Model Reduction for Design of Resistive Wall Mode Observers

David A. Maurer
Columbia University

with J. Bialek and G. A. Navratil

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Motivation

Optimize RWM and MHD control. Especially to clarify performance issues for low rotation burning plasmas in ITER

Consider coil geometry fixed (ITER external error field-RWM feedback coil set) and ask the question: How close to the ideal wall limit can we get?

Concentrate on feedback loop optimization to achieve the highest possible performance

- Control algorithm development
- Optimal control theory (O. Katsuro-Hopkins)
- Observer design using reduced models

Outline and Introduction

- Introduction to linear observers
- Model reduction based on eigenmodes
- Discuss VALEN circuit equations
- Results for HBT-EP and DIII-D
- Summary and future work

What Is An Observer?

An observer is a dynamical system that is picked to converge to the true “state” of the system using knowledge of the estimate, control input, and measurements of the true state.

$\hat{I}$ system estimate, V_f control input, and sensor fluxes Φ_s

$$\frac{d\hat{I}}{dt} = A\hat{I} + BV_f + F\Phi_s$$

Pick A, B, F such that $\left|\hat{I}(t) - I(t)\right| \to 0$ asymptotically, independent of the control input

Circuit Theory Observers

An observer uses a system model and real time measurements to estimate RWM amplitude and phase.

$$\frac{d\hat{I}}{dt} = \mathcal{L}^{-1}R\ \hat{I} + \mathcal{L}^{-1}V_f + K(\Phi_s - \mathcal{L}_s\hat{I})$$

Pick the observer gains using pole placement, LQG, or Kalman's prescription if stochastic

Want good model for $\mathcal{L}^{-1}R, \mathcal{L}^{-1}, \mathcal{L}_s$. And we want it "small"

Reduced Models

Various approaches

----Input-output relations

----Estimation techniques

----1st principle models

Can we construct a simple quantitative RWM model that retains the physics content of slab or cylindrical models that we like to use.(Fitzpatrick-Aydemir, Garofalo-Jensen, Okabayashi et al., etc…)

Start with full VALEN finite element model of the plasma, wall, feedback, and sensor coils.

Develop model of unstable mode first… then add sensors and other coils.

VALEN Circuit Equations

After including plasma stability effects the fluxes at the wall, plasma, and feedback coils are given by (J. Bialek)

$$\vec{\Phi}_w = \overleftrightarrow{\mathcal{L}}_{ww} \cdot \vec{I}_w + \overleftrightarrow{\mathcal{L}}_{wf} \cdot \vec{I}_f + \overleftrightarrow{\mathcal{L}}_{wp} \cdot I_d$$

$$\vec{\Phi}_f = \overleftrightarrow{\mathcal{L}}_{fw} \cdot \vec{I}_w + \overleftrightarrow{\mathcal{L}}_{ff} \cdot \vec{I}_f + \overleftrightarrow{\mathcal{L}}_{fp} \cdot I_d$$

$$\Phi = \overleftrightarrow{\mathcal{L}}_{pp} \cdot I_d + \overleftrightarrow{\mathcal{L}}_{pw} \cdot \vec{I}_w + \overleftrightarrow{\mathcal{L}}_{pf} \cdot \vec{I}_f$$

Using Faraday and Ohms law yields equations for system evolution

$$\begin{bmatrix} \overleftrightarrow{\mathcal{L}}_{ww} & \overleftrightarrow{\mathcal{L}}_{wf} & \overleftrightarrow{\mathcal{L}}_{pw} \\ \overleftrightarrow{\mathcal{L}}_{fw} & \overleftrightarrow{\mathcal{L}}_{ff} & \overleftrightarrow{\mathcal{L}}_{fp} \\ \overleftrightarrow{\mathcal{L}}_{pw} & \overleftrightarrow{\mathcal{L}}_{pf} & \overleftrightarrow{\mathcal{L}}_{pp} \end{bmatrix} \cdot \frac{d}{dt} \begin{Bmatrix} \vec{I}_w \\ \vec{I}_f \\ I_d \end{Bmatrix} = \begin{bmatrix} \overleftrightarrow{R}_w & 0 & 0 \\ 0 & \overleftrightarrow{R}_f & 0 \\ 0 & 0 & R_d \end{bmatrix} \begin{Bmatrix} \vec{I}_w \\ \vec{I}_f \\ I_d \end{Bmatrix} + \begin{Bmatrix} \vec{0} \\ \vec{V}_f \\ \vec{0} \end{Bmatrix}$$

This can easily be put in state space form…

Quantitative Reduced Models

Need to construct quantitative eigenmode models for observer design and feedback loop optimization

$$\mathcal{L}_{ww}\frac{dI_w}{dt}+\mathcal{L}_{wf}\frac{dI_f}{dt}=-R_wI_w$$

$$\mathcal{L}_{fw}\frac{dI_w}{dt}+\mathcal{L}_{ff}\frac{dI_f}{dt}=-R_fI_f+V_f$$

Use current representation rather than fluxes. How can we use large scale codes to generate small order reduced models.

Want to move away from simple slab or cylindrical approximations

Single Circuit Theory

The effective inductances of circuit theory can always be written as the sum of two distinct terms (Boozer PoP 1998 & 2004)

Example: wall effective inductance…

$$\mathcal{L}_{ww} = -L_w D(s) \quad \text{with,} \quad D(s) = \frac{1+s}{s}c - 1$$

RWM evolution given by

$$\frac{dI_w}{dt} = \frac{R_w}{L_w D(s)} I_w$$

with

Plasma-wall coupling: $c \equiv \dfrac{M_{pw}^2}{L_p L_w}$ Wall time: $1/\tau_w = \dfrac{R_w}{L_w}$

Use VALEN Spectral Decomposition

VALEN solves the genearalized eigenvalue problem

$$L_{ww} I_k = \lambda_k R_{ww} I_k$$

$$L_{ww} \Psi = R_{ww} \Psi \Lambda$$

No plasma: L_{ww}, R_{ww} sym. pos. definite
Passive stabilization: $\mathcal{L}_{ww}$ becomes sym. indefinite
Feedback: R_{ww} becomes nonnormal

$$\Psi = QT$$

$$\Psi^t R_{ww} \Psi = 1$$

$$\Psi^t L_{ww} \Psi = \Lambda$$

$$\Lambda^{-1} = \Psi^{-1} L_{ww}^{-1} R_{ww} \Psi$$

Contract or Project VALEN Matrices Using Eigenmodes

Evaluate equivalent VALEN matrices and identify the single circuit formula with the unstable eigenmode matrix elements

VALEN contains generalizations of the effective inductances and coupling constant and wall time mentioned in the context of single circuit theory.

$$\{I^u\} = \begin{Bmatrix} \{I_w^u\} \\ \{I_f^u\} \\ \{I_d^u\} \end{Bmatrix} = \begin{Bmatrix} \{\varsigma_w^u\} \\ \{\varsigma_f^u\} \\ \{\varsigma_d^u\} \end{Bmatrix} (\iota)$$

start with the unstable eigenmode

Insert back into matrix equations, neglecting the everything except for the wall currents

$$\left(\{\varsigma_w^u\}^t \{\varsigma_w^u\} - \frac{(1+s)}{s} \{\varsigma_w^u\}^t \left[L_{ww}^{-1} L_{wp} L_{pp}^{-1} L_{pw} \right] \{\varsigma_w^u\} \right) \frac{d(\iota)}{dt}$$

$$+ \{\varsigma_w^u\}^t \left[L_{ww}^{-1} R_{ww} \right] \{\varsigma_w^u\} (\iota) = 0$$

Matrix Contractions Yield Plasma Wall Coupling & Wall Time

Typically have one unstable mode and many damped modes of the coupled plasma wall system, say mode=1 is unstable, and modes 2,3,… are stable

Equate VALEN matrix elements with the simple single circuit equations and derive formula for the "effective" plasma-wall coupling and wall time constant

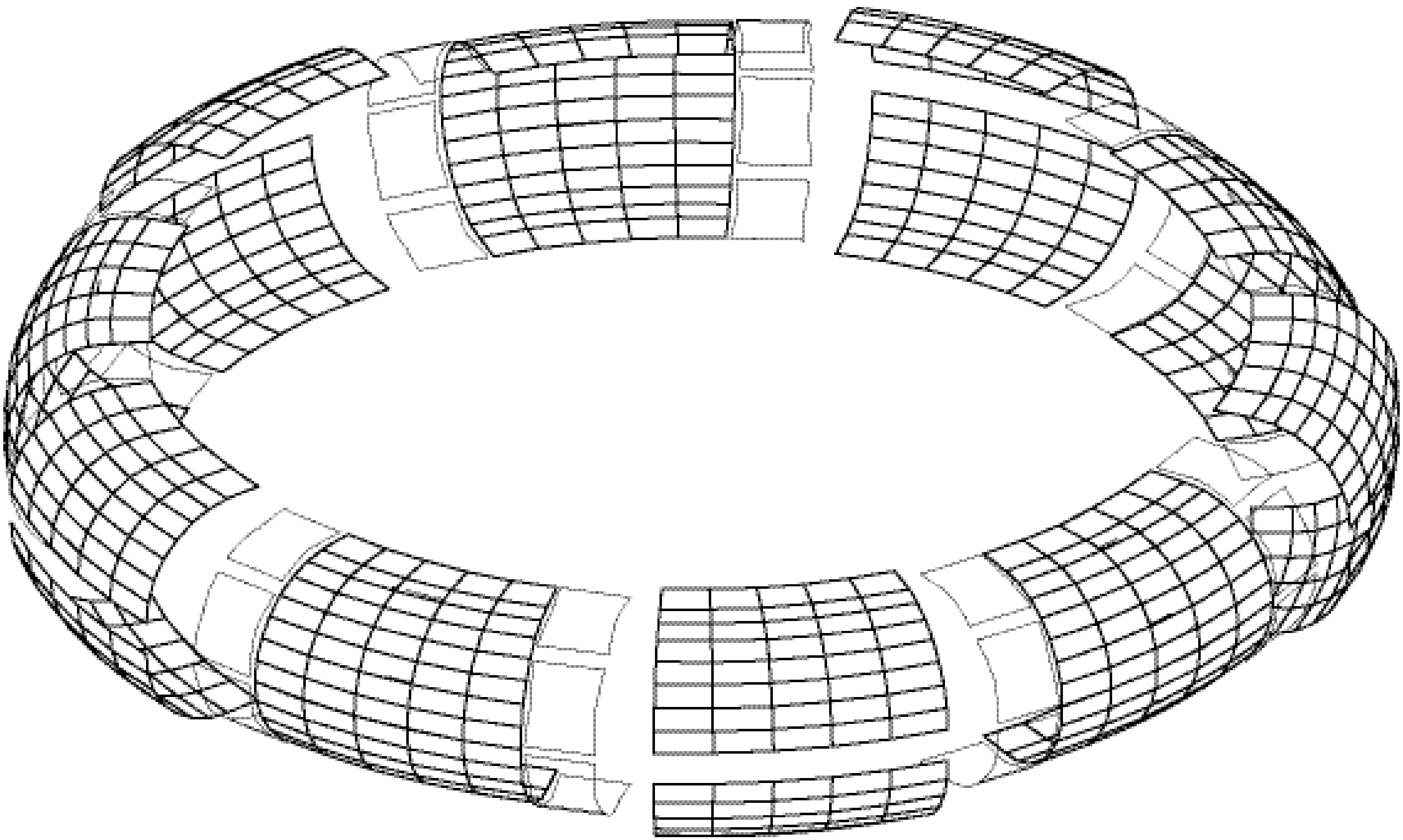
$$D(s) = \left(\frac{(1+s)}{s} c - 1 \right)$$

$$= (K)\left(\left\{ \varsigma_w^u \right\}^t \left\{ \varsigma_w^u \right\} - \frac{(1+s)}{s} \left\{ \varsigma_w^u \right\}^t \left[L_{ww}^{-1} L_{wp} L_{pp}^{-1} L_{pw} \right] \left\{ \varsigma_w^u \right\} \right)$$

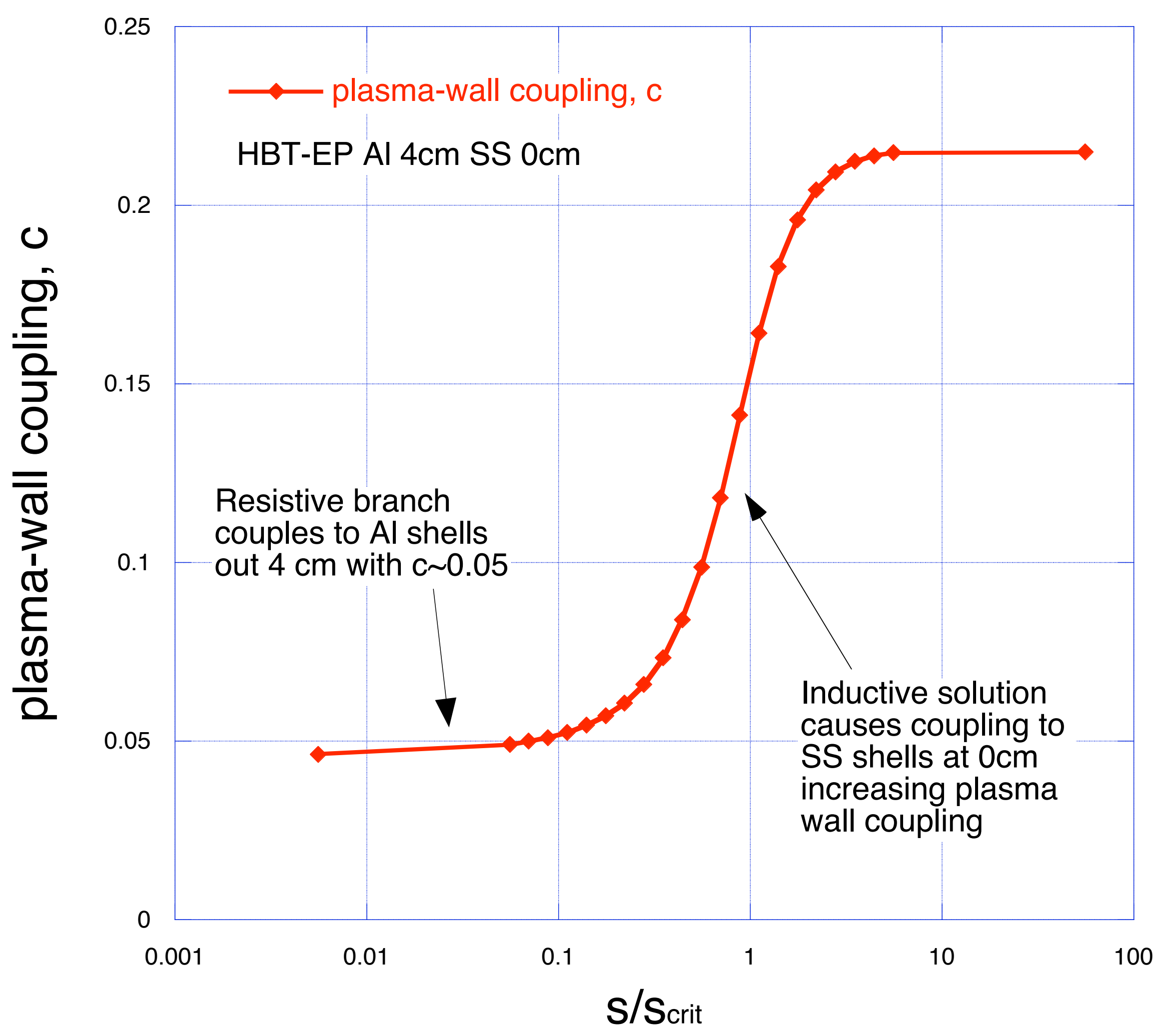
One can then define:

$$c = \frac{\left\{ \varsigma_w^u \right\}^t \left[L_{ww}^{-1} L_{wp} L_{pp}^{-1} L_{pw} \right] \left\{ \varsigma_w^u \right\}}{\left\{ \varsigma_w^u \right\}^t \left\{ \varsigma_w^u \right\}} \qquad 1/\tau_w = \frac{\left\{ \varsigma_w^u \right\}^t \left[L_{ww}^{-1} R_{ww} \right] \left\{ \varsigma_w^u \right\}}{\left\{ \varsigma_w^u \right\}^t \left\{ \varsigma_w^u \right\}}$$

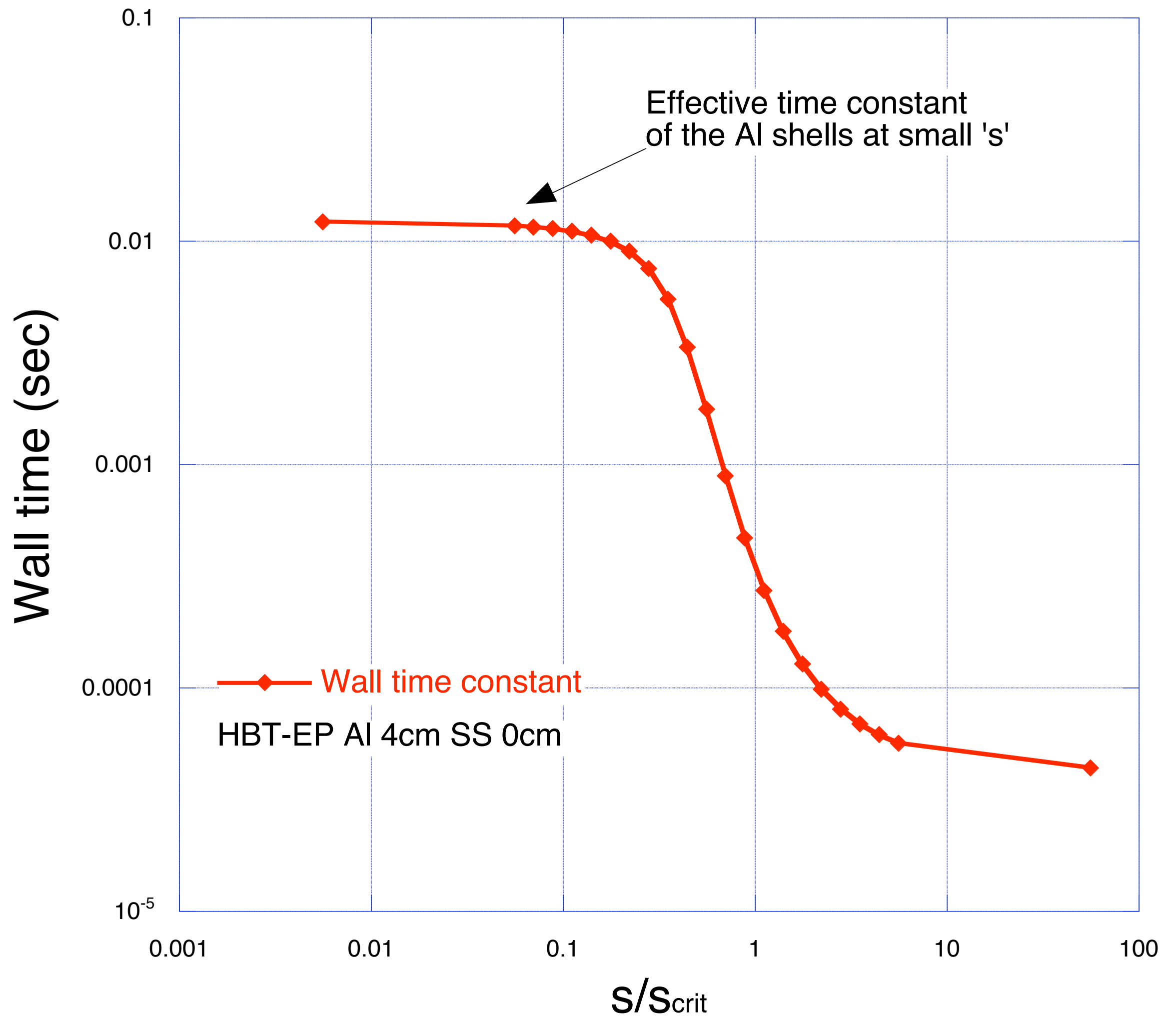
HBT-EP Conducting Wall Structure



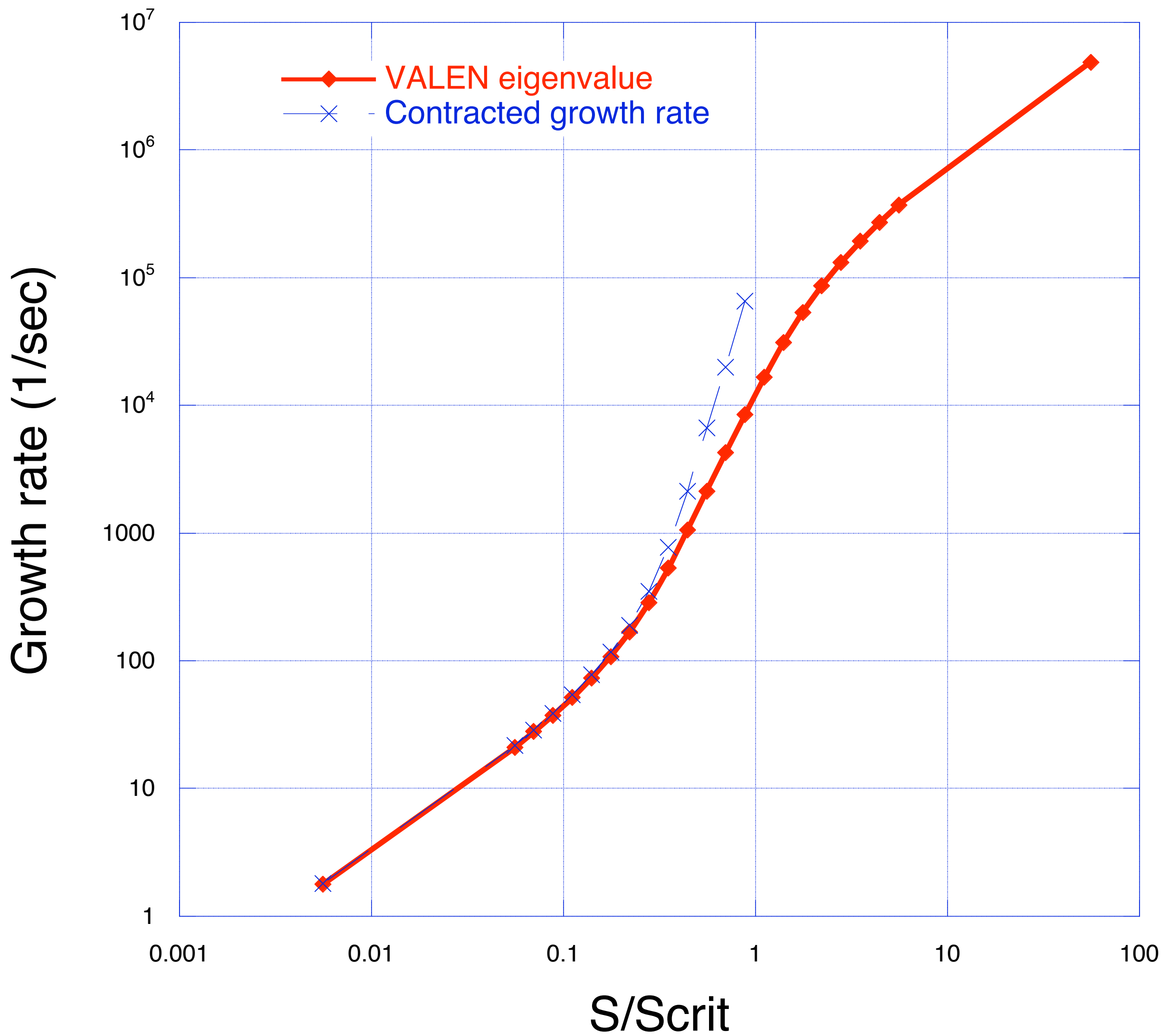
Plasma-Wall Coupling



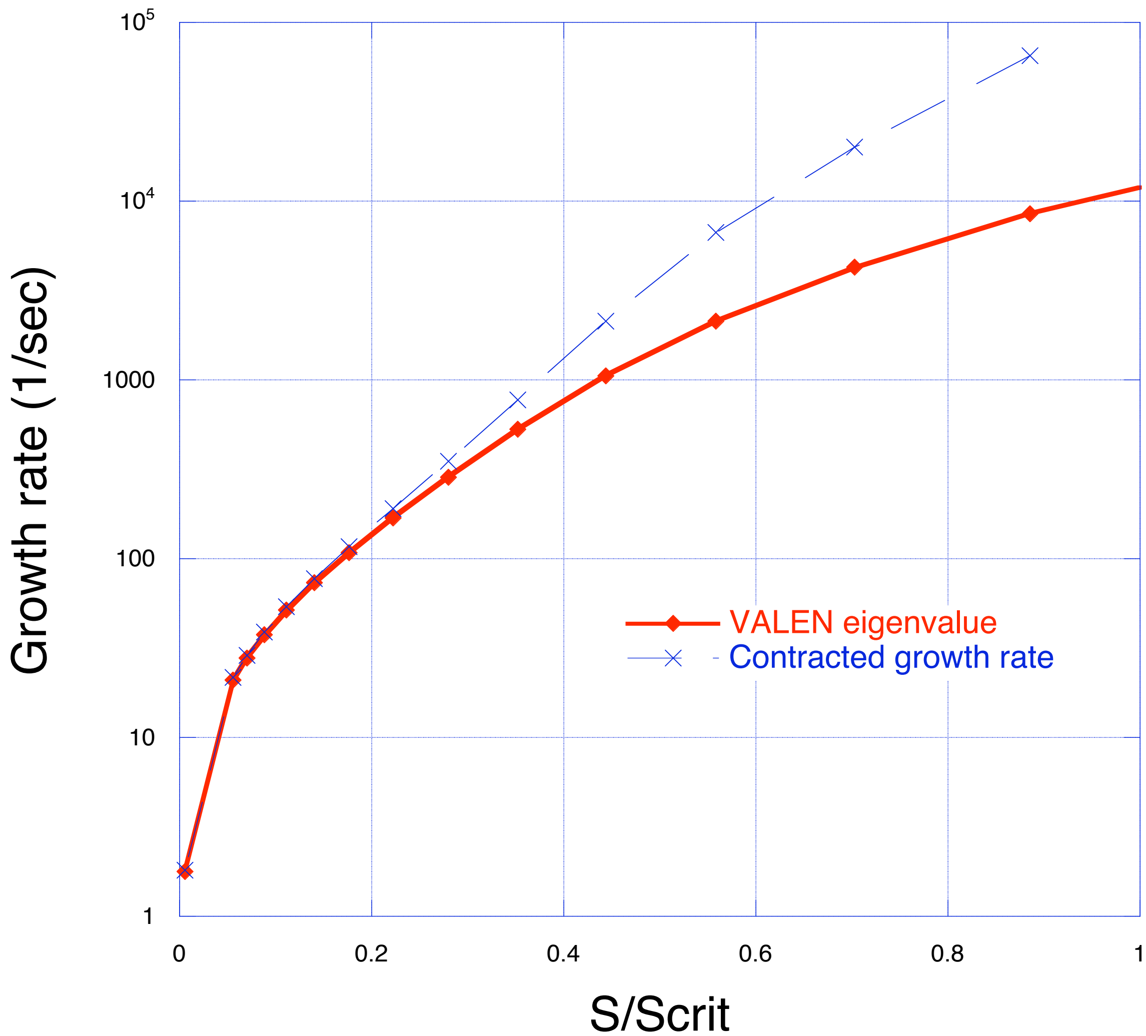
Wall Time Constant



Eigenvalue-Growth Rate Comparison



Poor Agreement Near the Ideal Limit



Include Effect of Ideal Mode and Feedback Coil

Evaluate equivalent VALEN matrices and identify the single circuit formula with the unstable eigenmode matrix elements using the entire mode currents

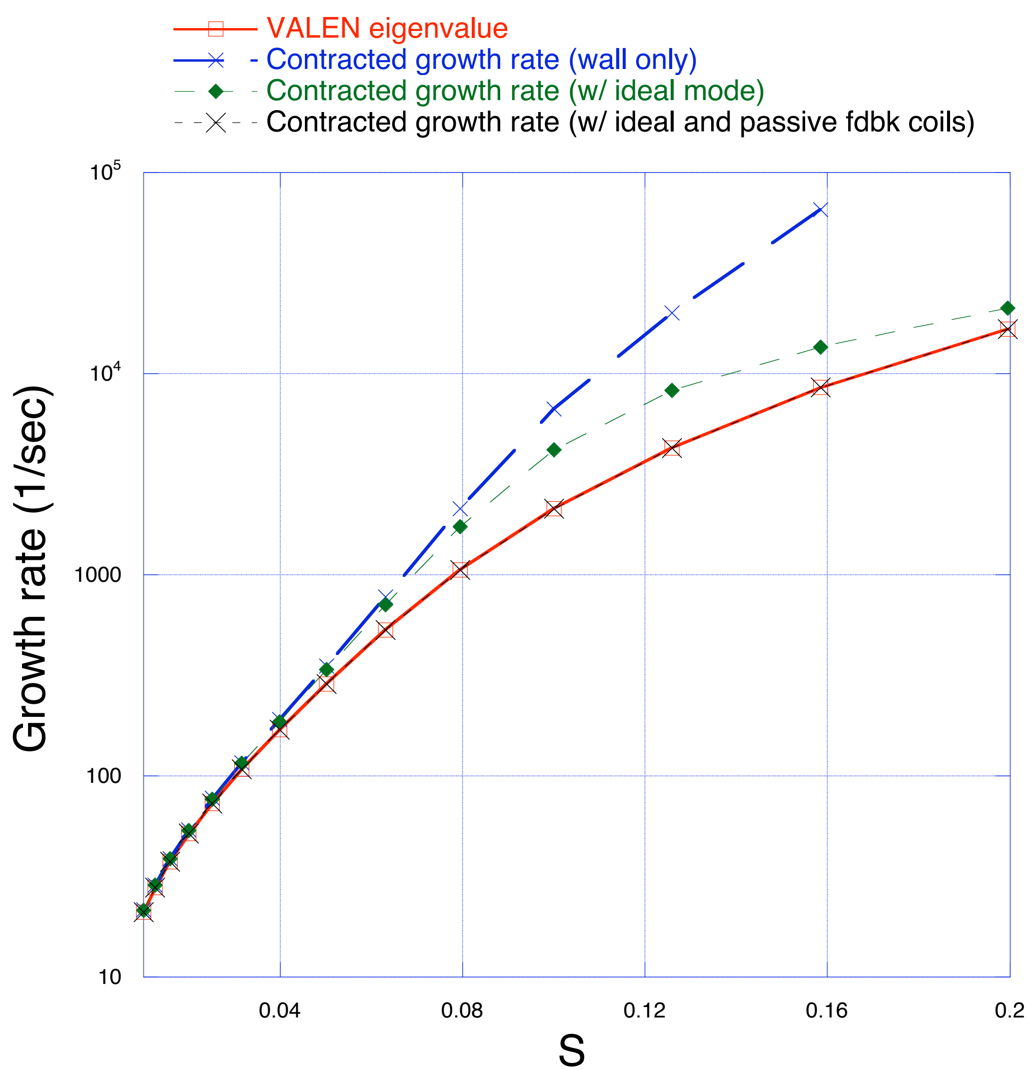
VALEN contains generalizations of the effective inductances and coupling constants mentioned in the context of single circuit theory.

$$\{\varsigma_w^u\}^t\{\varsigma_w^u\}\frac{d(\iota)}{dt}+\{\varsigma_w^u\}^t\left[L_{ww}^{-1}L_{wf}\right]\{\varsigma_f^u\}\frac{d(\iota)}{dt}+\{\varsigma_w^u\}^t\left[L_{ww}^{-1}L_{wp}\right]\{\varsigma_d^u\}\frac{d(\iota)}{dt}-$$

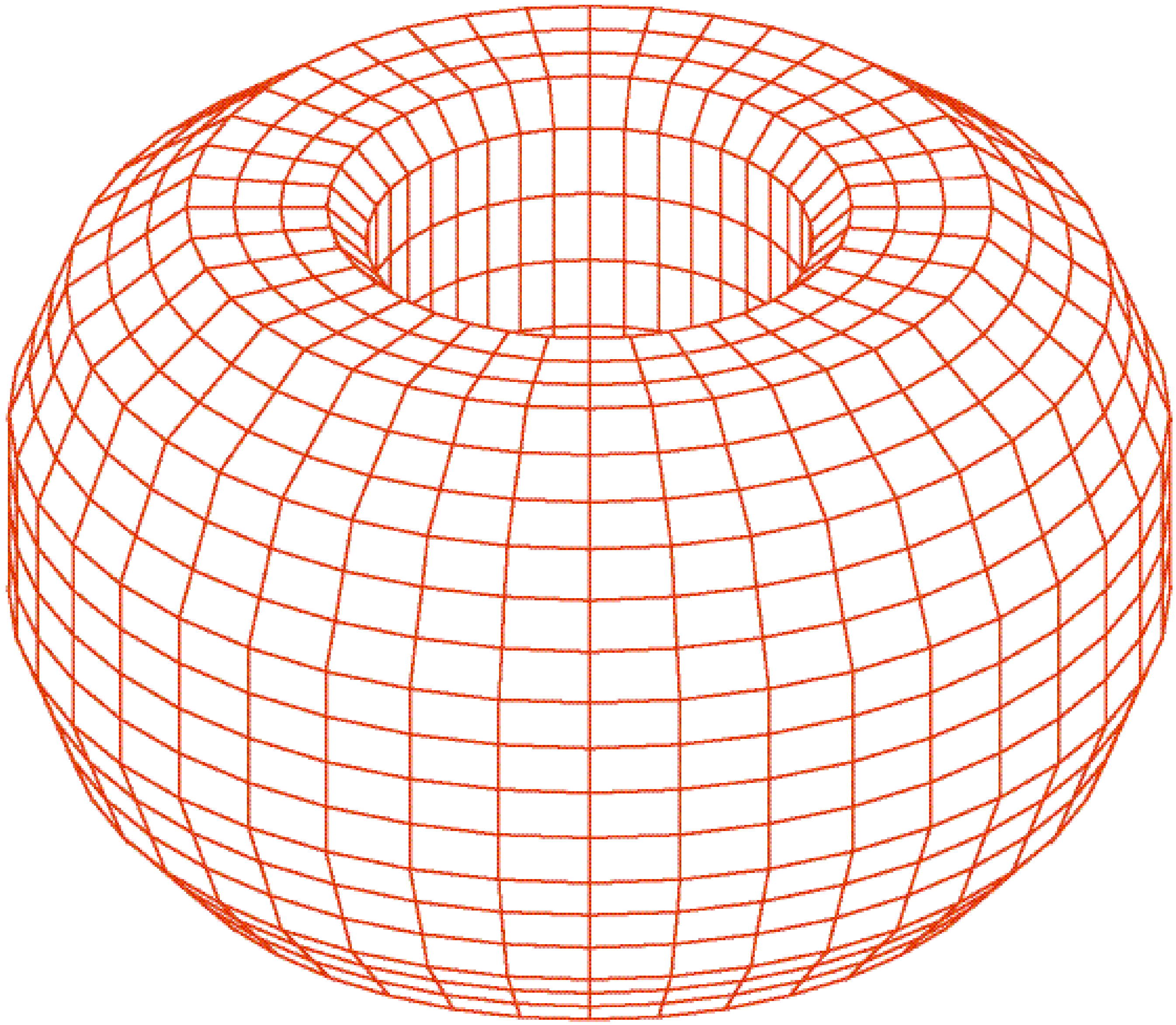
$$\frac{(1+s)}{s}\{\varsigma_w^u\}^t\left[L_{ww}^{-1}L_{wp}L_{pp}^{-1}L_{pw}\right]\{\varsigma_w^u\}\frac{d(\iota)}{dt}-\frac{(1+s)}{s}\{\varsigma_w^u\}^t\left[L_{ww}^{-1}L_{wp}L_{pp}^{-1}L_{pf}\right]\{\varsigma_f^u\}\frac{d(\iota)}{dt}$$

$$-\frac{(1+s)}{s}\{\varsigma_w^u\}^t\left[L_{ww}^{-1}L_{wp}L_{pp}^{-1}L_{pp}\right]\{\varsigma_d^u\}\frac{d(\iota)}{dt}+\{\varsigma_w^u\}^t\left[L_{ww}^{-1}R_{ww}\right]\{\varsigma_w^u\}(\iota)=0$$

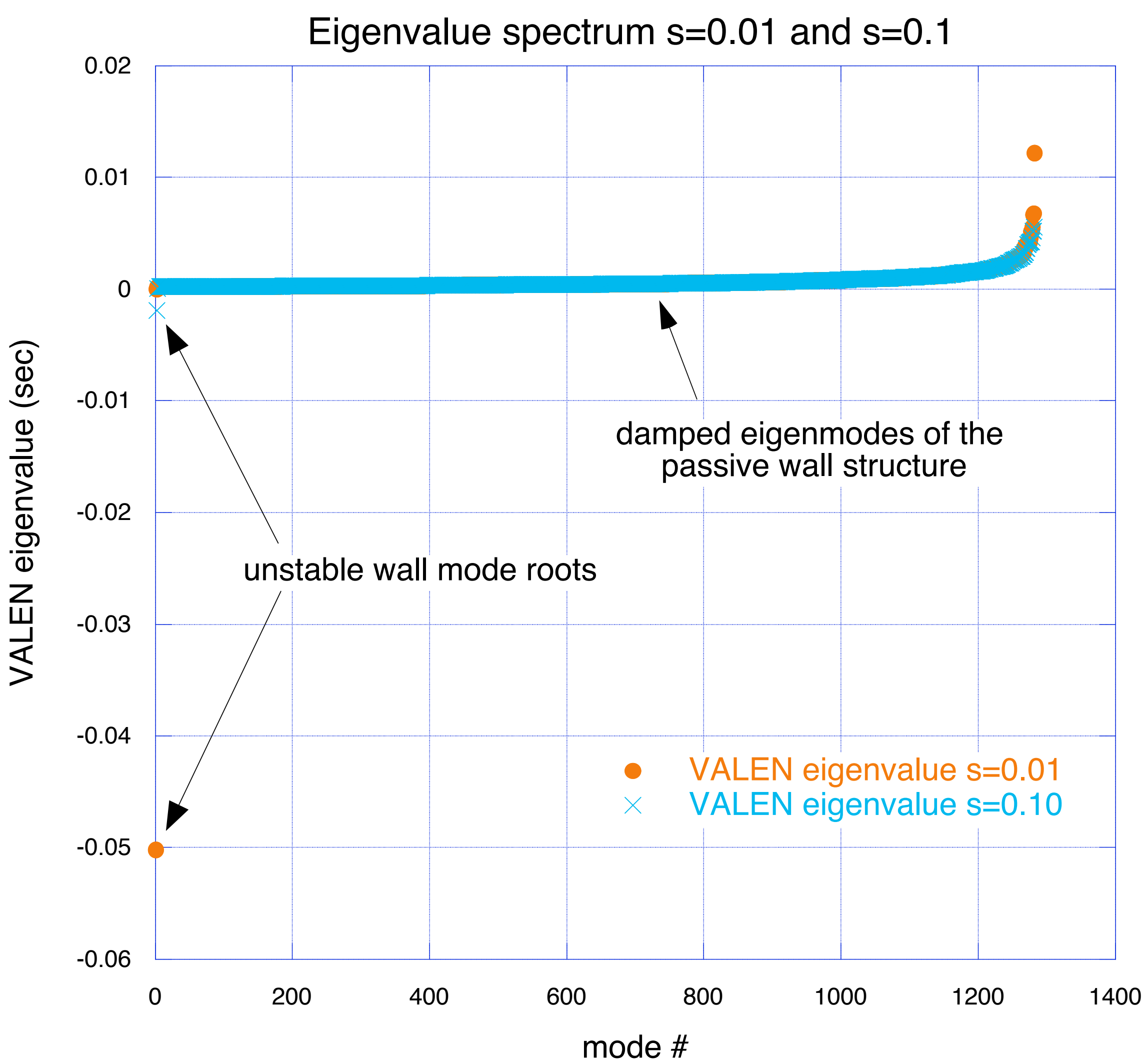
Good Agreement Including Ideal Mode And Feedback Coil Contributions



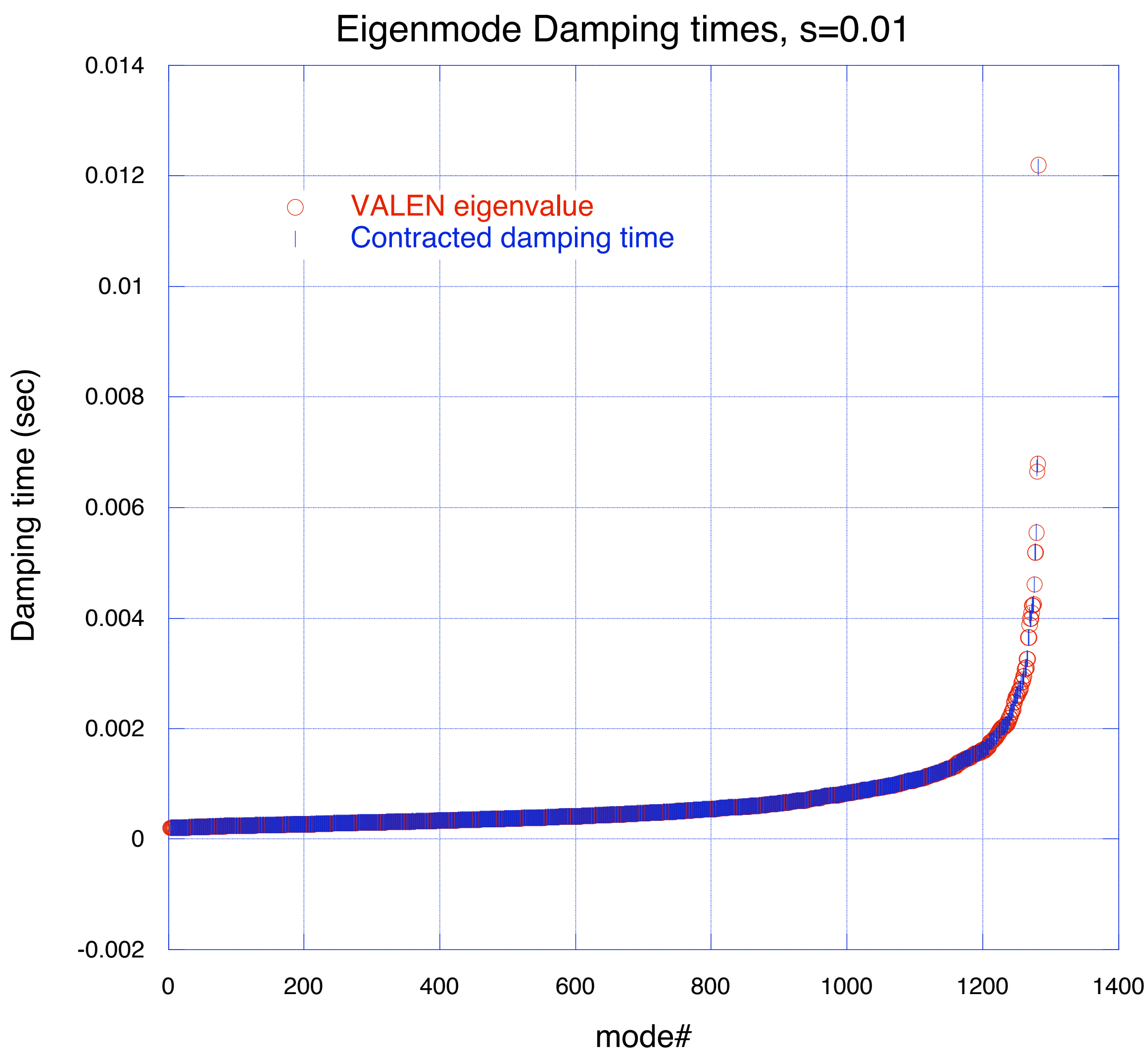
Symmetric DIII-D Vacuum Vessel Model



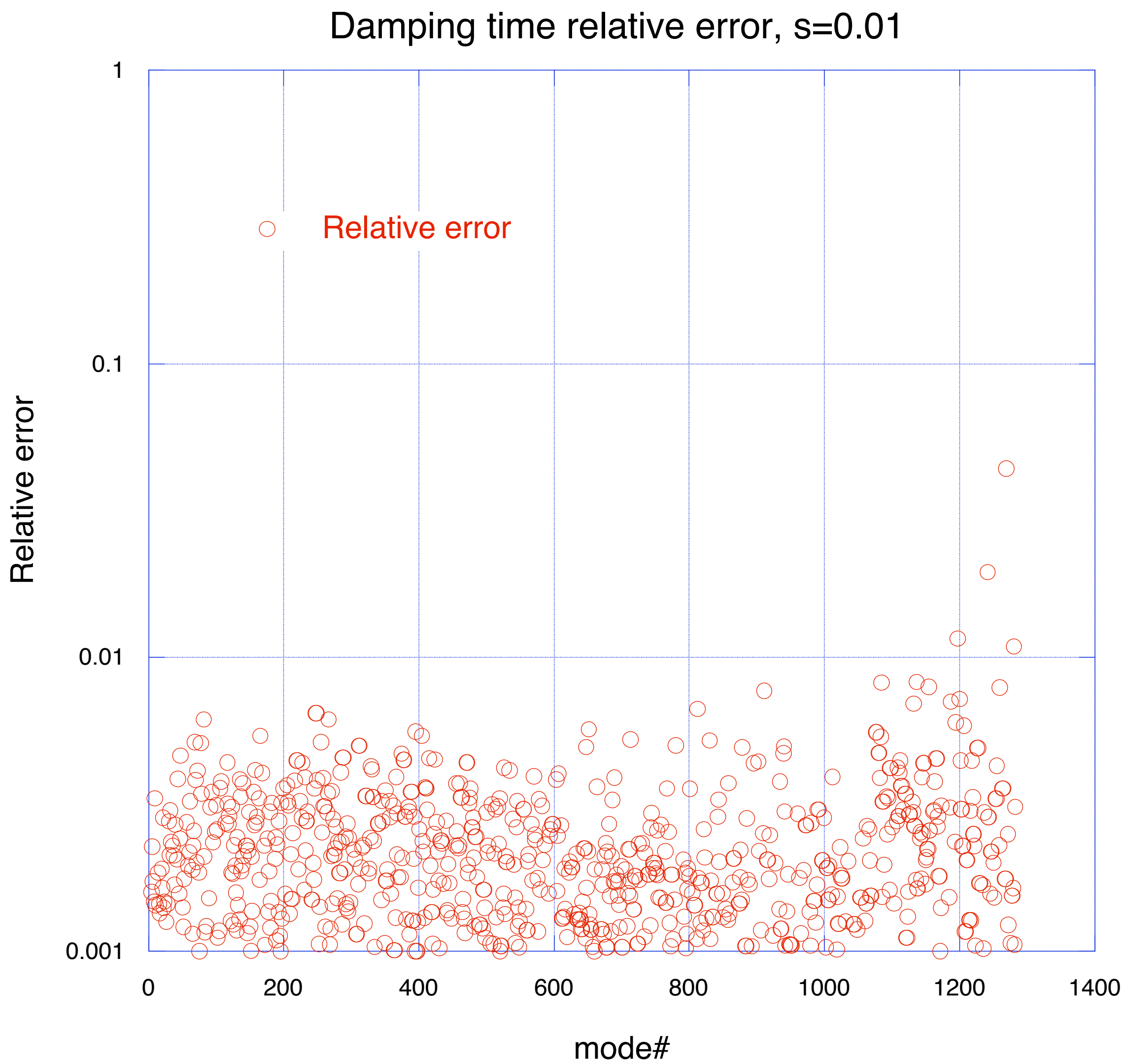
DIII-D Eigenmode Spectrum



Damped Modes Have Negligible Plasma Effects



Error in Damped Mode Calculation



Conclusions

• VALEN eigenmode calculations can be used to construct a simple ODE model of the RWM with beta dependent coefficients that allows accurate calculation of mode growth rates between the no wall and ideal wall limits for HBT-EP and DIII-D

• Each eigenmode can be characterized by two beta (plasma stability) dependent numbers: the plasma wall coupling and wall time for that mode.

• The plasma wall coupling is large for only the RWM and ideal mode. The coupling for the damped eigenmodes of the system is essentially zero, $c \sim 0$. Indicating that their eigenvalues are simply the damped modes of the wall.

• We have successfully incorporated the simulated VALEN ideal mode and feedback coils as passive elements into the model, and now have good numerical agreement up to the ideal limit for the passive case

Future Work

- Develop passive stabilization model for ITER with and without blankets

- Add sensor and feedback coils to develop full state space model that can be used to design observers and optimal controllers

- Incorporate more complicated plasma models (rotation, sin-cos components to account for mode phase, others…)