\mathbf{v_1}$ is a source term which bundles effects of field-errors and MHD noise.
- $\mathbf{v_2}$ is a white noise signal
- **z** is an optional vector expressing the desired performance
- *M* is a key factor for implementing process control and relates the MHD harmonics **x** to the merit vector **z**. (defines the control system objective).

Multiple-input multiple-output

$$\frac{dx}{dt} = Ax + Bu_{sys} + Nv_1$$

$$z = Mx$$

$$y_{sys} = Cx + v_2$$

Given measurements y_{sys}, an observer of z, presumably a subset of *important* MHD harmonics, yields the estimate $\hat{z}$, which is the inferred multivariable signal the control system reads (via model-based filtering, e.g. *Kalman*-filter).

MIMO implemented on T2R- How it is done. (dry plant)

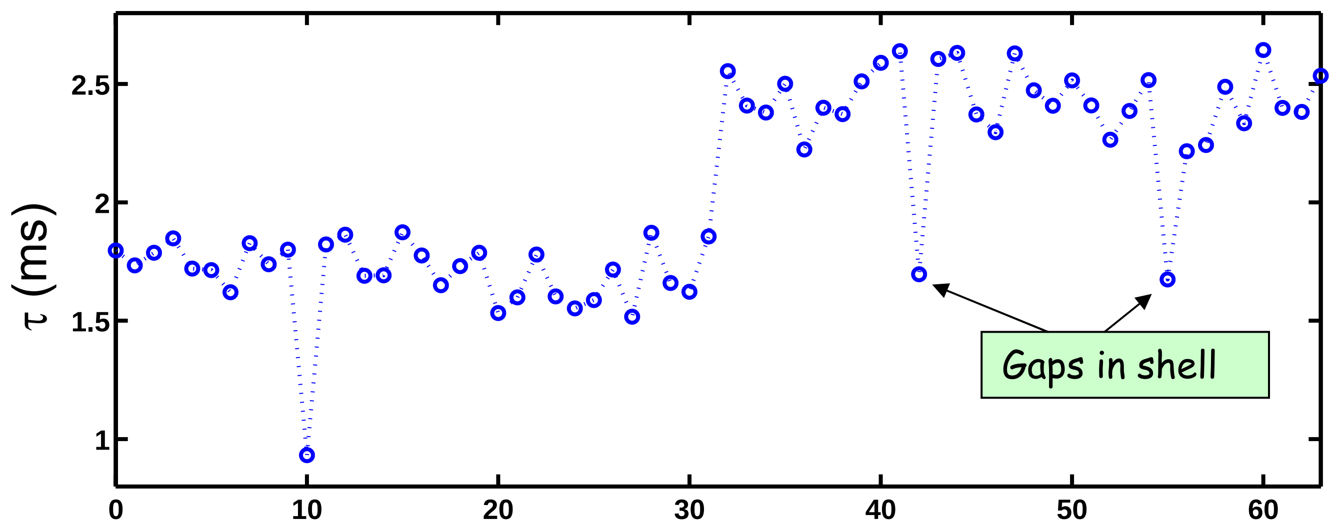
- Active coil, power amplifier and control system are aggregated in a model.
  - Coil rise time, pure delay, static gain, …
  - Open loop routing yields parameter estimation.
- Use *pseudo random binary sequence* (PRBS) input excitations for “dry” vacuum vessel identification via current controlled open loop routing to identify dry plant behaviour.
- Intermediate MISO step with “immediate neighbour (tri-diagonal)” inputs only to reduce size.
- Associated linear state space model parameter estimation problem then set up and the accompanying parameter-sensitivity DEs are set up and solved (numerical optimisation codes).
- Extend to full MIMO.
- Vacuum input-output mapping inverted and a tridiagonal input output mixing matrix is found to level-equalise input-output channels.

The dry plant is then characterised.

This can be represented in terms of something we recognise, namely the relevant wall penetration time for each active coil channel.

(A channel corresponds to an in-board out-board pair or an up-down pair. There are 32 toroidal positions which means 64 channels.)

Wall/vessel penetration time



0-31 (inboard-output pairs) 32-63 (up-down pairs)

Channel (toroidal position twice around torus)

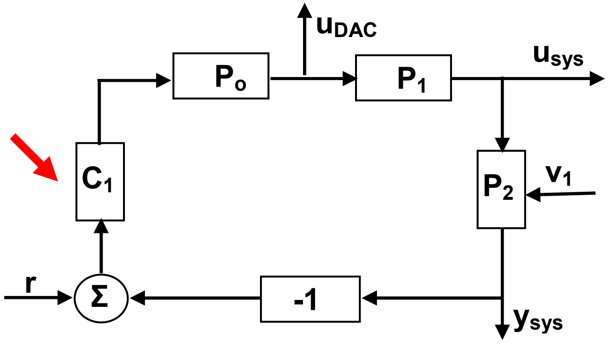
MIMO implemented on T2R- How it is done. (now with plasma)

ROYAL INSTITUTE OF TECHNOLOGY

C_1 is still a PID controller.

&

It is still an "Intelligent Shell" but now a somewhat more intelligent shell with modest MIMO capacity instead of a collection of localised SISO controllers.



The state-space representation is based on linear dynamics.

Design for *out-put tracking through IMC-SODUP nominal tuning.*

(Second order delayed unstable process)

Optimisation implies minimising the difference between the observed state and the reference state in order to tune *C_1*.

Revised IS feedback law

$$u_{DAC}(t) = M_{eq} F(s) \left[F_r(s) r(t) - y_{sys}(t) \right]$$

This is essentially a refurbished intelligent-shell, where

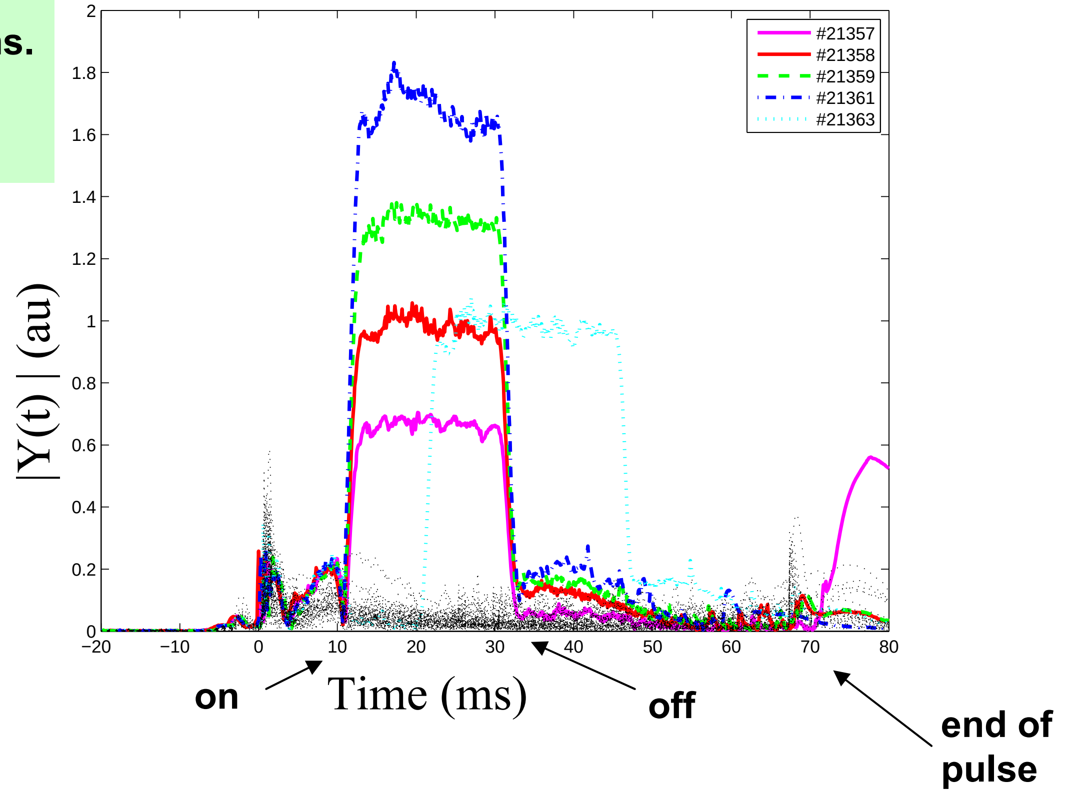
F(s) is a diagonally taylored PID
M_{eq} is tridiagonal
$F_r(s)$ represent parallel set-point filters
r(t) is the vector of reference values for the plant outputs
(the essence of *output tracking* is that r(t) is not zero.)
y_{sys} is the plant outputs

Experimental demonstration of output-tracking controller on EXTRAP T2R

Example 1. Select the reference amplitude for one mode.

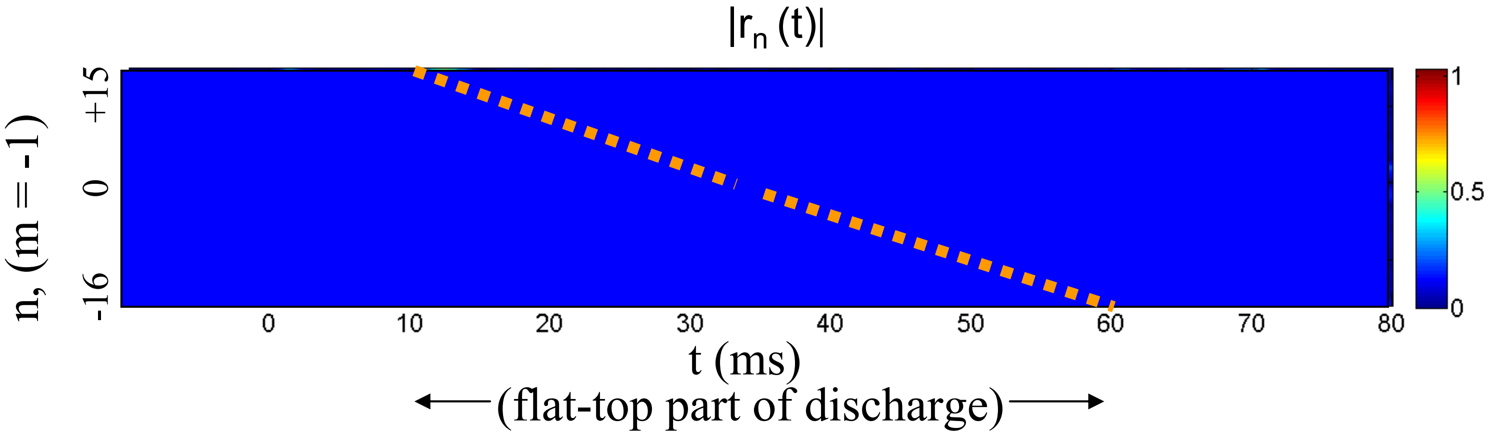
n = -12 harmonic with different reference gains.

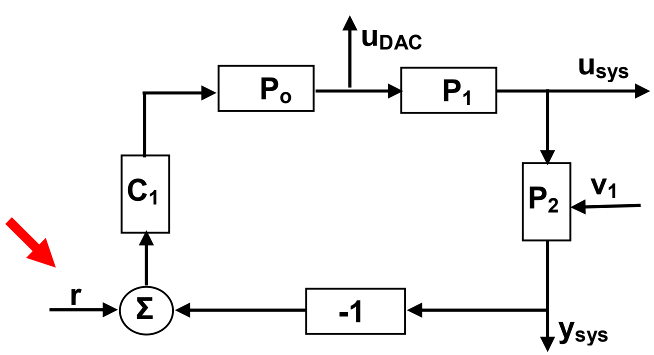
All other modes have reference zero.

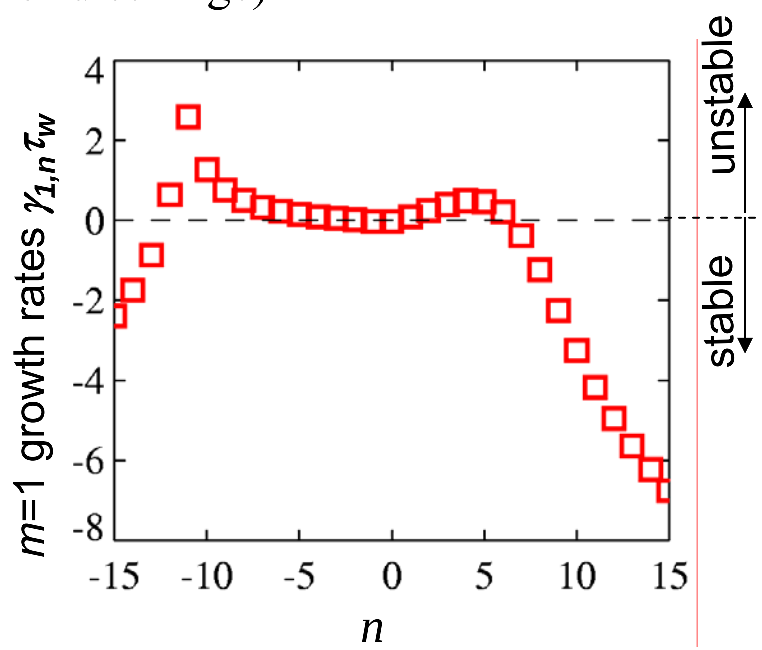


ROYAL INSTITUTE OF TECHNOLOGY

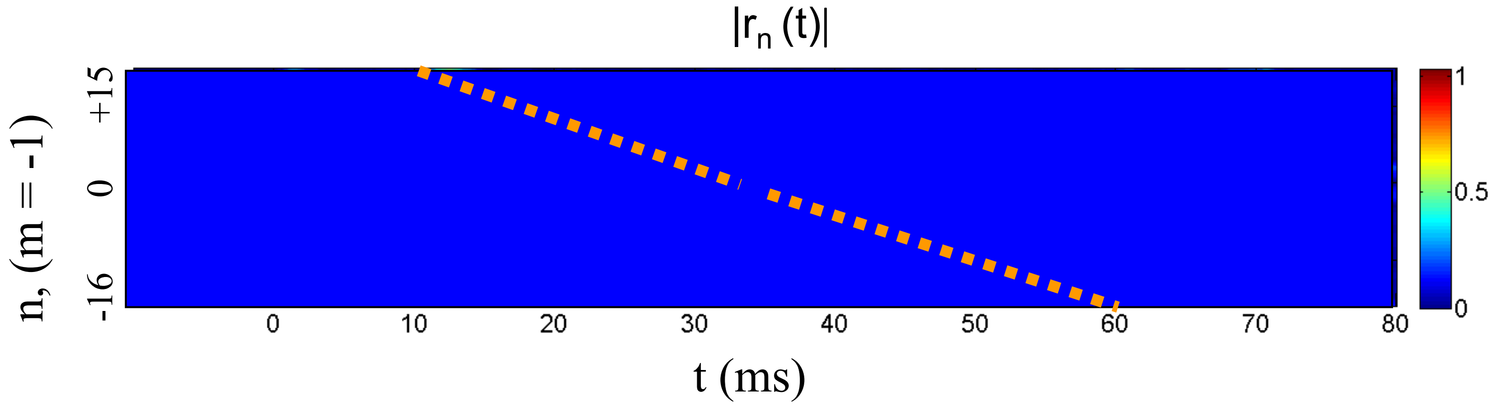
Example 2.
Reference spectrum is a scan from n=+15 to n=-16



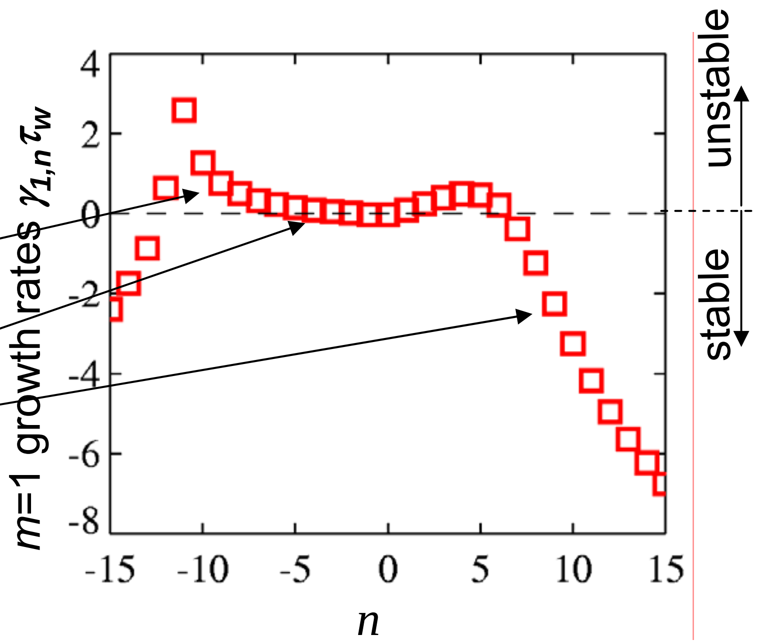




Example 2.
Reference RWM spectrum is a scan from n=+15 to n=-16



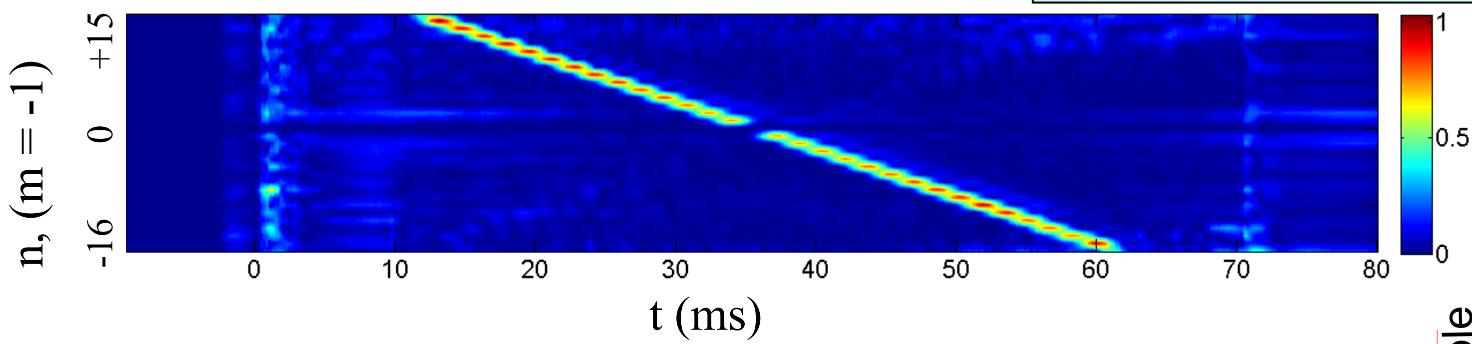
Scan includes resonant TMs (-16≤n≤-12) and non resonant modes (-11≤n≤+15) i.e.
- unstable RW modes
- marginally stable RW modes
- robustly stable RW modes.

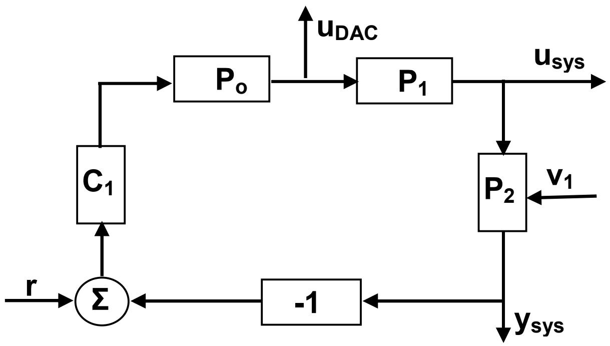


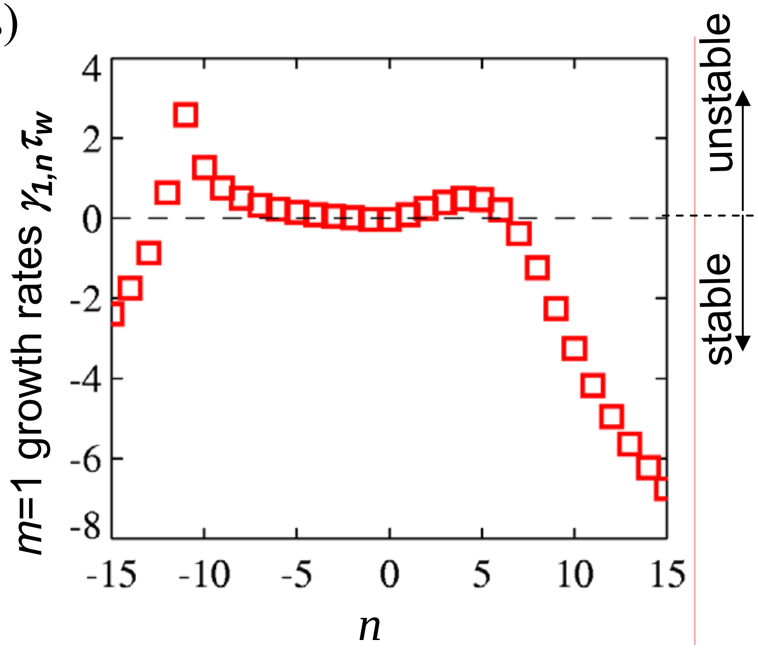
Example 2 (cont)
Reference RWM spectrum is a scan from n=+15 to n=-16

With plasma

Not pre-programmed time-dependent gain



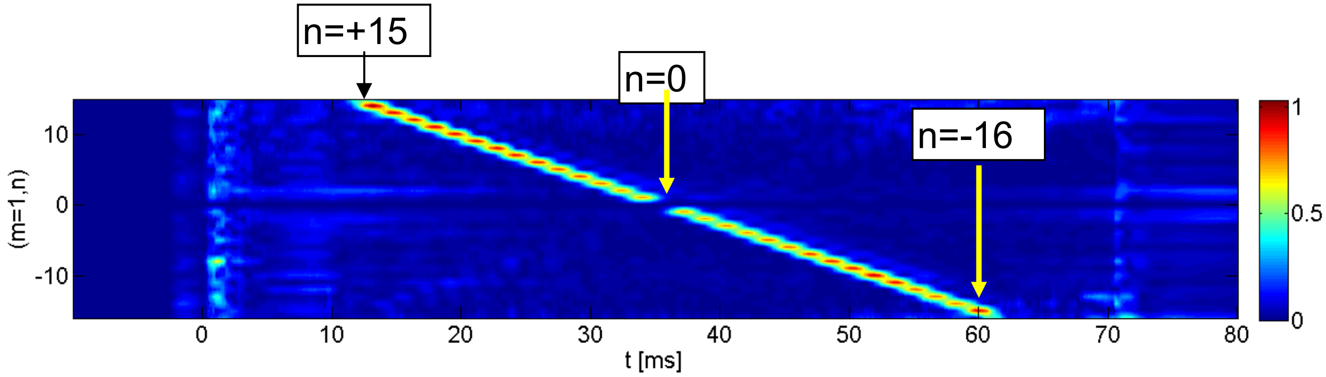




Spectrogram derived from sensor output

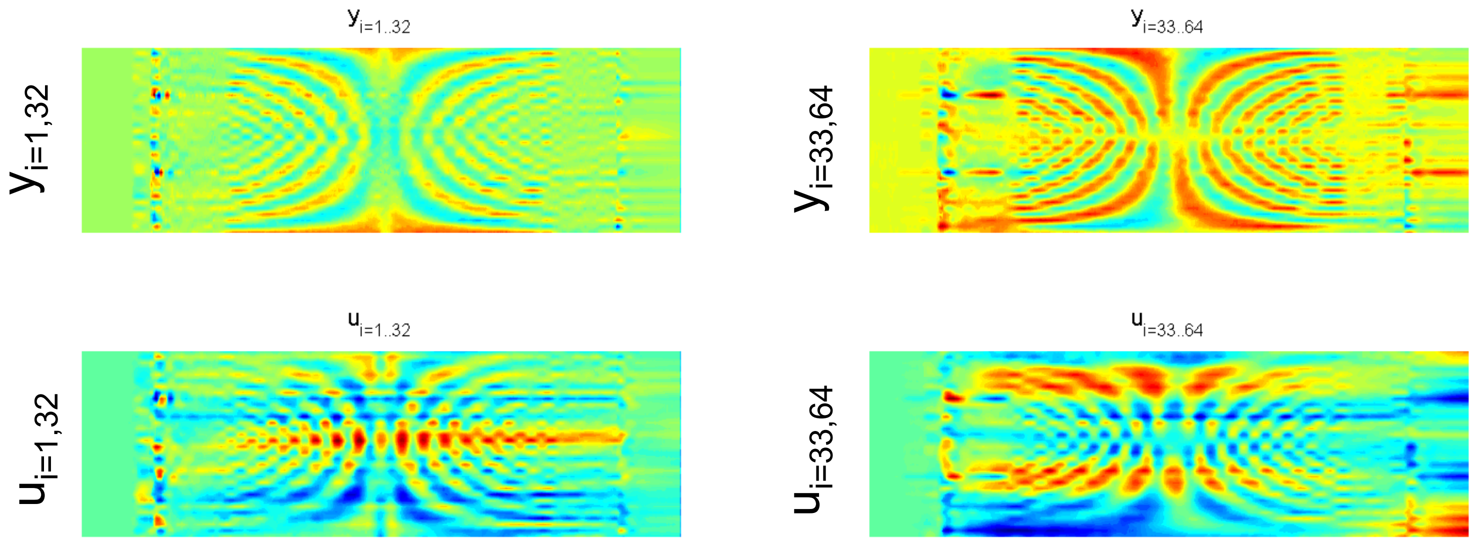
Example 2 (cont.)

Reference RWM spectrum is a scan from n=+15 to n=-16



Sensor signals

u_{sys} signals



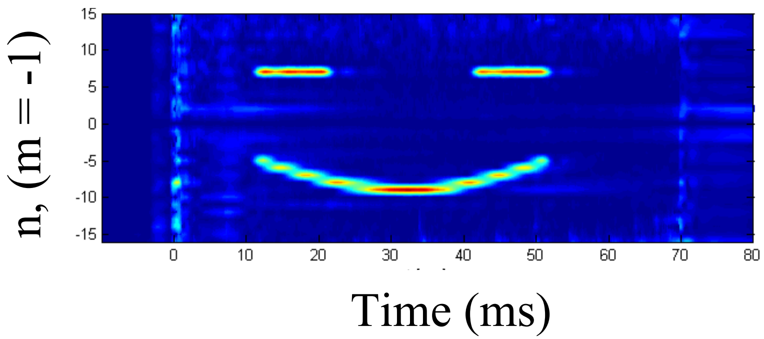
Same time intervals

Same time intervals

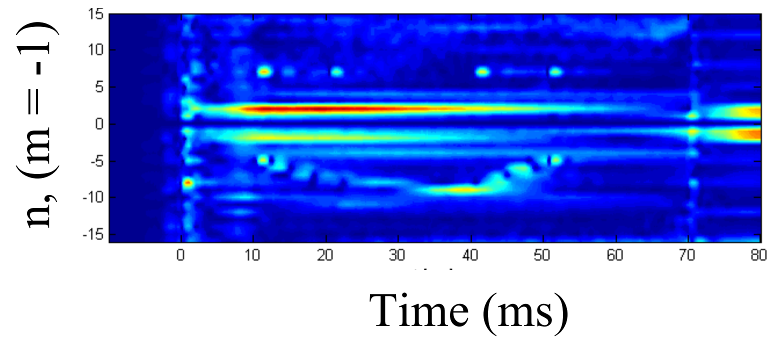
Example 3.
Reference RWM spectrum with simultaneous multiple harmonic amplitudes

ROYAL INSTITUTE OF TECHNOLOGY

$|Y_n|$ (spectrum from sensor signal)



$|U_n|$ (spectrum from coil currents)



Note that some features of the reference spectrum (seen in sensor signal) can be recognized in the spectrum of the actuator coil currents, but it is clear that the controller provides a "broader" spectrum to the actuator in order to reproduce the sharp reference spectrum

Recent advances
Optimization of IS gains

ROYAL INSTITUTE OF TECHNOLOGY

Modelling has also been used to optimize the baseline IS PID gains by particularly focusing on the implications of control system latency effects.

[Ref OLOFSSON, K.E.J., et al.,Proceedings of the 47th IEEE Conference on Decision and Control 2008 (to appear)].

The predicted optimal gains *(Kp,Ki,Kd)*, obtained through closed-loop eigenvalue minimization of a delay differential equation (DDE), were implemented in the controller and tested on the T2R device, which resulted in improved PID controller performance compared to the gains that had been empirically determined.

Improvements were in the sense of reduced magnetic field energy at the sensors, admittedly at the cost of higher control power requirements.

T2R Experimental results for classic IS with DEO-gains
[Ref: Erik Olofsson, CDC-2008]

Figures of merit:

$[J_y] = (mT)^2 \times 10^{-3}$

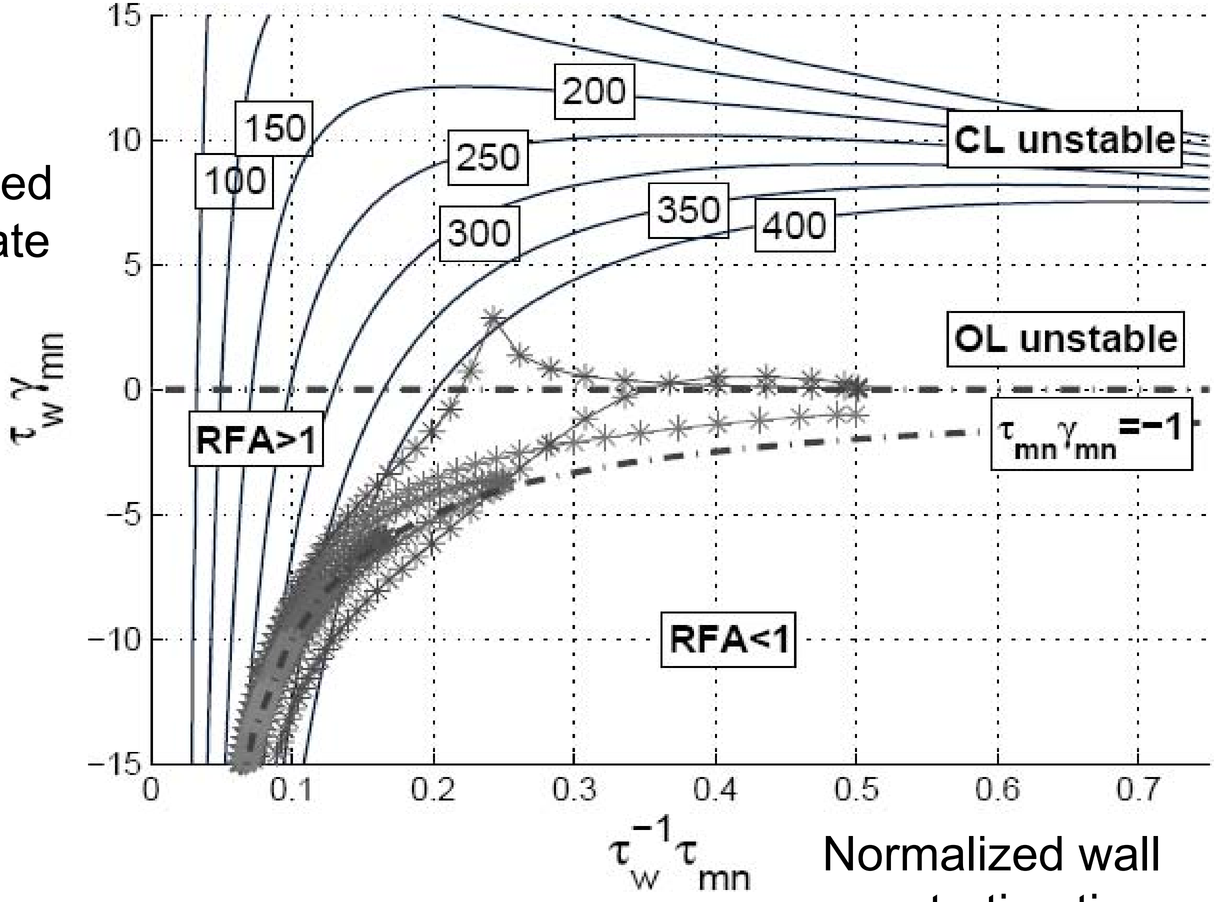
$[J_u] = A^2 \times 10^3$

Shot #	Kp	Ki	Kd	Jy (sensors) ± 10%	Ju (active coils) ± 10%	Remarks
20743	150	16000	0.05	1.04	1.66	Old gain
20746	106	37500	0.061	**0.581**	2.12	Opt *a)*
20835	106.8	39860	0.058	**0.645**	1.64	Opt *b)*

$$J_x(\theta) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} x^T(\tau,\theta) Q_x x(\tau,\theta) d\tau, \quad Q_x = I,\, x = y_{sys} \text{ and } u_{sys}$$

Latency time in μs

Normalized growth rate



Normalized wall penetration time

Controller for generic MHD studies.

Control an MHD phenomenon and study, possibly using the controller parameters needed to initiate the phenomenon as a diagnostic.

Control of rotating tearing mode amplitude using a FB-controlled stationary resonant magnetic perturbation. The ingredients in EXTRAP T2R are:

- *Intrinsic toroidal ambient plasma flow at high velocities (V_φ up to 30km/s).*
- *Intrinsic TM mode rotation at $\omega_n \leq nV_\varphi/R$*
- *Note there can be slippage: $\Delta\omega = (nV_\varphi/R - \omega_n)$.*

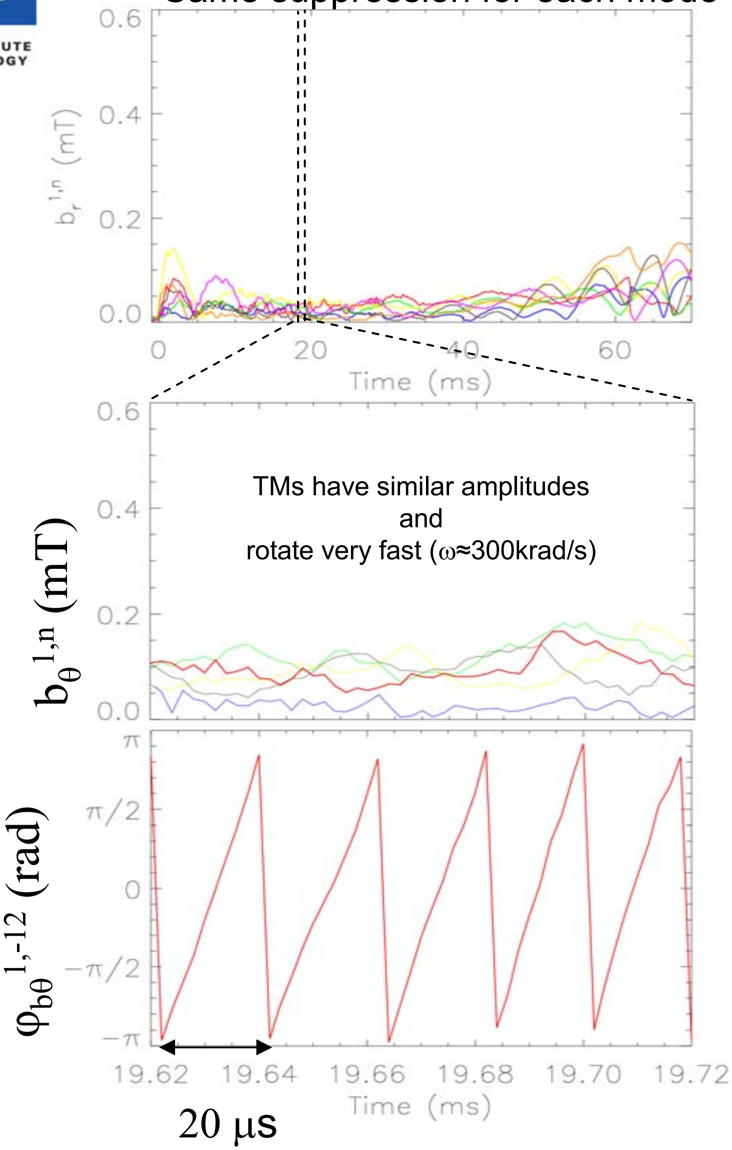
Torque balance determining ω_n.

- *Toroidal viscous torque from fluid to TM island (spin up).*
- *Drag due to eddy currents in the wall (spin down).*
    - *Dependent on $\delta b_{m,n}^2$(Fitzpatrick)*
- *Locking torque due to RMP (spin down).*
    - *Dependent on $\delta b_{m,n}\, \delta b_{RMP}$.*

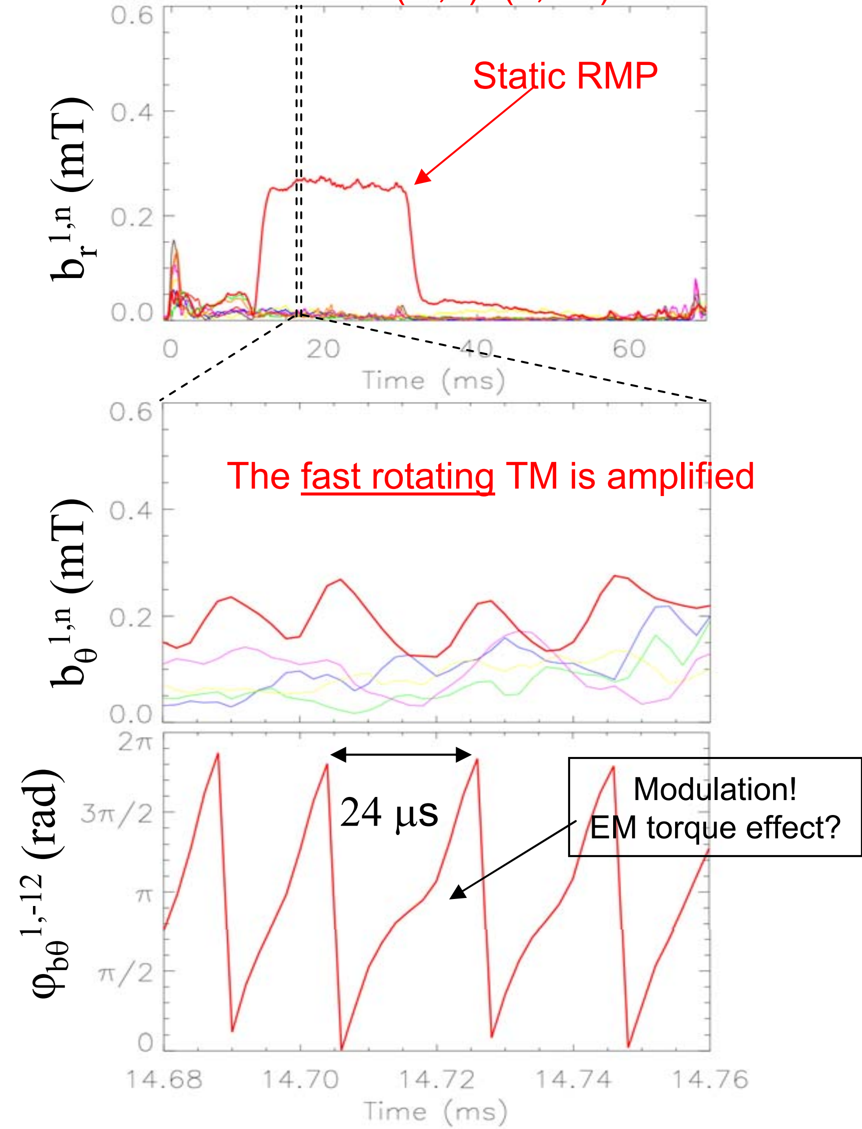
EXPERIMENTAL RESULTS

ROYAL INSTITUTE OF TECHNOLOGY

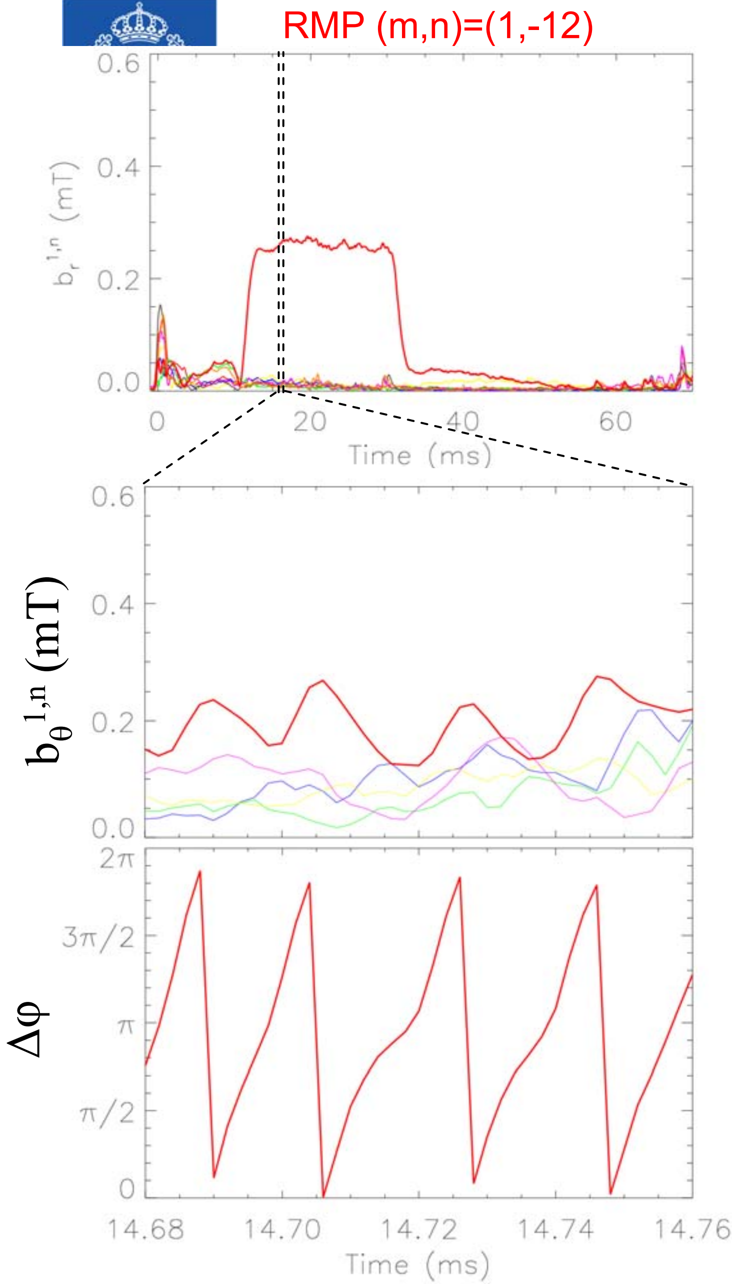
Same suppression for each mode

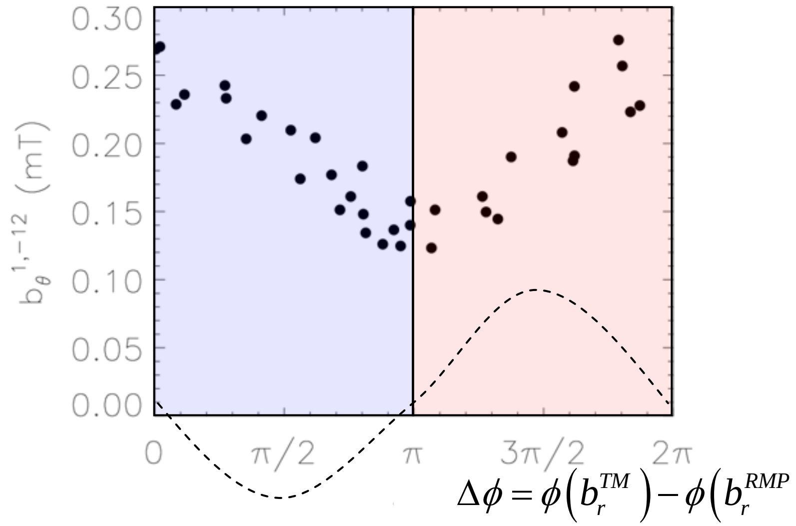


RMP (m,n)=(1,-12)



EXPERIMENTAL RESULTS





TM "in phase with RMP" ⇒ amplification
TM in "anti-phase with RMP" ⇒ suppression

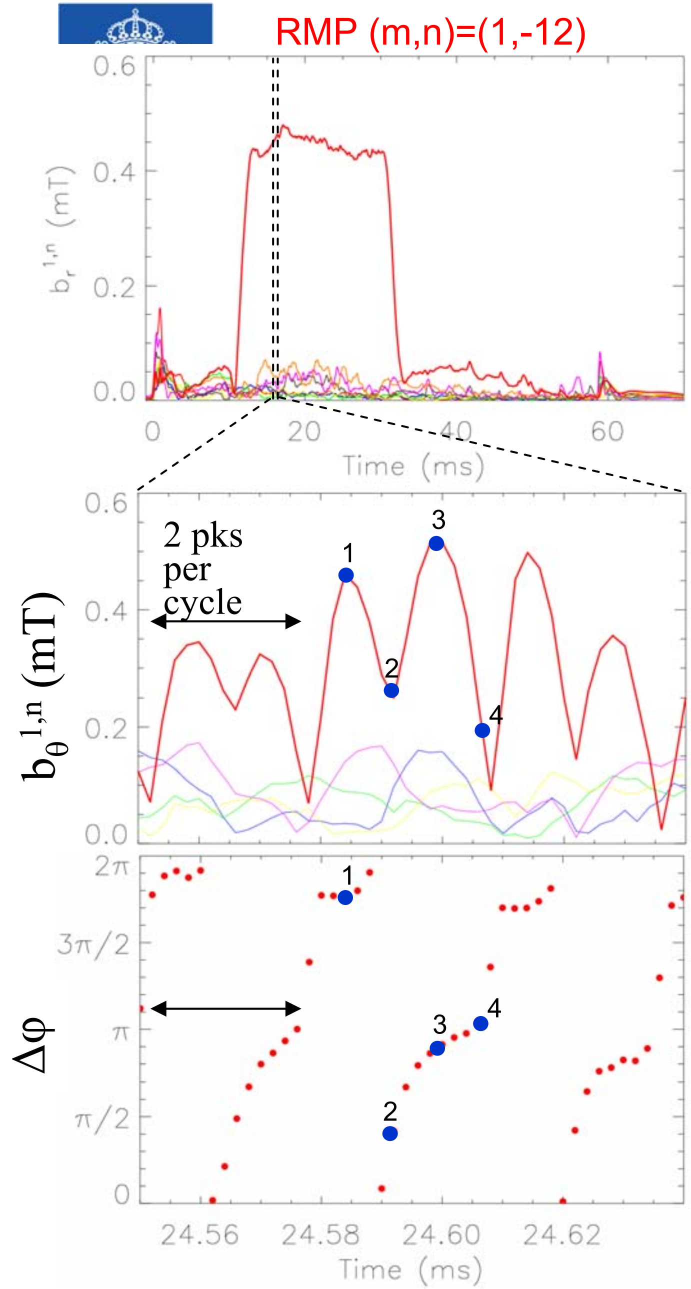
But:

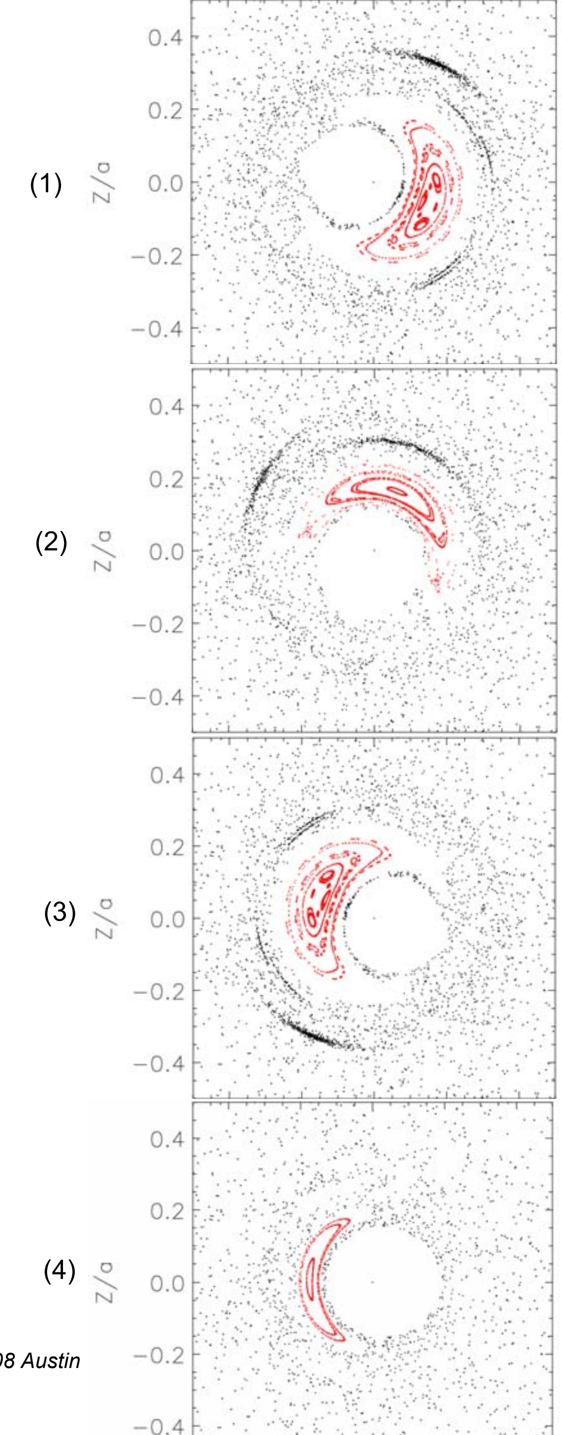
$$\tau_R \frac{d\Psi}{dt} = k_1 \left[k_2 \sqrt{\Psi} \log \left(\frac{k_3}{\sqrt{\Psi}} \right) - \Delta' \right] \sqrt{\Psi} - k_4 \frac{\epsilon}{\sqrt{\Psi}} \cos(\Delta\phi)$$

$$\epsilon = -i r_w b_r^{RMP}(r_w)$$

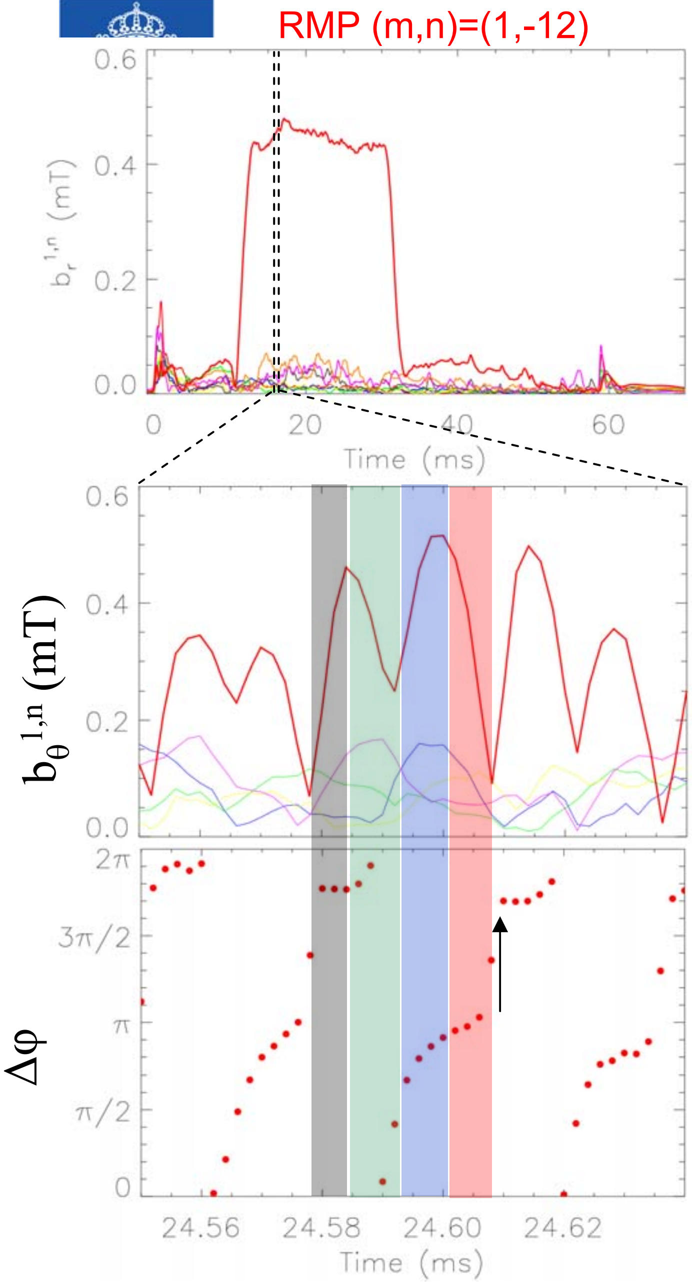
$$\Psi = -i r_{rs} b_r^{m,n}(r_{rs})$$

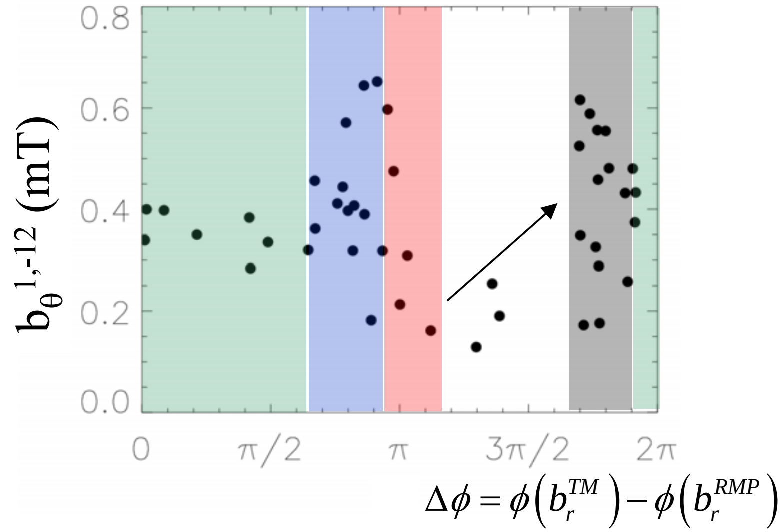
Based on
[Fitzpatrick, Phys. Plasmas 8, 4489 (2001)]


RMP (m,n)=(1,-12)
$b_r^{1,n}$ (mT)
Time (ms)
2 pks per cycle
$b_\theta^{1,n}$ (mT)
$\Delta\varphi$
1
2
3
4
Time (ms)


(1)
(2)
(3)
(4)
Z/a

EXPERIMENTAL RESULTS





2 peaks for each cycle

locked

acceleration

deceleration

pre-phase flip

Phase jump and the island re-born approximately in phase with the RMP

MODELLING

Based on
[Fitzpatrick, Phys. Plasmas 8, 4489 (2001)]

Three coupled partial differential equations:

1. TM evolution

$$\tau_R \frac{d\Psi}{dt} = k_1 \left[k_2 \sqrt{\Psi} \log\left(\frac{k_3}{\sqrt{\Psi}} \right) - \Delta' \right] \sqrt{\Psi} - k_4 \frac{\epsilon}{\sqrt{\Psi}} \cos(\Delta\phi)$$

$$\epsilon = -i r_w b_r^{RMP}(r_w)$$

$$\Psi = -i r_{rs} b_r^{m,n}(r_{rs})$$

if $\varepsilon >> 1$ $\tau_R \frac{d\Psi}{dt} = -k_4 \frac{\epsilon}{\sqrt{\Psi}} \cos(\Delta\phi)$

$$\Psi \approx \left[\frac{3k_4}{2\tau_R} \frac{\epsilon \sin(\omega t)}{\omega} \right]^{\frac{2}{3}}$$

$\Rightarrow$ The RMP is screened by the TM rotation

2. Torque balance

$$\rho \frac{\partial \Delta\Omega}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r\mu \frac{\partial \Delta\Omega}{\partial r} \right) + \frac{T_{EM}}{4\pi^2 R_0^3} \delta(r - r_s)$$

Viscous torque *EM torque*

with $$T_{EM} = \kappa_1 \frac{c_1 \omega \tau_w}{1 + c_2 \omega^2 \tau_w^2} \left| \Psi_{rs} \right|^2 + \kappa_2 \sin(\Delta\phi) \left| \Psi_{rs} \right| \left| \epsilon \right|$$

braking torque due to eddy currents In the wall generated by the rotating TM

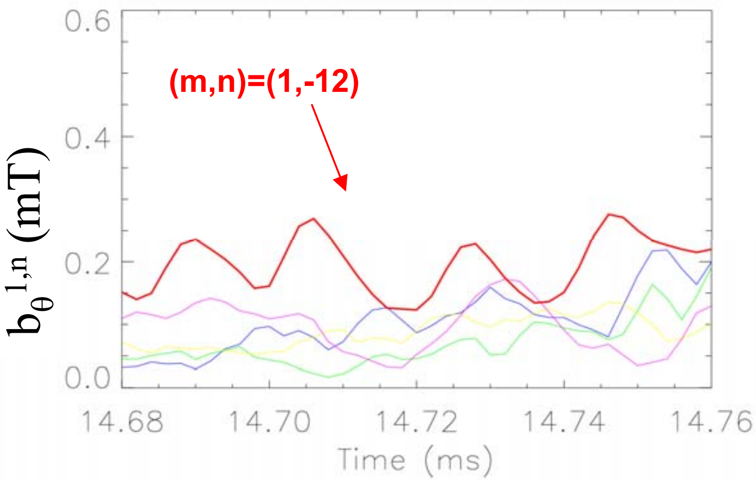
braking torque due to interaction of the rotating TM with the static RMP

3. Helical velocity

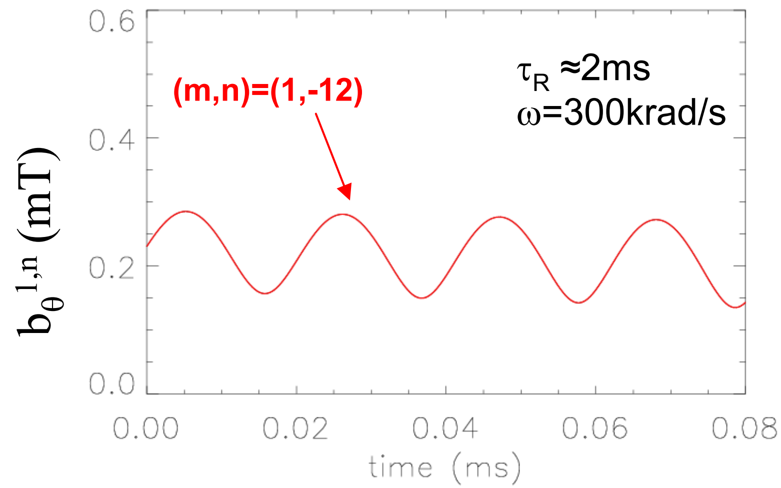
$$\Delta\dot{\phi} = \Delta\Omega + v_0$$

MODELLING

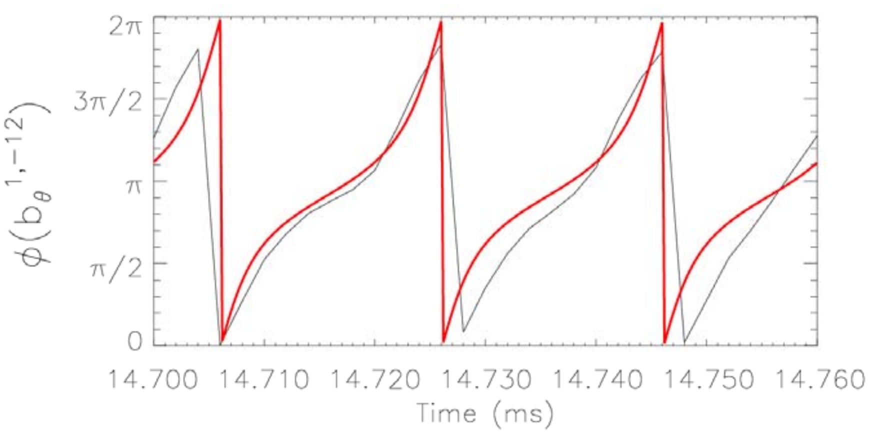
Experiment with
b_r^{RMP} =0.25mT



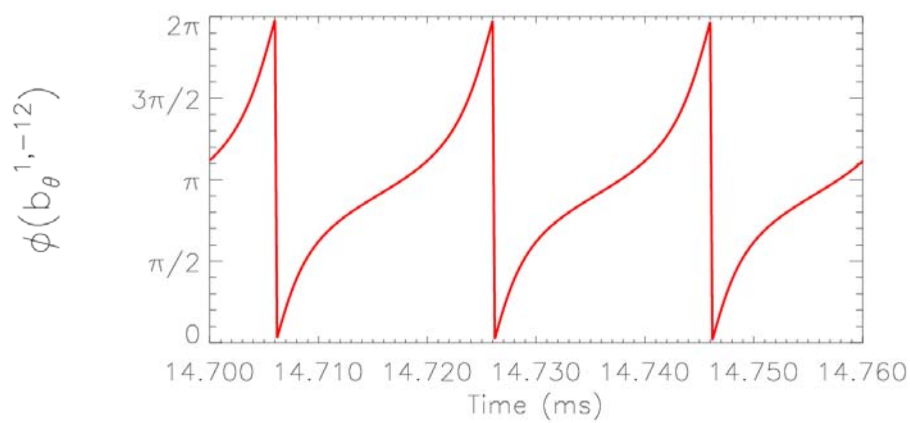
Simulation with
b_r^{RMP} =0.25mT



Experiment with
b_r^{RMP} =0.4mT



Simulation



Future studies

- MIMO controller optimisation.
  - Expand optimisation to include coil array parameters, geometry, distance, sparse arrays, etc.

- MHD experiments.
  - Erik Olofsson will give an example in a presentation to come.
  - There is more to do concerning action of a stationary RMP applied to a rotating resonant tearing mode with slippage (i.e. shielding of RMP effect on an island at a resonant surface by a rotating plasma).
  - Controlled slow rotation of non-resonant RWM with a rotating plasma fluid.