Feedback stabilization of tearing modes in RFPs with a resistive wall ABOVE the ideal wall limit

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*J. M. Finn, "Control of magnetohydrodynamic modes with a resistive wall above the wall stabilization limit", *Phys. Plasmas* **13**, 082504 (2006).

Background

- Known: resistive wall modes can be unstable when PC wall stabilizes
- Earlier work: feedback: sensing $\widetilde{B}_r$ or $\widetilde{B}_\theta$ (normal or tangential)
- Tokamaks or RFPs – increasing or decreasing $q(r)$ profile
- New: we sense a **linear combination** of $\widetilde{B}_r$ and $\widetilde{B}_\theta$
- New: stability of *tearing modes* is possible **ABOVE** the wall stabilization regime. For *ideal* modes? No.
- Related to *virtual wall* of Bishop (1989)? For $Re(\omega) = 0$, sort of. Otherwise, no.
- Applications: stabilizing the $m = 0$ or the $m = 1$ modes in RFPs for single RFPs; possibly control the amplitude of neoclasical tearing modes in tokamaks.

RADIAL AND TANGENTIAL SENSING (REVIEW)

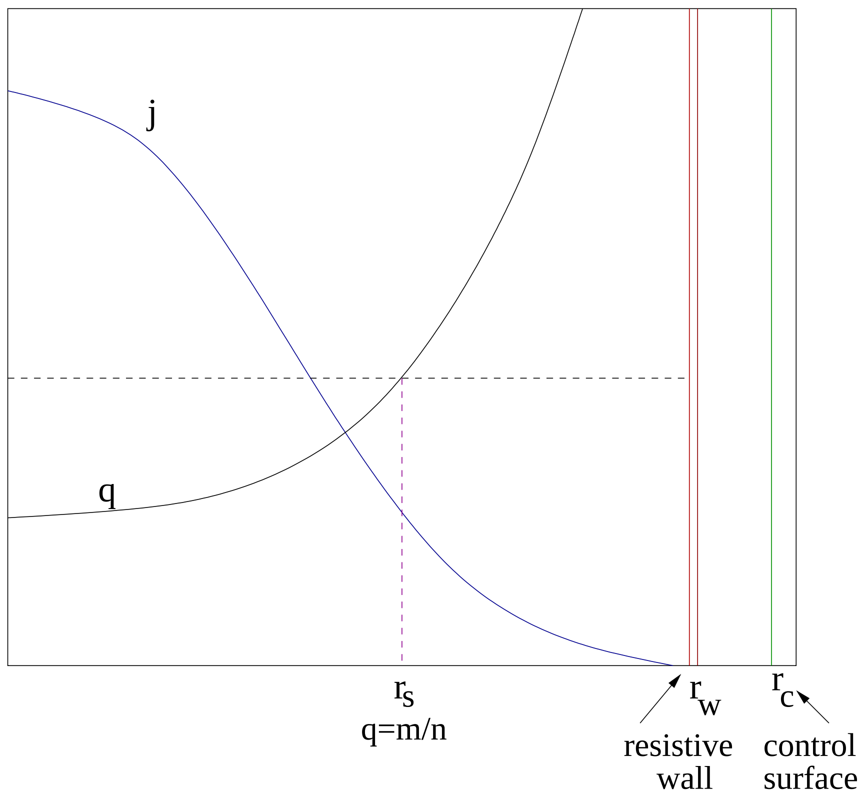
✔ Tangential ($\widetilde{B}_\theta$) sensing was found to work better than radial ($\widetilde{B}_r$) sensing [Y. Q. Liu, A. Bondeson et al., *Phys. Plasmas* **2000**; C. M. Fransson et al., *Phys. Plasmas* **2000**; A. Bondeson et al., *Nucl. Fusion* **2002**; M. S. Chu et al., *Nucl. Fusion* **2003**; M. S. Chu et al., *Phys. Plasmas* **2004**, J. M. Finn, *Phys. Plasmas* **2004**].

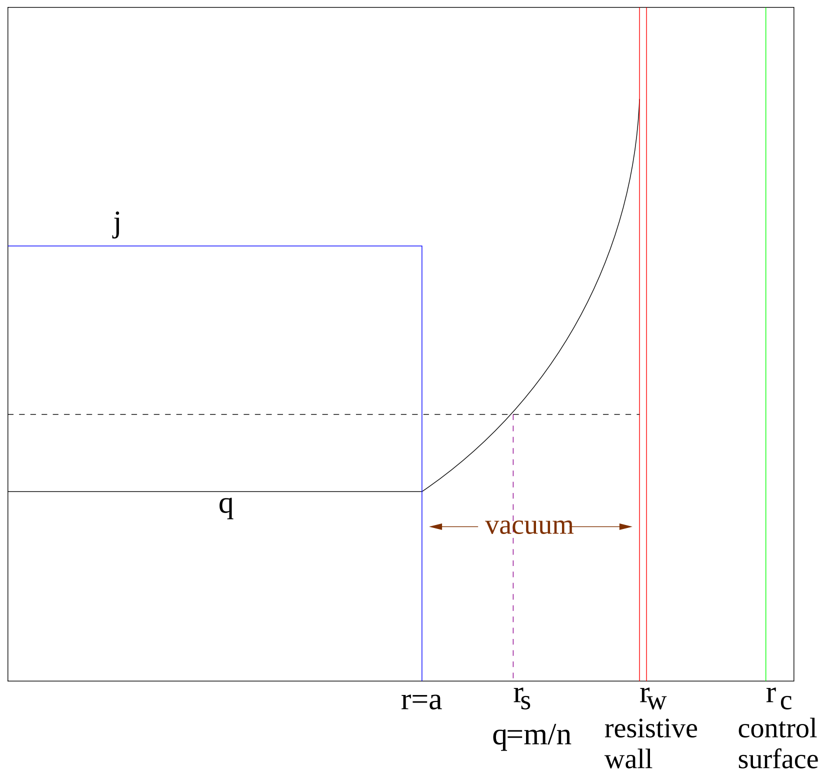
✔ Tangential appears to work better because it is less sensitive to $m \rightarrow m \pm 1$ coupling due to coils

✔ Works better in DIII-D too.

MODEL

- Cylindrical tokamak model, reduced ($R/a \gg 1$) resistive MHD equations, resistive wall (RW) at plasma edge, control flux (single m, n) applied at r_c





Resistive wall tearing mode – reduced MHD

$$\mathbf{B} = \nabla\psi \times \widehat{z} + B_0\widehat{z} \quad \mathbf{j} = j_z\widehat{z} = -\nabla_\perp^2\psi$$

$$\mathbf{B}\cdot\nabla\widetilde{j_z} + \widetilde{\mathbf{B}}\cdot\nabla j_z = \rho d\omega/dt \qquad \nabla_\perp^2\widetilde{\psi} - \frac{m}{r}\frac{j_z'}{\mathbf{k}\cdot\mathbf{B}}\widetilde{\psi} = 0 \quad vac.$$

Matching at $r = r_s$ ($q = m/n$) – visco-resistive *ct. psi* tearing matching condition:

$$[\psi']_{r_t} = \gamma\tau_t\psi(r_t) \quad \tau_t \sim \mu^{1/6}/\eta_p^{5/6}(k\cdot B)'^{1/3}$$

Matching at $r = r_w$ – resistive wall – thin wall (ct. psi) matching condition:

$$[\psi']_{r_w} = \gamma\tau_w\psi(r_w) \quad \tau_w \sim r_w\delta/\eta_w$$

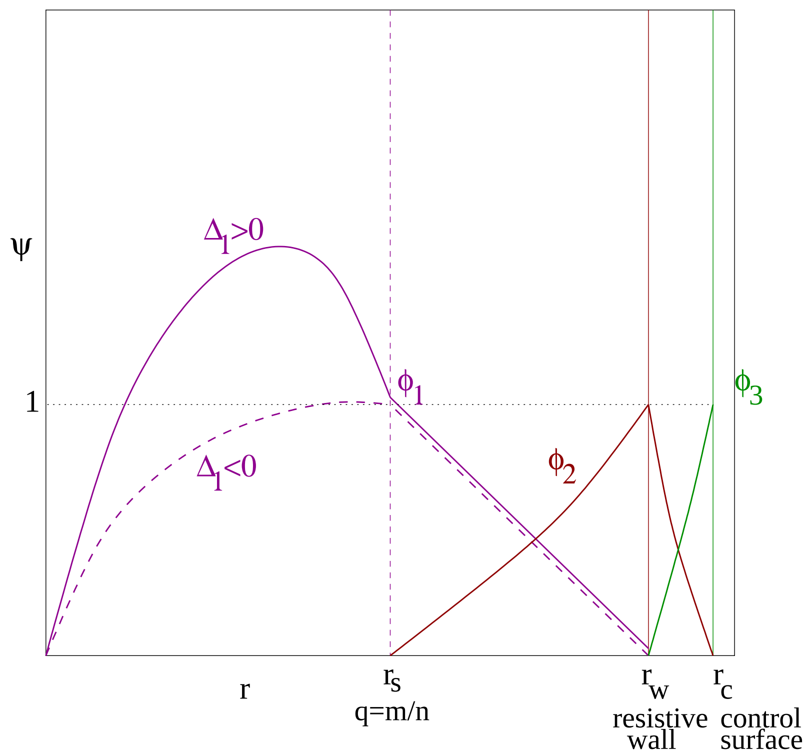
FEEDBACK MODEL

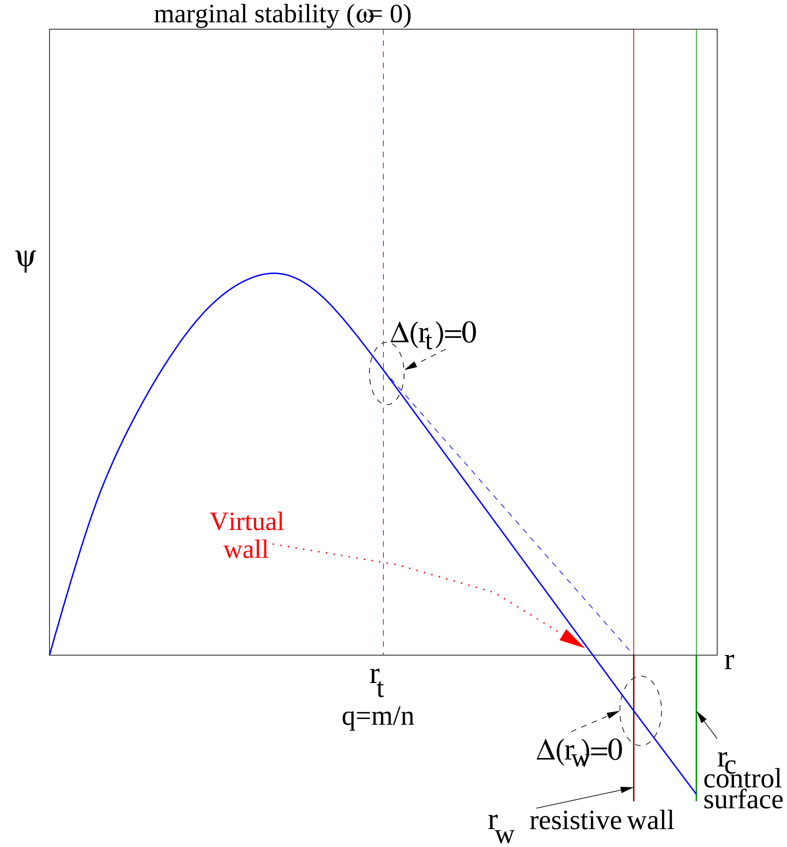
Feedback – linear combination of radial field $\psi(r_c) = -G\psi(r_w)$ and poloidal field $\psi(r_c) = K\psi'(r_w-)$ [$\widetilde{B}_r = im\psi/r$; $\widetilde{B}_\theta = -\partial\psi/\partial r$]

$$\psi(r_c) = -G\psi(r_w) + K\psi'(r_w-) \leftarrow - - -$$

$$\widetilde{B}_r(r_w) \propto \psi(r_w) \; \ldots \; \widetilde{B}_\theta(r_w-) \propto \psi'(r_w-)$$

RW TM (cont)





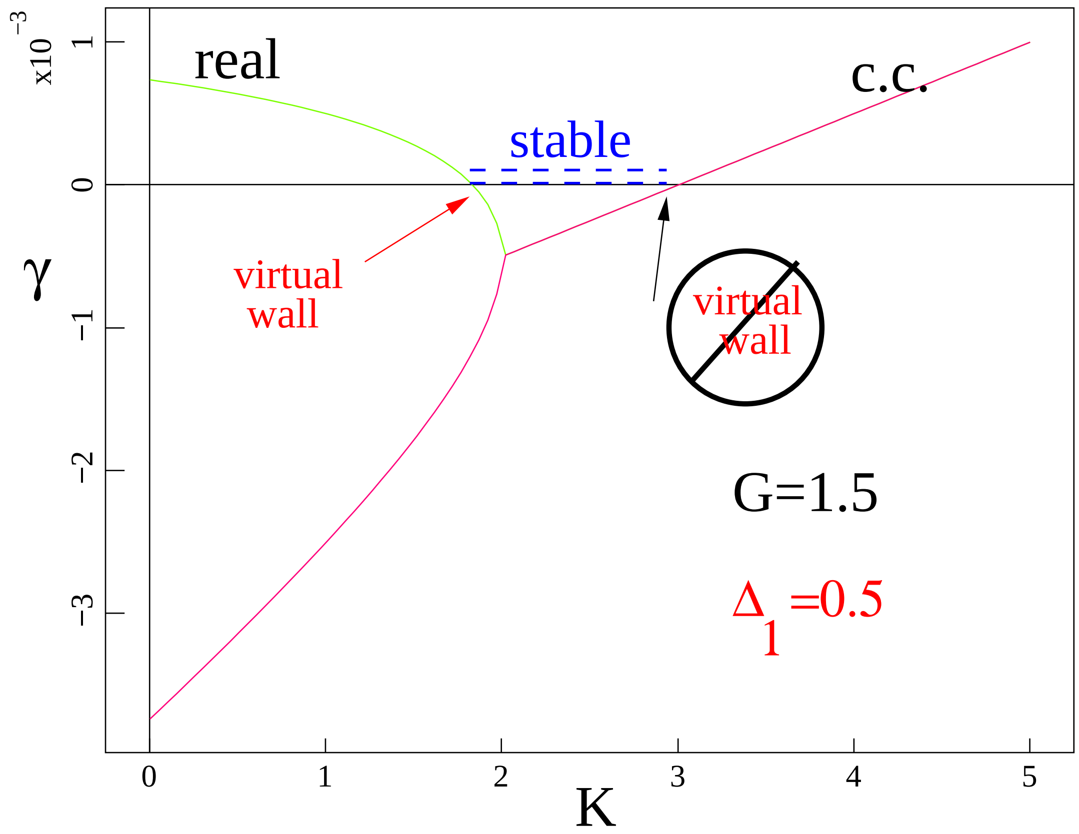
$\Delta_1 < 0 \quad \Longrightarrow$ TM is stable with PC (perfectly conducting) wall at r_w

$\Delta_1 > 0 \quad \Longrightarrow$ TM is unstable with PC wall at r_w

$$\begin{bmatrix} \Delta_1 - \gamma\tau_t & l_{12} & 0 \\ l_{21} & \Delta_2 - \gamma\tau_w & l_{23} \\ -Kl_{21} & -G + Kl_{22}^{(-)} & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = 0$$

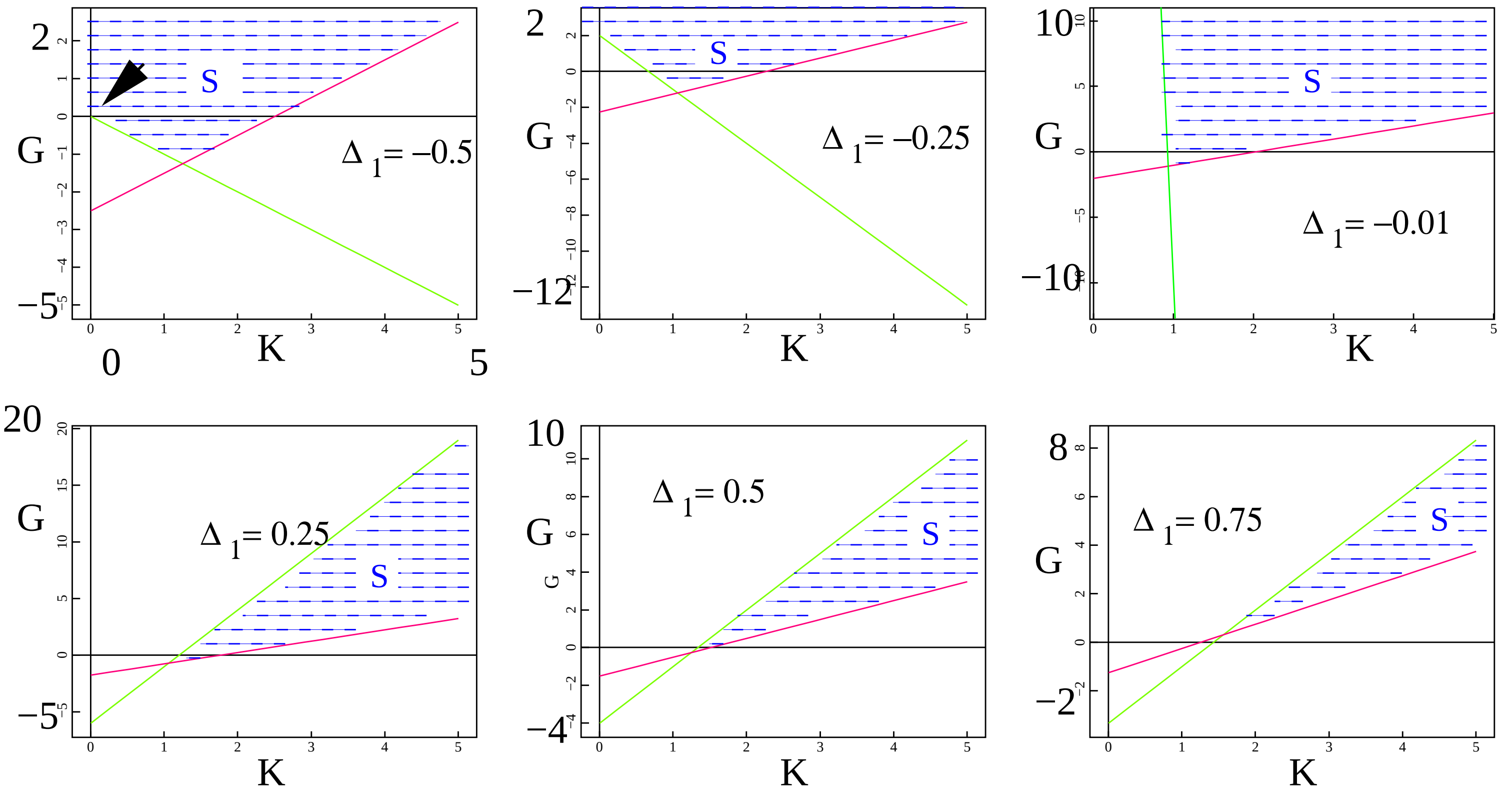
$$\begin{bmatrix} \frac{\Delta_1}{\tau_t} - \gamma & \frac{l_{12}}{\tau_t} \\ \frac{l_{21}(1-Kl_{23})}{\tau_w} & \frac{\Delta_2 - l_{23}(G - Kl_{22}^{(-)})}{\tau_w} - \gamma \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = 0$$

Stability conditions



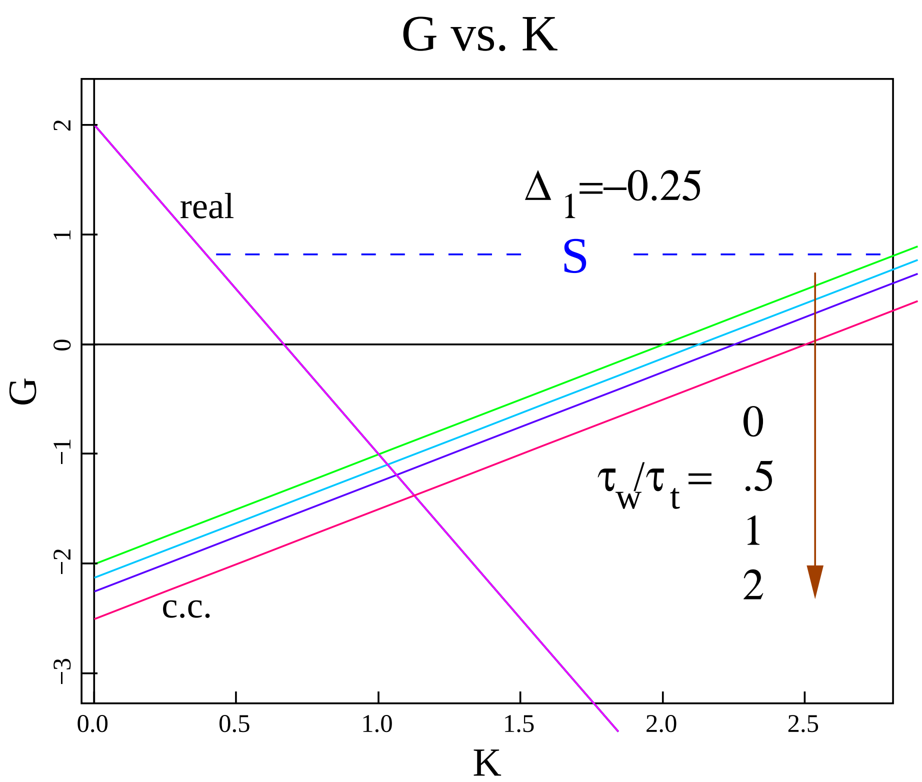
Stability regimes in G, K

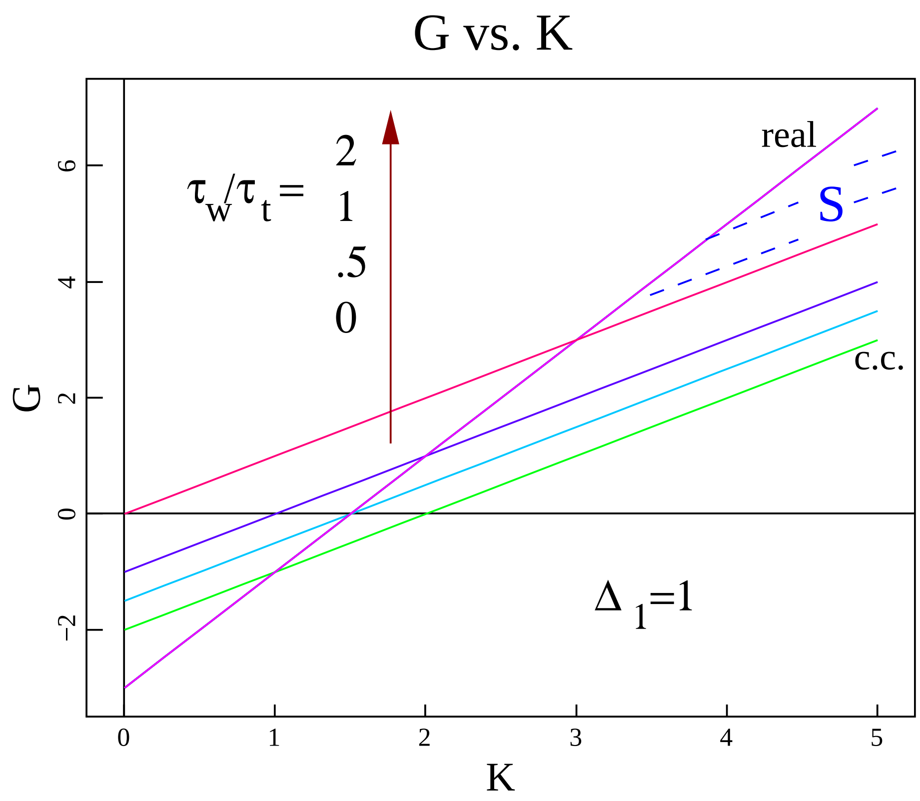
$\tau_w/\tau_t = 1$: Real marginal ... C.c. marginal ... Stable region



Always a stability window! Thinner, with large G, K, for large Δ_1. For Δ_1 large enough, G and K are both required.

The effect of τ_w/τ_t





The effect of τ_w/τ_t:

Making resistive wall thicker or less resistive (τ_w larger)

- τ_w/τ_t helps in the wall stabilization regime ($\Delta_1 < 0$). Slow penetration of flux from mode in the plasma slows mode.

- Harms above wall stabilization regime ($\Delta_1 > 0$). Prevents penetration of the feedback flux.

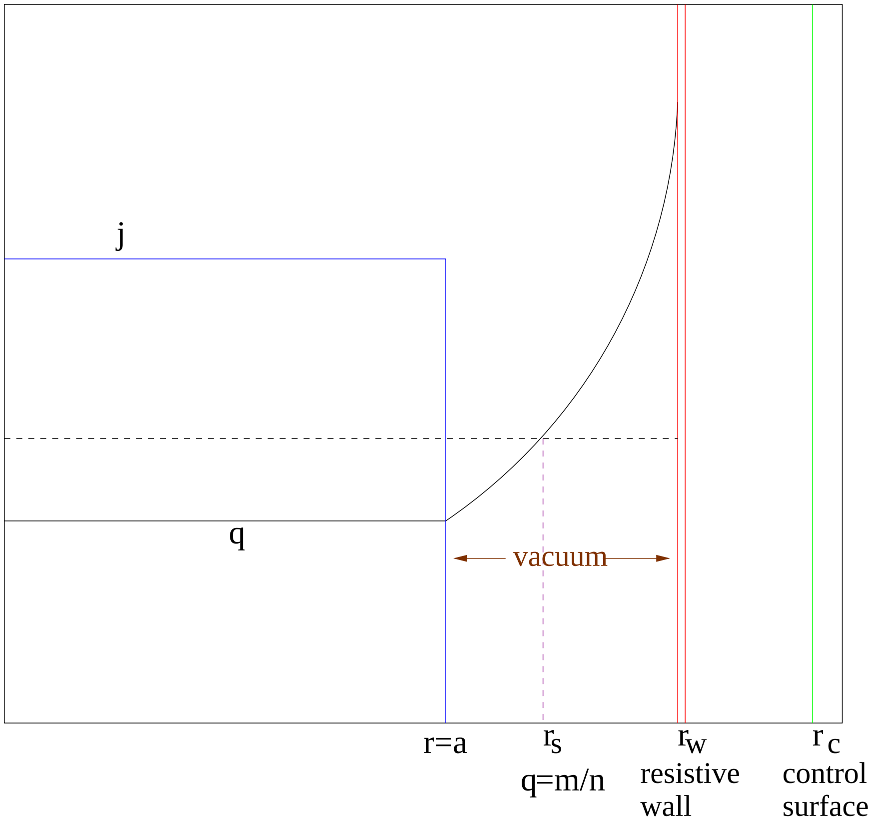
Feedback based on tangential field outside the resistive wall

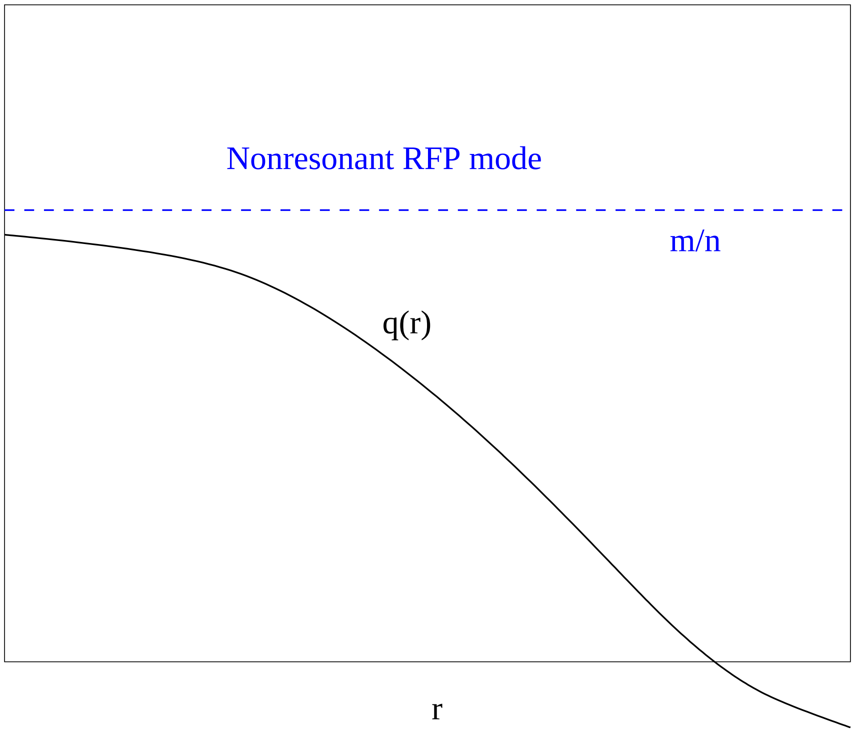
$$\psi(r_c) = -G\psi(r_w) + K\psi'(r_w+)$$

$$\Uparrow$$

No stabilization above $\Delta_1 = 0$, where tearing mode is wall stabilized.

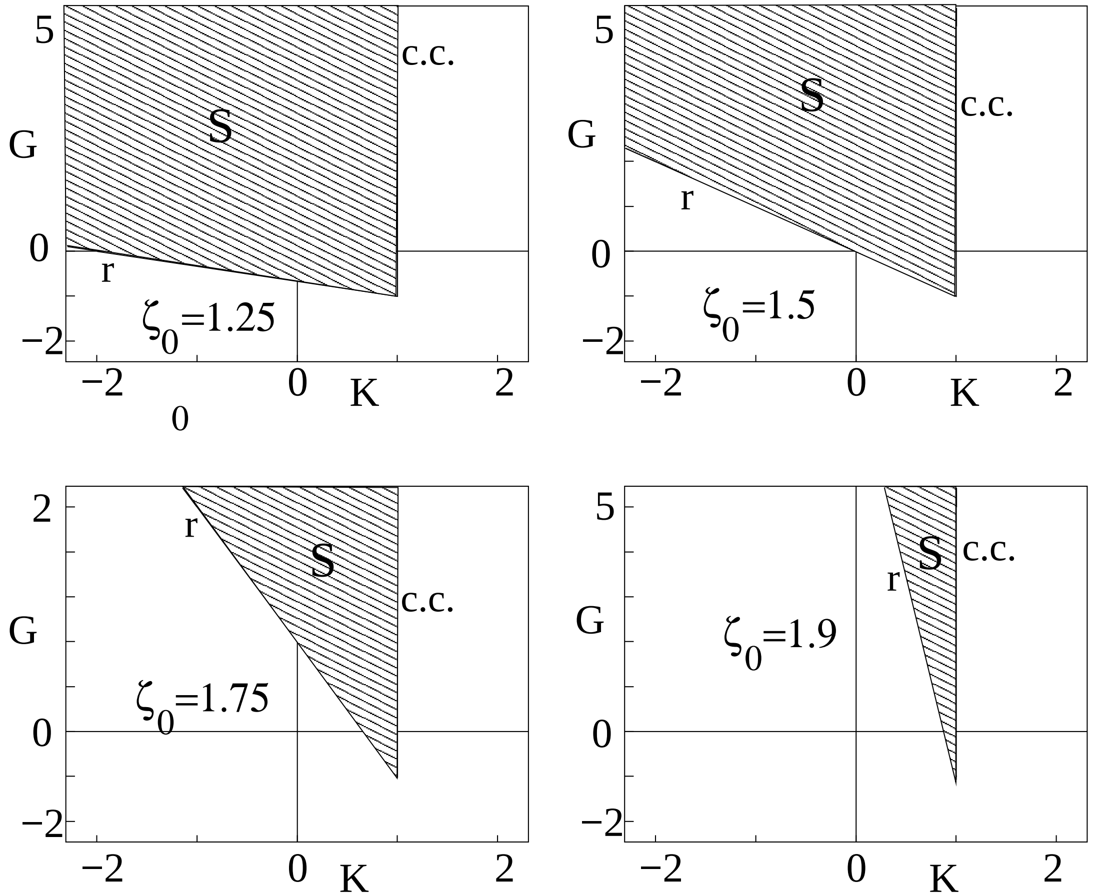
Ideal plasma nonresonant external modes – mode rational surface in vacuum. Non-resonant RFP modes





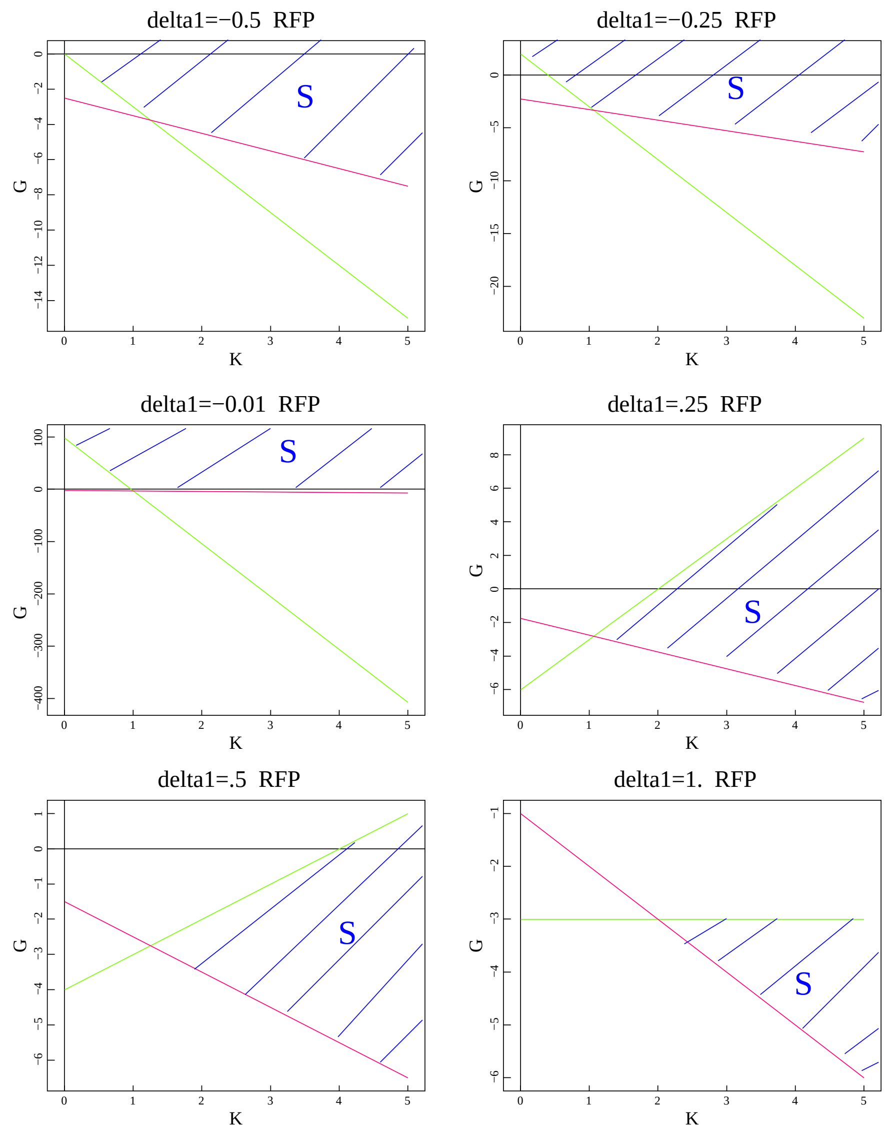
Wall stabilization: $1.5 < \zeta_0 < 2.0$

Ideal external kink (tokamak)



Tokamak-like geometry with mode rational surface in the vacuum. Unstable $\zeta_0 > 2$.

RFP-like q profiles – tearing



Finally, ideal plasma resonant modes (RFP)

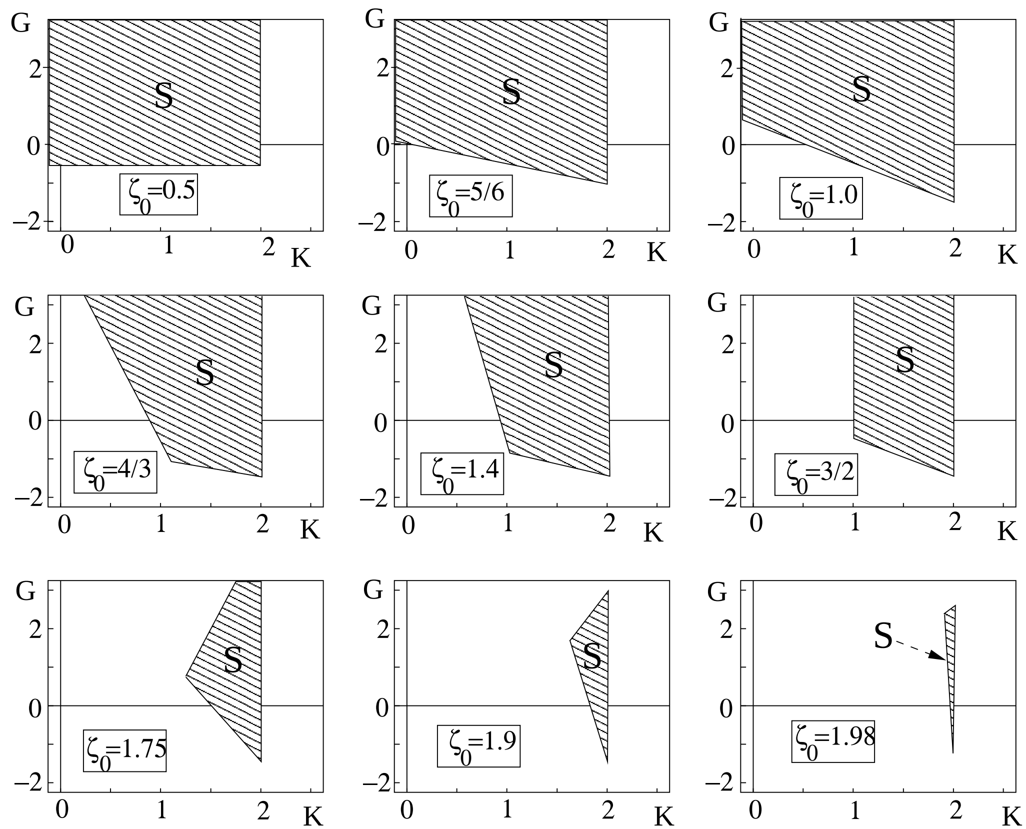
Plasma resistive layer at $r = r_t$

Jump in j or in p at $r = a$

Resistive wall at $r = r_w$

Control at $r = r_c$

$$\begin{bmatrix} \Delta_1 - \gamma\tau_t & l_{12} & 0 \\ l_{21} & \Delta_2 + \frac{\zeta_0}{1+\rho\gamma^2/F_a^2} & l_{23} \\ 0 & l_{32}(1 - l_{34}K) & \Delta_3 - \gamma\tau_w - l_{34}(G - l_{33}^{(-)}K) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = 0$$



Stable region of (K, G) space vanishes for $\zeta_0 > 2$ (ideal mode with PC wall). A stable region exists **BELOW** this.

Summary

- Tearing modes (tokamak or RFP): linear feedback sensing radial magnetic field and tangential field can stabilize the tearing mode below **and above** the threshold for stability with a PC wall. Some relation with virtual wall of Bishop ('89) when $Re(\omega) = 0$.

- Increasing τ_w/τ_t (making wall more conducting) increases the range (K, G) of stability for $\Delta_1 < 0$ ... **decreases(!)** the range for $\Delta_1 > 0$. Easy to understand.

- Using radial field and **external** tangential field can stabilize for $\Delta_1 < 0$ but not for $\Delta_1 > 0$.

- Resonant or non-resonant ideal modes: can stabilize below threshold for stability with PC wall, but **not** above.

- Applications: stabilizing $m = 1$ or $m = 0$ modes in RFPs; perhaps controlling the amplitude of neoclassical tearing modes in tokamaks.