Computational analysis of advanced control methods applied to RWM control in tokamaks

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Magnetic Islands and Rotation

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Control theory terminology used in this talk

Advanced controller

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State-space based controller

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Linear Quadratic Gaussian Controller (**LQG**)

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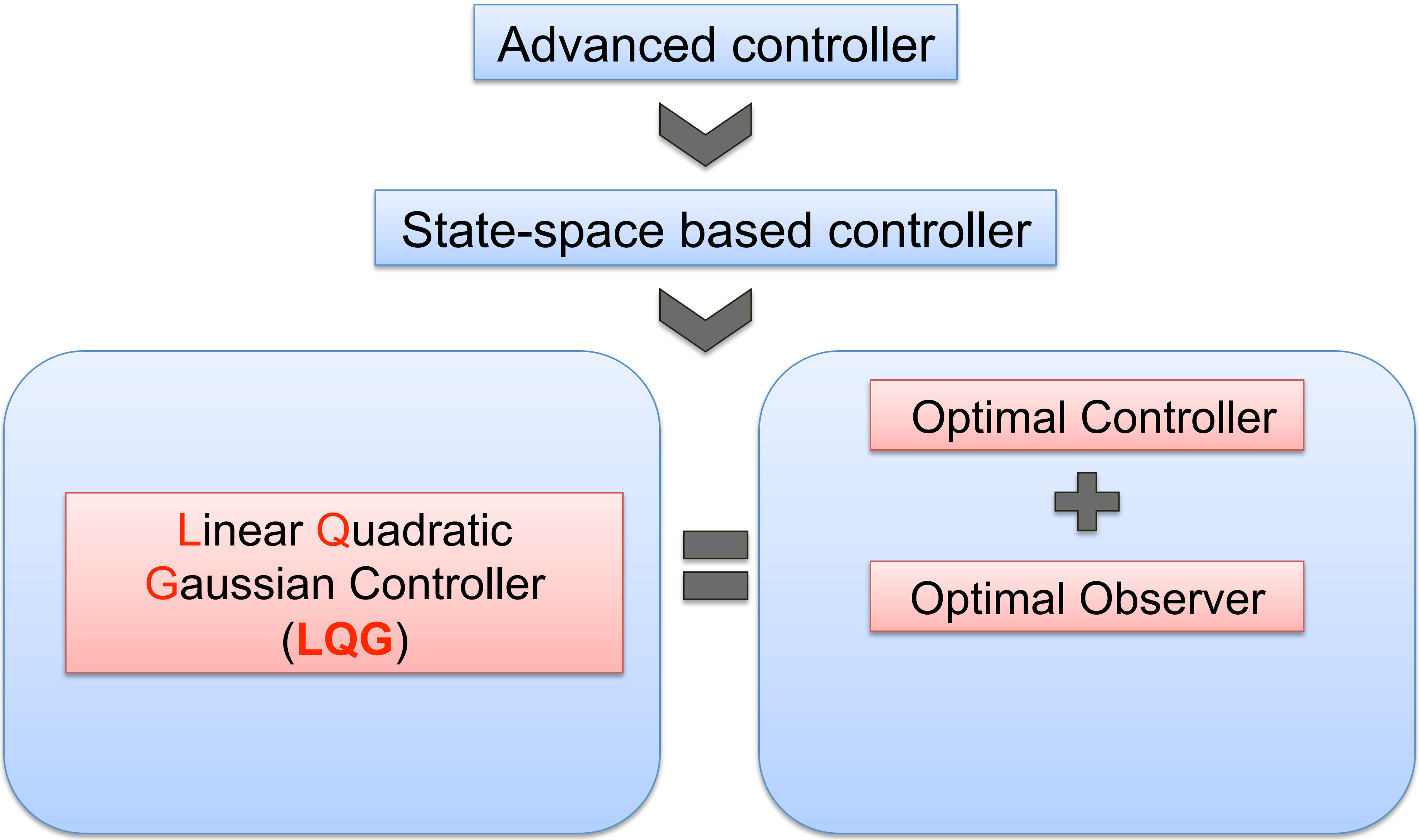

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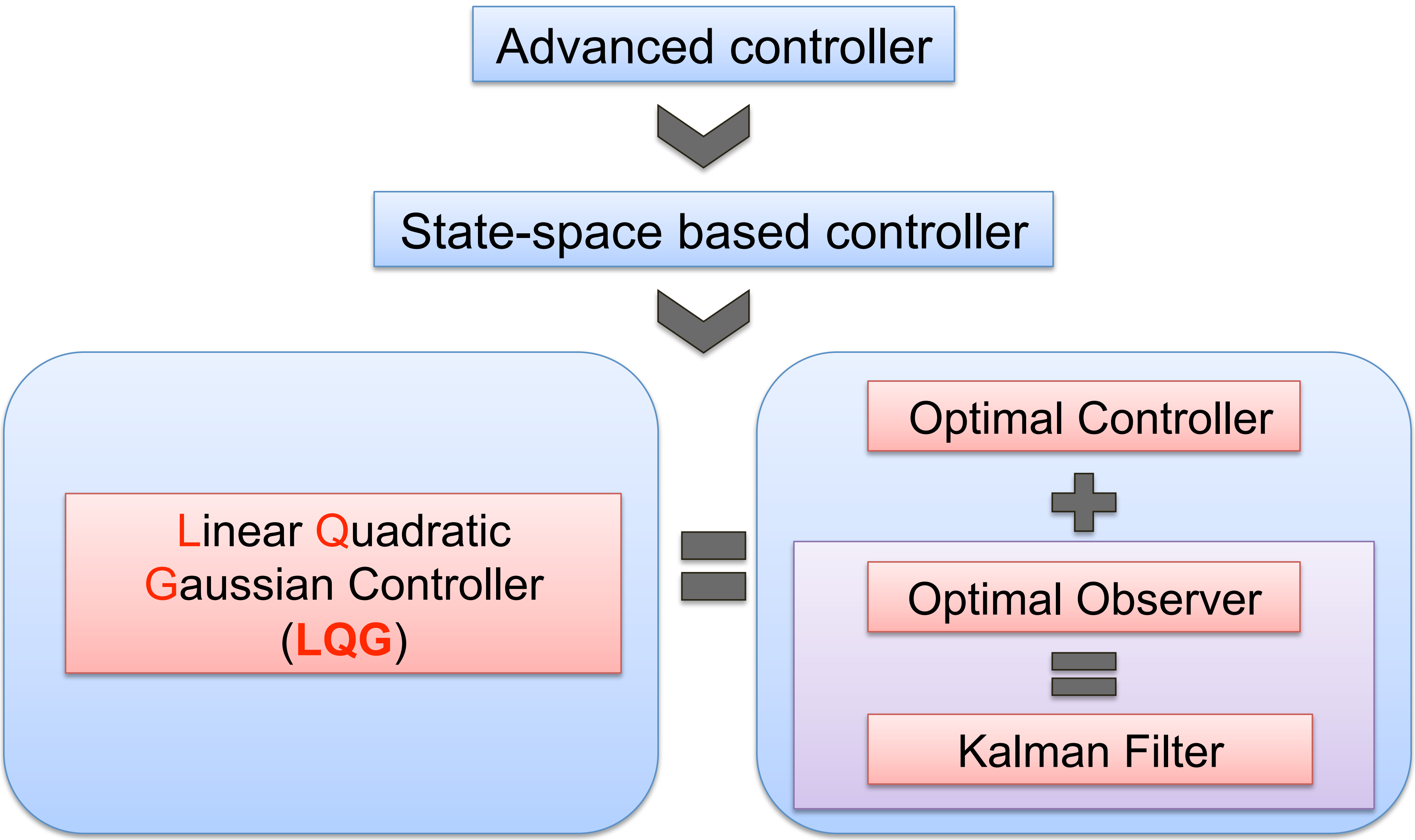
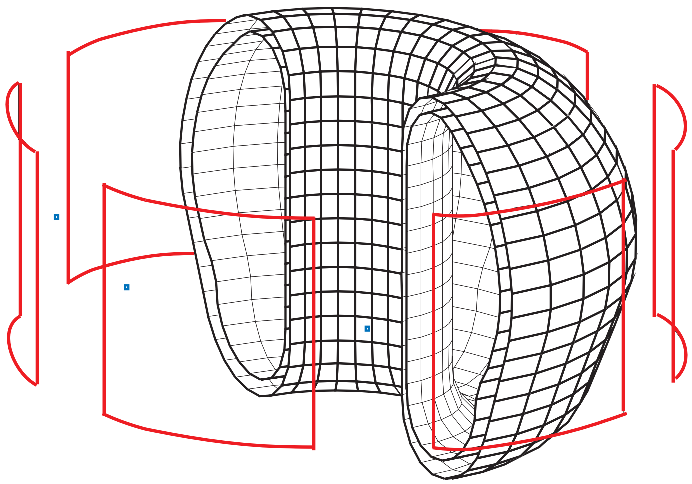


LQG controller is capable to enhance resistive wall mode control system

- Motivation
  - To improve RWM feedback control in NSTX with present external RWM coils
- Outline
  - Advantages of the LQG controller
  - VALEN state space modeling with mode rotation and control theory basics used in the design of LQG
  - Application of the advanced controller techniques to NSTX

Limiting β_n RWM in ITER can be improved with LQG controller* and external field correction coils



- Simplified ITER model includes
  - double walled vacuum vessel
  - 3 external control coil pairs
  - 6 magnetic field flux sensors on the midplane (z=0)

*Nucl. Fusion 47 (2007) 1157-1165

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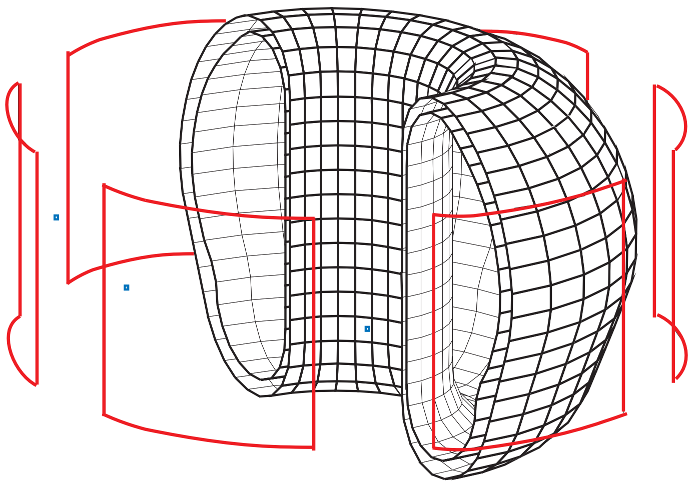

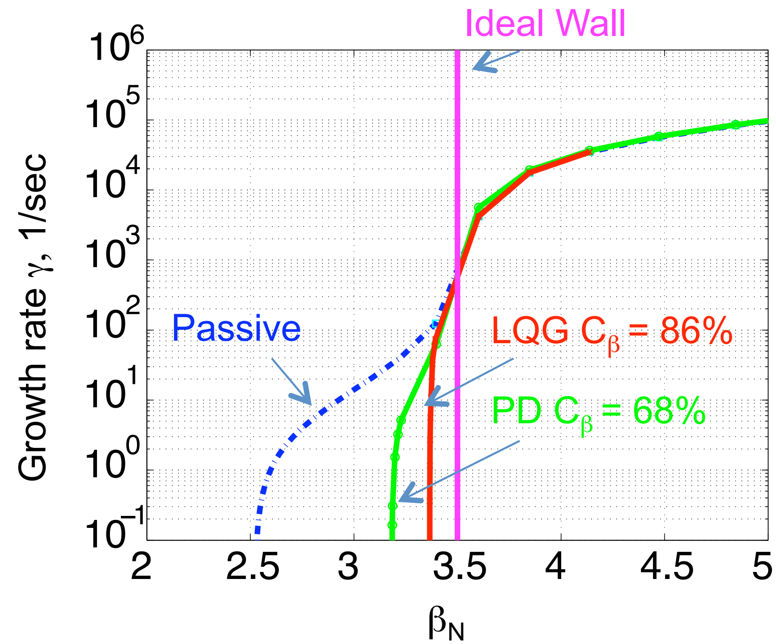

- Simplified ITER model includes
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- 10 Gauss sensor noise RWM
- LQG is robust for all $C_\beta < 86\%$ with respect to β_N

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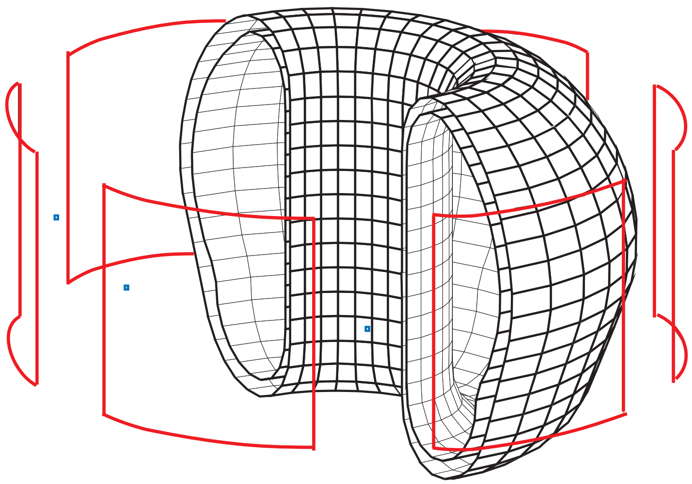

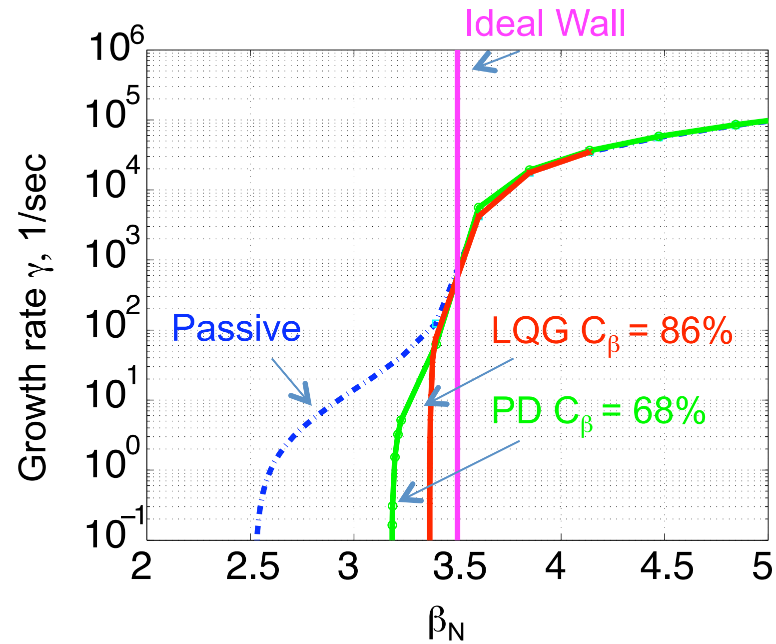

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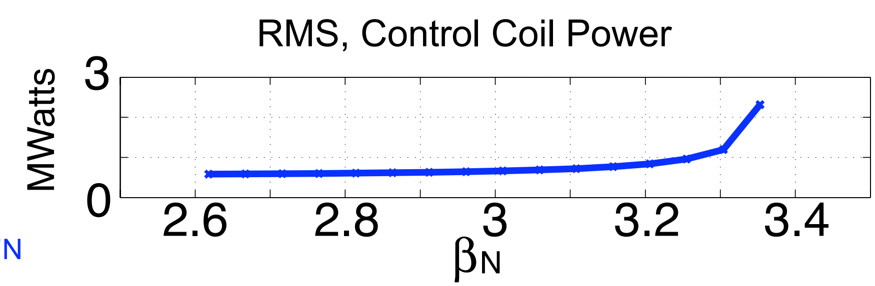

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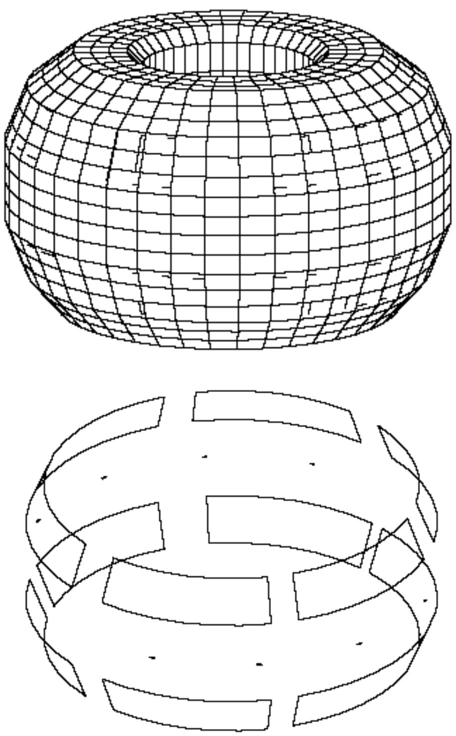
DIII-D LQG is robust with respect to β_N and stabilizes RWM up to ideal wall limit



DIII-D with internal control coils

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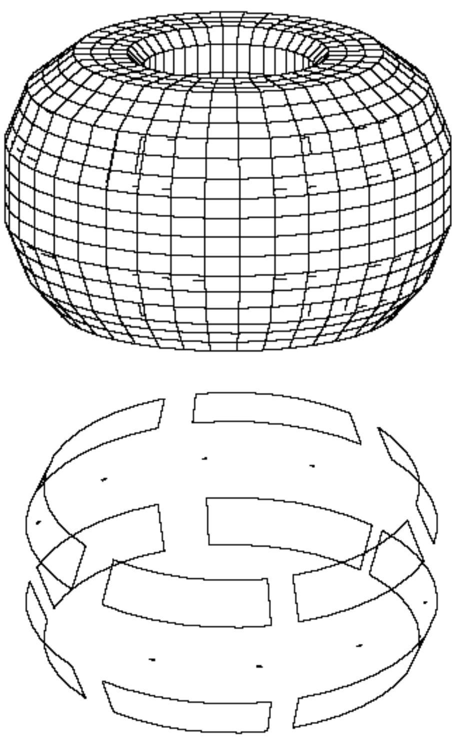

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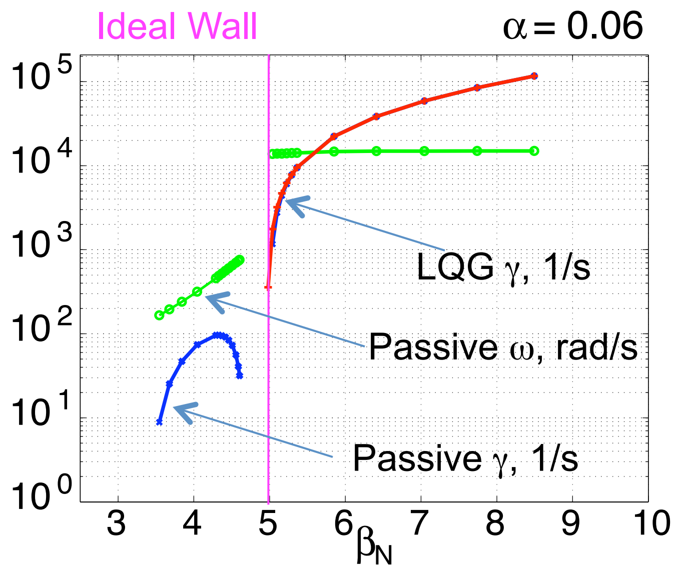

- LQG is robust with respect to β_N and stabilize RWM up to ideal wall limit for $0.01 < \text{torque} < 0.08$

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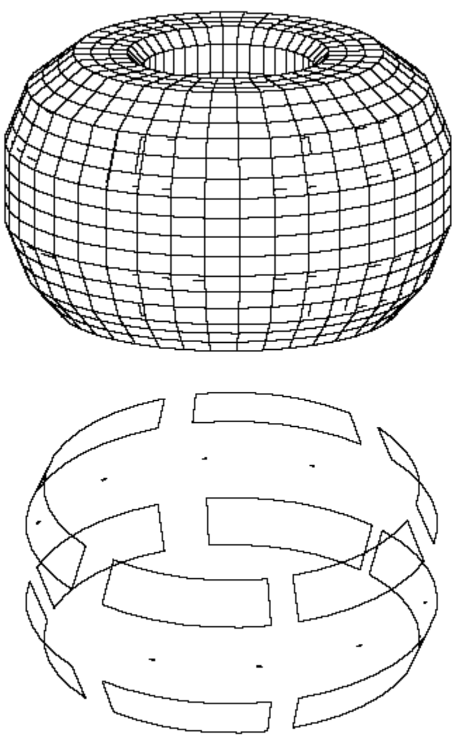

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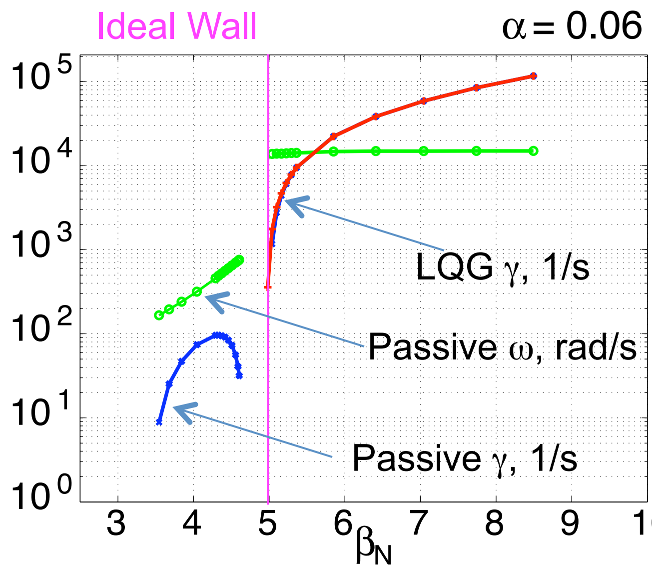
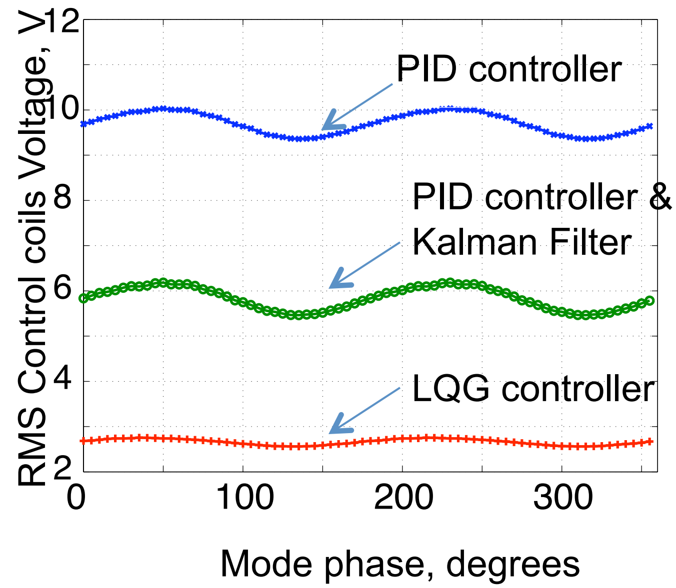
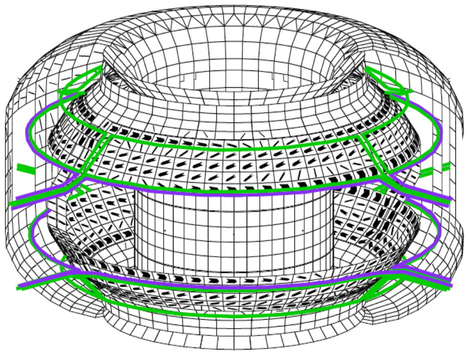




- LQG is robust with respect to β_N and stabilize RWM up to ideal wall limit for $0.01 < \text{torque} < 0.08$
- LQG provides better reduction of current and voltages compared with proportional gain controller

Initial results using advanced Linear Quadratic Gaussian (LQG) controller in KSTAR yield factor of 2 power reduction for white noise*

n=1 RWM passive
stabilization currents



- Conducting hardware, IVCC set up in VALEN-3D* based on engineering drawings
- Conducting structures modeled
  - Vacuum vessel with actual port structures
  - Center stack back-plates
  - Inner and outer divertor back-plates
  - Passive stabilizer (PS)
  - PS Current bridge

* IAEA FEC 2008 TH/P9-1 O. Katsuro-Hopkins

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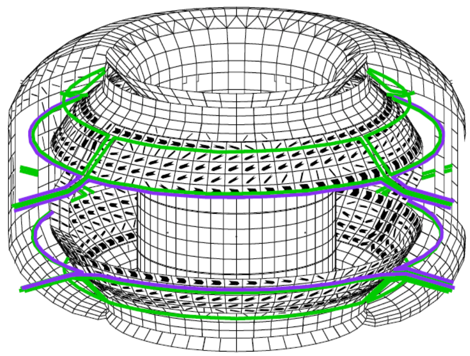

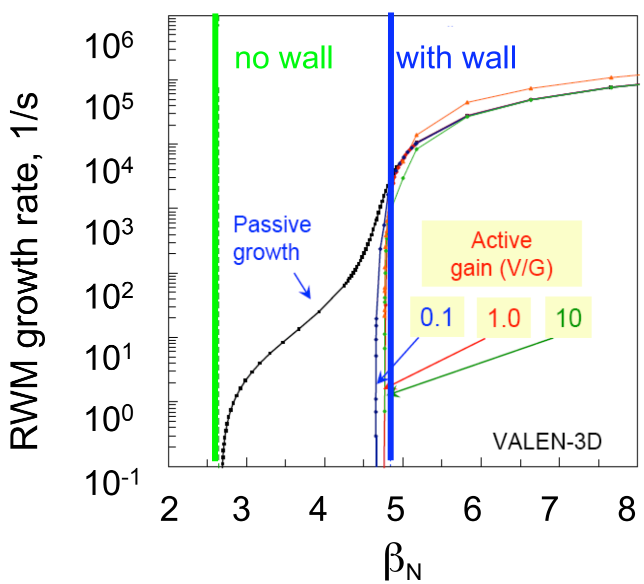

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- IVCC allows active n=1 RWM stabilization near ideal wall β_n limit, for proportional gain and LQG controllers

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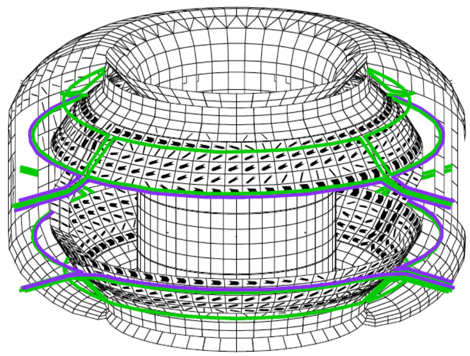

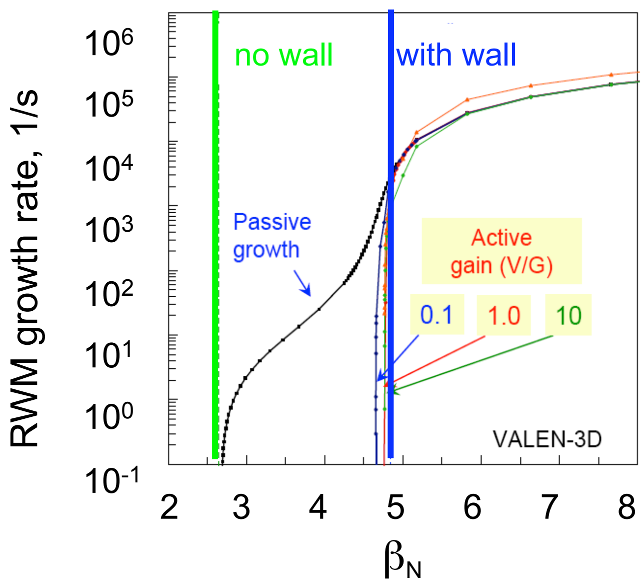

White noise (1.6-2.0G RMS)

(RMS values)

C_β	I_{IVCC}(A)	V_{IVCC}(V)	P_{IVCC}(W)	
80%	3%	50%	47%	Unloaded IVCC L/R=12.8ms
95%	15%	51%	58%	
80%	38%	75%	47%	FAST IVCC circuit L/R=1.0ms
95%	15%	73%	58%	

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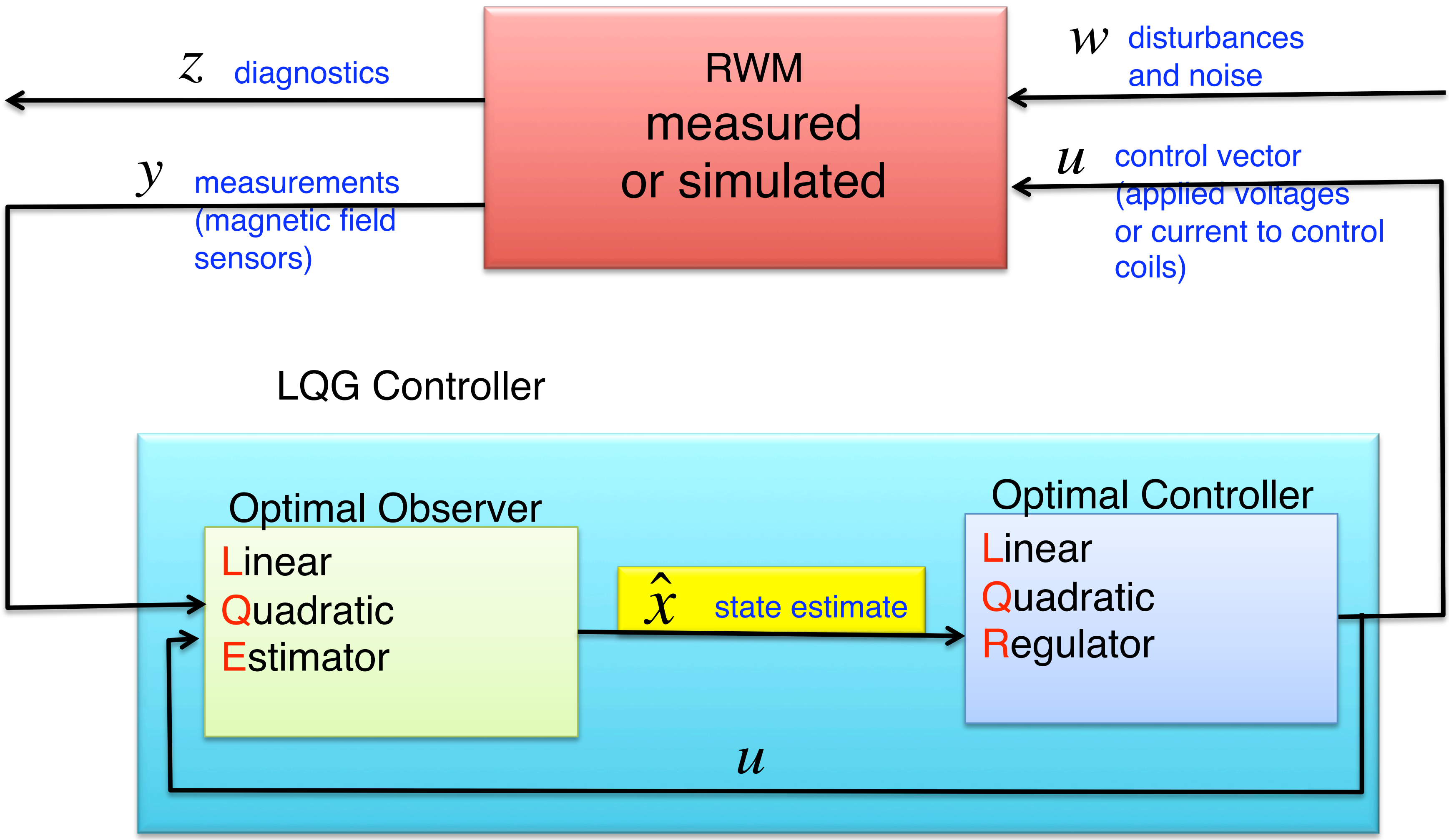
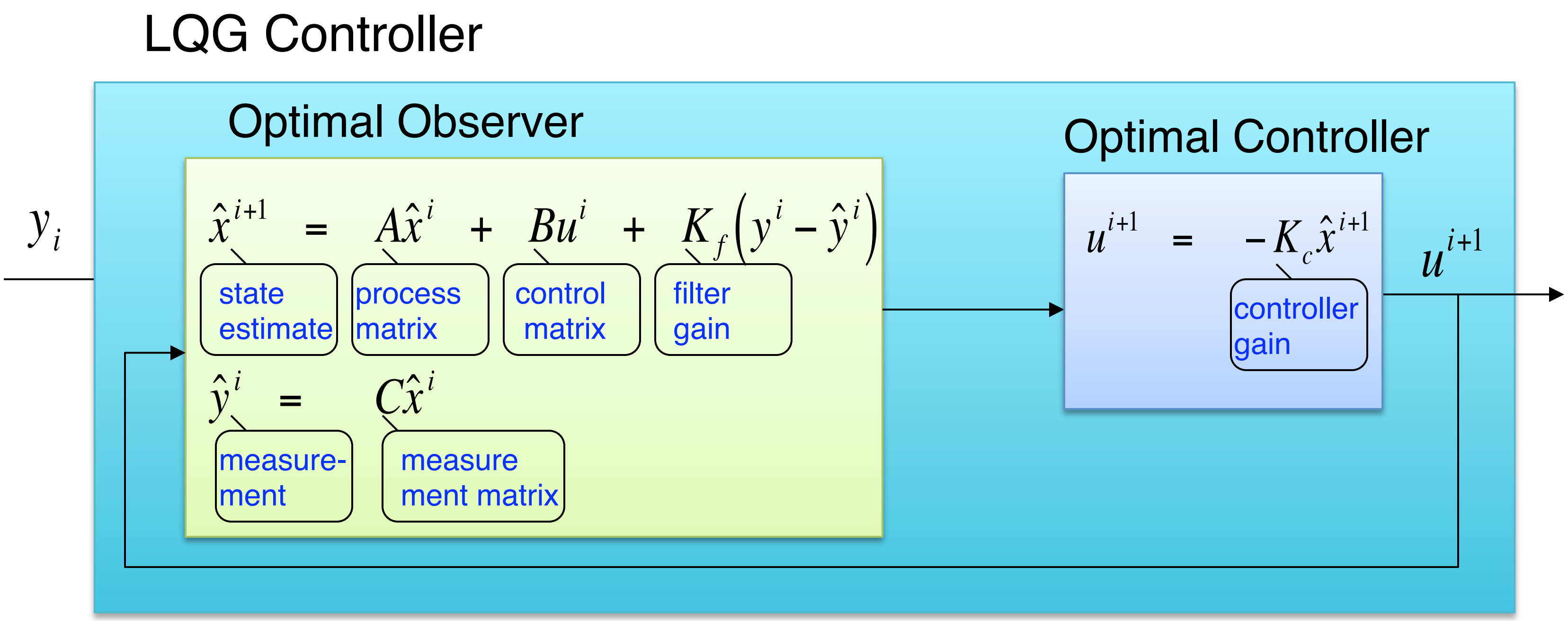
Tutorial
on selected control theory topics

Digital LQG controller proposed to improve stabilization performance



Detailed diagram of digital LQG controller



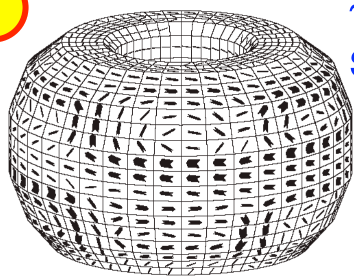
- State estimate stored in observer provides information about amplitude and phase of RWM and takes into account wall currents
- Dimensions of LQG matrices depends on
  - State estimate (reduced balanced VALEN states)~10-20
  - Number of control coils ~3
  - Number of sensors ~ 12-24
- All matrixes in LQG calculated in advance using VALEN state-space for particular 3-D tokamak geometry, fixed plasma mode amplitude and rotation speed

Balanced truncation significantly reduces VALEN state space

Full VALEN computed wall currents:

I

~3000 states



$\vec{x} = \left(\vec{I}_w \;\; \vec{I}_f \;\; I_d\right)^T; \qquad \vec{u} = \vec{V}_f;$

$\overleftrightarrow{A} = -\overleftrightarrow{L}^{-1} \cdot \overleftrightarrow{R}; \qquad \overleftrightarrow{B} = \overleftrightarrow{L}^{-1} \cdot \vec{I}_{cc};$

$\vec{y} = \vec{\Phi}; \qquad \overleftrightarrow{C} = \overleftrightarrow{M}$

sensor fluxes

mutual inductance matrix

Balancing transformation

$\hat{x} = \overleftrightarrow{T}\vec{x}$

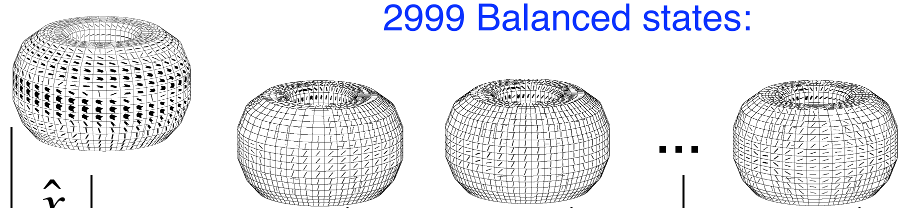
II

Balanced realization:

RWM fixed phase

$$\left[\begin{array}{c|c} \overleftrightarrow{T}\overleftrightarrow{A}\overleftrightarrow{T}^{-1} & \overleftrightarrow{T}\overleftrightarrow{B} \\ \hline \overleftrightarrow{C}\overleftrightarrow{T}^{-1} & \overleftrightarrow{0} \end{array}\right] = \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & 0 \end{array}\right] = \left[\begin{array}{cc|c} \tilde{A}_{11} & \tilde{A}_{12} & \tilde{B}_1 \\ \tilde{A}_{21} & \tilde{A}_{22} & \tilde{B}_2 \\ \hline \tilde{C}_1 & \tilde{C}_2 & 0 \end{array}\right]$$

2999 Balanced states:



$\hat{x}_1$ $\hat{x}_2$ $\hat{x}_3$ … $\hat{x}_{3000}$

Truncation

III State reduction:

$$\left[\begin{array}{c|c} \tilde{A}_{11} & \tilde{B}_1 \\ \hline \tilde{C}_1 & 0 \end{array}\right] \equiv \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$$

~3-20 states

- I What is VALEN state-space?
- II How to find balancing transformation $\overleftrightarrow{T}$?
- III How to determine number of states to keep ?

State-space control approach may allow superior feedback performance

I

- VALEN circuit equations after including unstable plasma mode. Fluxes at the wall, feedback coils and plasma are

$$\vec{\Phi}_w = \overleftrightarrow{L}_{ww} \cdot \vec{I}_w + \overleftrightarrow{L}_{wf} \cdot \vec{I}_f + \overleftrightarrow{L}_{wp} \cdot I_d$$

$$\vec{\Phi}_f = \overleftrightarrow{L}_{fw} \cdot \vec{I}_w + \overleftrightarrow{L}_{ff} \cdot \vec{I}_f + \overleftrightarrow{L}_{fp} \cdot I_d$$

$$\Phi_p = \overleftrightarrow{L}_{pw} \cdot \vec{I}_w + \overleftrightarrow{L}_{pf} \cdot \vec{I}_f + \overleftrightarrow{L}_{pp} \cdot I_d$$

- Equations for system evolution

$$\begin{pmatrix} \overleftrightarrow{\mathcal{L}}_{ww} & \overleftrightarrow{\mathcal{L}}_{wf} & \overleftrightarrow{\mathcal{L}}_{wp} \\ \overleftrightarrow{\mathcal{L}}_{fw} & \overleftrightarrow{\mathcal{L}}_{ff} & \overleftrightarrow{\mathcal{L}}_{fp} \\ \overleftrightarrow{\mathcal{L}}_{pw} & \overleftrightarrow{\mathcal{L}}_{pf} & \overleftrightarrow{\mathcal{L}}_{pp} \end{pmatrix} \cdot \frac{d}{dt} \begin{pmatrix} \vec{I}_w \\ \vec{I}_f \\ I_d \end{pmatrix} = \begin{pmatrix} \overleftrightarrow{R}_w & 0 & 0 \\ 0 & \overleftrightarrow{R}_f & 0 \\ 0 & 0 & \overleftrightarrow{R}_d \end{pmatrix} \cdot \begin{pmatrix} \vec{I}_w \\ \vec{I}_f \\ I_d \end{pmatrix} + \begin{pmatrix} \vec{0} \\ \vec{V}_f \\ 0 \end{pmatrix}$$

- In the state-space form

$$\dot{\vec{x}} = \overleftrightarrow{A}\vec{x} + \overleftrightarrow{B}\vec{u}$$
$$\vec{y} = C\vec{x}$$

where $\vec{x} = \left(\vec{I}_w \;\; \vec{I}_f \;\; I_d\right)^T$; $\overleftrightarrow{A} = -\overleftrightarrow{L}^{-1} \cdot \overleftrightarrow{R}$; $\overleftrightarrow{B} = \overleftrightarrow{L}^{-1} \cdot \overleftrightarrow{I}_{cc}$; $\vec{u} = \vec{V}_f$

& measurements $\vec{y} = \vec{\Phi}_s$ are sensor fluxes. State-space dimension ~1000 elements!

- Classical control law with proportional gain defined as $\vec{u} = -\overleftrightarrow{G}_p \vec{y}$

VALEN formulation with rotation allows inclusion of mode phase into state space*

- VALEN uses two copies of a single unstable mode with $\pi/2$ toroidal displacement of these two modes
- The VALEN uses two dimensionless parameter normalized torque "α" and normalized energy "s"

$$s = -\frac{\delta W}{L_B I_b^2/2} = -\frac{\text{energy required with plasma}}{\text{energy required WITHOUT plasma}}$$

$$\alpha = \frac{\text{torque}}{L_B I_b^2/2} = \frac{\text{torque on mode by plasma}}{\text{energy required WITHOUT plasma}}$$

- The VALEN parameters 's' and 'α' together determine growth rate γ and rotation Ω of the plasma mode
- LQG is optimized off line for best stability region with respect to 's' and 'α' parameters.

*Boozer PoP Vol6, No. 8, 3190 (1999)

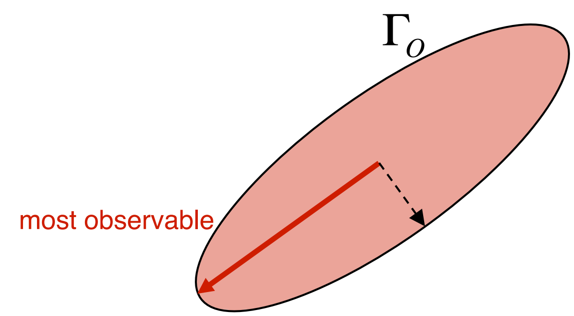
Measure of system controllability and observability is given by controllability and observability grammians

II

- Given stable Linear Time-Invariant (LTI) Systems $\left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$

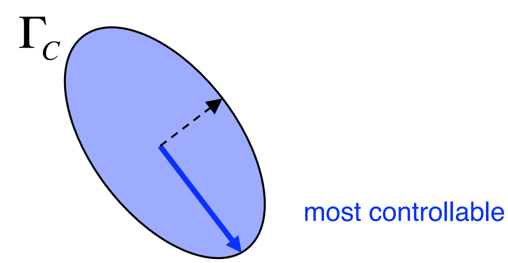
- Observability grammian, $\Gamma_o = \int_0^\infty e^{A^T\tau} C^T C e^{A\tau} d\tau$, can be found be solving continuous-time Lyapunov equation, $A^T\Gamma_o + \Gamma_o A + C^T C = 0$, provides measure of output energy: $\|y\|_2^2 = x_0^T \Gamma_O x_0$

- $\Gamma_O = U\Lambda_O U^T$ defines an "observability ellipsoid" in the state space with the longest principal axes along the most observable directions



- Controllability grammian, $\Gamma_c = \int_0^\infty e^{A\tau} BB^T e^{A^T\tau} d\tau$, can be found be solving continuous-time Lyapunov equation, $A\Gamma_c + \Gamma_c A^T + BB^T = 0$, provides measure of input(control) energy: $\|u\|_2^2 = x_0^T \Gamma_C^{-1} x_0$

- $\Gamma_C = V\Lambda_C V^T$ defines a "controllability ellipsoid" in the state space with the longest principal axes along the most controllable directions



Balanced realization exists for every stable controllable and observable system

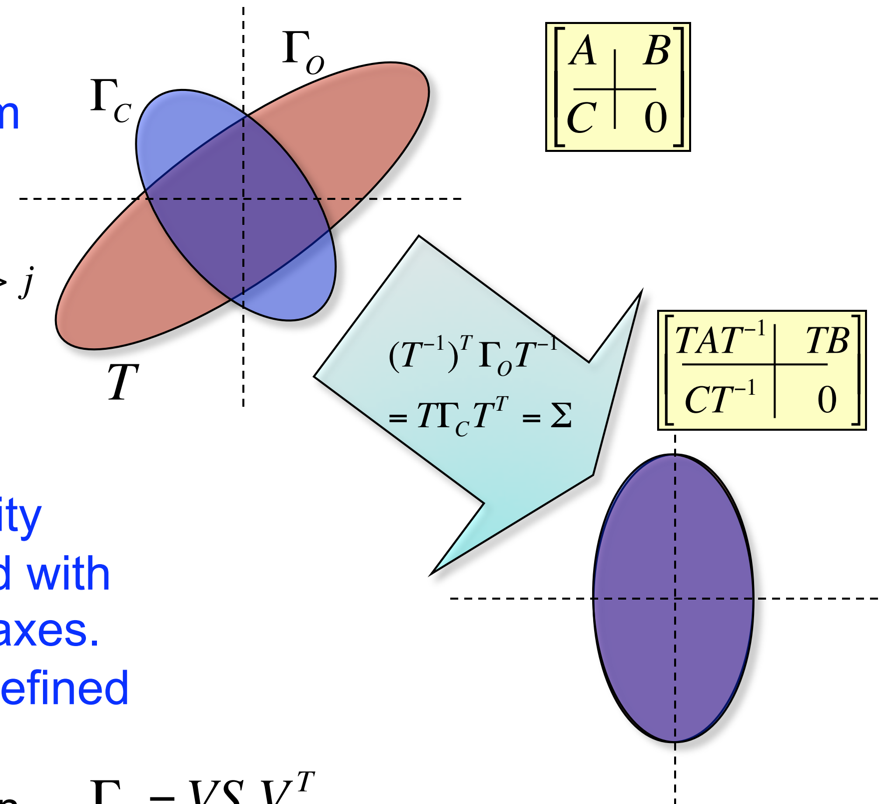
II

- Controllable, observable & stable system called balanced if

$$\tilde{\Gamma}_C = \tilde{\Gamma}_O = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_n \end{pmatrix} = \Sigma, \text{ where } \sigma_i > \sigma_j \text{ for } i > j$$

  σ_i - Hankel Singular Values

- Balancing similarity transformation transforms observability and controllability ellipsoids to an identical ellipsoid aligned with the principle axes along the coordinate axes.
- The balanced transformation T can be defined in two steps:
  - Start with SVD of controllability grammian $\Gamma_c = VS_cV^T$ and define the first transformation as $T_1 := VS_c^{1/2}$
  - Perform SVD of observability grammian in the new basis: $\tilde{\Gamma}_o = T_1^T \Gamma_o T_1 = US_oU^T$ the second transformation defined as $T_2 := US_o^{-1/4}$
  - The final transformation matrix is given by: $T := T_1T_2 = VS_c^{1/2}US_o^{-1/4}$



Determination of optimal controller gain for the dynamic system

For given dynamic process: $\dot{\vec{x}} = \overleftrightarrow{A}\vec{x} + \overleftrightarrow{B}\vec{u}$

Find the matrix $\overleftrightarrow{K}_c$ such that control law: $\vec{u} = -\overleftrightarrow{K}_c\vec{x}$

minimizes Performance Index: $J = \int_t^T (\hat{\vec{x}}'(\tau)\overleftrightarrow{Q}_r(\tau)\hat{\vec{x}}(\tau) + \vec{u}'(\tau)\overleftrightarrow{R}_r(\tau)\vec{u}(\tau))d\tau \rightarrow \min$

where tuning parameters are presented by $\overleftrightarrow{Q}_r$, $\overleftrightarrow{R}_r$ - state and control weighting matrixes,

Solution:

Controller gain for the steady-state can be calculated as $\overleftrightarrow{K}_C = \overleftrightarrow{R}^{-1}\overleftrightarrow{B}_r^T\overleftrightarrow{S}$

Where $\overleftrightarrow{S}$ is solution of the controller Riccati matrix equation

$$\overleftrightarrow{S}\overleftrightarrow{A}_r + \overleftrightarrow{A}_r^T\overleftrightarrow{S} - \overleftrightarrow{S}\overleftrightarrow{B}_r\overleftrightarrow{R}_r^{-1}\overleftrightarrow{B}_r^T\overleftrightarrow{S} + \overleftrightarrow{Q}_r = 0$$

Determination of optimal observer gain for the dynamic system

III

For given stochastic dynamic process: $\dot{\vec{x}} = \vec{A}_r \vec{x} + \vec{B}_r \vec{u} + \vec{v}$

with measurements: $\vec{y} = \vec{C}_r \vec{x} + \vec{\omega}$

Find the matrix $\vec{K}_f$ such that observer equation $\dot{\hat{\vec{x}}} = \vec{A}_r \hat{\vec{x}} + \vec{B}_r \vec{u} + \vec{K}_f \left(\vec{y} - C_r \hat{\vec{x}} \right)$

minimizes error covariance:

$$E\left\{ (\vec{x} - \hat{\vec{x}})(\vec{x} - \hat{\vec{x}})^T \middle| \vec{y}(\tau), \tau \le t \right\} \to \min$$

where tuning parameters are presented by $\vec{V}$, $\vec{W}$ - plant and measurement noise covariance matrix

Solution:

Observer gain for the steady-state can be calculated as $\vec{K}_f = \vec{P}\vec{C}'\vec{W}^{-1}$

Where $\vec{P}$ is solution of the observer Riccati matrix equation

$$\vec{A}_r \vec{P} + \vec{P}\vec{A}_r^T - \vec{P}\vec{C}_r^T \vec{W}^{-1} \vec{C}_r \vec{P} + \vec{V}^T = 0$$

Closed system equations with optimal controller and optimal observer based on reduced order model

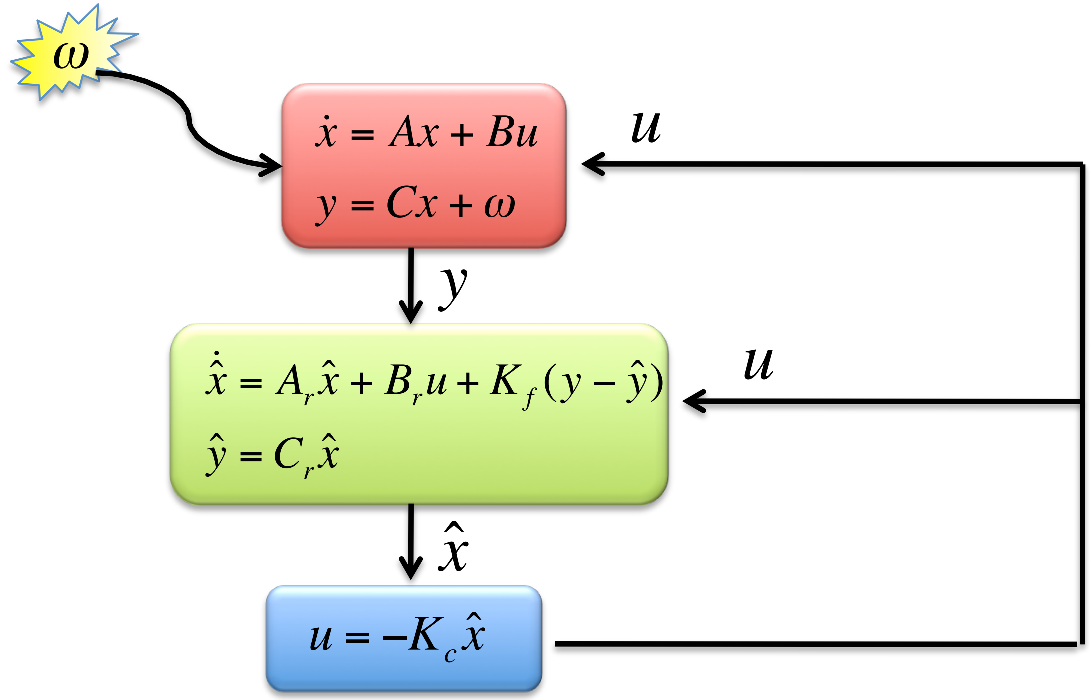
III

Measurement noise

Full order VALEN model

Optimal observer

Optimal controller



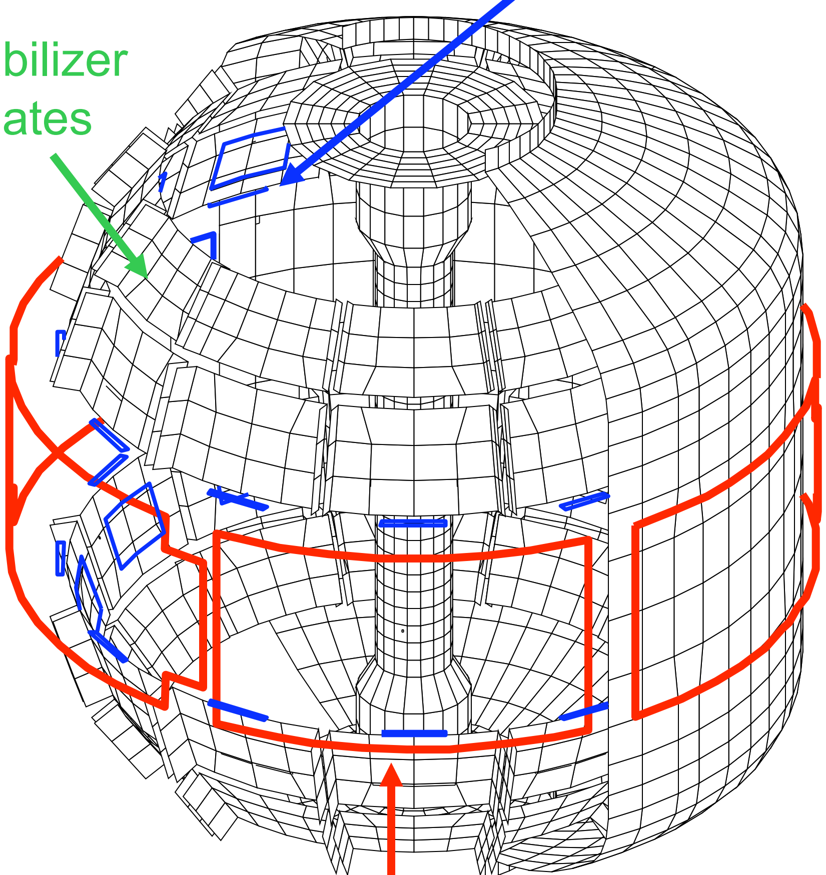
$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK_c \\ K_f C & F \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} + \begin{pmatrix} 0 \\ K_f \end{pmatrix} \omega$$

$$F = A_r - K_f C_r - B_r K_c$$

⇐ Closed loop continuous system allows to

- Test if Optimal controller and observer stabilizes original full order model and defined number of states in the LQG controller
- Verify robustness with respect to β_n
- Estimate RMS of steady-state currents, voltages and power

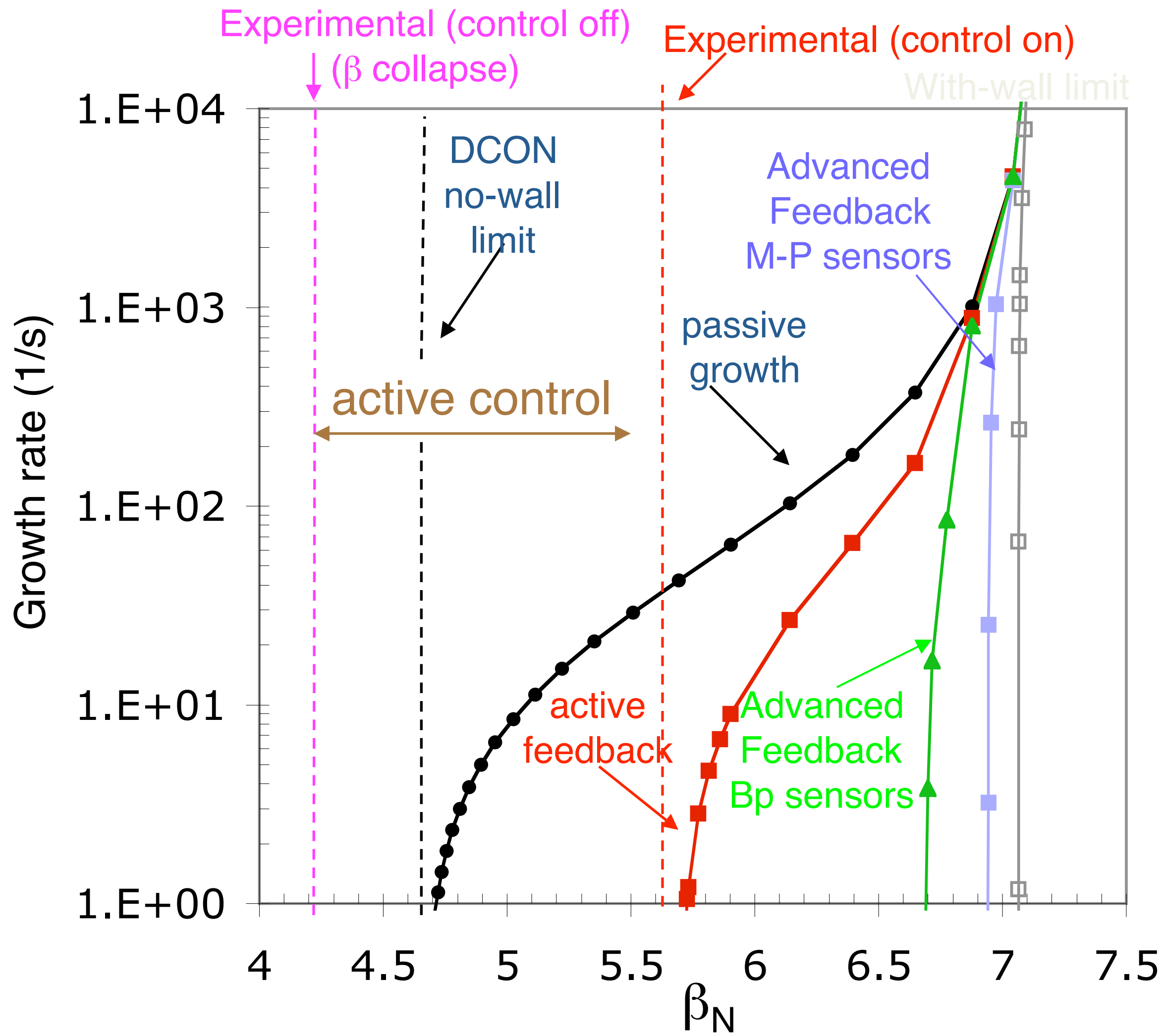
Advanced controller methods planned to be tested on **NSTX** with future application to KSTAR



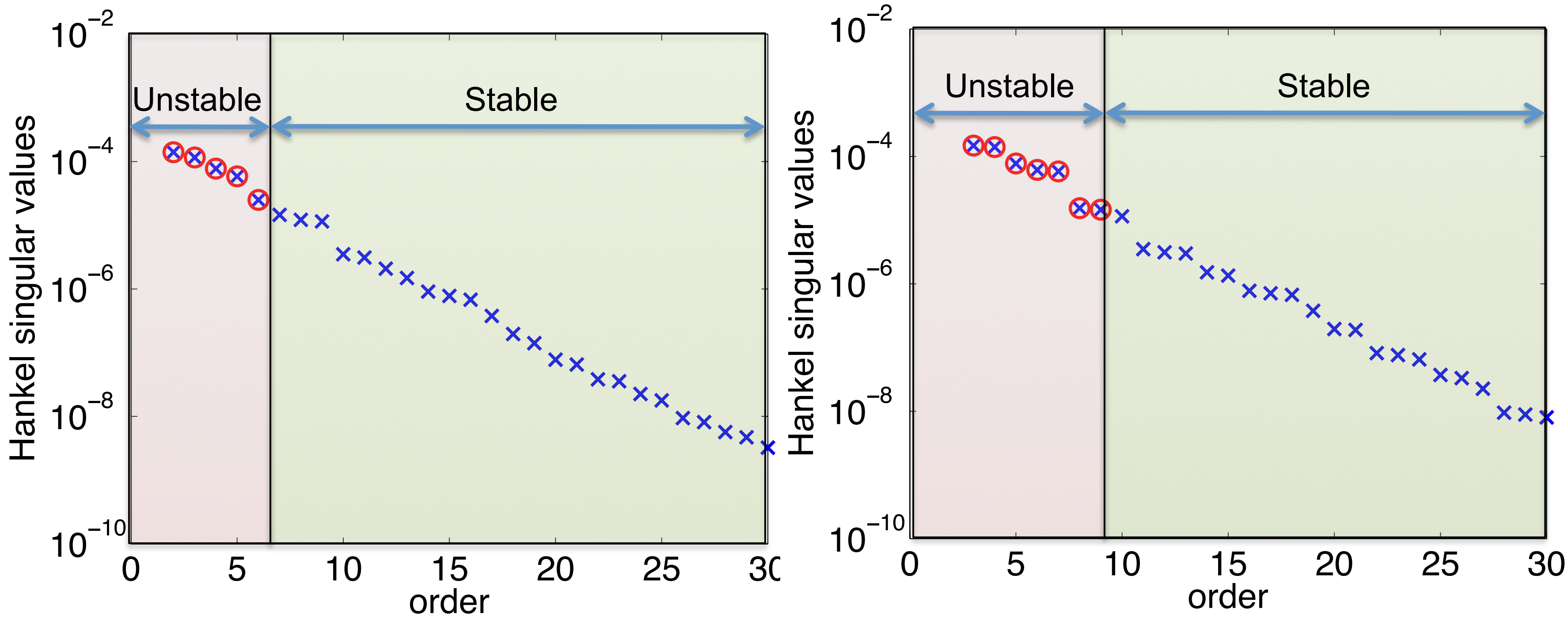
- VALEN NSTX Model includes
  - Stabilizer plates
  - External mid-plane control coils closely coupled to vacuum vessel
  - Upper and lower Bp sensors in actual locations
  - Compensation of control field from sensors
  - Experimental Equilibrium reconstruction (including MSE data)
- Present control system on NSTX uses Proportional Gain

Advanced control techniques suggest significant feedback performance improvement for NSTX up to β_n/β_n^{wall}= 95%

- **Classical proportional feedback methods**
  - VALEN modeling of feedback systems agrees with experimental results
  - RWM was stabilized up to β_n = 5.6 in experiment.
- **Advanced feedback control may improve feedback performance**
  - Optimized state-space controller can stabilize up to C_β=87% for upper Bp sensors and up to C_β=95% for mid-plane sensors
  - Uses only 7 modes for LQG design



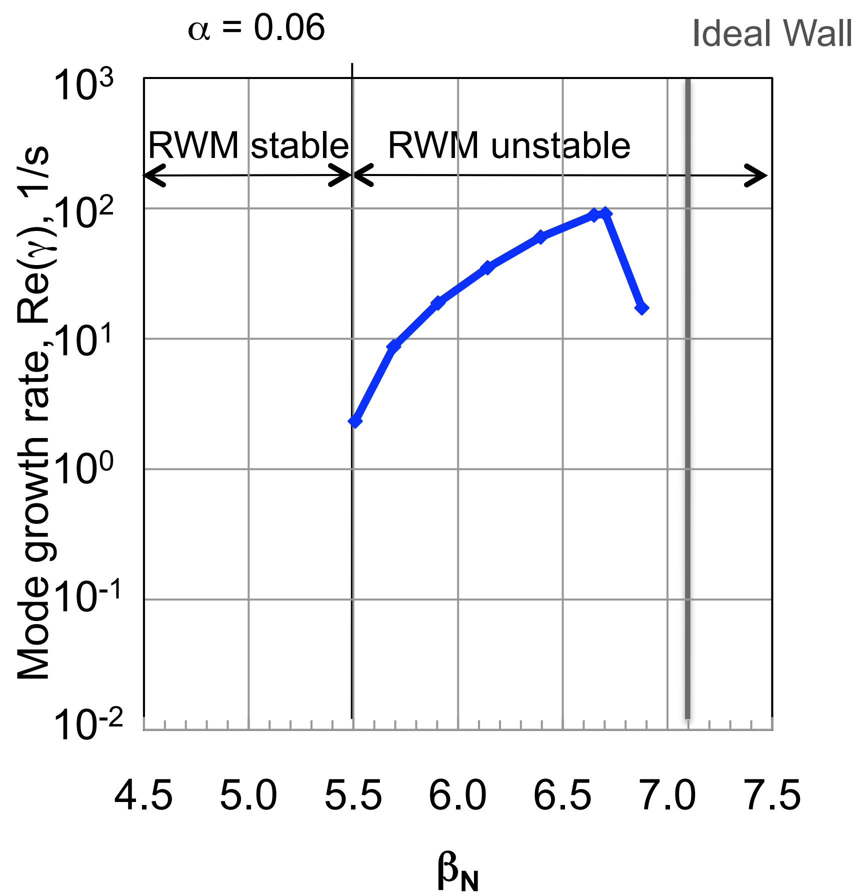
LQG with mode phase needs only 9 modes using B_p upper and lower sensors sums and differences



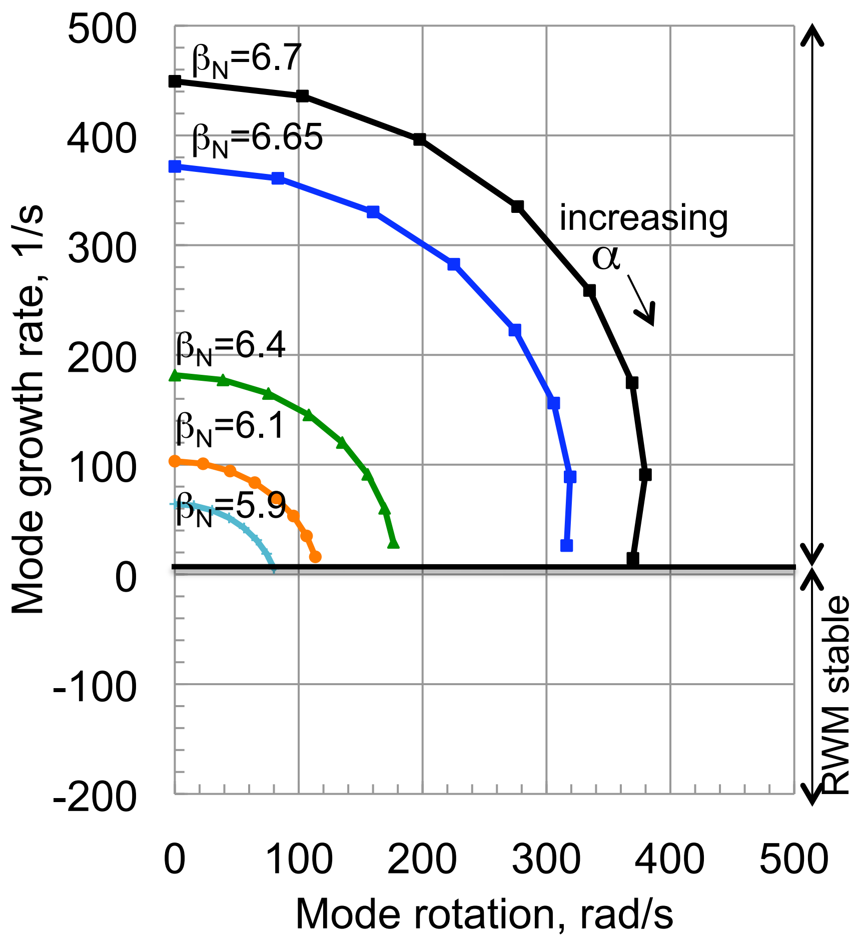
- Fixed mode (fixed phase)
- 7 states (1 unstable mode + 6 stable balanced states)
- LQG(β_N=6.7,N=7)

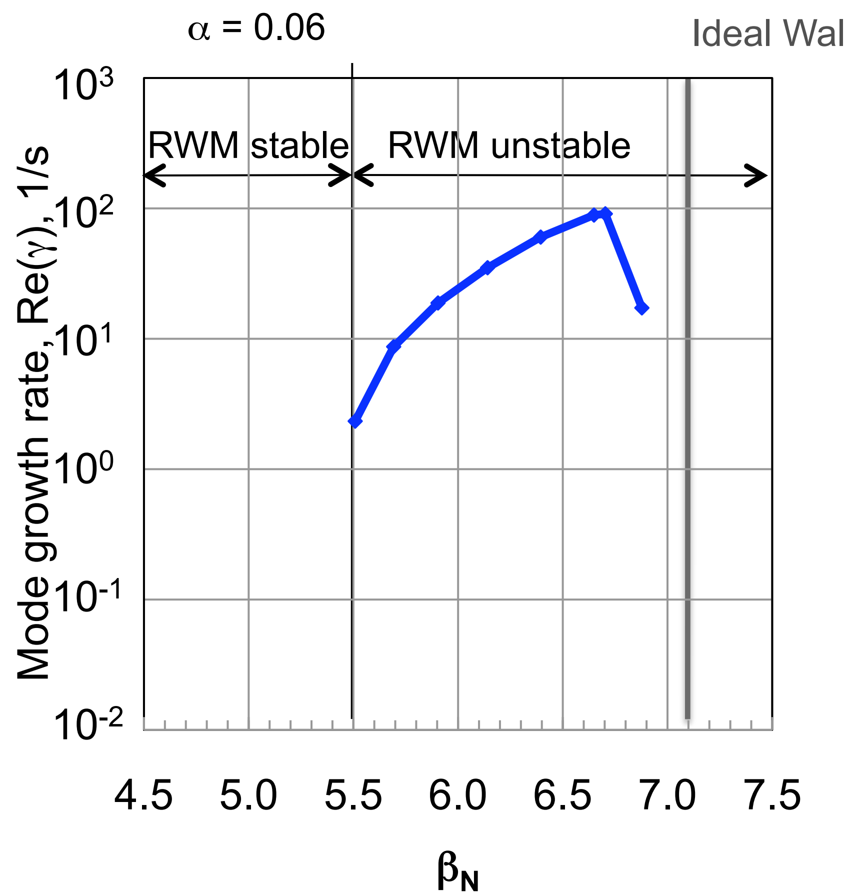
- Rotating mode (phase included)
- 9 states (2 unstable modes + 7 stable balanced states)
- LQG(α=0,β_N=6.7,N=9)

LQG(0,6.7,9) stabilizes slow rotating RWM mode up to β_N<6.7

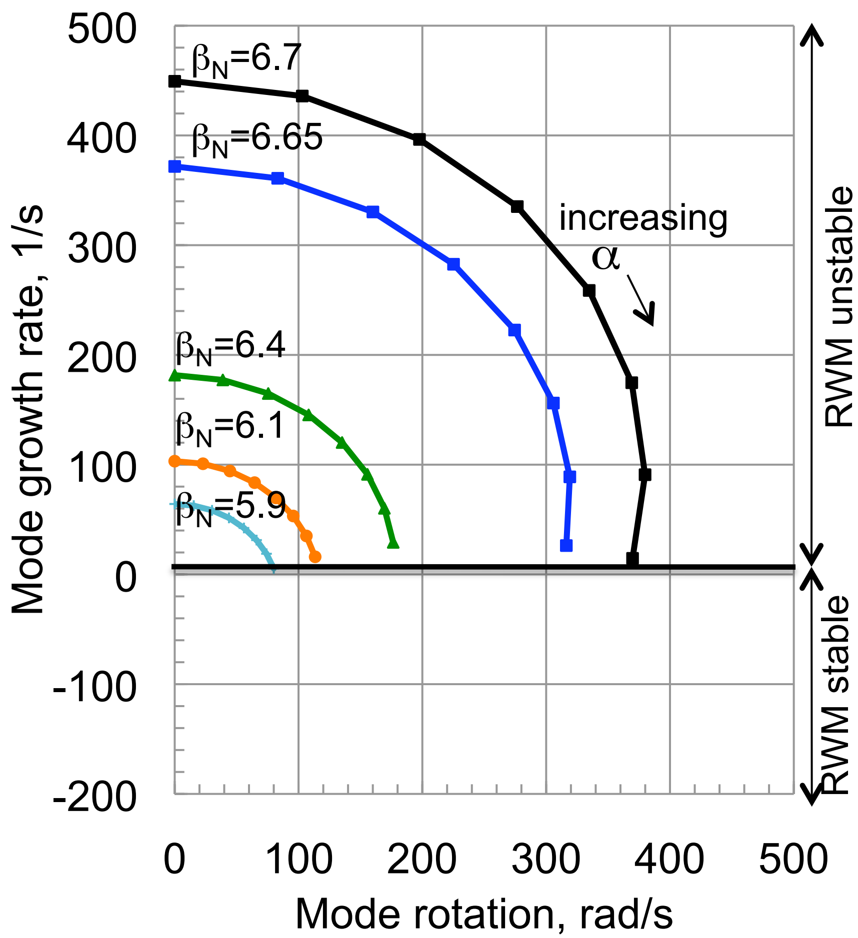


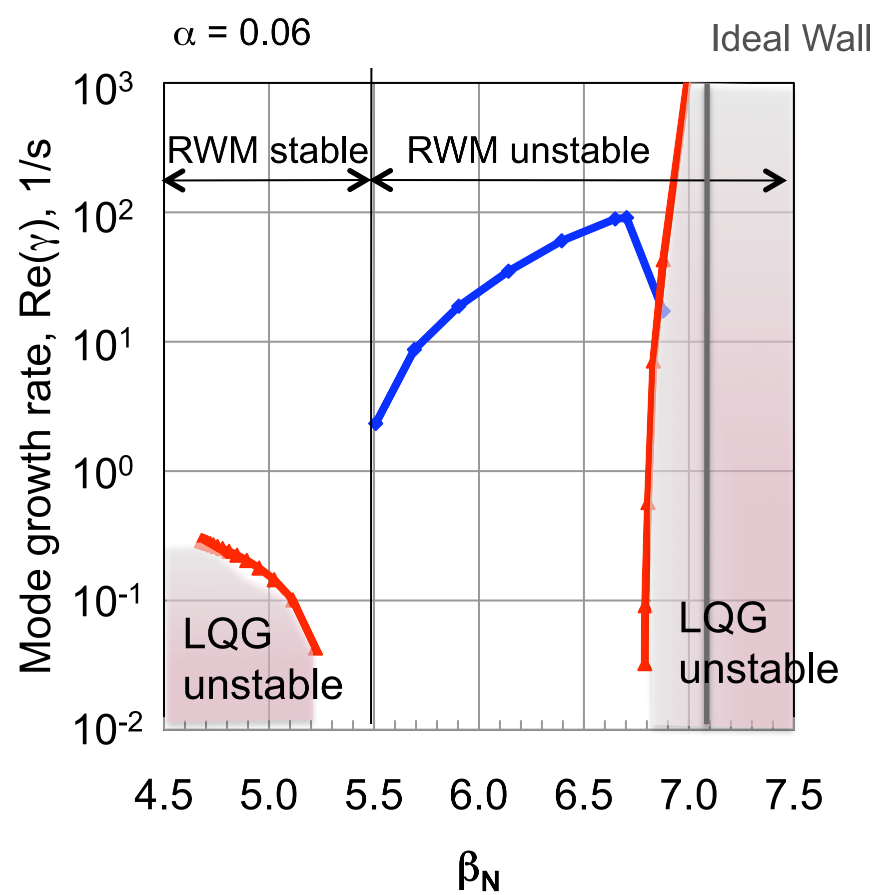
LQG(0,6.7,9) stabilizes slow rotating RWM mode up to β_N<6.7



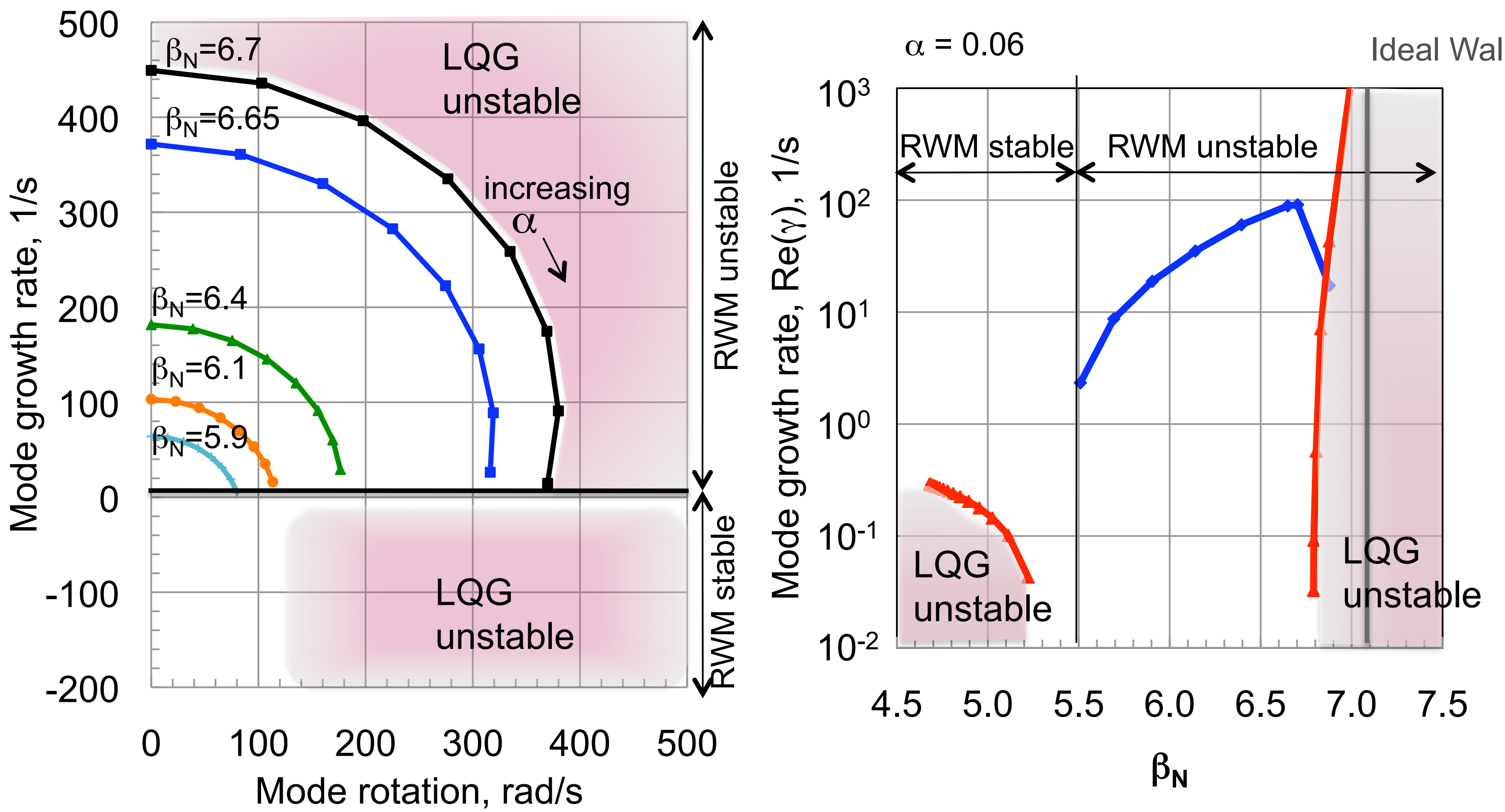


LQG(0,6.7,9) stabilizes slow rotating RWM mode up to β_N<6.7



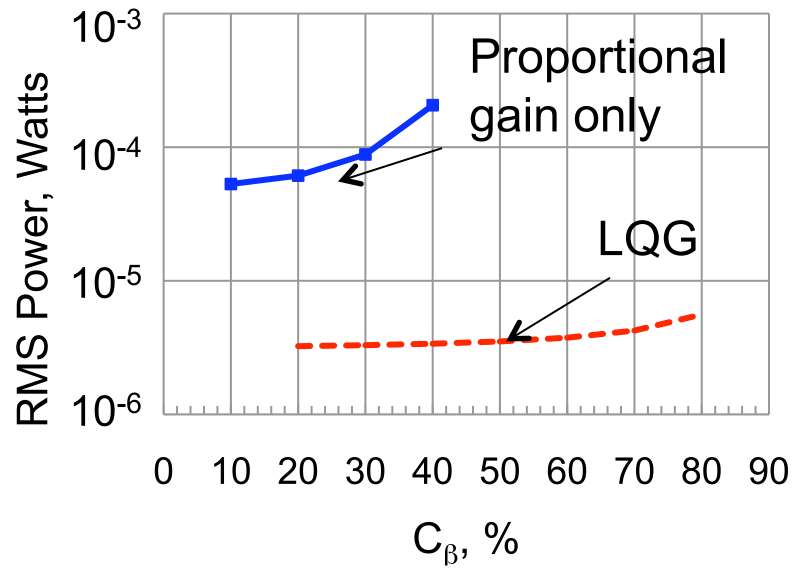


LQG(0,6.7,9) stabilizes slow rotating RWM mode up to β_N<6.7



>90% power reduction for white noise driven time evolution of the controlled RWM

- White noise 7 gauss in amplitude with 5kHz sampling frequency
- Power is proportional to white noise amplitude^2 and sampling frequency^{-1}
- No filters on B_p sensors in proportional controller was used in this study



C_β	RMS values I_{CC}, A	RMS values V_{CC}, V	RMS values P, Watts	Peak Values I_{CC}, A	Peak Values V_{CC}, V	Peak Values P, Watts
10%	70%	84%	94%	66%	84%	93%
20%	73%	85%	95%	68%	84%	94%
30%	77%	86%	96%	73%	85%	95%
40%	85%	90%	98%	82%	89%	98%

Advanced controller study continuing…

- Conclusions
  - NSTX advanced controller with mode phase has been tested numerically, 9 modes needed, large stability region for slow rotating mode. (RWM is usually locked in the NSTX experiments.)

- Next Steps
  - Study LQG with different torque, to improve robustness with respect to β_N and mode rotation speed.
  - Off line comparison for mode phase and amplitude calculated with reduced order optimal observer and the present measured RWM sensor signal data SVD evaluation of n = 1 amplitude and phase from the experimental data
  - Redesign state-space for control coil current controller input
  - Analyze time delay effect on LQG performance

Thank you!