Neoclassical effects on error-field penetration in ohmic tokamak plasmas

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Motivation: resolving error-field penetration threshold disagreement

- Theory model (Fitzpatrick-Hender): resonant surface experiences error-field induced EM torque which is resisted by $\mu_\perp$-torque. Steady-state torque balance equation

$$T_{EM} + T_{\mu_\perp} = 0$$

  fails to have a solution above critical error-field strength $\longrightarrow$ bifurcation, magnetic reconnection, locked islands...

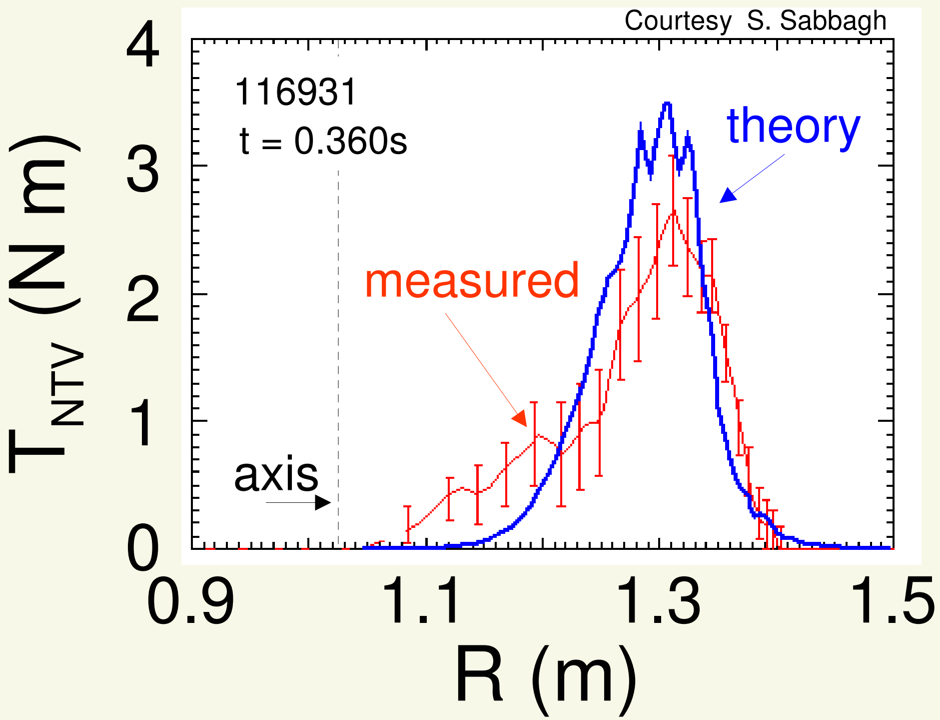
- Theory scaling for ITER suggest more sensitive than current machines.

- Experimental studies disagree: suggest ITER no worse than current devices.

Experimental evidence suggests both R and NR fields matter

- M. Schaffer[1] noted that sum of non-resonant [NR] field errors can be factor 10 larger than resonant [R] field errors in DIIID, i.e. $\tilde{B}_{NR}^{eff}/B_\phi \sim 10^{-3}$.

- Sabbagh et. al[a] have measured toroidal momentum dissipation (NSTX) and found profile and decay time agree with torque arising from Neoclassical Toroidal Viscosity [NTV].



[1]Error-field workshop-post 2005 DPP.
[a]PRL **96**, 225002 (2006).

T_{NTV} fundamentally different than $T_{\mu_\perp}$ and T_{EM}

- Toroidal damping rate induced by NR field errors in the plateau and collisional regimes [2, 3] is of the form

$$\frac{\partial V_\phi}{\partial t} \sim \omega_{ti} q \sum_{mn} \frac{\left| n\tilde{B}_{mn}/B_\phi \right|^2}{|n - m/q| + \nu_{eff}} \left(V_\phi - V_*^{NC}\right) \equiv \omega_{ti} q \left| \frac{\tilde{B}_{NR}^{eff}}{B_\phi} \right|^2 \left(V_\phi - V_*^{NC}\right)$$

Salient features:

- Non-local: acts throughout plasma volume.
- Acts to maintain plasma flowing at V_*^{NC} not lock to stationary coil/wall frame as T_{EM} and $T_{\mu_\perp}$ do.

Incorporate above equation into large aspect-ratio cylindrical tokamak model equilibrium $\longrightarrow$ modified Fitzpatrick-Hender model.

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[2] K. Shaing et. al, Phys. Fluids **29**, 521 (1986).
[3] K. Shaing, Phys. Fluids B **5**, 3841 (1993).

Determine flow profile with R and NR field error effects

- Need steady-state toroidal flow profile:

$$\frac{1}{r}\frac{d}{dr}\left(\mu_\perp(r) r \frac{dV_\phi(r)}{dr}\right) - \nu_\| b^2(r)\left(V_\phi(r) - V_*^{NC}(r)\right) = -F_{E,M}\delta(r-r_s) - F_0$$

where $b^2(r) \equiv \left|\frac{\tilde{B}_{NR}^{eff}}{B_\phi}\right|^2$, and $\nu_\| \equiv \omega_{ti} q$.

Limiting solutions:

- Axisymmetric: $F_{E,M} = b^2(r) = 0$ yields $V_0(r)$.

- Weak neoclassical effects: $\nu_\| b^2(r) \rightarrow 0$, old Fitzpatrick-Hender solution.

- Strong neoclassical effects: $\nu_\| b^2(r_s) \gg \frac{\mu_\perp}{r_s^2}$, localized WKB-type solution. NTV acts to enhance $\mu_\perp$, but relaxes V_ϕ to V_*^{NC} not 0.

Resonant layer torque balance equation

- Integrating force balance equation across resonant layer in a high-temperature tokamak yields:

$$\left|\frac{b^{vac}_{r,nm}}{B_\phi}\right|^2 \frac{\mathrm{Im}(\Delta(V))}{|\Delta(V)|^2} = \frac{P^{eff}}{S}\left(\tilde{V}_0 - V\right)$$

where $\tilde{V}_0$ (V) is the toroidal velocity in the presence of NR (NR + R) error-fields harmonics, S the Lundquist number, and τ_H the hydromagnetic timescale (all at resonant surface).

Two limiting cases:

- $\nu_\parallel \tau_V b^2(r_s) \ll 1 \longrightarrow \frac{P^{eff}}{S} \sim \frac{P}{S}$ and $\tilde{V}_0 = V_{E\times B}$ (F-H)
- $\nu_\parallel \tau_V b^2(r_s) \gg 1 \longrightarrow \frac{P^{eff}}{S} \sim \frac{P}{S}\sqrt{\nu_\parallel \tau_V} b(r_s)$ and $\tilde{V}_0 = V_*^{NC} \sim V_{*,i}$

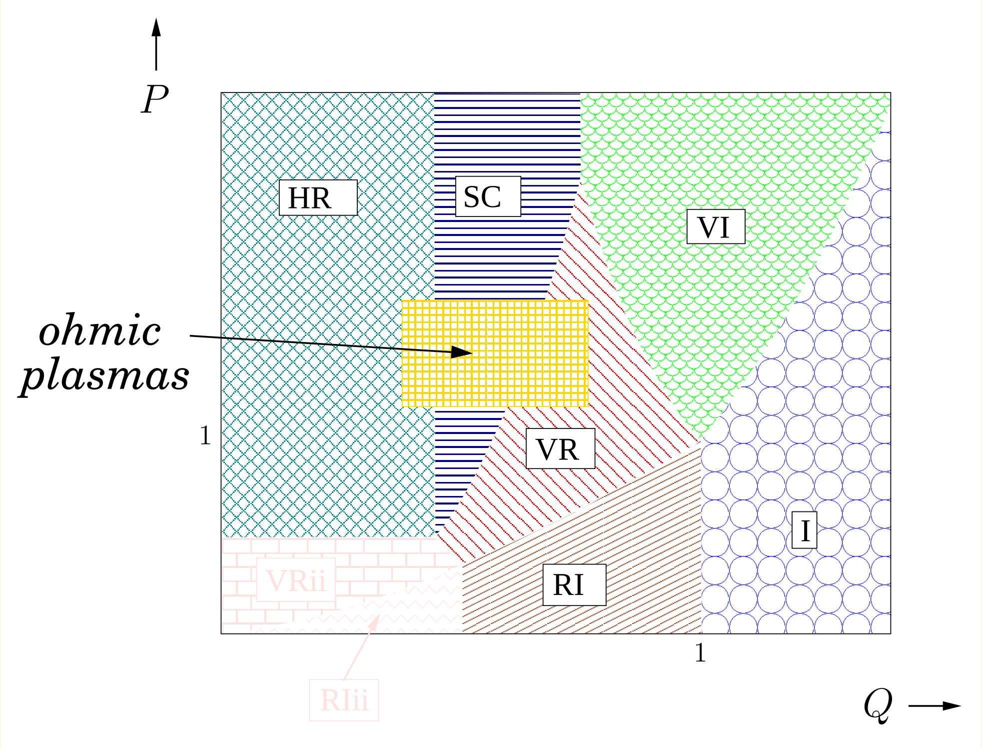
Here $\tau_V = r_s^2 \rho_m(r_s)/\mu_\perp(r_s)$ viscous timescale.

Two-fluid layer response regimes relevant to ohmic tokamak

- SC[4, 5] regime: perp. viscosity, resistivity, kinetic-Alfvén effects dominant in resonant layer.

$$\left|\frac{b_{r,nm}^{vac}}{B_\phi}\right|_{crit}^2 \sim \frac{P}{S^{1/2}} \frac{V_*^{5/2}}{\rho_*} \left[\frac{1+\lambda\Gamma_s+\lambda^2\Gamma_s^2}{\lambda+\lambda^2\Gamma_s}\right]$$

- $\rho_* = \rho_s(r_s)/R_0$ using T_e
- $\Gamma_s \sim \sqrt{\nu_\| \tau_V}\, b(r_s)$
- $$\lambda = \int_{r_s}^{a} \frac{\mu(r_s)}{\mu(r)} \frac{dr}{r}$$



[4]A. Cole and R. Fitzpatrick, Phys. Plasmas **13**, 032503 (2006).
[5]F.L. Waelbroeck, Phys. Plasmas **10**, 4040 (2003).

Limiting cases

- $\sqrt{\nu_{\parallel}\tau_V}b(r_s) << 1 \longrightarrow$ old two-fluid SC regime:

$$\left|\frac{b^{vac}_{r,nm}}{B_\phi}\right|^2_{crit} \sim \frac{P}{S^{1/2}}\frac{V_*^{5/2}}{\rho_*}$$

- $\sqrt{\nu_{\parallel}\tau_V}b(r_s) >> 1 \longrightarrow$ new neoclassical SC regime:

$$\left|\frac{b^{vac}_{r,nm}}{B_\phi}\right|^2_{crit} \sim \frac{P}{S^{1/2}}\frac{V_*^{5/2}}{\rho_*}\sqrt{\nu_{\parallel}\tau_V}\left|\frac{\tilde{B}^{eff}_{NR}(r_s)}{B_\phi}\right|$$

Recall $b(r_s) = \left|\frac{\tilde{B}^{eff}_{NR}(r_s)}{B_\phi}\right|$.

Threshold scaling: aspect ratio, profile fixed, $T_i \sim T_e$

By definition

$$P \sim \frac{R_0^2 T_e^{3/2}}{\tau_M} \qquad V_* \sim \omega_* \tau_H \sim \frac{T_e \sqrt{n_e}}{R_0 B_\phi^2}$$

$$S \sim \frac{B_\phi T_e^{3/2} R_0}{\sqrt{n_e}} \qquad \rho_* \sim \frac{T_e^{1/2}}{R_0 B_\phi} \qquad \nu_\parallel \sim \frac{T_e^{1/2}}{R_0}$$

Find

$$\left|\frac{b^{vac}_{r,nm}}{B_\phi}\right| \sim n_e^{3/2}\ R_0^{-1/2}\ B_\phi^{-9/2}\ T_e^3\ \tau_M^{-1/2} \left|\frac{\tilde{B}^{eff}_{NR}}{b^{vac}_{r,nm}}\right|$$

Summary and conclusions

- Have constructed new model of error-field penetration that includes NTV damping torque—accounting for R and NR field-error.

- When NTV dominates $T_{\mu_\perp}$ in bulk plasma, resonant surface rotates at $V_*^{NC} \sim V_{*,i}$, and

- anomalous perpendicular viscosity enhanced:

$$\mu_\perp(r_s) \rightarrow \mu_\perp(r_s)\sqrt{\nu_\| \tau_V} \left| \frac{\tilde{B}_{NR}^{eff}(r_s)}{B_\phi} \right|$$

A giant leap of faith

- $\tau_E \sim \tau_M \longrightarrow$ eliminate via ohmic power balance: $\tau_M \sim \frac{n_e T_e^{5/2} R_0^2}{B_\phi^2}$
- Further assume $\left|\frac{\tilde{B}_{NR}^{eff}}{b_{r,nm}^{vac}}\right| \sim 1$

Yields:

$$\left|\frac{b_{r,nm}^{vac}}{B_\phi}\right| \sim n_e \; R_0^{-3/2} \; B_\phi^{-7/2} \; T_e^{7/4}$$

compared to previous two-fluid result:

$$\left|\frac{b_{r,nm}^{vac}}{B_\phi}\right| \sim n_e^{1/4} \; R_0^{-1} \; B_\phi^{-5/4} \; T_e^{1/8}$$

and experimentally[6], one expects:

$$\left|\frac{b_{r,nm}^{vac}}{B_\phi}\right| \sim n_e \; R_0^{\alpha_R} \; B_\phi^{-1} \qquad 0.5 < \alpha_R < 1.25$$

[6] S. Wolfe, et. al, PoP **12**, 056110 (2005)