Interaction Between Magnetic Islands and Electrostatic Turbulence

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Outline

- Basic Principles: from Standard to Reduced Physical Model
- The Numerical Codes
- Magnetic Island affecting Turbulence
- Turbulence affecting Magnetic Island
- Summary and Conclusions

Standard Electromagnetic Model

- A Standard 2D slab, three-field model with curvature terms with constant T_e and cold ions can describe both magnetic reconnection and turbulence:

$$\frac{\partial U}{\partial t} + [\varphi, U] = [J, \psi] + G\partial_y (n - \varphi) + \mu \nabla^2 U$$

$$\frac{\partial n}{\partial t} + [\varphi, n] = [J, \psi] + G\partial_y (n) + D\nabla^2 n$$

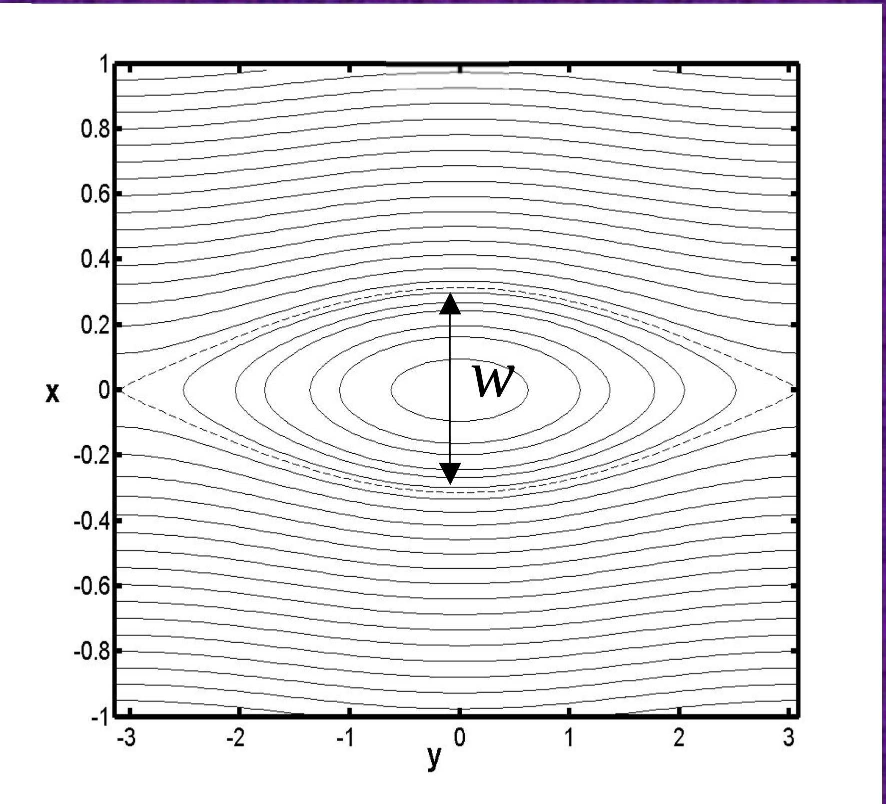
$$\frac{\partial \psi}{\partial t} + [\varphi, \psi] = [n, \psi] + CJ$$

$$U = \nabla^2 \varphi$$

Parametrization of the Magnetic Island

$$\psi = -\frac{x^2}{2} + \tilde{\psi}(t)\cosh(x)\cos(y)$$

$$w = 4\sqrt{\tilde{\psi}}$$
$$y = \xi - \omega t$$



- How does electrostatic turbulence affect the evolution of w and ω ?
- The method could be generalized to more complex island structures: the detailed shape is not important, the topology is!

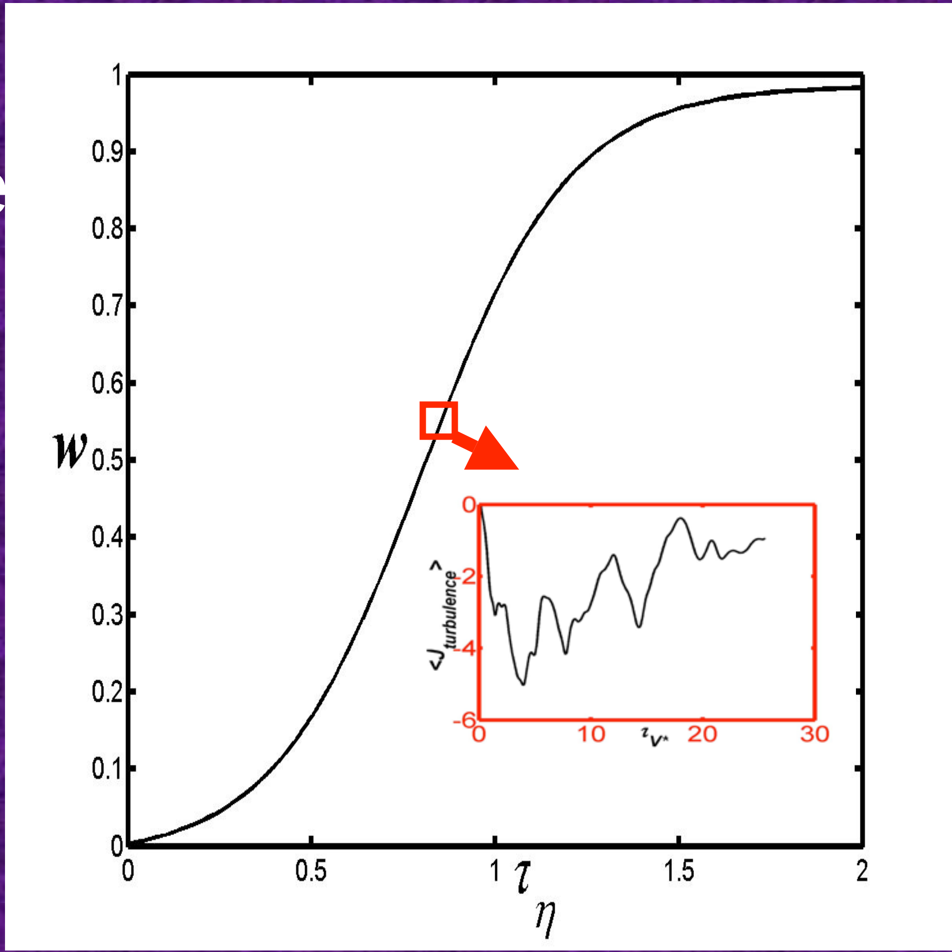
Electrostatic Approximation

- If $\partial_t \tilde{\psi} / \tilde{\psi} << \omega_*$ the turbulence time scale is faster than evolution of the magnetic field.
- The two time scale can be separated.

IN THE TIME SCALE OF TURBULENCE <u>ISLAND WIDTH</u> IS ASSUMED <u>FIXED</u>

⇩

w = CONST



Reduced Electrostatic Model

- We use an extended version of the Hasegawa-Wakatani equations in the island frame of reference.

$$\frac{\partial U}{\partial t}+[\varphi,U]=\frac{1}{C}[[n-\varphi,\psi],\psi]+G\partial_y(n)+\mu\nabla^2 U$$

$$\frac{\partial n}{\partial t}+[\varphi,n]=\frac{1}{C}[[n-\varphi,\psi],\psi]+G\partial_y(n-\varphi)+D\nabla^2 n$$

$$J=\frac{1}{C}[n-\varphi,\psi], \qquad \psi=-\frac{x^2}{2}+\tilde{\psi}\cosh(x)\cos(y)$$

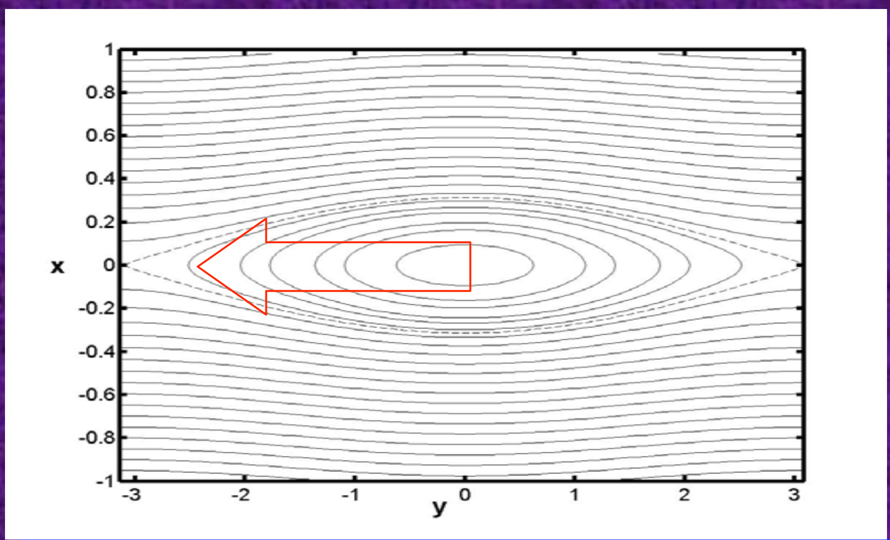
$$U=\nabla^2\varphi$$

The Island Frame of Reference

- Like in a wind tunnel, we keep the island still while the plasma flows at a velocity, V.
- The island rotation frequency, ω, is calculated self-consistently.

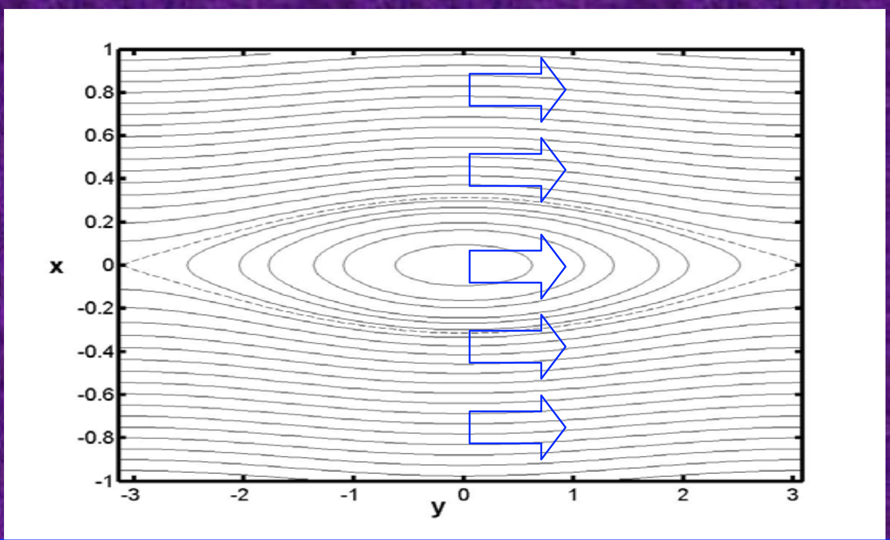
Laboratory frame of reference:

Island moving at velocity: $V=\omega/k_y$



Island frame of reference

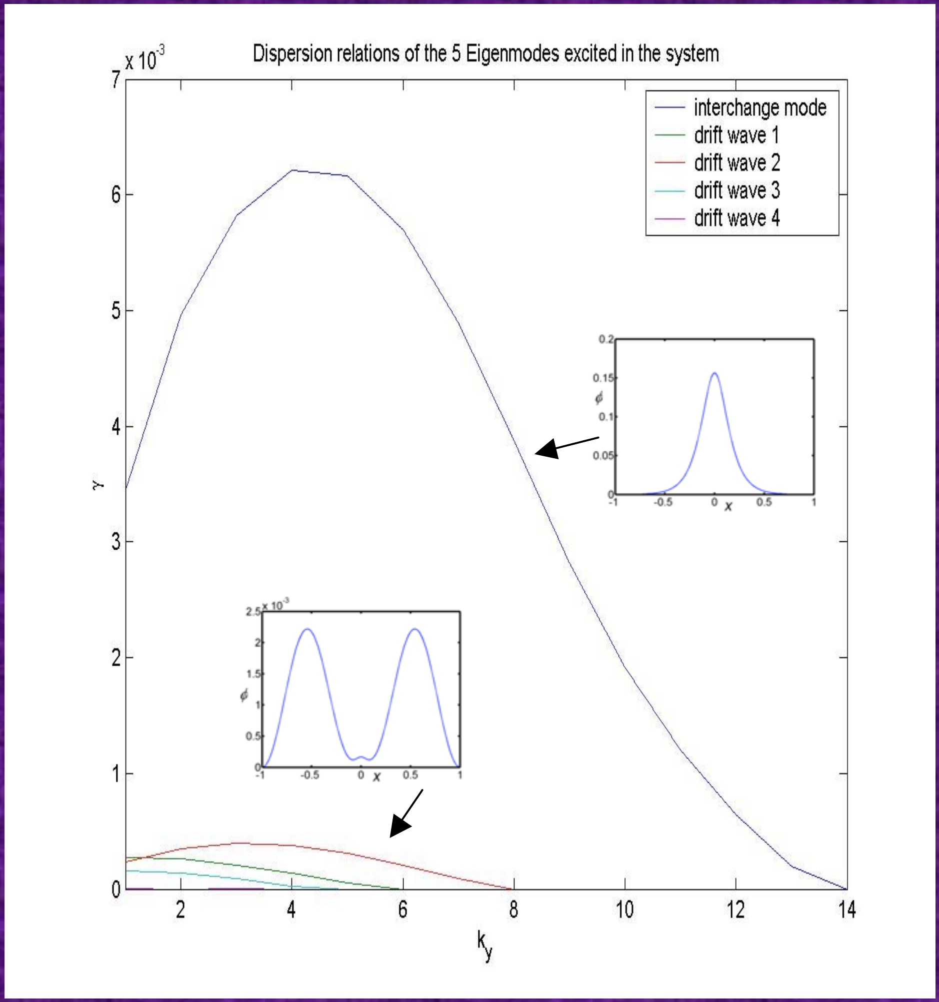
Plasma velocity $-V=-\omega/k_y$



Linear Code mode dispersion relation

- Linear stability investigated with the linear code LinHWDF.
- When $G>0$, the interchange mode is unstable and drives the turbulence.

C	G	D	μ
1	0.45	0.02	0.02



Nonlinear Code HasWak

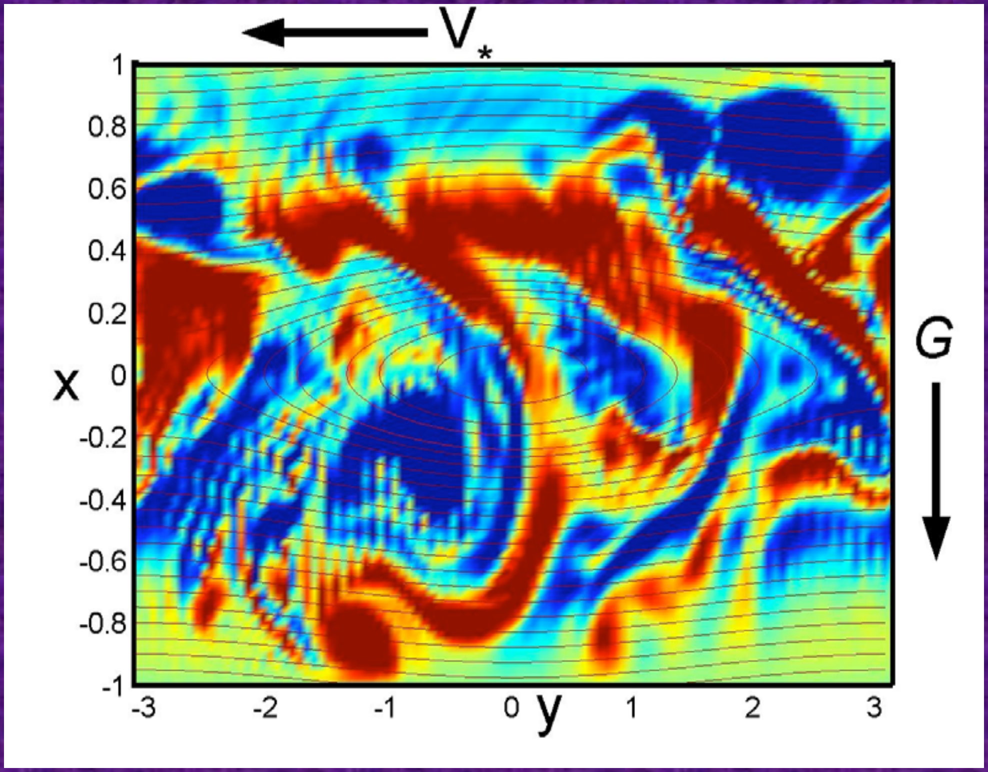
- Equations solved using an initial value, finite-difference (4°-order in space, 2°-order in time)
- The code employs fully-implicit, multi-grid, time-stepping algorithm constructed using PETSc.
- It runs on a two-processor machine at the IFS and on NERSC.

Turbulence Simulation Set Up

- Initial Condition: random turbulence localized with a Gaussian envelope.
- Adiabatic Fields ($\tilde{n} = \tilde{\varphi}$).
- Turbulent relaxation and driven Turbulence (by means of interchange instability) investigated.
- Simulations in the island frame of reference, island rotation frequency calculated using force balance in poloidal direction.

Turbulence Simulation Set Up

- Periodicity in the y-direction.
- *G* pointing downward, Electron Diamagnetic velocity from left to right.
- Density gradient in the *x*-direction

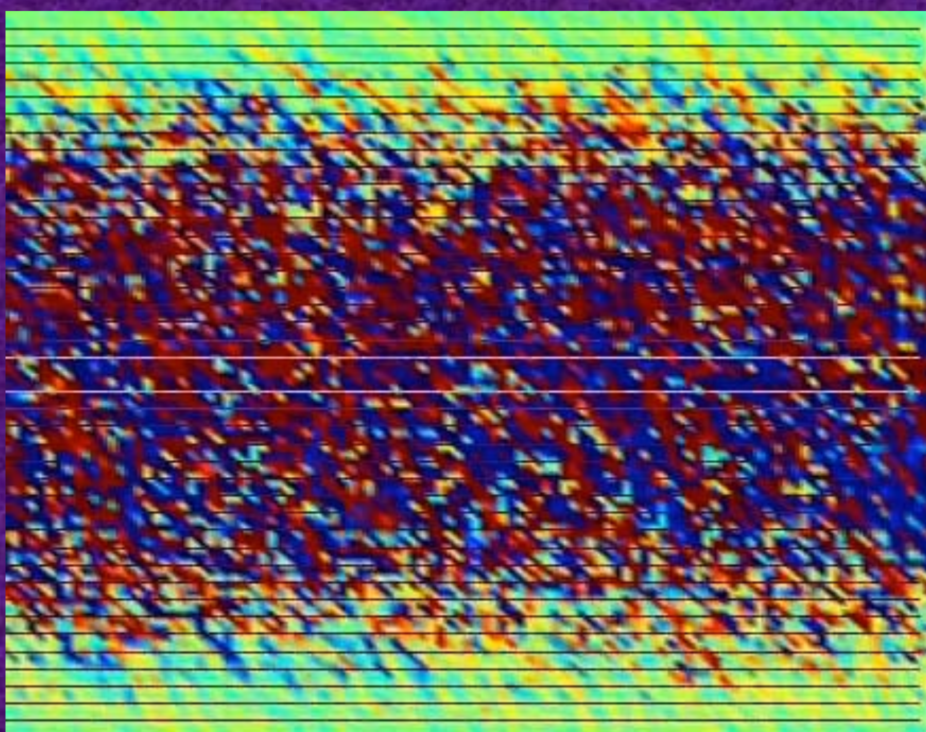


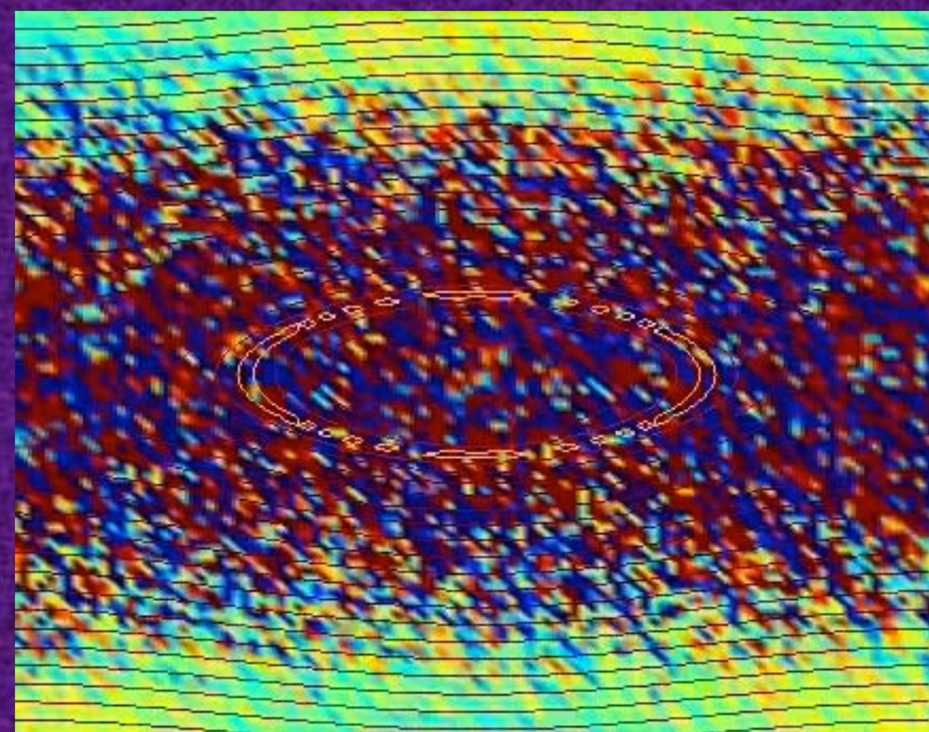
Turbulent Relaxation

G=0

Vorticity Field

Sheared Magnetic Field **Magnetic Island**





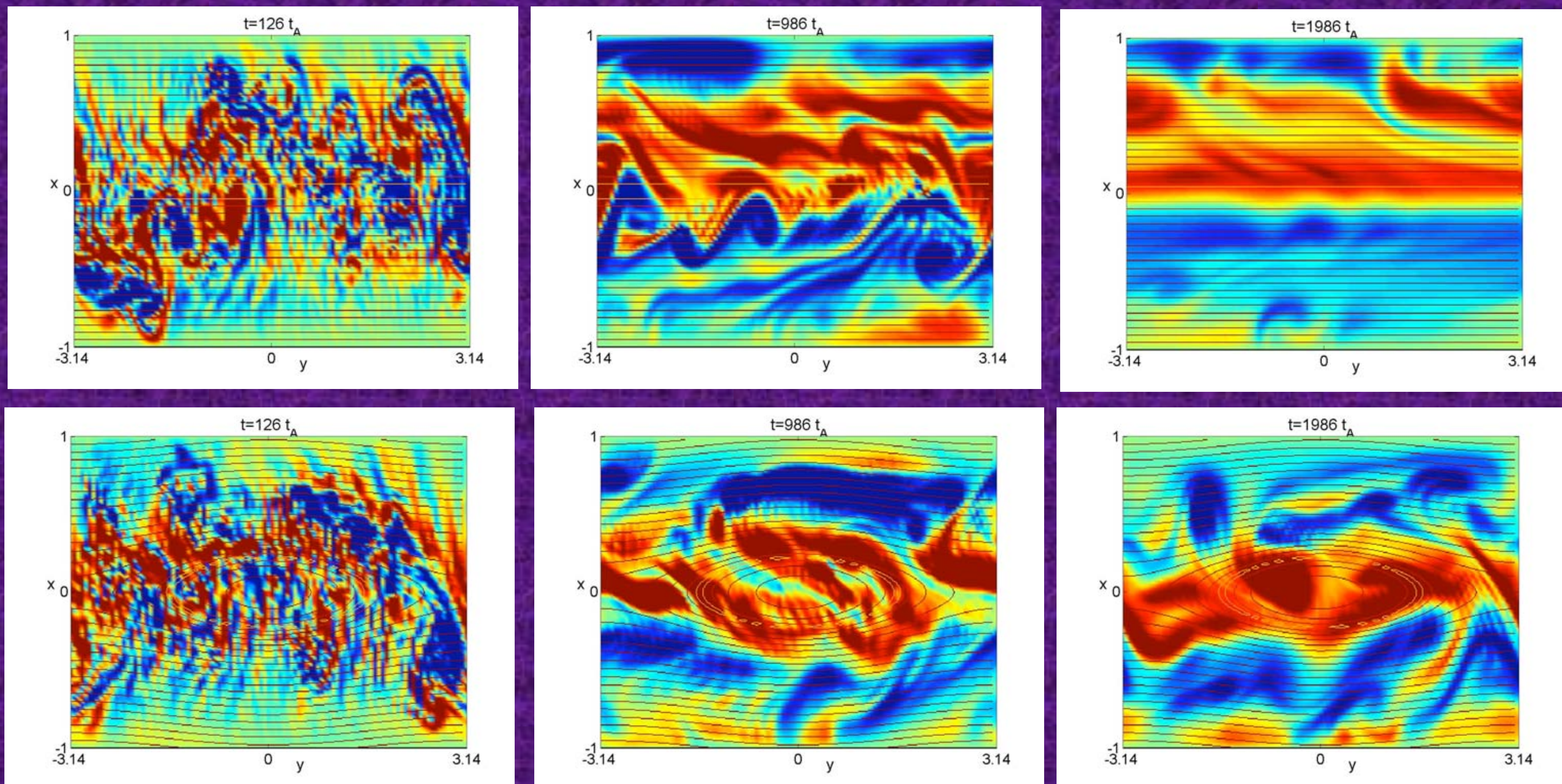
Turbulent Relaxation

G=0

Vorticity Field

Sheared Field

Magnetic Island

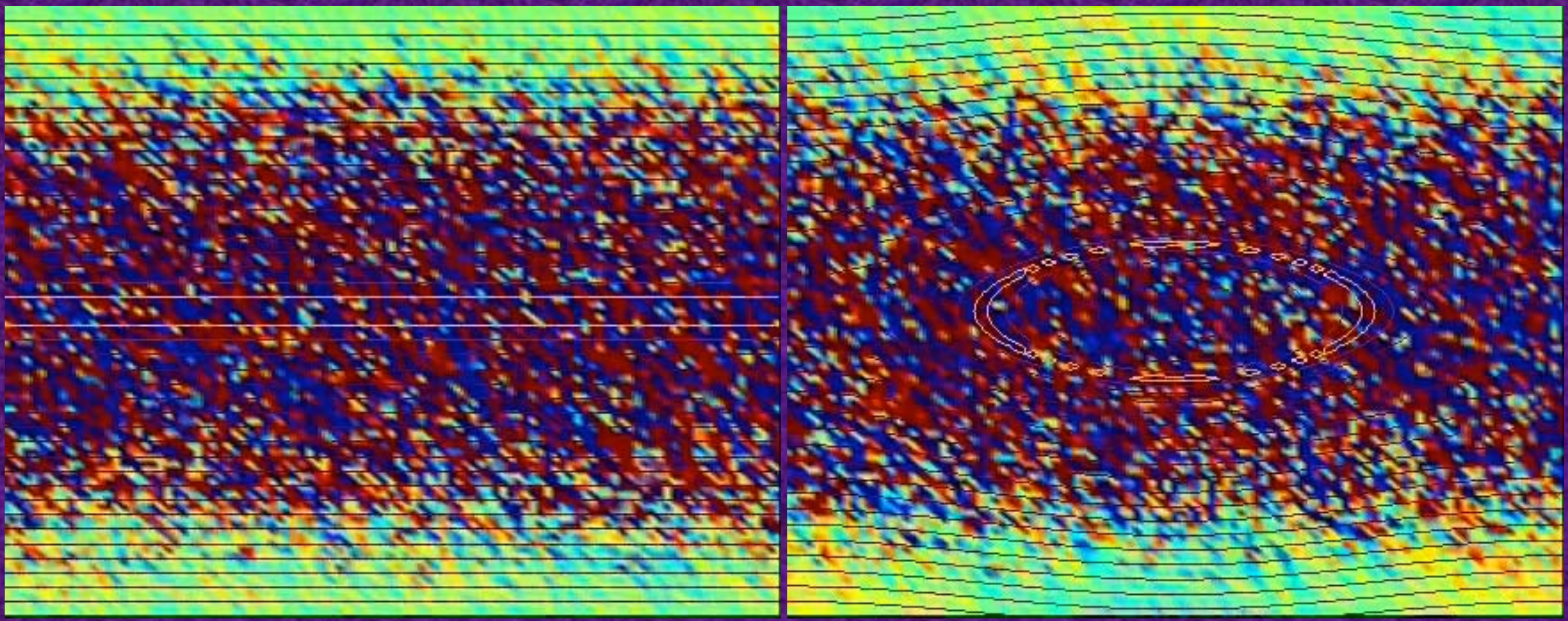


Driven Interchange Turbulence

G≠0

Vorticity Field

Sheared Magnetic Field **Magnetic Island**



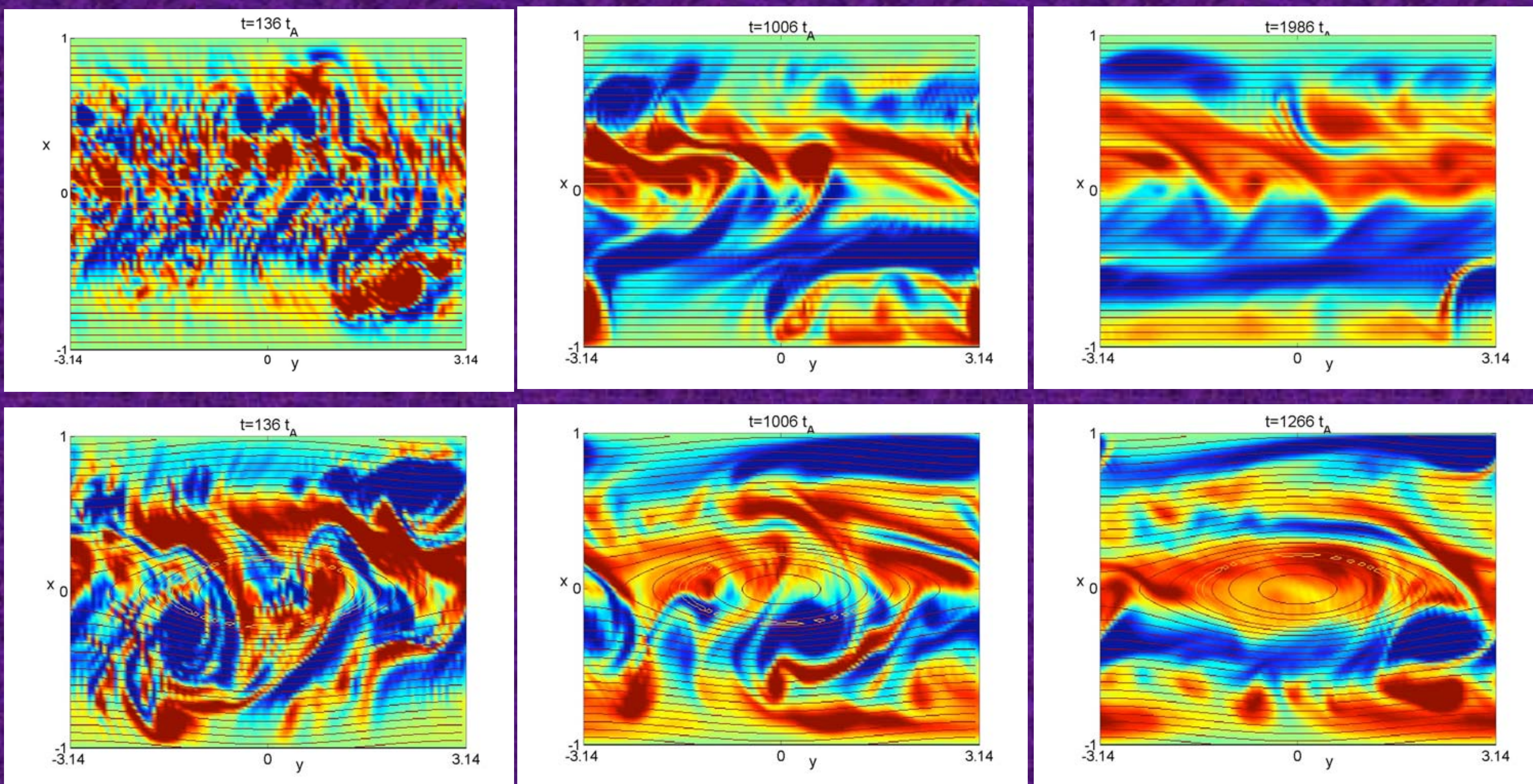
Driven Interchange Turbulence

G≠0

Vorticity Field

Sheared Field

Magnetic Island

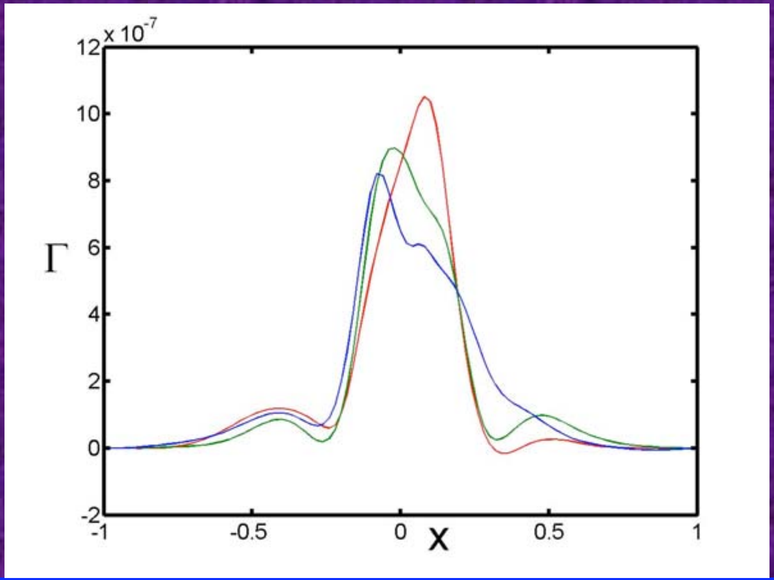


Turbulent Particle Flux

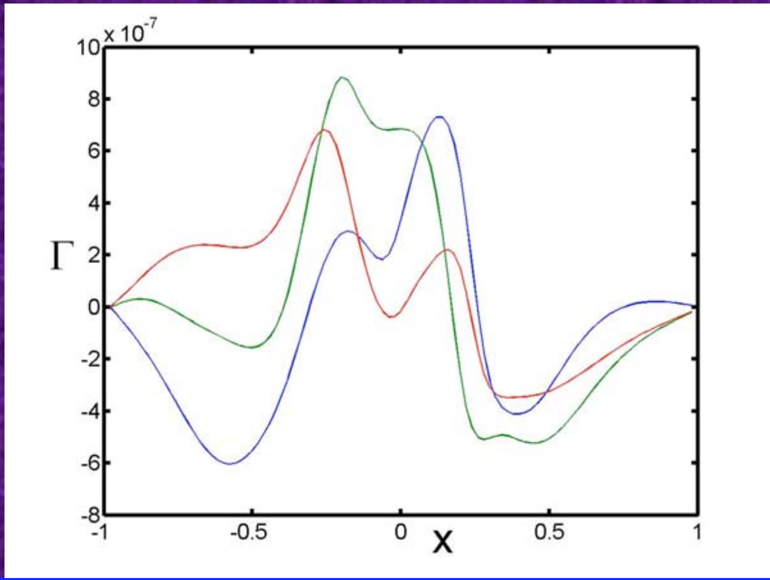
- The presence of a magnetic island changes the behavior of the turbulent diffusion (D_T~$10D$):

$$\Gamma = \frac{1}{2\pi}\oint dy\tilde{n}V_x = D_T\frac{dn_0}{dx}$$

w=0



w=0.4



Effect of the turbulence on the Island I

- The effect of the turbulence on the island growth is mesured by Δ'_T:

Standard Rutherford Equation: $w=w(\Delta'$,equilibrium,Bootsrtap current)

$$f\left(\frac{\partial w}{\partial t}, w; \eta, \omega_*, \ldots\right) = \Delta' - \frac{1}{2\pi\tilde{\psi}}\int dx \oint dy J_{turbulence}\cos(y)$$

New term due to electrostatic turbulence, $-\Delta'_T$

Effect of the turbulence on the Island II

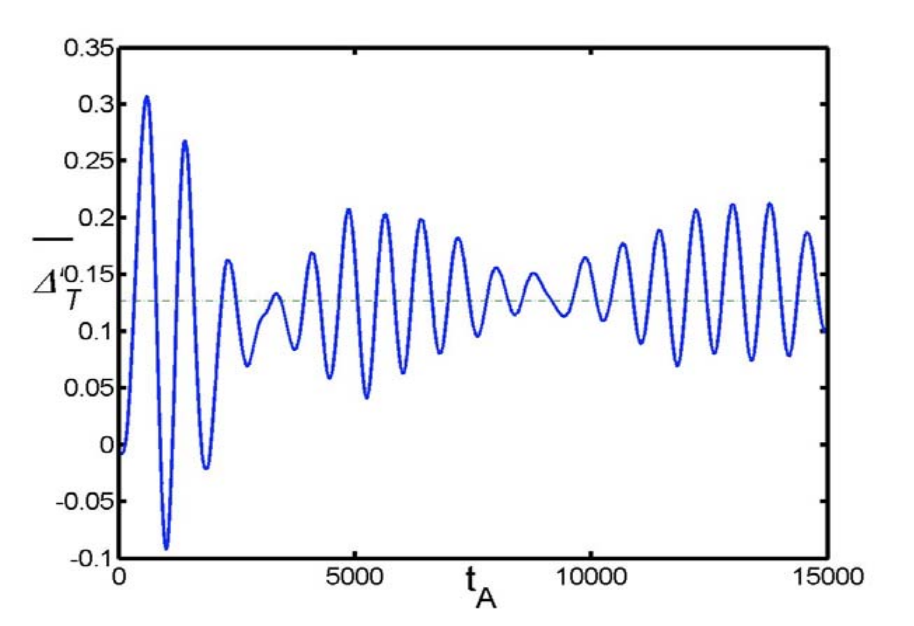
- The island magnetic field and the turbulent currents produce an electromagnetic force on the island and induce island rotation.
- The island rotation frequancy, ω, is determined using the poloidal momentum conservation equation.

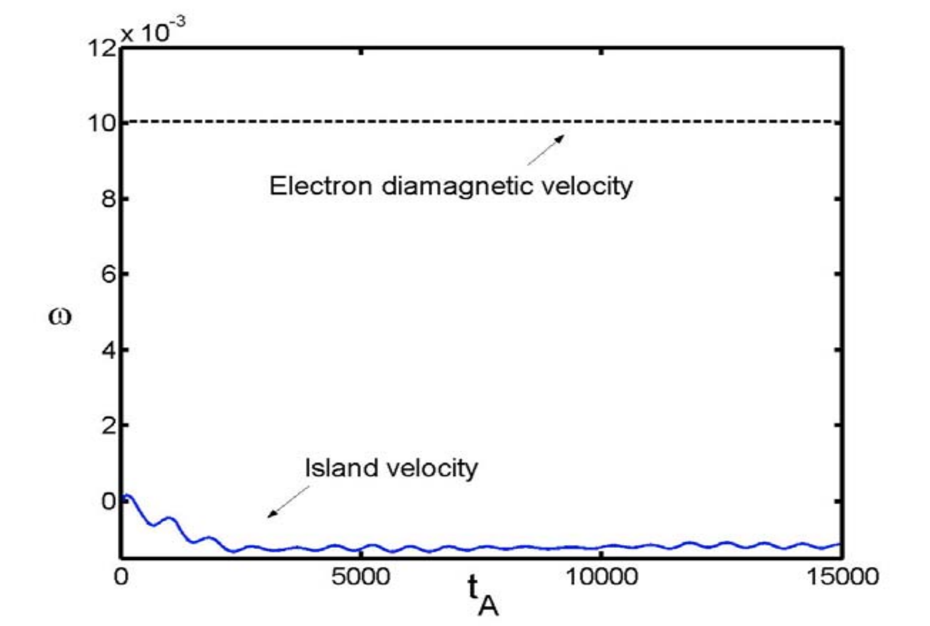
$$\rho \frac{d\vec{V}}{dt} = -\nabla p + c^{-1}\vec{J}\times\vec{B} + \mu\nabla^2\vec{V}$$

$$\frac{d\omega}{dt} = \frac{\tilde{\psi}}{2\pi L_x}\int dx \oint dy J_{turbulence} \sin(y)$$

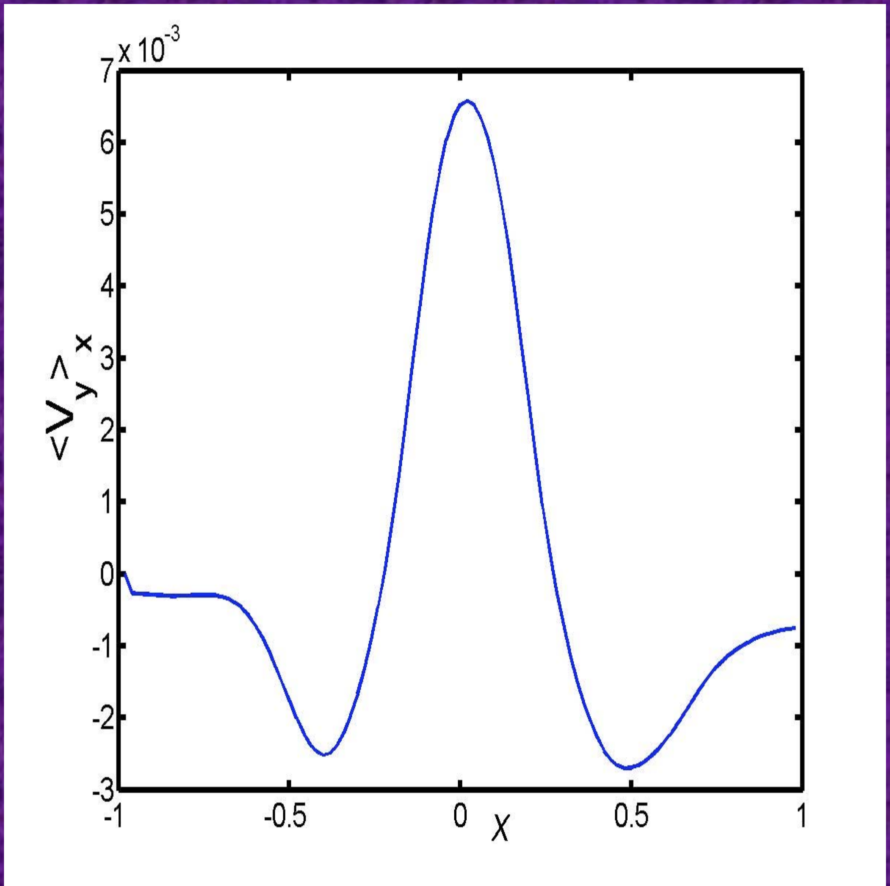
Time evolution of ω and Δ'_T

- The island velocity matches that of the coherent turbulent structures.
- Δ'_T is positive (*destabilizing*) and oscillates around a constant value.





Island rotation Frequency, an interpretation

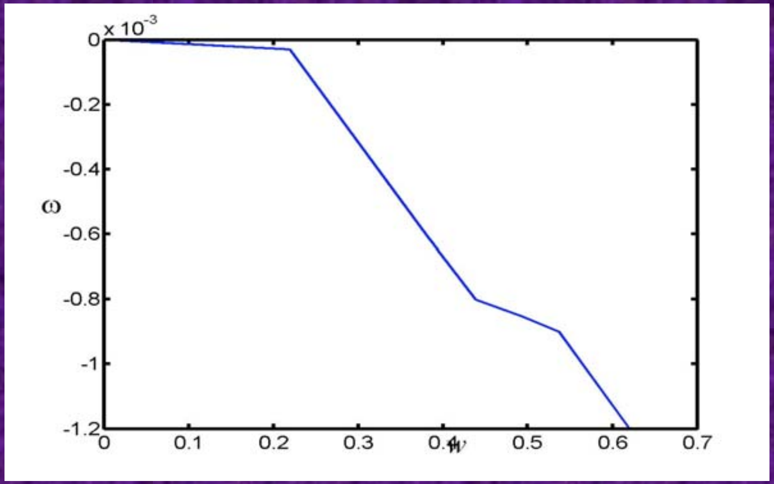


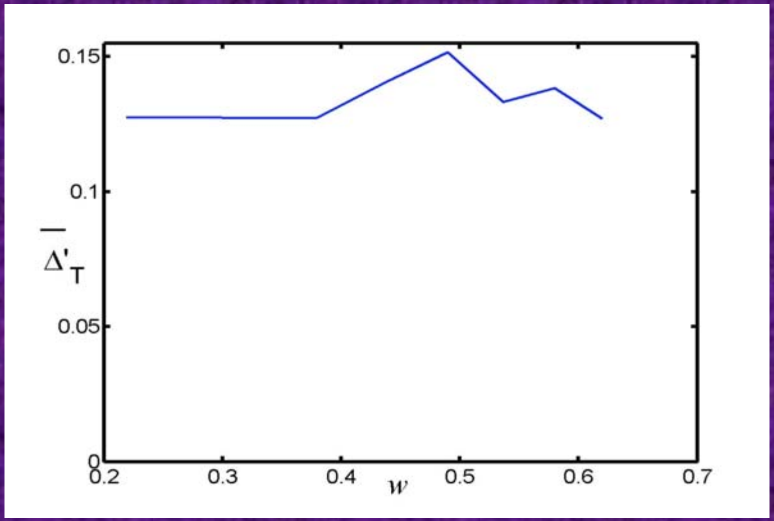
- Turbulence produces zonal flows.
- Coherent structures move in the zonal flow field.
- The island is “dragged” by the coherent structures.

MI affected by Turbulence

- We did a scan of Δ'_T and ω for different w.
- In this case, Turbulence produces a *destabilizing* effect and reduces the poloidal rotation frequency of the island.

$$\Delta'_T = \frac{\beta (L_S / L_n)^2}{\rho} \overline{\Delta}'_T$$





Summary and Conclusions

- We have investigated the interaction between turbulence and magnetic islands in both “directions”
- We find that turbulence is strongly affected by a reconnected magnetic field.
- The magnetic islands are destabilized by turbulence and their rotation frequency is significantly modified.
- Further work is planned to understand how the turbulence and the magnetic island react to different set of parameters.