A New Approach to Resistive Wall Mode Feedback Control Using Optimal Control Techniques based on VALEN State-Space

Presented by

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  Results
\+ Conclusions

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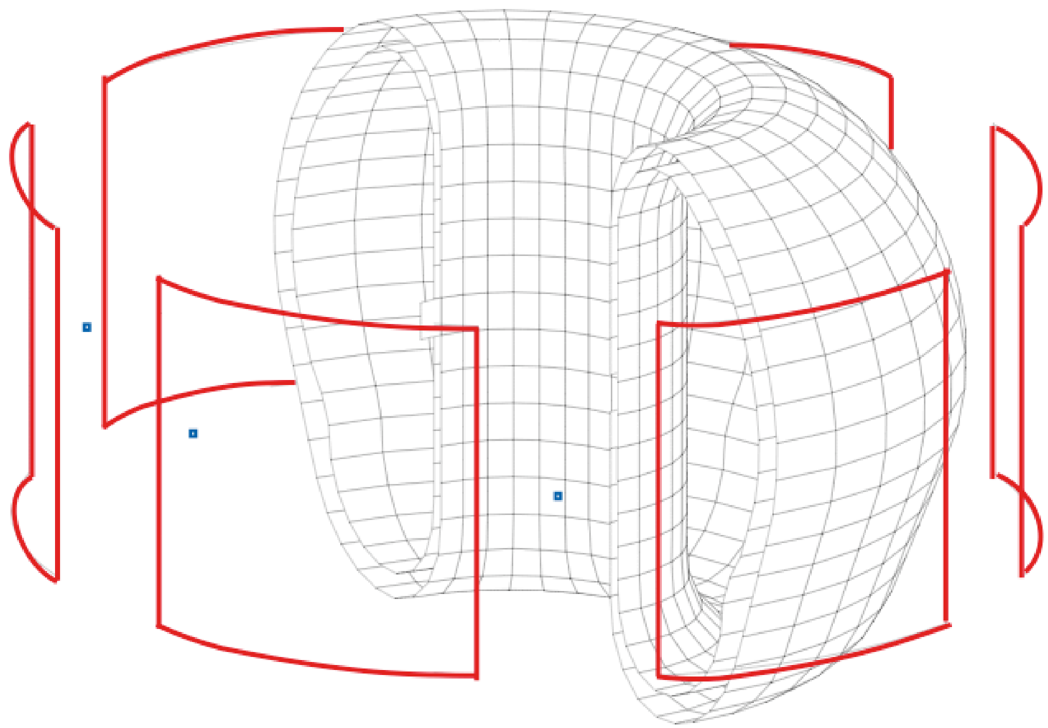
  Motivation

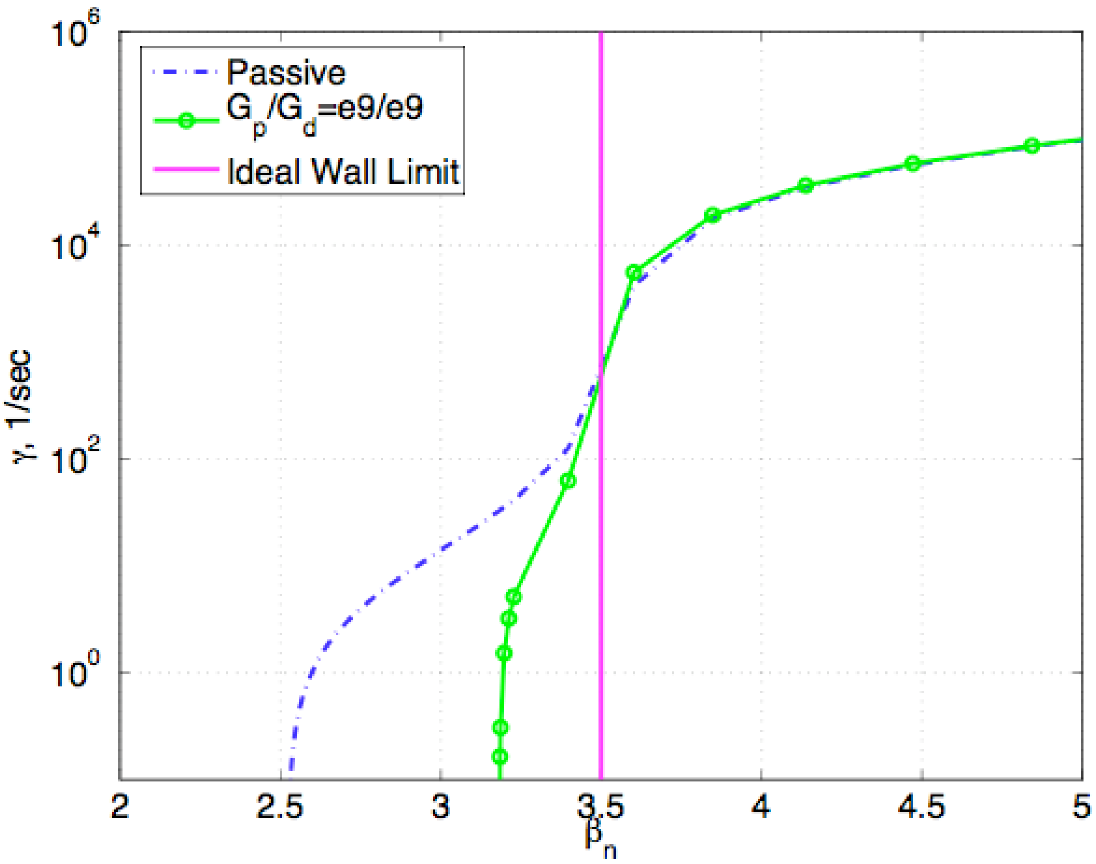
Mr. Brown Upside Down!

Results: Improved Stabilization of ITER RWM with New Optimal Controller and Observer

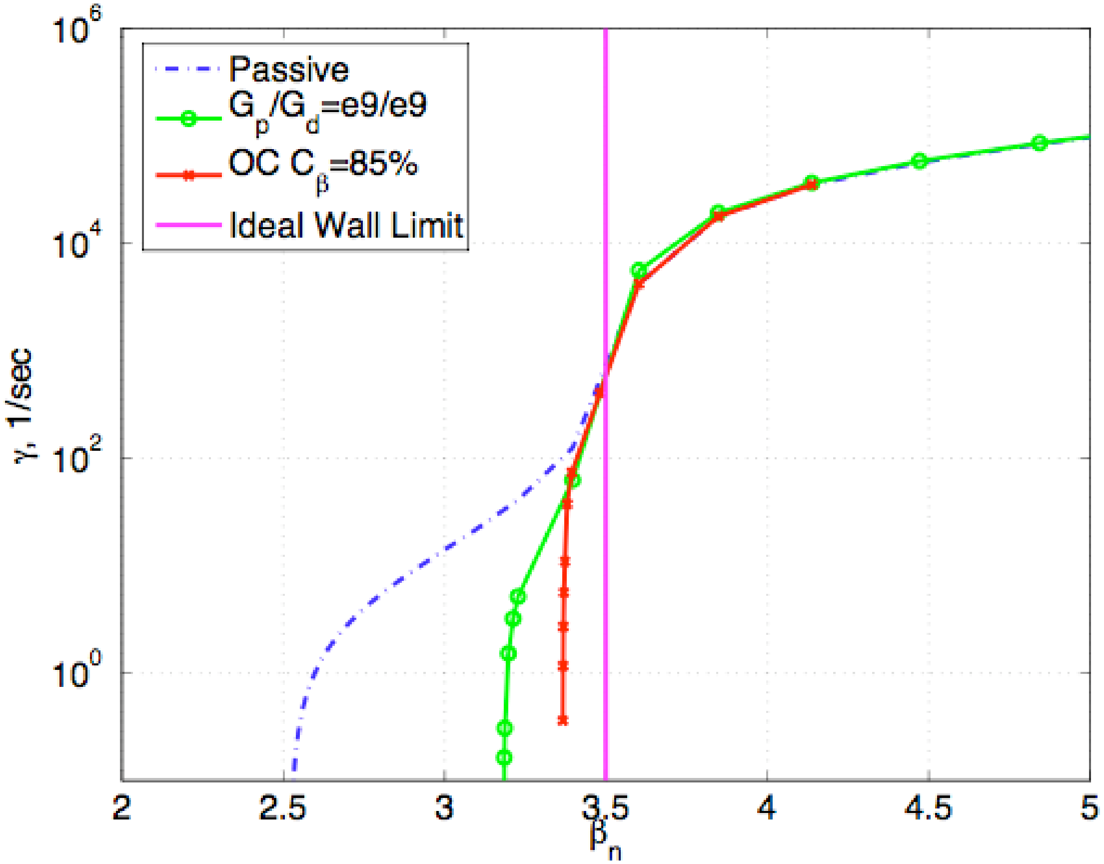


VALEN model of ITER includes double walled vacuum vessel, 3 external control coil pairs and 6 magnetic field flux sensors on the midplane (z=0).

Old Result: Classical PD controller stabilizes RWM up to $\mathbf{C_\beta = 68\%}$

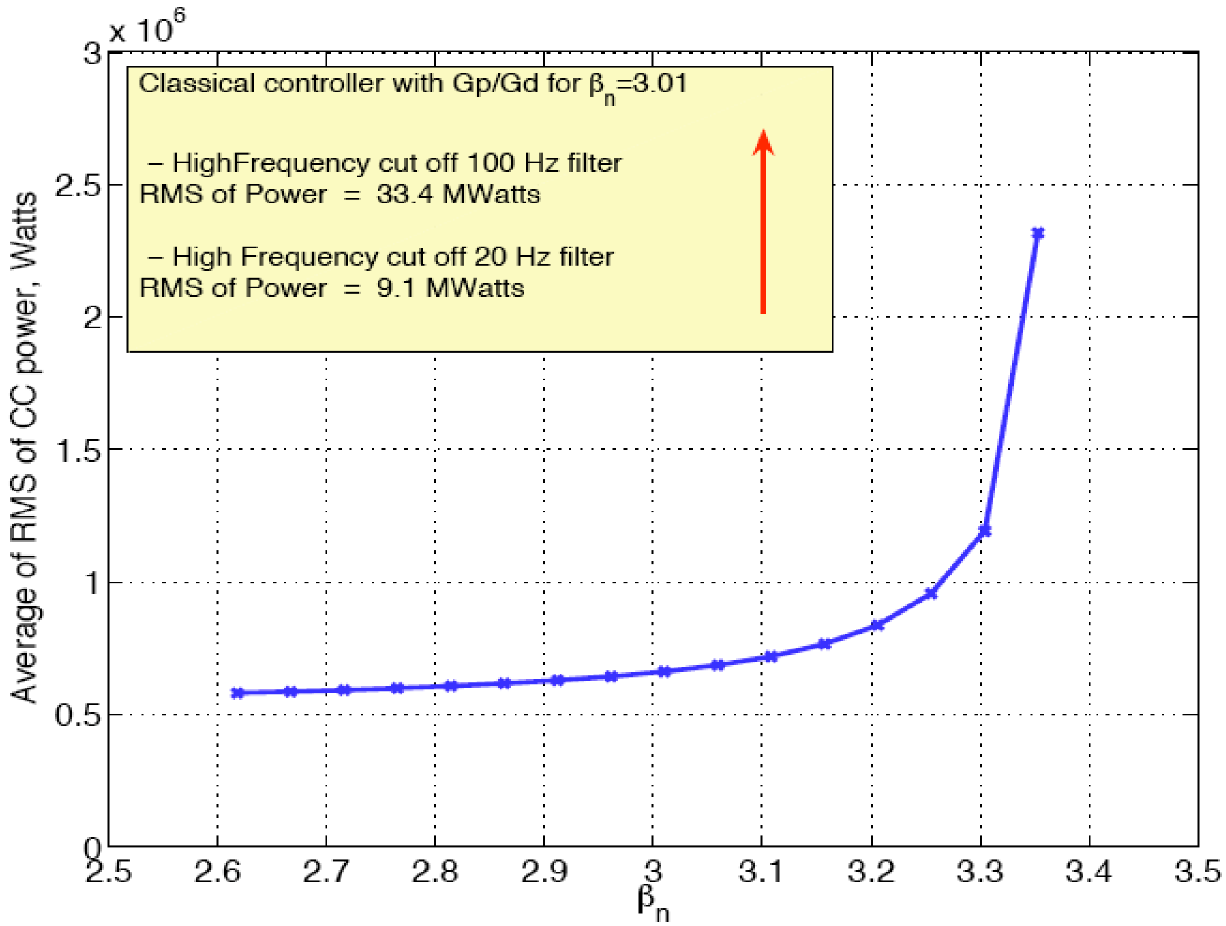


New Result: Optimal Controller and Observer stabilize RWM up to $\mathbf{C_\beta = 86\%}$



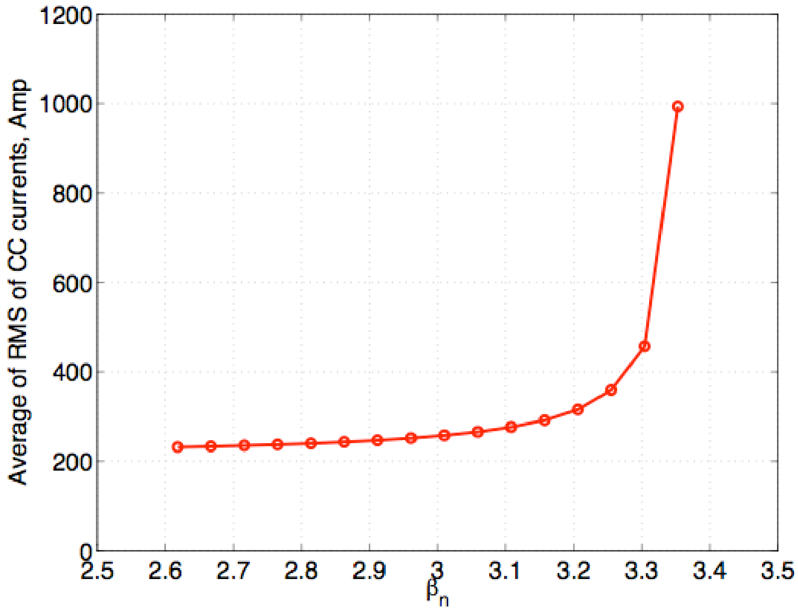
- In the presence of 10 Gauss sensor noise RWM was stabilized up to $\mathbf{C_\beta = 86\%}$, as compared to $\mathbf{C_\beta = 68\%}$ reached by classical PD controller.
- Optimal Controller and Observer **is Robust** for all $\mathbf{C_\beta < 86\%}$.

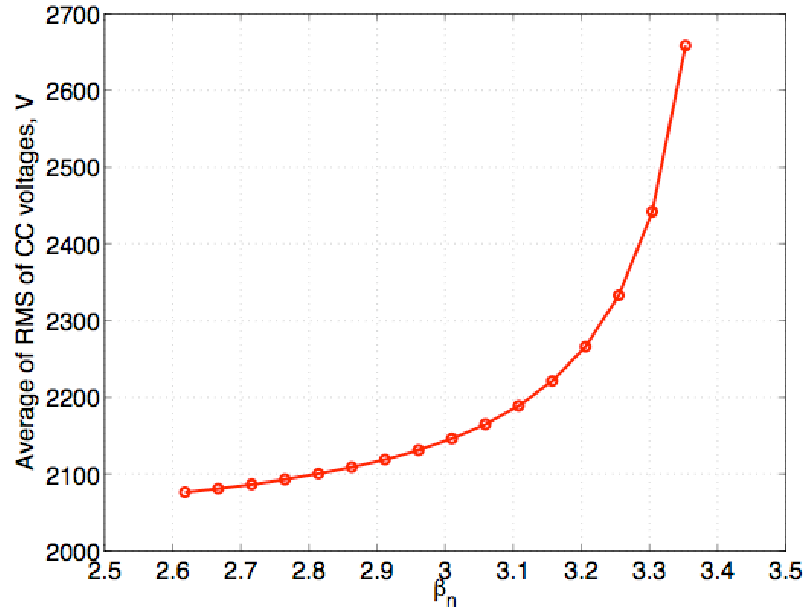
Results: Improved Stabilization of ITER RWM with New Optimal Controller and Observer



- **No derivative Gain** is needed with Optimal Controller and Observer
- **Significantly reduction in power requirements** for Optimal Controller and Observer.

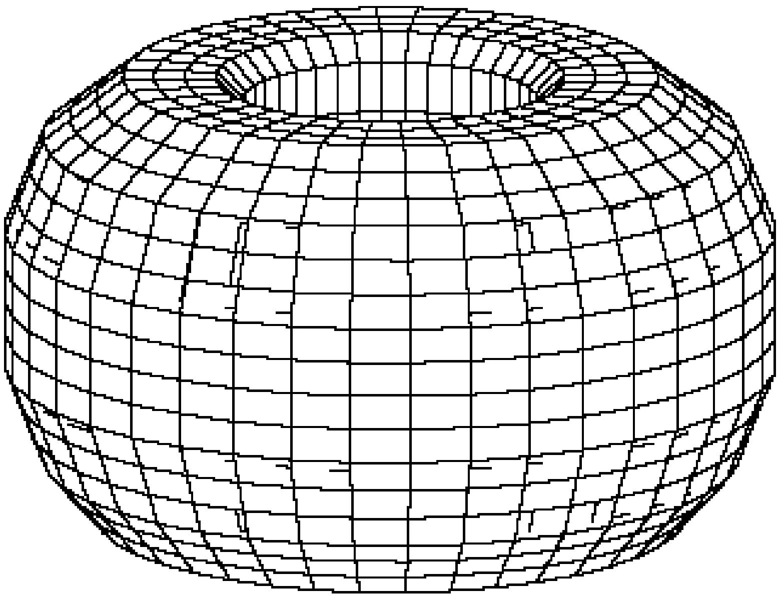
RMS values of Control Coils Current and Applied Voltages for ITER RWM with New Optimal Controller and Observer.

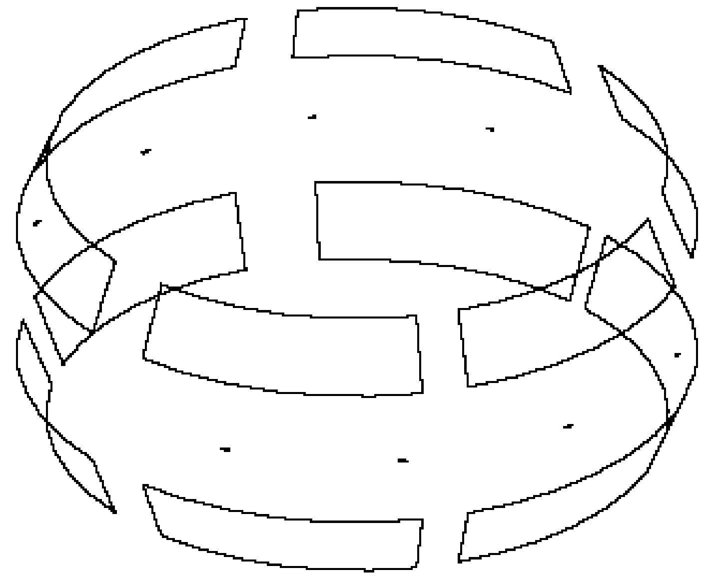




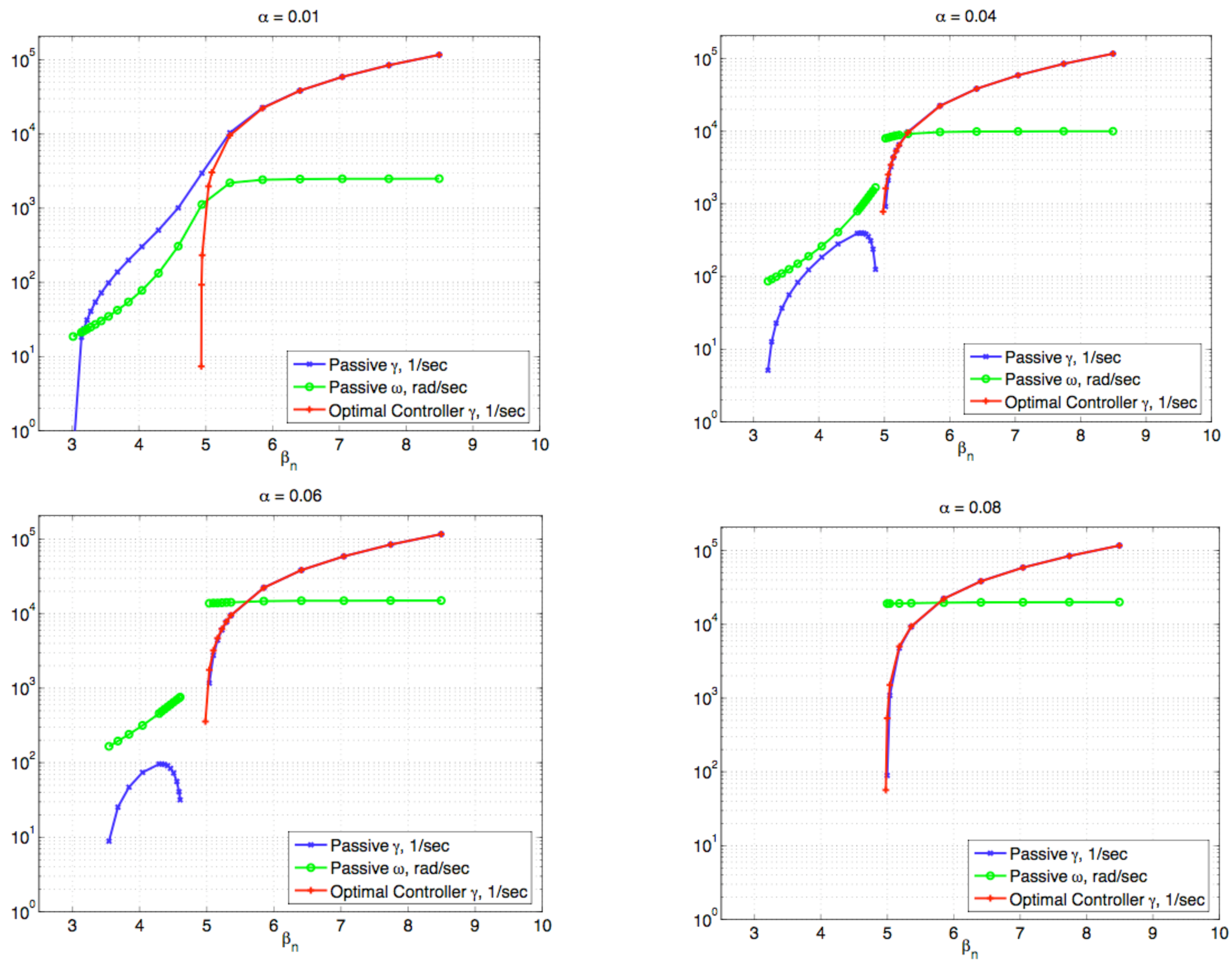
Solution of continuous algebraic Lyapunov equation for closed loop system provides means to estimate steady-state RMS values of control coil currents and voltages.

Results for DIII-D using Internal Control Coils.





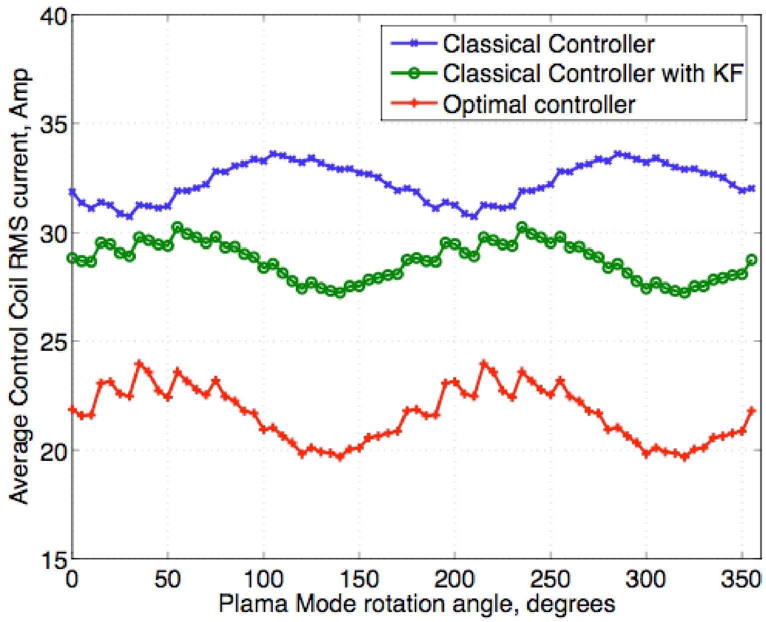
DIII-D optimal controller and observer is robust and stabilize RWM up to ideal wall limit for all torques - α.

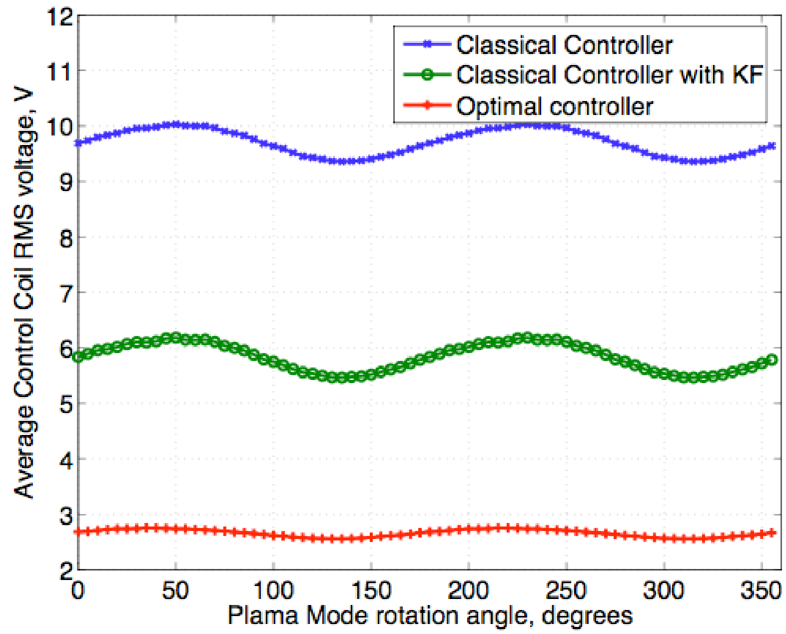


C_β=84%

Optimal Controller and Observer provides better reduction of current and voltages compared with classical controller with or without Kalman Filter.

C_β=84%





RMS values are calculated for fixed plasma rotated toroidally as solution of Lyapunov equation for closed loop continuous system with measurement noise.

Conclusions

- Optimal Controller and Observer designed based on reduced order VALEN model of ITER stabilizes RWM for higher β_n values than classical controller with proportional and derivative gains
- It also requires significantly less power to operate in steady-state regime
- The Optimal Controller and Observer designed for DIII-D with I-coils and plasma rotation stabilizes upto ideal wall limit as classical controller.
- Advantage of using new controller for DIII-D with I-coils is in significant reduction of control coil voltages and currents. Using Kalman filter and classical controller with proportional gain does not guarantee reduction of control coil currents.
- In both cases controller and observer have constant coefficients and robust.

What is an Optimal Controller and Observer, based on reduced VALEN state-space?

THING
THING
What is that thing?

VALEN Circuit Equations

After including plasma stability effects the fluxes at the wall, plasma, and feedback coils are given by (J.Bialek, et al., PoP 2001)

$$\vec{\Phi}_w = \overleftrightarrow{\mathcal{L}}_{ww} \cdot \vec{I}_w + \overleftrightarrow{\mathcal{L}}_{wf} \cdot \vec{I}_f + \overleftrightarrow{\mathcal{L}}_{wp} \cdot I_d$$

$$\vec{\Phi}_f = \overleftrightarrow{\mathcal{L}}_{fw} \cdot \vec{I}_w + \overleftrightarrow{\mathcal{L}}_{ff} \cdot \vec{I}_f + \overleftrightarrow{\mathcal{L}}_{fp} \cdot I_d$$

$$\Phi = \overleftrightarrow{\mathcal{L}}_{pw} \cdot \vec{I}_w + \overleftrightarrow{\mathcal{L}}_{pf} \cdot \vec{I}_f + \overleftrightarrow{\mathcal{L}}_{pp} \cdot I_d$$

Using Faraday and Ohms law yields equations for system evolution

$$\begin{pmatrix} \overleftrightarrow{\mathcal{L}}_{ww} & \overleftrightarrow{\mathcal{L}}_{wf} & \overleftrightarrow{\mathcal{L}}_{wp} \\ \overleftrightarrow{\mathcal{L}}_{fw} & \overleftrightarrow{\mathcal{L}}_{ff} & \overleftrightarrow{\mathcal{L}}_{fp} \\ \overleftrightarrow{\mathcal{L}}_{pw} & \overleftrightarrow{\mathcal{L}}_{pf} & \overleftrightarrow{\mathcal{L}}_{pp} \end{pmatrix} \cdot \frac{d}{dt} \begin{pmatrix} \vec{I}_w \\ \vec{I}_f \\ I_d \end{pmatrix} = \begin{pmatrix} \overleftrightarrow{R}_w & 0 & 0 \\ 0 & \overleftrightarrow{R}_f & 0 \\ 0 & 0 & \overleftrightarrow{R}_d \end{pmatrix} \cdot \begin{pmatrix} \vec{I}_w \\ \vec{I}_f \\ I_d \end{pmatrix} + \begin{pmatrix} \vec{0} \\ \vec{V}_f \\ 0 \end{pmatrix}$$

Rotation in VALEN State-Space.(A.Boozer PoP 1999).

Single mode equations:

$$\vec{\Phi}_w = \overleftrightarrow{L}_{ww}\vec{I}_w + \overleftrightarrow{L}_{wf}\vec{I}_f + \overleftrightarrow{L}_{wp}\vec{I}_d + \overleftrightarrow{L}_{wp}\vec{I}_p$$

$$\vec{\Phi}_f = \overleftrightarrow{L}_{fw}\vec{I}_w + \overleftrightarrow{L}_{ff}\vec{I}_f + \overleftrightarrow{L}_{fp}\vec{I}_d + \overleftrightarrow{L}_{fp}\vec{I}_p$$

$$\vec{\Phi}_p = \overleftrightarrow{L}_{pw}\vec{I}_w + \overleftrightarrow{L}_{pf}\vec{I}_f + \overleftrightarrow{L}_{pp}\vec{I}_d + \overleftrightarrow{L}_{pp}\vec{I}_p$$

where

$$\overleftrightarrow{L}_{pp}\vec{I}_p = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} s & \alpha \\ -\alpha & s \end{pmatrix}\right]\vec{\Phi}_p \quad \text{with} \quad s = \frac{-\delta W}{L_B I_B^2/2} \quad \text{and} \quad \alpha = \frac{torque}{L_B I_B^2/2}$$

plasma properties summarized by normalized mode energy ‘s’, and normalized mode torque ‘α’. Two copies of the same mode are used, the toroidal displacement of these two modes is π/2 (for n=1).

$\vec{I}_p$ can be eliminated from the equations and hence effective inductance matrixes are calculated as function of parameters s and α.

VALEN State Space and classical P&D controller

In the state-space form:

$$\dot{\vec{x}} = \overleftrightarrow{A}\vec{x} + \overleftrightarrow{B}\vec{u}$$

where

$$\vec{x} = \left(\vec{I}_w \;\; \vec{I}_f \;\; I_d\right)^T; \quad \overleftrightarrow{A} = \overleftrightarrow{\mathcal{L}}^{-1} \cdot \overleftrightarrow{R}; \quad \overleftrightarrow{B} = \overleftrightarrow{\mathcal{L}}^{-1} \cdot \overleftrightarrow{I}_{cc}; \quad \vec{u} = \vec{V}_f$$

with $\overleftrightarrow{I}_{cc}$ - control coil position matrix (1's at the coil location and zeros everywhere else)

classical control law with proportional and derivative gain defined as

$$\vec{u} = -\overleftrightarrow{G}_p \vec{y} - \overleftrightarrow{G}_d \dot{\vec{y}}$$

Where measurements are sensor fluxes and given by $\vec{y} = \overleftrightarrow{C} \cdot \vec{x}$

$\overleftrightarrow{G}_p$ and $\overleftrightarrow{G}_d$ are proportional and derivative gains calculated to insure stability of closed loop equations:

$$\dot{\vec{x}} = \left[\left(\overleftrightarrow{I} + \overleftrightarrow{B} \cdot \overleftrightarrow{G}_d \cdot \overleftrightarrow{C}\right)^{-1} \left(\overleftrightarrow{A} - \overleftrightarrow{B} \cdot \overleftrightarrow{G}_p \cdot \overleftrightarrow{C}\right)\right] \cdot \vec{x}$$

VALEN State Space and Modern Control Theory

Plus: Modern Control Theory uses state-space, not transfer function. VALEN equations have a natural built in state space structure.

Minus: High dimensions in VALEN models! Varies from 1000 upto 5000 equations

Solution: Reduction of state space, using balanced realization method and matched DC gain truncation.

Steps for calculating VALEN Reduced Order Model*

- Unstable mode is isolated from the system
- Balanced realization and Hankel Singlur Values (HSV) are computed for the stable portion of the system.
- Unstable mode has infinite HSV and added back to the balanced system as the first mode.
- Small HSVs correspond to states that can be removed to simplify the model, while retaining the most important input-output characteristics of the original stable system.
- Model reduction is performed by matching DC gains (steady-state response) between full order and reduced system.

*M.M.M. Al-Husari and et. al., IEEE Conference on Decision and Control, 1165-1170 (1991)
D.J.N. Limebeer and et. al., "Recent advances in tokamak modelling" IEEE Symposium on Industrial Electronics, Durban, South Africa, 21-27 (1998)

Controllability and Observability Grammians for Linear Stable and Time-Invariant Systems*

- Continuous-time state-space model:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

- Controllability Grammian: $$\Gamma_c = \int_0^\infty e^{A\tau} BB^T e^{A^T\tau} d\tau$$

  is calculated by solving continuous-time Lyapunov equation $$A\Gamma_c + \Gamma_c A^T + BB^T = 0$$

- Observability Grammian: $$\Gamma_o = \int_0^\infty e^{A^T\tau} C^T C e^{A\tau} d\tau$$

  is calculated by solving continuous-time Lyapunov equation $$A^T\Gamma_o + \Gamma_o A + C^T C = 0$$

*B.Friedland, "Control System Design: An Introduction to State-Space Methods," Dover 3rd Ed, 2005.

Balanced Realization Exists for Every Controllable Observable Stable System*

- Controllable, observable and stable system called a **balanced** system if

$$\Gamma_c = \Gamma_o = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_n \end{pmatrix}, \quad \sigma_i > \sigma_j \ \text{ for } i > j$$

σ_i - **Hankel singular values**

- The balanced transformation can be defined in two steps: $(A,B,C) \rightarrow (TAT^{-1}, TB, CT^{-1})$

1) Start with SVD of controllability grammian and define the first transformation as

$$\Gamma_c = VS_cV^T$$
$$T_1 = VS_c^{1/2}$$

2) Perform SVD of observability grammian in the new basis:

$$\tilde{\Gamma}_o = T_1^T \Gamma_o T_1 = US_oU^T$$

the second transformation defined as

$$T_2 = US_o^{-1/4}$$

The final transformation matrix is given by:

$$T = T_1T_2 = VS_c^{1/2}US_o^{-1/4}$$

*B.C.Moore, IEEE Trans. On Automatic Control, Vol AC-26, No 1, 17-32, Feb (1981).

Model Order Reduction of Combined Balanced System and Unstable Mode by Method of Matched DC Gains

Continuous time model is

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}$$

$$\vec{y} = C\vec{x} + D\vec{u}$$

State partition

$$\vec{x} = \left(\vec{x}_1, \vec{x}_2\right)^T$$

$$\begin{pmatrix} \dot{\vec{x}}_1 \\ \dot{\vec{x}}_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \vec{x}_1 \\ \vec{x}_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \vec{u}$$

$$\vec{y} = \begin{pmatrix} C_1 & C_2 \end{pmatrix} \begin{pmatrix} \vec{x}_1 \\ \vec{x}_2 \end{pmatrix} + D\vec{u}$$

Set $\frac{d}{dt}\vec{x}_2 = 0$ and eliminate $\vec{x}_2$ from the system:

$$\dot{\vec{x}}_1 = \left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)\vec{x}_1 + \left(B_1 - A_{12}A_{22}^{-1}B_2\right)\vec{u} = A_r\vec{x}_1 + B_r\vec{u}$$

$$\vec{y} = \left(C_1 - C_2A_{22}^{-1}A_{21}\right)\vec{x}_1 + \left(D - C_2A_{22}^{-1}B_2\right)\vec{u} = C_r\vec{x}_1 + D_r\vec{u}$$

Next Steps are

- **Design an Optimal Controller and Observer based on VALEN Reduced Order Model:** Steady-state optimal controller and observer gains are calculated for system containing one unstable mode and reduced stable part.

- **Verify it's robustness for the full order model:** Robustness of controller and observer designed for fixed plasma strength parameter **s** are verified for different β_n, by calculation of eigen values of closed loop system.

Optimal Controller Based on Reduced Order Model*

Minimize Performance Index:

$$J = \int_t^T \left(\hat{\vec{x}}'(\tau) Q_r(\tau) \hat{\vec{x}}(\tau) + \vec{u}'(\tau) R_r(\tau) \vec{u}(\tau) \right) d\tau$$

Q_r - state weighting matrix,

R_r - control weighting matrix.

Controller gain for the steady-state can be calculated as $K_c = R^{-1} B_r^T S$, where S is solution of the controller Riccati equation

$$SA_r + A_r^T S - SB_r R_r^{-1} B_r^T S + Q_r = 0$$

Optimal Observer - Kalman Filter* modeled using Reduced VALEN State-Space

Observer equation is

$$\dot{\hat{x}} = A_r \hat{x} + B_r u + K_f \left(y - \hat{y} \right)$$

$$\hat{y} = C_r \hat{x} + D_r u$$

where $K_f = PC'W^{-1}$ is Kalman Filter gain

and P is solution of observer Riccati equation

$$A_r P + P A_r^T - P C_r^T W^{-1} C_r P + V^T = 0$$

V, W plant and measurement noise covariance matrix.

Closed System Equations with Optimal Controller and Kalman Filter based on Reduced Order Model

Full Order Plant Model:	$x = Ax + Bu$ $y = Cx + \omega$
Reduced Order Observer:	$\hat{x} = A_r\hat{x} + B_r u + K_f(y - \hat{y})$ $\hat{y} = C_r\hat{x} + D_r u$
Optimal Control Law:	$u = -K_c\hat{x}$

Closed system equations

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK_c C_r \\ K_f C & F \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} + \begin{pmatrix} 0 \\ K_f \end{pmatrix} \omega$$

$$F = A_r - K_f C_r - (B_r - K_f D_r) K_c C_r$$

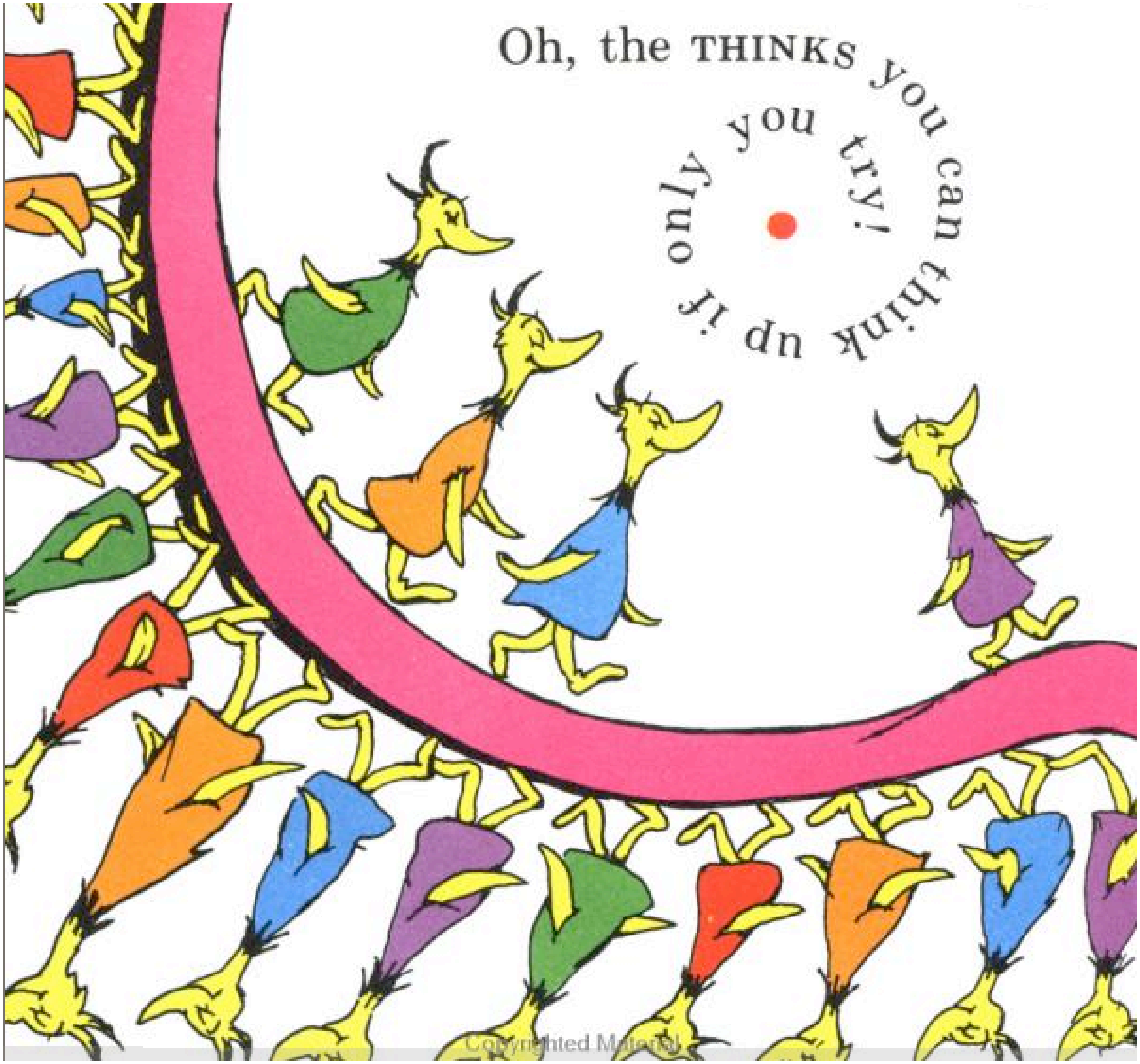
$$eig(\tilde{A}) = eig\begin{pmatrix} A & -BK_c C_r \\ K_f C & F \end{pmatrix}$$

- stability and robustness test

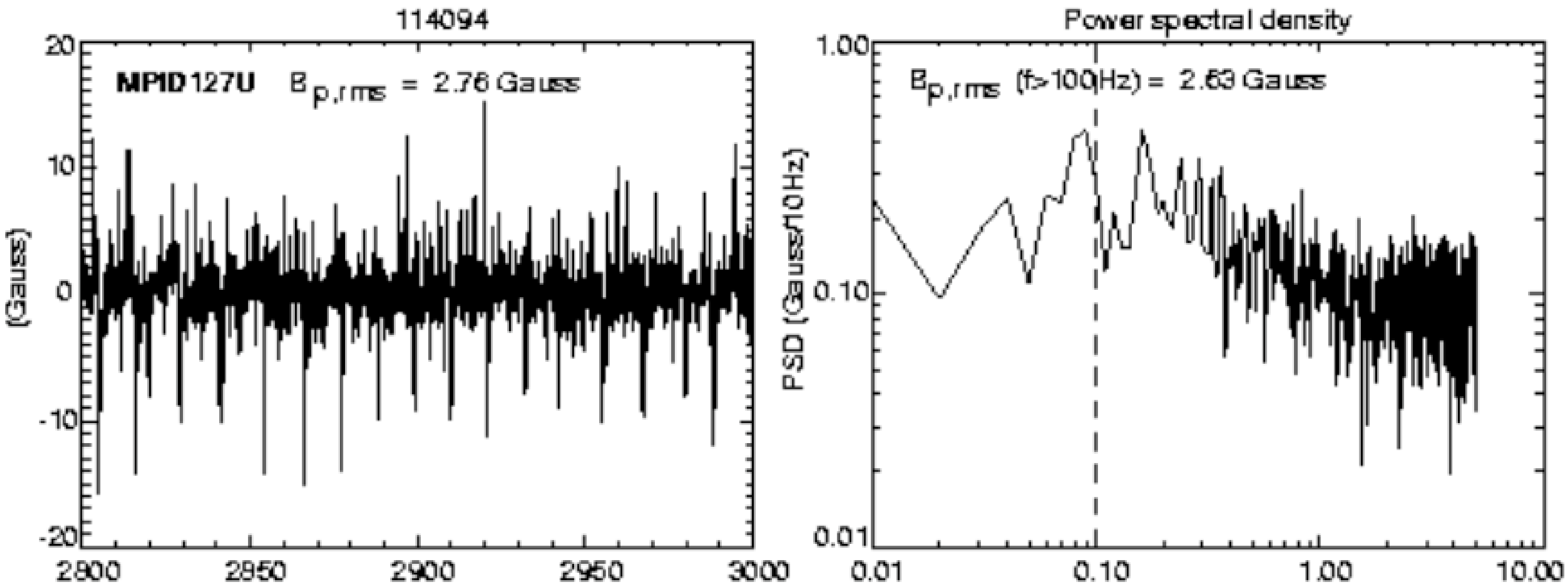
$$\tilde{A} x_{RMS}^2 + x_{RMS}^2 \tilde{A}^T + \tilde{D} W \tilde{D}^T = 0$$

$$\tilde{D}^T = \begin{pmatrix} 0 & K_f \end{pmatrix}$$

- RMS values of state (currents) estimated from Lyapunov equation

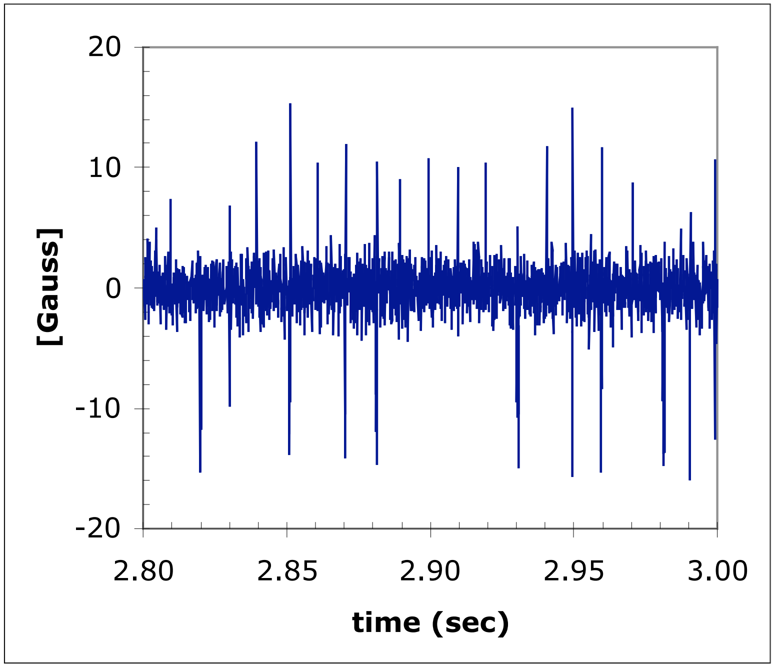

Oh, the THINKS you can think up if only you try!

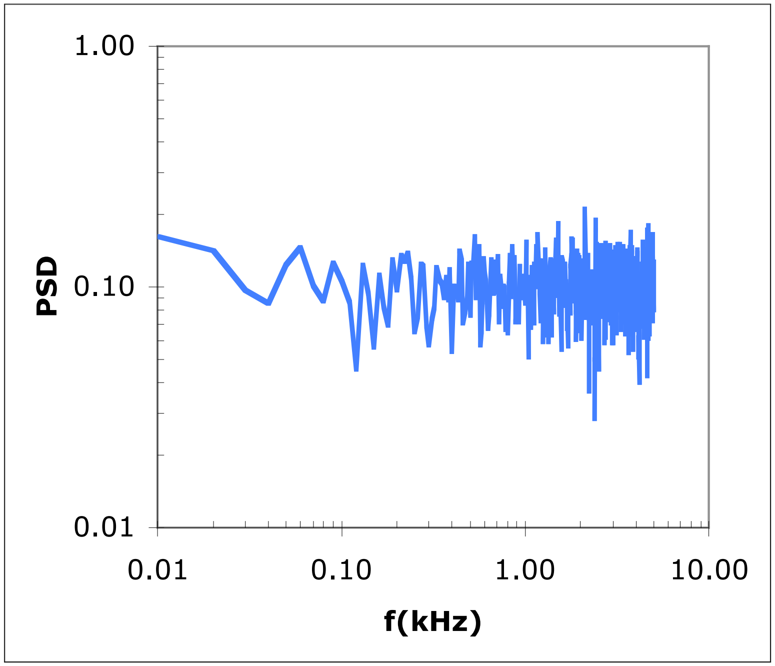
RWM Noise Data on DIII-D



Noise on the poloidal field sensors in the midplane. The signals are corrected for DC offsets. The power spectral density is shown as root-mean square amplitude per 10Hz frequency bin.

Feedback Power Determined by Noise on DIII-D Poloidal Sensors: Broadband and ELMs

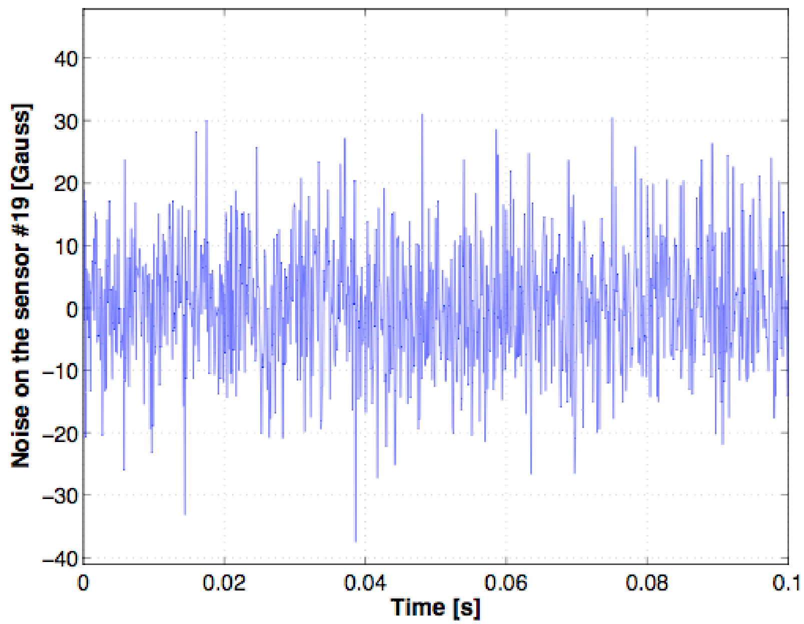


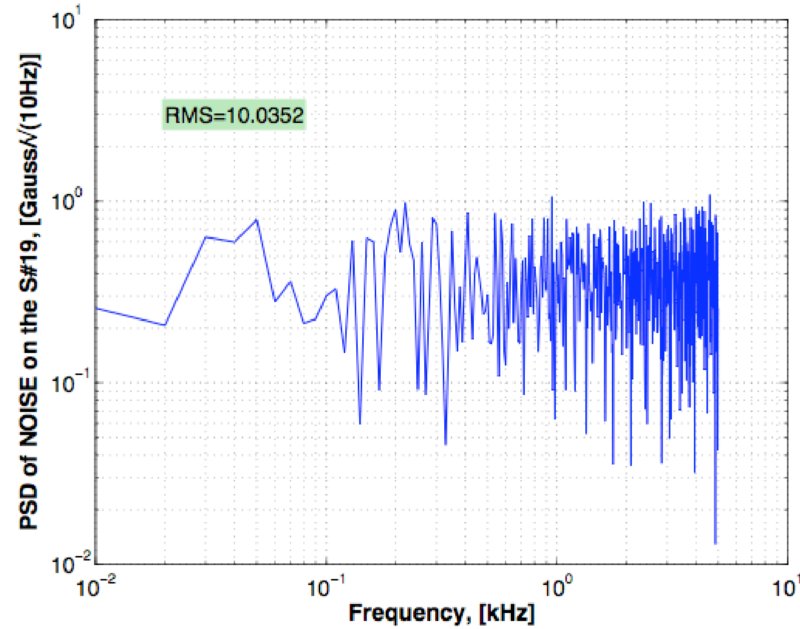


Broadband noise was modeled as Gaussian random number with standard deviation 1.5 G about 0 mean and frequency 10kHz.

To the broadband noise ELMs (Edge Localized Modes) were added as additional Gaussian random distribution from 6 G to 16 G approximately every 10 msec with +/- chosen with 50% probability and ElMs duration of 200 μsec.

Modeling of Noise on ITER RWM Control



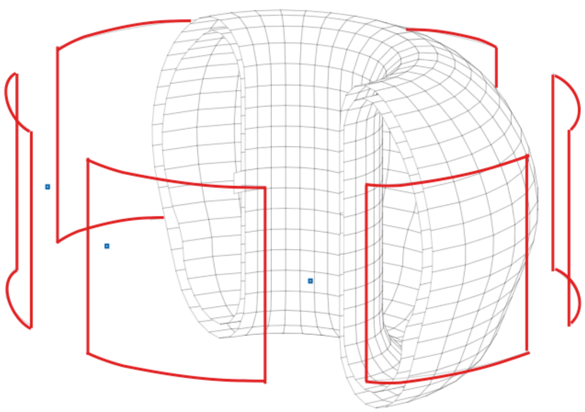


- Use DIII-D observed noise in RWM feedback loop as basis for extrapolation to ITER with $\delta B_{noise} \sim I_p$.

  Broadband noise was modelled with amplitude of 10 gauss [about 7 times level in DIII-D - ratio of I_{ITER}/I_{DIII-D}].No ELMs in applied noise spectrum

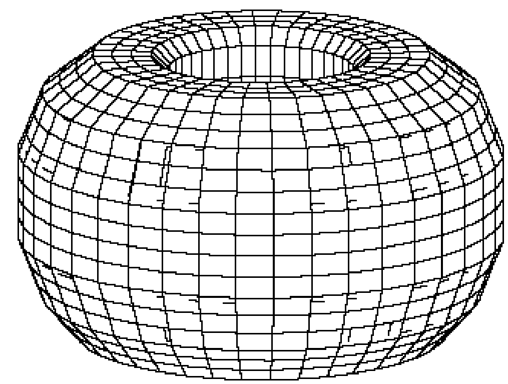
Parameters for Optimal Controller and Observer designed for ITER.

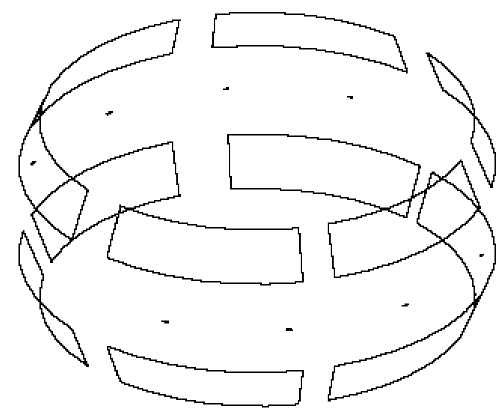
- ITER VALEN model without blankets 3 control coil pairs and 6 sensors located inside vacuum vessel for **$C_\beta = 50\%$** and **$C_\beta = 85\%$**. was reduced to **20 elements**. Number 20 was chosen by trial and error method to ensure stabilization of controller when applied to the full order model.
- Control weighting matrix **R** was chosen to be unity matrix and state weighting matrix $\mathbf{Q=(T_r^{-1})^T Q_0 T_r^{-1}}$. Where $\mathbf{T_r^{-1}}$ reduced matrix of balanced transformation and $\mathbf{Q_0}$ zero **N x N** matrix except diagonal elements that corresponded to control coil currents, that were set to be **1**.
- Continuous measurement noise covariance **W = 1e-18**, that corresponds discrete white noise spectrum of 10 Gauss at 10kHz and sensor area 1cm^2 ; Plant noise covariance was set to be $\mathbf{V = B_r G_p W G_p^T B_r^T}$ that would correspond to input noise of control coils.
- Note: **Q** and **V** , are two parameters that can be modified to improve performance of the Optimal Controller and Observer



Parameters for Optimal Controller and Observer designed for DIII-D (with rotation).

- DIII-D VALEN model 6 control coil pairs and 8 sensors located inside vacuum vessel for $\mathbf{C_\beta = 84\%}$ and $\boldsymbol{\alpha} = \mathbf{0.0001}$ was reduced to **25 elements**. Number 25 was chosen to provide robustness of controller for different plasma rotation angle.
- Control weighting matrix **R** and state weighting matrix **Q** were set to be unity matrix
- Continuous measurement noise covariance **W = 1.08e-11**, that corresponds discrete white noise spectrum of 1.5 Gauss at 10kHz and sensor area 7.2 cm^2 ; Plant noise covariance was set to be $\mathbf{V = B_r G_p W G_p^T B_r^T}$ that would correspond to input noise of control coils.
- Note: **Q** and **V** , are two "tuning" parameters that can be modified to improve performance of the Optimal Controller and Observer. Number of modes in the reduced system is also can be varied to achieve robustness and smallest RMS values.





Summary and Future Work

- Large scale VALEN model was successfully reduced using methods of Balanced Realization and matched DC gain truncation.
- Optimal Controller and Observer designed based on reduced order VALEN model not only stabilize RWM for higher β_n values than classical controllers but also require significantly less power to operate in steady-state regime.
- The next step would be to study Optimal Controller and Observer in real experiments like DIII-D
- Further ITER modeling including blanket modules with Optimal Controller and Observer.

References:

1. J. Bialek, et al., Physics of Plasmas, Vol. 8, No. 5, 2170 (2001).
2. A. H. Boozer, Physics of Plasmas, Vol. 5, No. 9, 3350, (1998)
3. A. H. Boozer, Physics of Plasmas, Vol. 6, No. 8, 3180, (1999)
4. A. K. Sen, et al., Physics of Plasmas, Vol. 10, No.11, 4350 (2003).
5. M.M.M. Al-Husari and et. al., IEEE Conference on Decision and Control, 1165-1170 (1991)
6. D.J.N. Limebeer and et. al., "Recent advances in tokamak modelling" IEEE Symposium on Industrial Electronics, Durban, South Africa, 21-27 (1998)
7. B.C.Moore, IEEE Trans. On Automatic Control, Vol AC-26, No 1, 17-32, Feb (1981).
8. Kalman, R. E. "A New Approach to Linear Filtering and Prediction Problems," Transactions of the ASME - Journal of Basic Engineering Vol. 82: pp. 35-45 (1960).
9. Kalman, R. E., Bucy R. S., "New Results in Linear Filtering and Prediction Theory", Transactions of the ASME - Journal of Basic Engineering Vol. 83: pp. 95-107 (1961).
10. B.Friedland, "Control System Design: An Introduction to State-Space Methods," Dover 3rd Ed, 2005.
11. A.P.Sage and C.C.White III, "Optimum Systems Control" , Prentice-Hall, Englewoods Cliffs, NJ, 1977
12. Dr Seuss, “Hop on Pop” and “Oh, the thinks you can think”, Beginner Books