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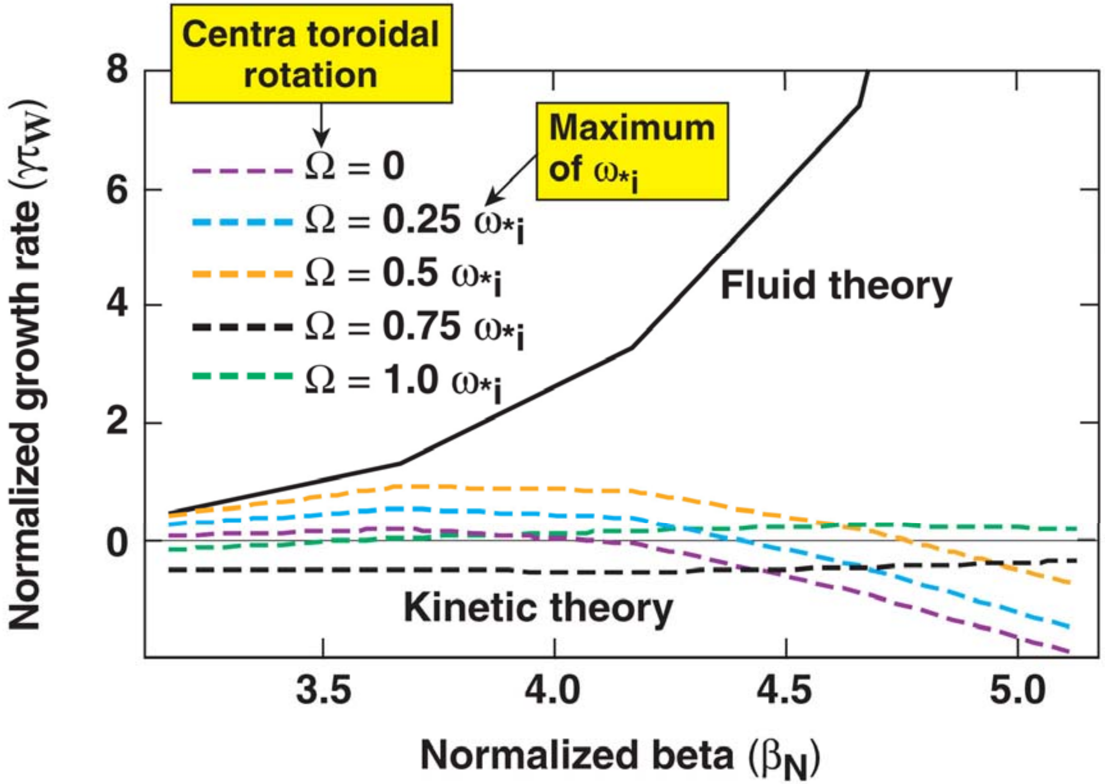
Kinetic Effects on MHD modes



R. Betti
University of Rochester
Laboratory for Laser Energetics

MHD Workshop
Princeton Plasma Physics Laboratory
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Slide 1

b1 betti, 11/1/2006

Collaborators

Bo Hu

University of Rochester

Janardhan Manickam

Princeton Plasma Physics Laboratory

OUTLINE

- The low-frequency energy principle

- The low-frequency energy principle application to MHD modes

- The qualitative picture of RWM interaction with trapped particles

- The PEST kinetic postprocessor

For kinetic analysis of RWM, see
Hu and R. Betti, Phys. Rev. Lett. (2004)
B.Hu, R. Betti and J. Manickam, Phys. Plasmas (2005)

The kinetic contributions to the macroscopic plasma stability is included in the kinetic energy principle.

$$\omega^2 K_{MHD} = \delta W = \delta W_{MHD} + \delta W_K$$

$$\delta W_K = -\frac{1}{2}\int dV \left\{ \left(\delta P_{\parallel} - \delta P_{\perp} \right) \kappa \bullet \xi + \delta P_{\perp} \nabla \bullet \xi_{\perp} \right\}$$

Kinetic effects from solution of drift-kinetic equation

$$\delta P_{\parallel,\perp} = \frac{1}{2} m \int_{trapped} d\vec{\mathbf{v}} \left(\mathrm{v}_{\parallel,\perp}^2 \delta f \right) + \frac{1}{2} m \int_{circulating} d\vec{\mathbf{v}} \left(\mathrm{v}_{\parallel,\perp}^2 \delta f \right)$$

In most cases, the kinetic contribution is treated as a perturbation of the fluid one.
Ideal MHD yields the mode structure

$$\omega^2 K_{MHD}(\xi,\xi^*) = \delta W_{MHD}(\xi,\xi^*) + \cancel{\delta W_K(\xi,\xi^*)}$$

AEs

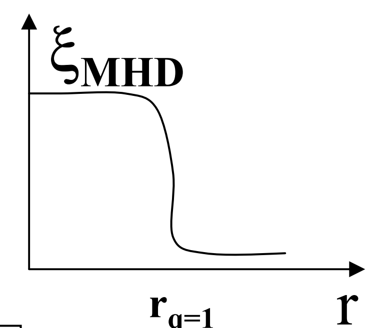
$$\omega_{real}^2 = \delta W_{MHD}(\xi,\xi^*)$$

Find ω_{real}, ξ_{MHD}

Internal Kink

$$0 = \delta W_{MHD}(\xi,\xi^*)$$

Find ξ_{MHD}



Perturbation analysis does not apply to the EPM

In the perturbative analysis, the kinetic contribution is calculated using $\xi = \xi_{MHD}$ and used to determine the growth rate

$$\omega^2 K_{MHD}(\xi,\xi^*) = \delta W_{MHD}(\xi,\xi^*) + \delta W_K(\xi,\xi^*)$$

AEs

$$2\omega_{real}\gamma K_{MHD}(\xi,\xi^*) = \Im[\delta W_K(\xi,\xi^*)]$$

Find γ

internal kink

Find ω_r and γ

"Fishbone" branch

$\gamma > 0,\ \omega_r \approx \omega_D$

"MHD" branch

$\gamma > 0,\ \omega_r \approx 0$

The imaginary part of δW_K is determined by the mode resonance with the particle motion

- **Resonances for low frequency modes $\omega_r \ll \Omega_{ci,e}$**

$$\omega_{doppler}^{E\times B} \equiv \omega - \omega_{E\times B}$$

$$\omega_{doppler}^{E\times B} = \omega_D$$

$$\omega_D \sim \frac{r_L}{a}\frac{v_{th}}{R}$$

Resonance with precession drift of banana orbits of trapped particles

$\ll$

$$\omega_{doppler}^{E\times B} = \ell\,\omega_{bounce}$$

$$\omega_{bounce} \sim \sqrt{\frac{a}{R}}\frac{v_{th}}{R}$$

Resonance with bounce frequency of trapped particles

$<$

$$\omega_{doppler}^{E\times B} = \omega_{transit}$$

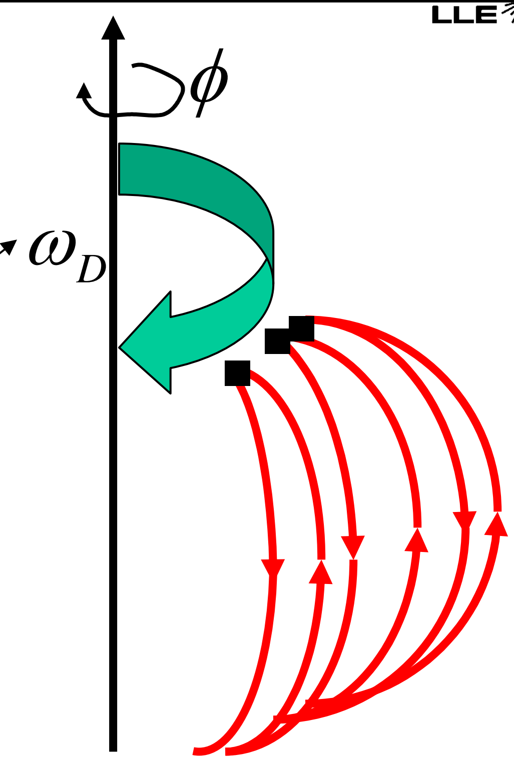
$$\omega_{transit} \sim \frac{v_{th}}{R}$$

Resonance with transit frequency of circulating particles

The precession drift frequency is the slowest frequency. It provides an efficient resonance for very low frequency modes

Precession or magnetic drift frequency

$$\omega_D = \left\langle k_\perp \bullet \left[V_{\nabla B} + V_\kappa \right] \right\rangle$$



Precession motion of the trapped particle banana orbits

Even a zero frequency mode such as the n=1 RWM can resonate with the precession drift frequency in slow rotating plasmas and with bounce/transit frequencies in fast rotating plasmas

$$\omega - \omega_{E\times B} = \omega_D, \omega_b, \omega_t$$

Resonance condition

$$\omega_{E\times B} = \Omega_{rot} - \omega_{*i}$$

Ion force balance equation

$$\omega - \Omega_{rot} + \omega_{*i} = \omega_D, \omega_b, \omega_t$$

(the ω term is crossed out)

Resonance condition

For $\Omega_{rot} \gg \omega_{*i} > \omega_D$

$$-\Omega_{rot} = \omega_b, \omega_t$$

Resonance in fast plasmas

For $\Omega_{rot} \sim \omega_{*i}$

$$-\Omega_{rot} + \omega_{*i} = \omega_D$$

Resonance in slow plasmas

Kinetic effects are not restricted to wave-particle interaction. Trapped particles with a fast precession velocity adiabatically conserve the magnetic flux enclosed by the precession motion. Their effect is stabilizing.

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Trapped particle Real contribution $\blacklozenge << \blacklozenge_D \rightarrow$ STABILIZING

$$\Re[\delta W_K] \sim \int dV \beta \int_{\Lambda_{trap}} d\Lambda \hat{\tau}_{bounce} \left| \langle \kappa \bullet \xi \rangle \right|^2 \frac{\omega_*}{\omega_D} > 0$$

It requires:

$$\omega - \Omega_{rot} + \omega_{*i} < \omega_D$$

Easily satisfied for energetic particles. m=1 hot ion stabilization

Requires $\Omega_{rot} \approx \omega_{*i}$ for thermal particles and $\omega \approx 0$ (i.e. RWM)

The low-frequency energy principle yields the RWM stability threshold

$$\gamma\tau_w = -\frac{\delta W_{MHD}^{\infty} + \delta W_K}{\delta W_{MHD}^{b} + \delta W_K}$$

$$\delta W_K = \delta W_K^i + \delta W_K^e + \delta W_K^{\alpha}$$

$$\delta W_K = \Re\left[\delta W_K\right] + i\Im\left[\delta W_K\right]$$

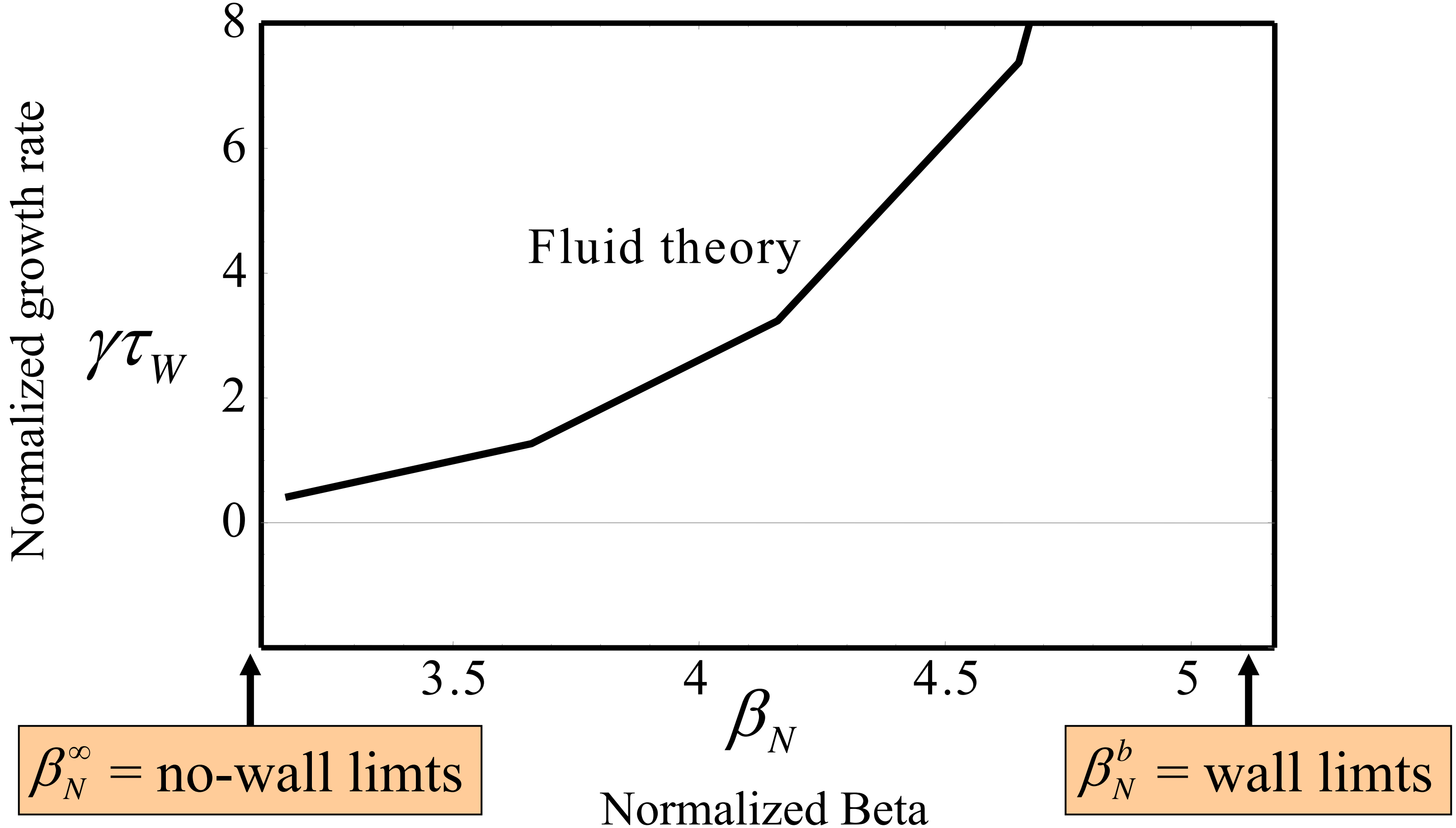
Instability condition

$$-\delta W_{MHD}^{b}\delta W_{MHD}^{\infty} > \left|\delta W_K\right|^2 + \Re\left[\delta W_K\right]\left(\delta W_{MHD}^{b} + \delta W_{MHD}^{\infty}\right)$$

MHD energy principle for RWM: Haney, Freidberg Phys. Fluids B 1, 1637 (1989)

Without kinetic effects, the RWM is unstable between the wall and no-wall beta limits

• Normalized RWM growth rate without kinetic effects



Qualitative analysis of the instability condition

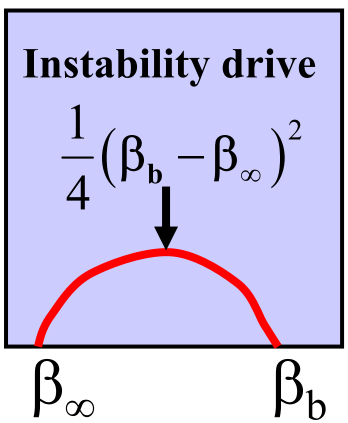
$$\delta W_{MHD}^{\infty} \sim \beta_\infty - \beta$$

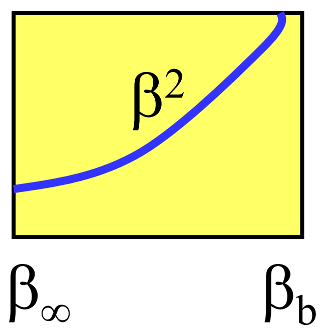
$$\delta W_{MHD}^{b} \sim \beta_b - \beta$$

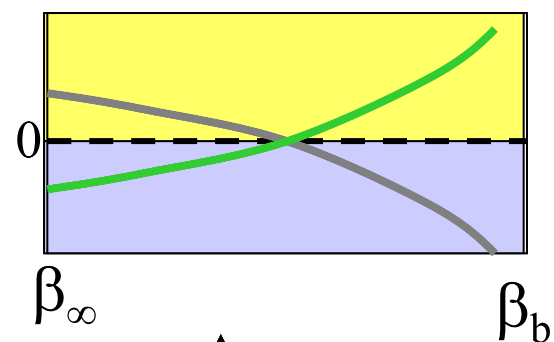
$$\delta W_K \sim \beta(X + iY)$$

→ **stabilizing**

→ **destabilizing**

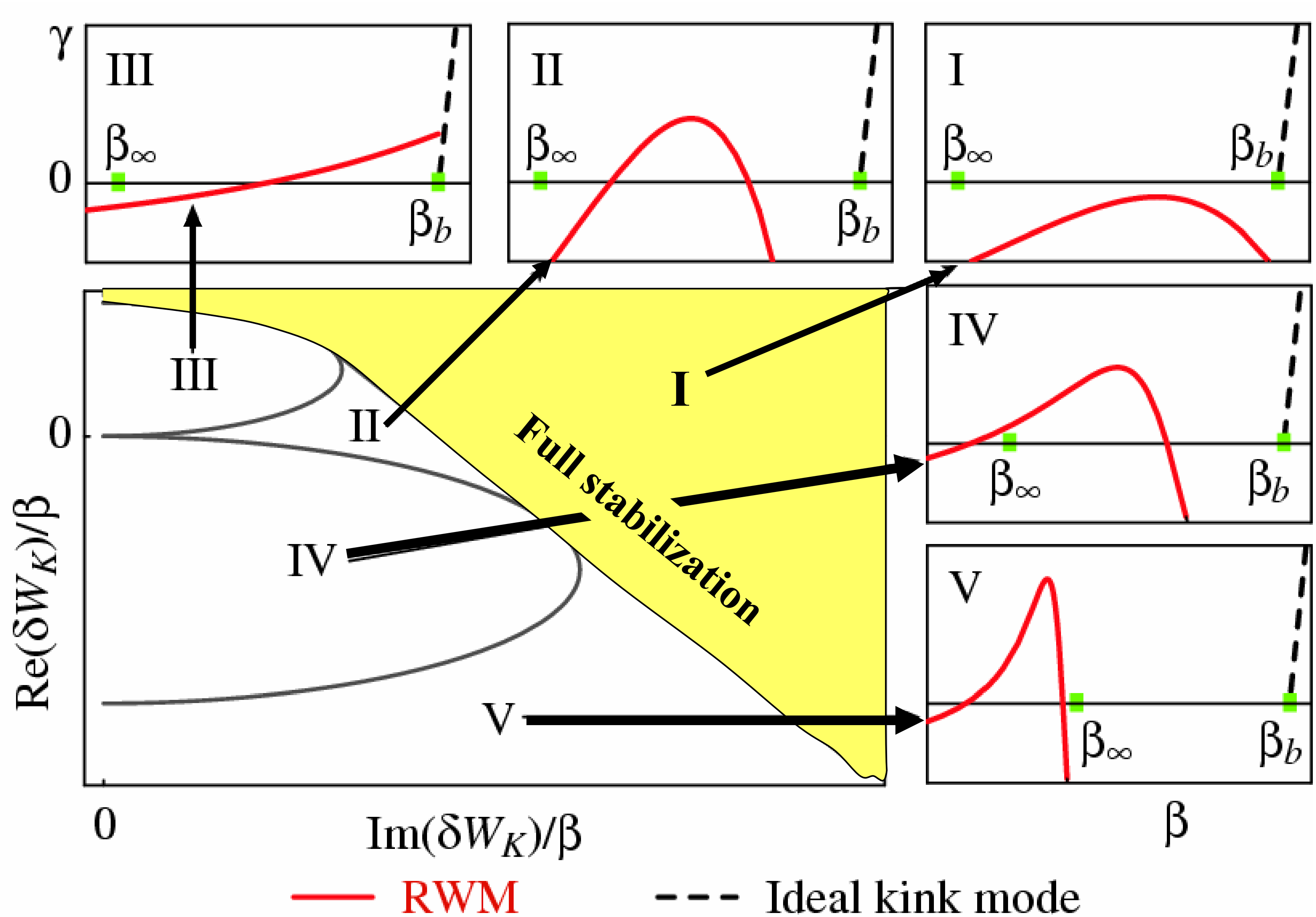






$$-\delta W_{MHD}^{b}\,\delta W_{MHD}^{\infty} > \left|\delta W_K\right|^2 + \Re\left[\delta W_K\right]\left(\delta W_{MHD}^{b} + \delta W_{MHD}^{\infty}\right)$$

Five regimes of RWM stability/instability



Necessary condition for stabilization requires relatively small ♎W_K

$$\delta W_{MHD}^{\infty/b} \square \beta_{\infty/b} - \beta$$

instability drive δW_{MHD}^{Drive}

$$\delta W_K \square \beta(X + iY)$$

kinetic effects

- **Kinetic stabilization enhanced for strong dissipation. Full suppression requires**

$$\left|\delta W_K\right| > 0.29\delta W_{MHD}^{Drive} \quad \Rightarrow \quad \delta W_K^{Minimum} = \beta(0.08 + i0.28)$$

- **Kinetic stabilization in the absence of dissipation (Y=0) requires X>0.5:**

$$\delta W_K > 0.5\beta \quad \Rightarrow \quad \left|\delta W_K\right| > 0.5\delta W_{MHD}^{Drive}$$

Low-frequency kinetic theory of the RWM: approximations

- RWM frequency: $\omega \sim 1/\tau_w$ - 50/ τ_w (τ_w =wall time)

- $\omega << \omega_D$, ω_{*i} → zero mode frequency

  ω_D magnetic drift frequency

  ω_{*i} ion diamagnetic drift frequency
- $\Omega_{rot} \sim \omega_{*i}$ → quasi-stationary plasma
- For large rotation frequencies ($\Omega_{rot} >> \omega_{*i}$), resonances with Alfven and sound continuum are important (not included here)

- In the presence of enough dissipation, the RWM can be fully suppressed for fast plasma rotation (not included here)

[4] Bondeson and Chu, Phys. Plasmas 3, 3013 (1996);
[5] Liu et al, Nuc. Fusion 44, 232 (2004)
[6] A. D. Turnbull et al, Phys. Rev. Lett. 74, 718 (1995)
[7] A. Bondeson and D.J. Ward, Phys. Rev. Lett. 72, 2809 (1994): Betti and Friedberg, PRL, (1995)
[8] A.M. Garofalo et atl, Phys. Plasmas 9, 1997 (2002)
[9] R. Fitzpatrick, Phys. Plasmas 9, 3459 (2002)

A resonance occurs between the precession frequency and the ExB Doppler-shifted mode frequency

$$\delta W_{Kj} \sim \int dV \beta_j \int_{\Lambda_{trap}} d\Lambda \hat{\tau}_{bounce} \left| \langle \kappa \bullet \xi \rangle \right|^2 I_\varepsilon(\Lambda, \vec{r})$$

$$\hat{\varepsilon} = \frac{\varepsilon}{T_j}$$

$$I_\varepsilon = \int_0^\infty \hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} \frac{\omega_{*i}(\hat{\varepsilon} - 3/2) + \omega_{E\times B}}{\bar{\omega}_{Dj}\hat{\varepsilon} - \omega_{Doppler}^{E\times B}} d\hat{\varepsilon}$$

$$\omega_{Doppler}^{E\times B} \equiv \omega - \omega_{E\times B}$$

$$\omega_{Doppler}^{E\times B} \equiv \bar{\omega}_{Dj}\hat{\varepsilon}$$

Resonance condition

The strongest resonance comes from **suprathermal** particles (Φ_{rot}=0 case)

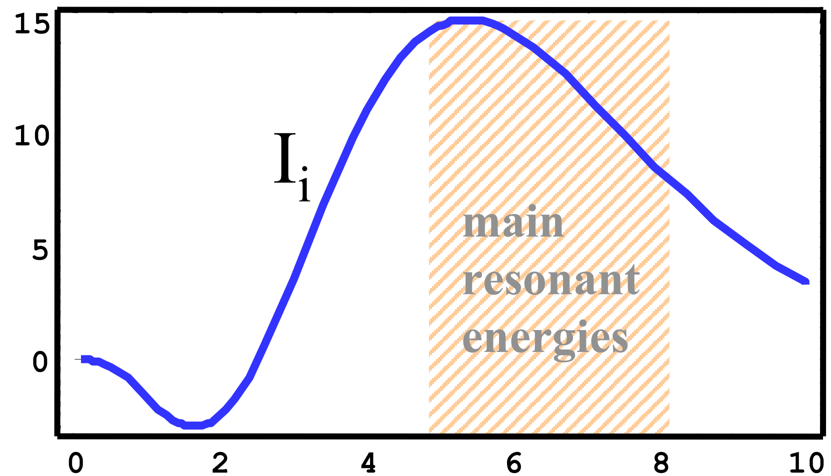
$$I_\varepsilon = \int_0^\infty \frac{\hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} (\hat{\varepsilon} - 5/2)}{\dfrac{\hat{\varepsilon}}{r_\omega} - 1} d\hat{\varepsilon} = \mathrm{I}_r + i\mathrm{I}_i$$

$$\hat{\varepsilon} = \frac{\varepsilon}{T_j}$$

$$\mathrm{I}_i = \pi r^{7/2} (r - 5/2) \exp(-r)$$

Large aspect ratio approximation

$$\hat{\varepsilon}_{res} = r_\omega \equiv \frac{\omega_{*i}}{\bar{\omega}_{Di}^{th}} \sim 5 - 8$$



$$r_\omega = \hat{\varepsilon}_{res}$$

Fast and thermal ions can resonate even in the absence of plasma rotation

Resonance condition for RWM

$$-\Omega_{rot} + \omega_{*i} = \omega_D$$

$$\omega_{*i} = \frac{r_L}{a}\frac{1}{\in}\omega_t$$

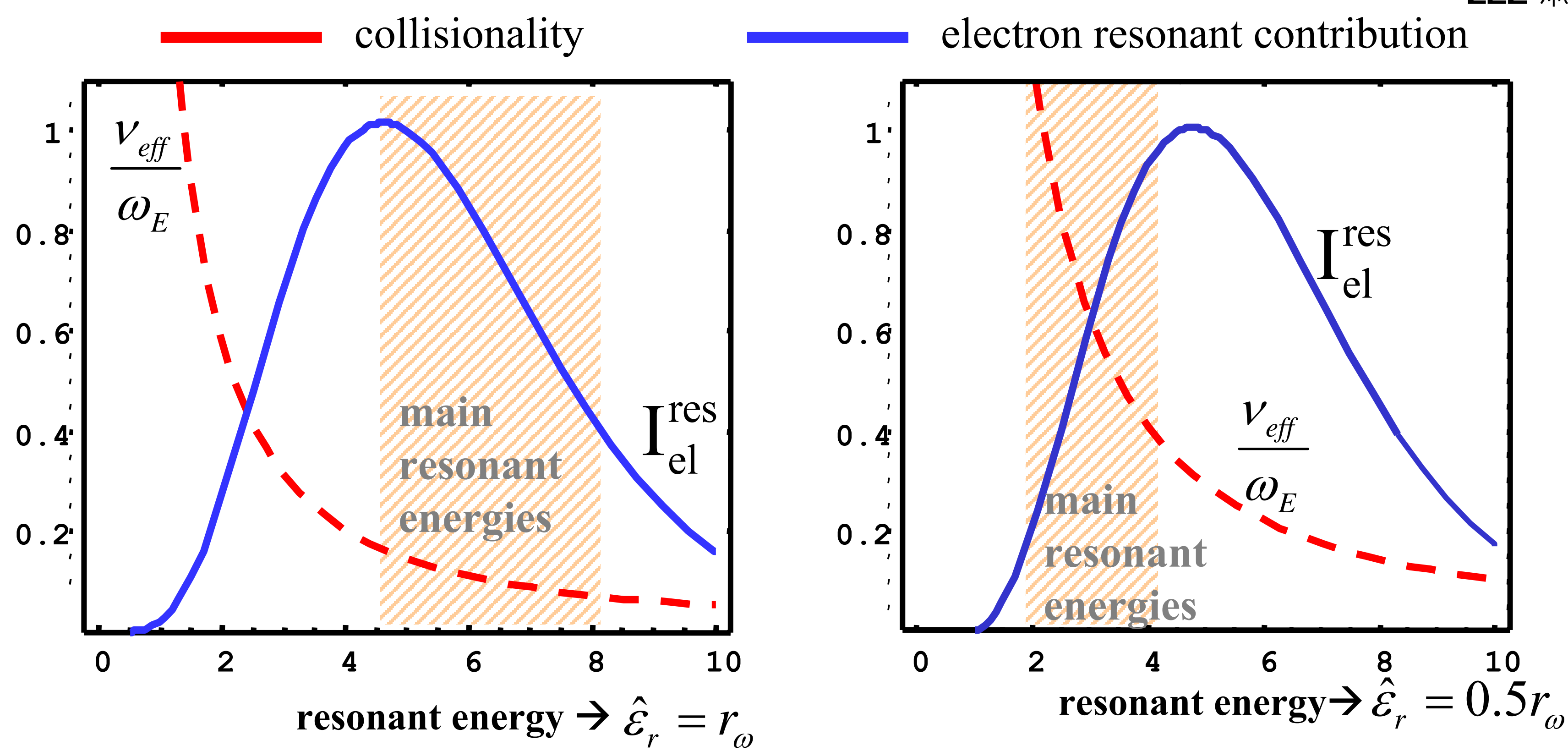
$$\omega_{Di} = \in \frac{\varepsilon}{T}\omega_{*i}$$

The resonances in MARS are very weak

$$\Omega_{rot} << \omega_{transit}$$

$$\Omega_{rot} << \omega_{bounce}$$

Electrons can resonate for ⌖$_{rot}$>◆$_{*i}$. Collisionality reduces the resonant interaction but does not eliminate it.



$$\omega_E = \omega_{*i}$$
$$\Omega_{rot} = 2\omega_{*i}$$

$$r_\omega \equiv \frac{\omega_{*i}}{\bar{\omega}_{De}^{th}} \sim 5\text{-}8, \quad \hat{\varepsilon} = \frac{\varepsilon}{T_e}$$

$$\omega_E = 0.5\omega_{*i}$$
$$\Omega_{rot} = 1.5\omega_{*i}$$

In a burning plasma, nonresonant and resonant alpha particles contribute to the stabilization.

$$\delta W_{K\alpha} \sim \int dV \int_{\Lambda_{trap}} d\Lambda \hat{\tau}_{bounce} \left| \langle \kappa \bullet \xi \rangle \right|^2 \beta_\alpha \frac{\omega_{*\alpha}}{\bar{\omega}_{D\alpha}} I_\varepsilon(\Lambda, \vec{r})$$

$$I_\varepsilon = \int_0^1 \frac{\hat{\varepsilon}}{\hat{\varepsilon} - \left(\omega_E / \bar{\omega}_{D\alpha}\right)} d\hat{\varepsilon}$$

- **In high-β plasmas, ω_D can be significantly smaller than large-aspect-ratio prediction → strong resonance can occur between ω_E and ω_D ,**
- **Nonresonant contribution always stabilizing and enhanced by ratio $\omega_* / \omega_D >> 1$**
- **Alpha contribution is significant since ∇p_α is large where the RWM eigenfunction is large.**

Analytic predictions are not reliable based on the variability of ω_D with respect to ω_{ExB} (drift reversal, rotation profile, eigenfunction shape). Quantitative assessment requires a stability code.

PEST[6] → δW_{MHD}^{∞}

PEST[6] → δW_{MHD}^{b}

PEST[6] → ξ

- ξ from PEST is used to compute

$$\delta W_K = \delta W_K^i + \delta W_K^e + \delta W_K^\alpha$$

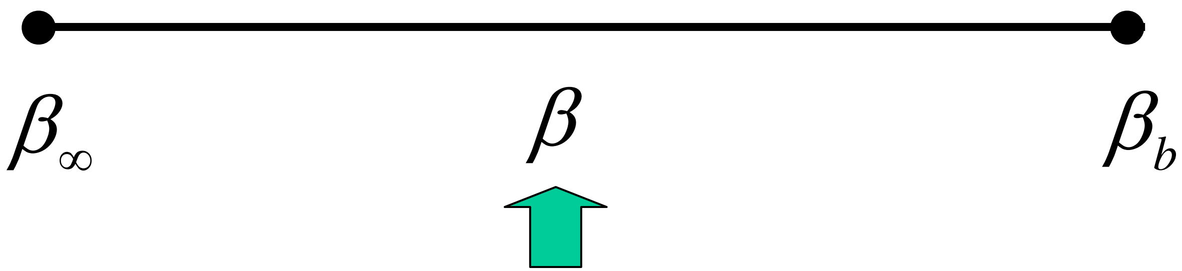
- The RWM energy principle is used to compute the RWM growth rate

$$\gamma \tau_w = -\frac{\delta W_{MHD}^{\infty} + \delta W_K}{\delta W_{MHD}^{b} + \delta W_K}$$

[6] Preische, Manickam, Johnson, Computer Physics Communications 76, 318 (1993)

The RWM plasma-eigenfunction ξ is approximated by the ideal MHD ξ from PEST for marginally stable wall position.

RWM is ideal MHD unstable



- **For each equilibrium with $\beta_\infty < \beta < \beta_b$ the wall is moved closer to the plasma till marginal stability is reached**
- **Since the RWM is essentially zero frequency, the corresponding plasma eigenfunction computed with PEST is used to approximate the RWM eigenfunction**
- **Such an eigenfunction is used to compute the all fluid and kinetic terms**

The low-frequency energy principle applied to wall modes show the possibility of RWM growth reduction/suppression at low rotation frequencies

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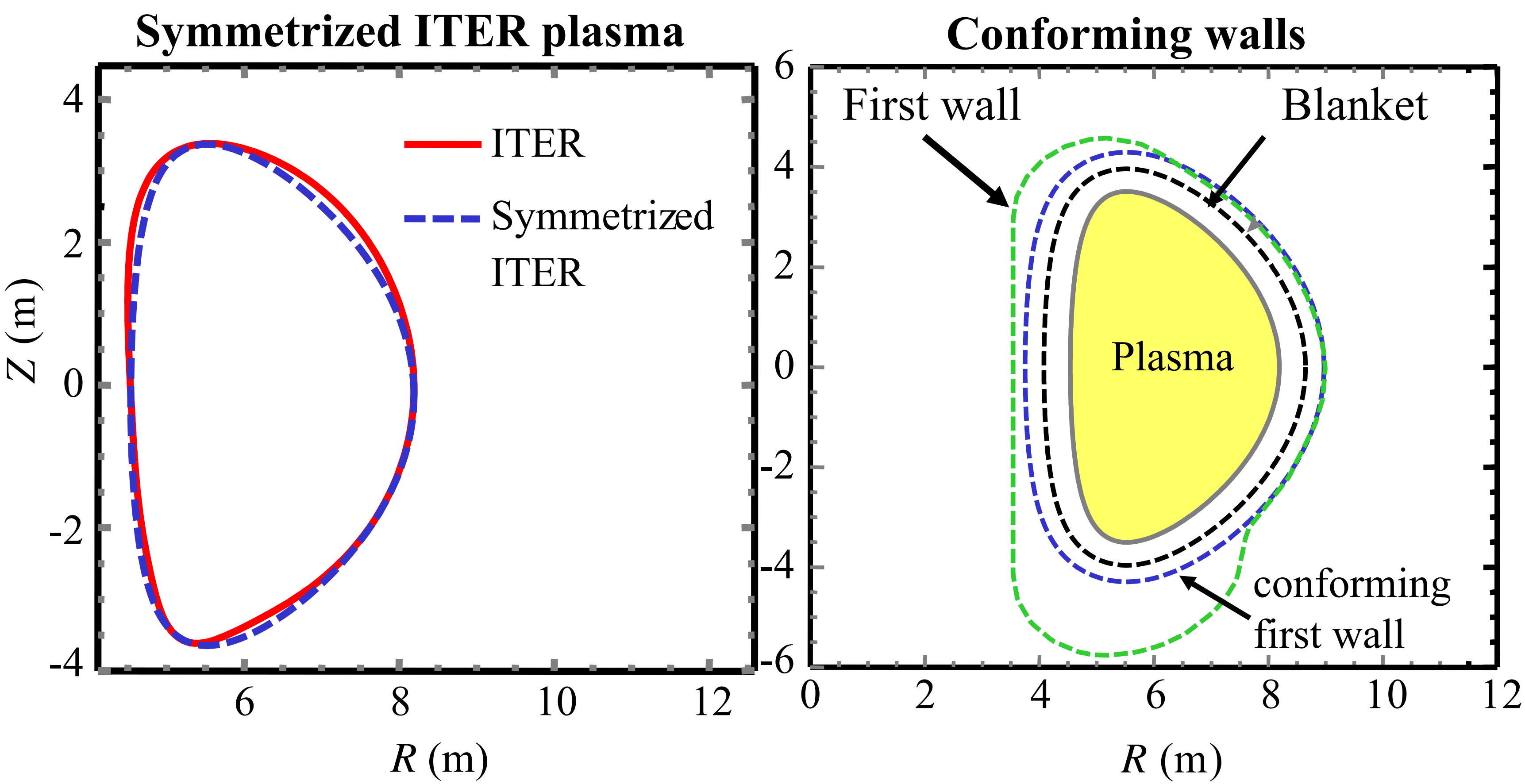
- The low-frequency energy principle is applied to RWMs and m=1 by post-processing the PEST output to compute the trapped particle contribution

- The resonance with the precession drift frequency of trapped particles (thermal ion and electron, as well as fast ions) provides a source of dissipation at low rotation frequencies for the RWM

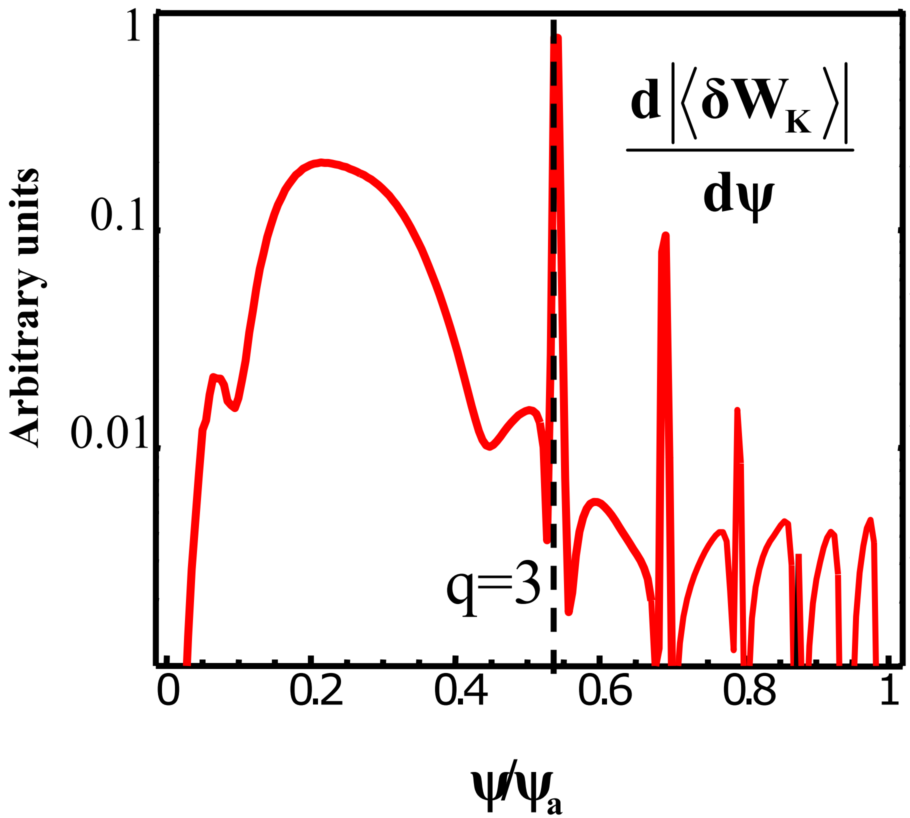
- When all the kinetic species are included [see next B. Hu] (alpha, ions and electrons), the RWM growth is strongly reduced or fully suppressed in the low rotation regime for ITER-like plasmas

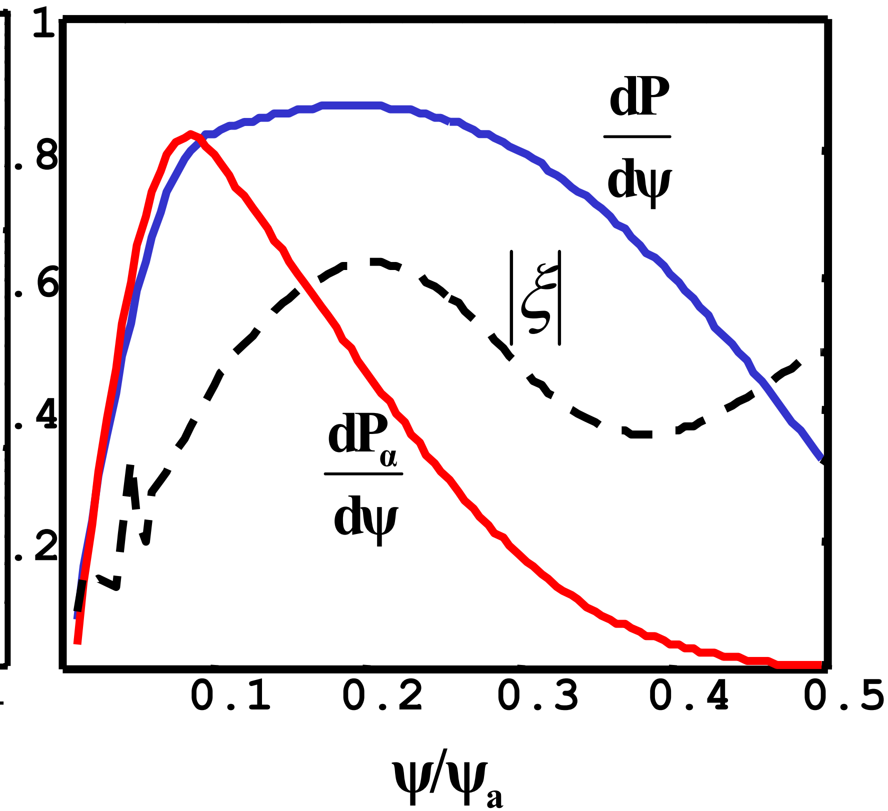
The wall mode in ITER-like plasmas

The RWM stability of an ITER Advanced-Tokamak scenario is studied using a symmetrized plasma and conforming wall



Most of the kinetic contribution comes from within the q=3 surface

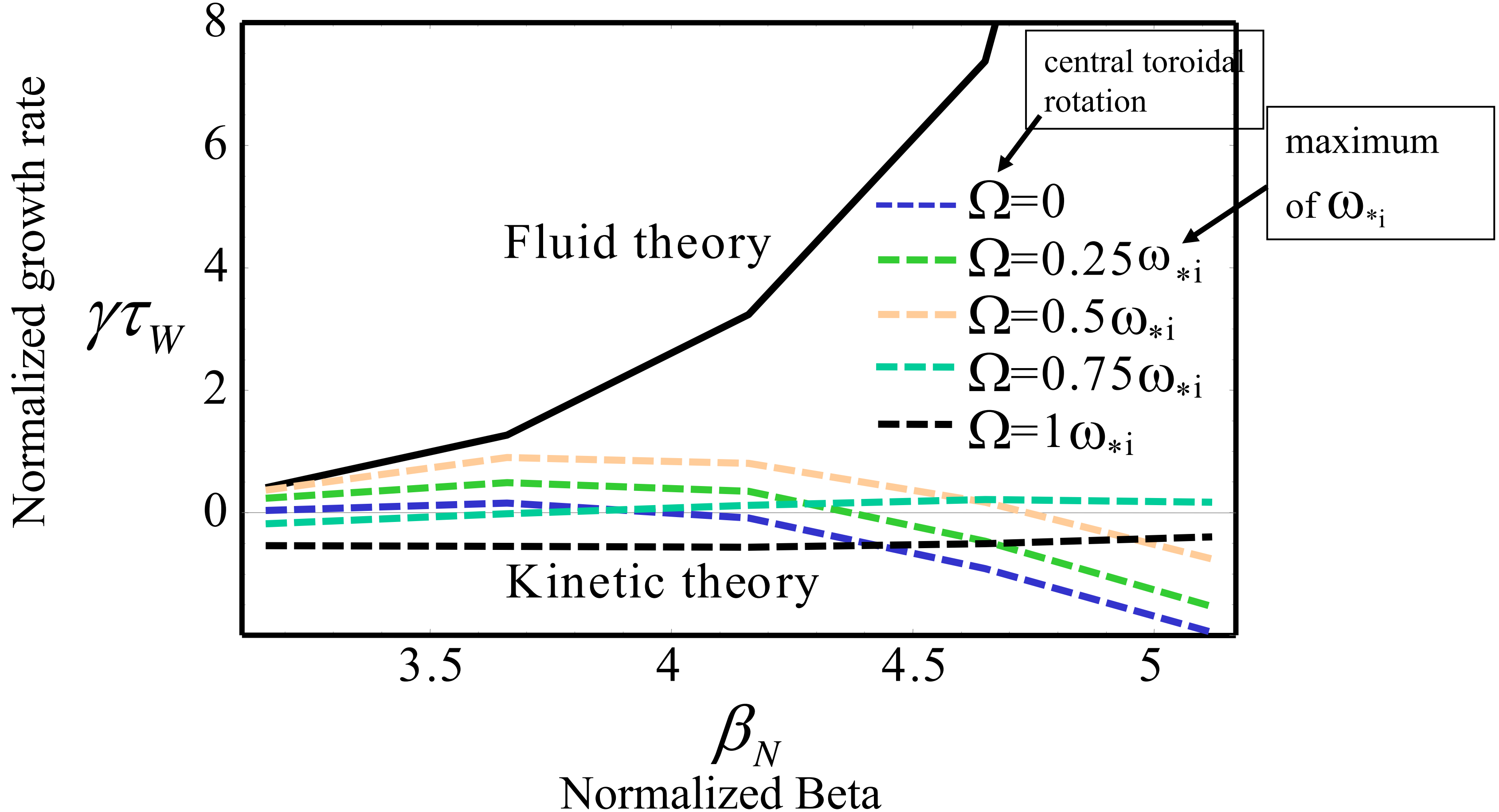




- **Alpha contribution is significant since ∇p_α is large where the RWM eigenfunction is large.**
- **Alpha contribution is comparable to ions and electrons for $\langle \beta_\alpha \rangle = 0.1 - 0.15 \langle \beta \rangle$**

The growth of the RWM is strongly reduced or fully suppressed by the kinetic effects in slow-rotating ITER-like plasmas

- RWM normalized growth rate with and without kinetic effects and varying plasma rotation frequencies $\Omega(0)$

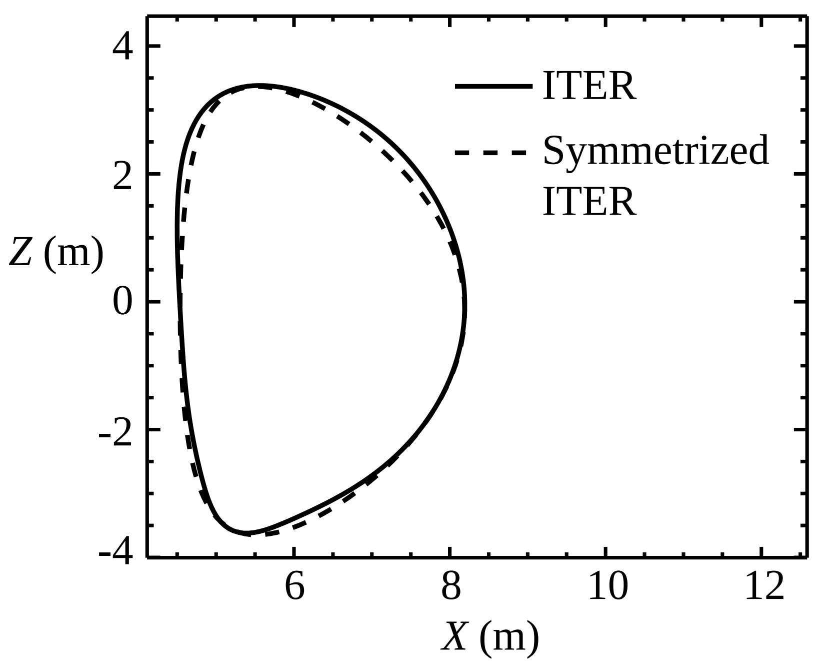


Resistive Wall Mode in ITER

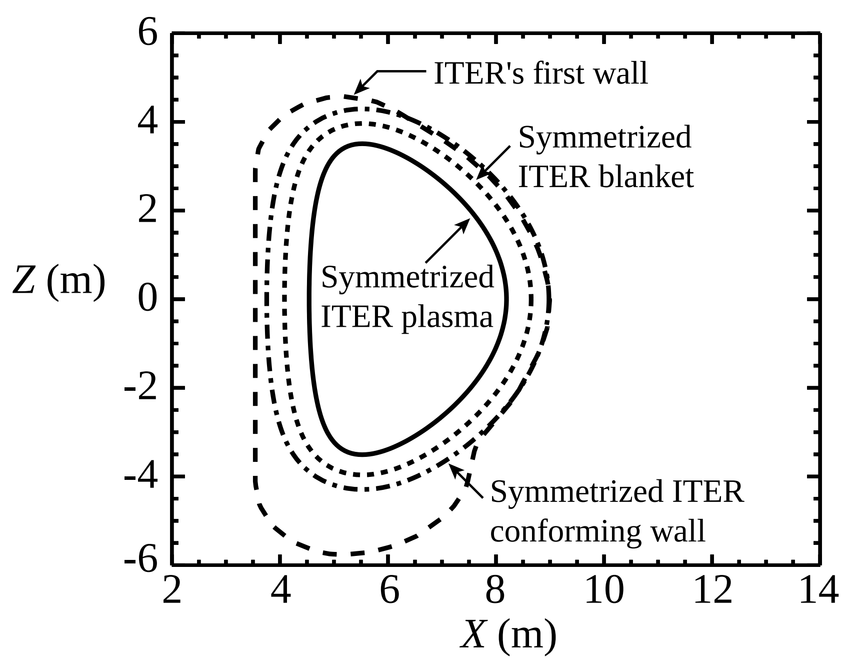
for high-β advanced scenario

The RWM stability of an ITER-advanced tokamak scenario is studied using a symmetrized plasma and comforming wall

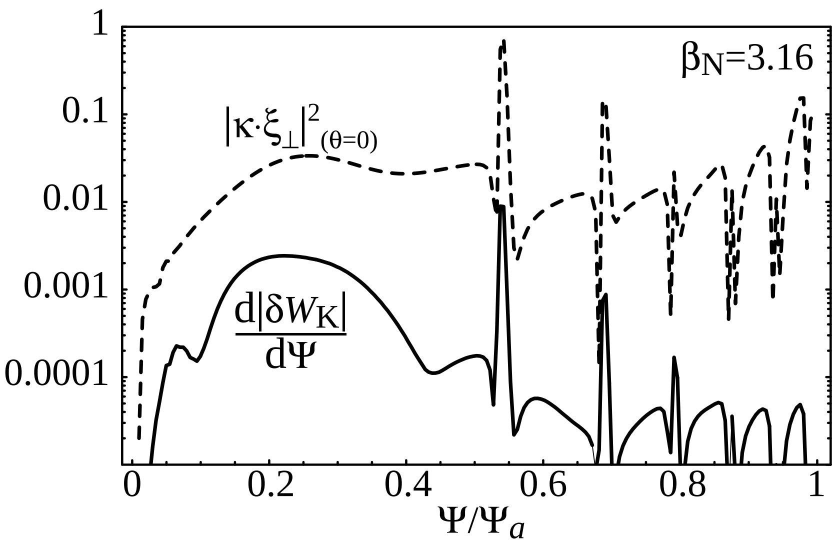
Symmetrized ITER plasma

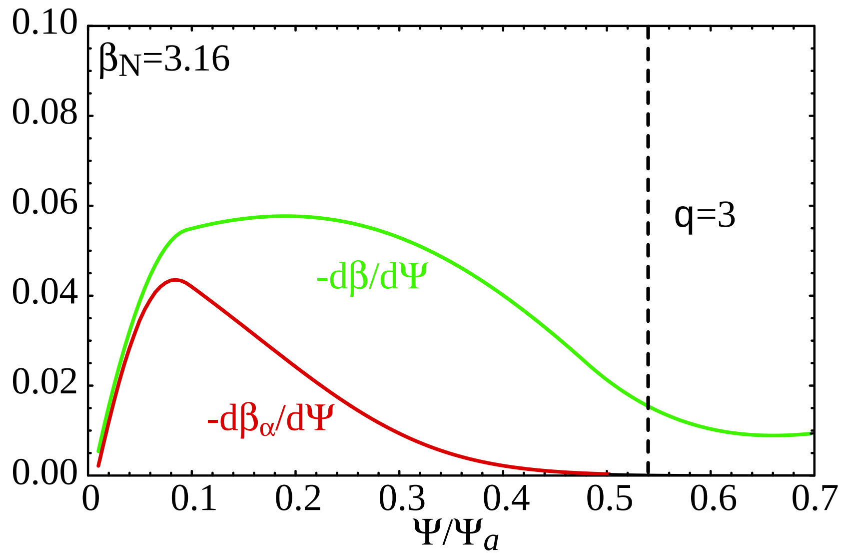


Conforming wall



Most of the kinetic contribution is produced within the $q = 3$ surface

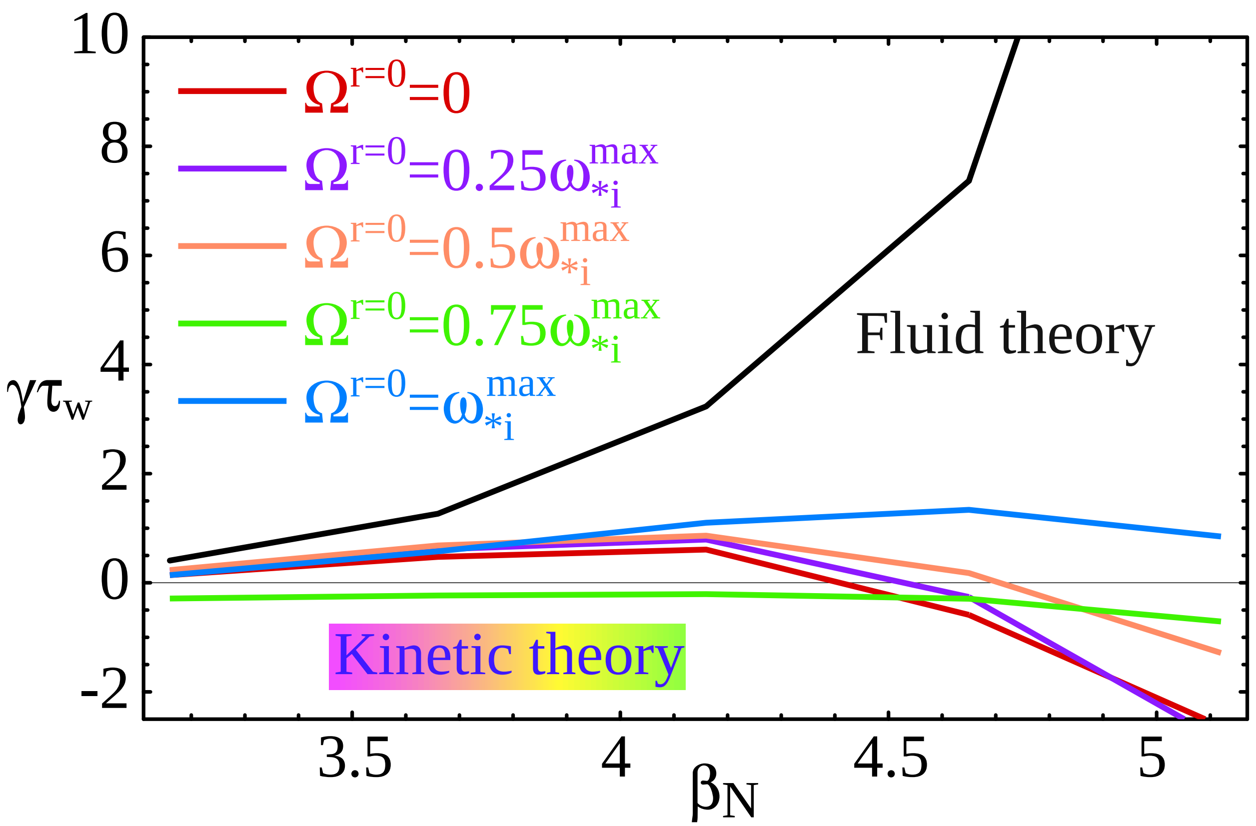




- **α particle contribution is significant since ∇p_α is large where the RWM eigenfunction is large.**
- **α contribution can be comparable to ions and electrons for $\langle \beta_\alpha \rangle = 0.15 \langle \beta \rangle$.**

The growth of the RWM is strongly reduced or fully suppressed by the kinetic effects in slow-rotating ITER-like plasmas

Normalized RWM growth rate with and without kinetic effects and varying plasma rotation frequencies $\Omega^{r=0}$



Internal Kink Mode in ITER

for standard operation baseline scenario

Internal Kink Energy Principle

- **Conventional dispersion relation**

$$\sqrt{-\mathbf{3}(\omega-\omega_E)(\omega-\omega_E-\omega_{*\mathrm{i}})}\Big|_{q=1} = -\delta\hat{W}(\omega) \qquad \mathbf{3 = 1 + 2q^2}$$

$$\Downarrow$$

- **Kinetic bulk ion inertia enhancement Π replaces adiabatic one**

$$\sqrt{-[(\omega-\omega_E)(\omega-\omega_E-\omega_{*\mathrm{i}})+\Pi(\omega)]}\Big|_{q=1} = -\delta\hat{W}(\omega)$$

$$\Pi = -\delta W_{\mathrm{K}}^{\mathrm{ion}}|_{q\sim 1}/I_\perp, \qquad I_\perp = (1/2)\int dV\rho|\boldsymbol{\xi}_\perp|^2$$

$\boldsymbol{\Pi/\omega^2 \sim 2q^2\epsilon_r^{-1/2}}$, and is stabilizing for the fishbone branch.

Parameters

ITER's parameters

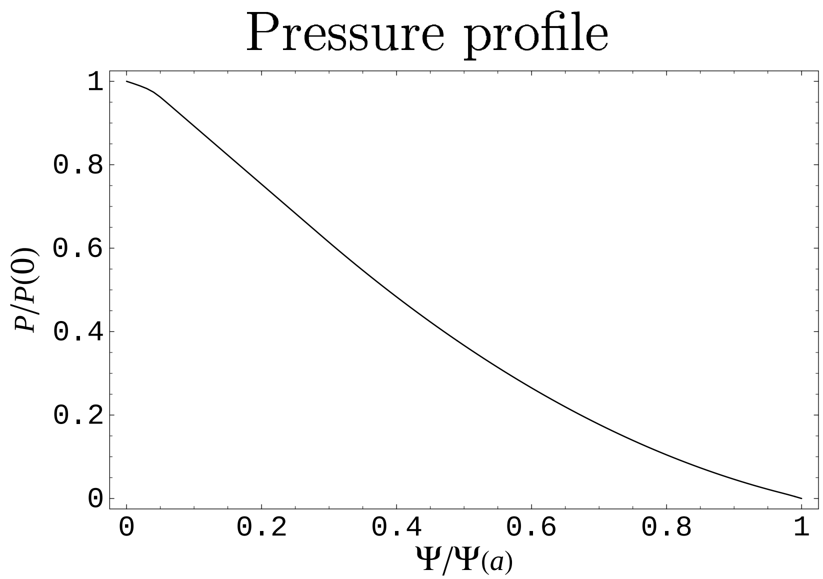
R	6.2 m
a	2.0 m
κ	1.8
δ	0.44
B	5.3 T
I_{p}	15 MA
$N_{\mathrm{e,i}}(0)$	10^{20} m^{-3}
$T_{\mathrm{e}}(0)$	23 keV
$T_{\mathrm{i}}(0)$	19 keV
β_{N}	1.8
$\langle\beta_\alpha\rangle / \langle\beta\rangle$	7%
ω_{A}	10^6 rad/s

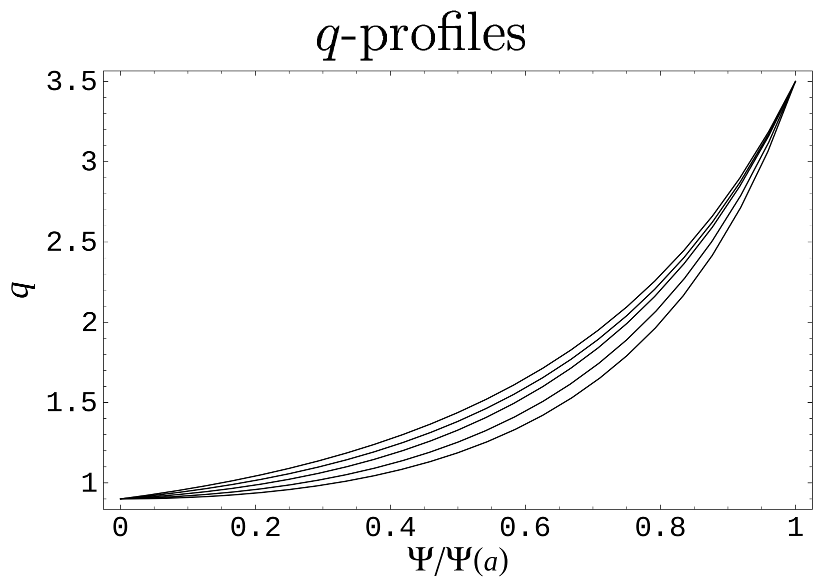
Frequencies

$\lvert\omega_{*,\mathrm{th}}\rvert$	$10^{-3}\omega_{\mathrm{A}}$	10^3 rad/s
$\lvert\omega_E\rvert$	$10^{-3}\omega_{\mathrm{A}}$	10^3 rad/s
$\lvert\Omega\rvert$	$10^{-3}\omega_{\mathrm{A}}$	10^3 rad/s
$\nu^{\mathrm{e}}_{\mathrm{eff}}$ $(\hat{\varepsilon} \sim 3)$	$10^{-3}\omega_{\mathrm{A}}$	10^3 rad/s
$\omega^{\mathrm{i}}_{\mathrm{b}}$	$2 \times 10^{-2}\omega_{\mathrm{A}}$	3×10^4 rad/s
$\omega^{\alpha}_{\mathrm{D}}$	$2 \times 10^{-2}\omega_{\mathrm{A}}$	3×10^4 rad/s
ω_{r} (fishbone)	$2 \times 10^{-2}\omega_{\mathrm{A}}$	3×10^4 rad/s
ω_{r} (MHD)	$3 \times 10^{-3}\omega_{\mathrm{A}}$	4×10^3 rad/s

Pressure and q profiles with moderate shear

$s_1 \sim 0.3$

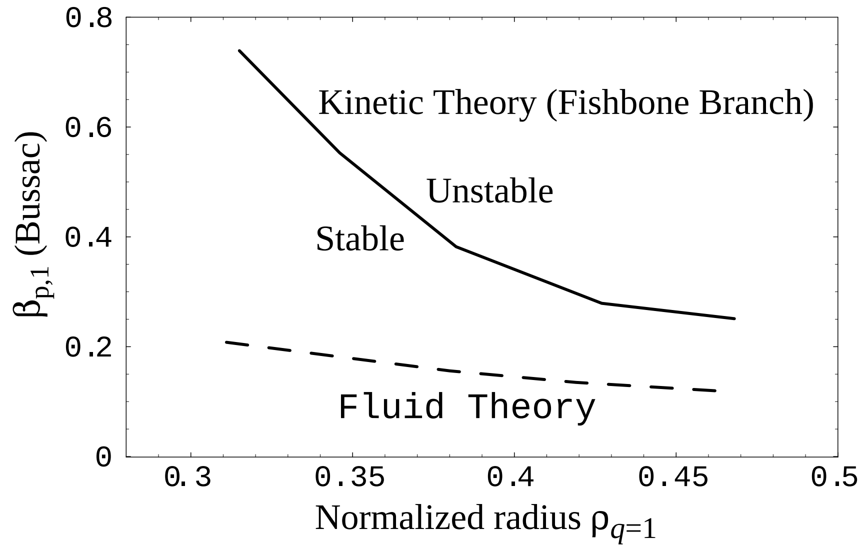




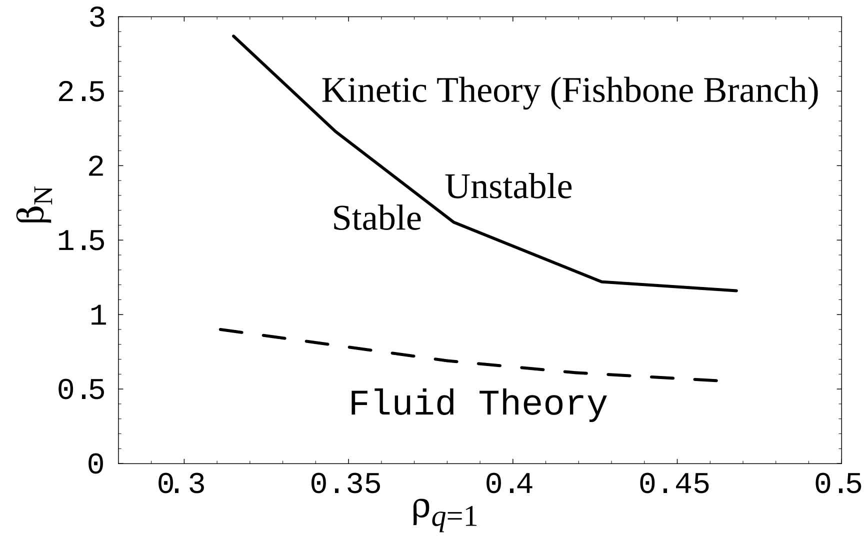
- q-profiles with $q_0 = 0.9$ and moderate magnetic shear ($s_1 \sim 0.3$).
- Boundary condition: ideal wall away from plasma boundary by 25% of minor radius.

Stability thresholds for moderate shear

In terms of $\beta_{\mathrm{p},1}$

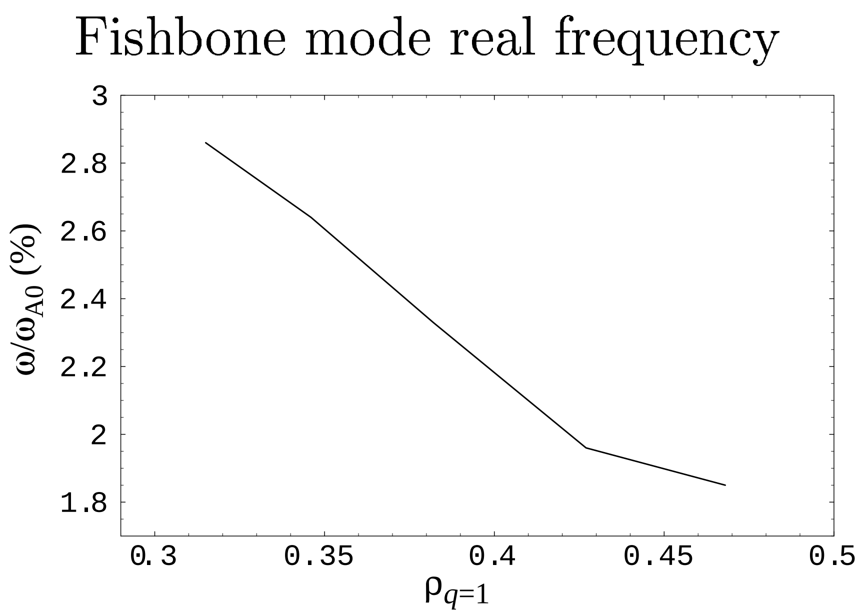


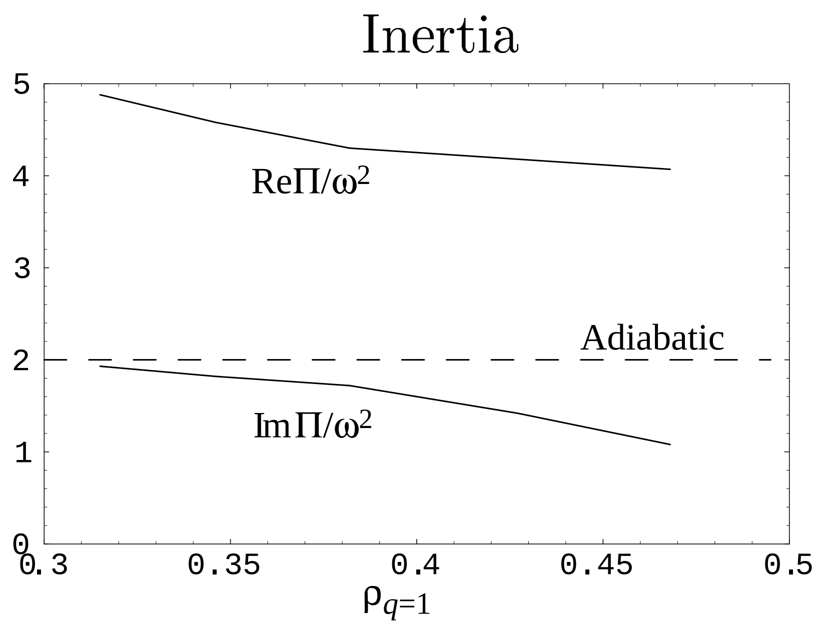
In terms of β_{N}



- MHD branch is fully stabilized by the kinetic effects.
- Fishbone branch is destabilized at high β or large $\rho_{q=1}$, and threshold is almost independent of the fluid instability drive.
- Stability threshold β_{cr} is a decreasing function of $\rho_{q=1}$ since $\beta_{\mathrm{cr}} \propto \omega_{\mathrm{D}}^{\alpha}$, and $\omega_{\mathrm{D}}^{\alpha} \propto 1/r$.

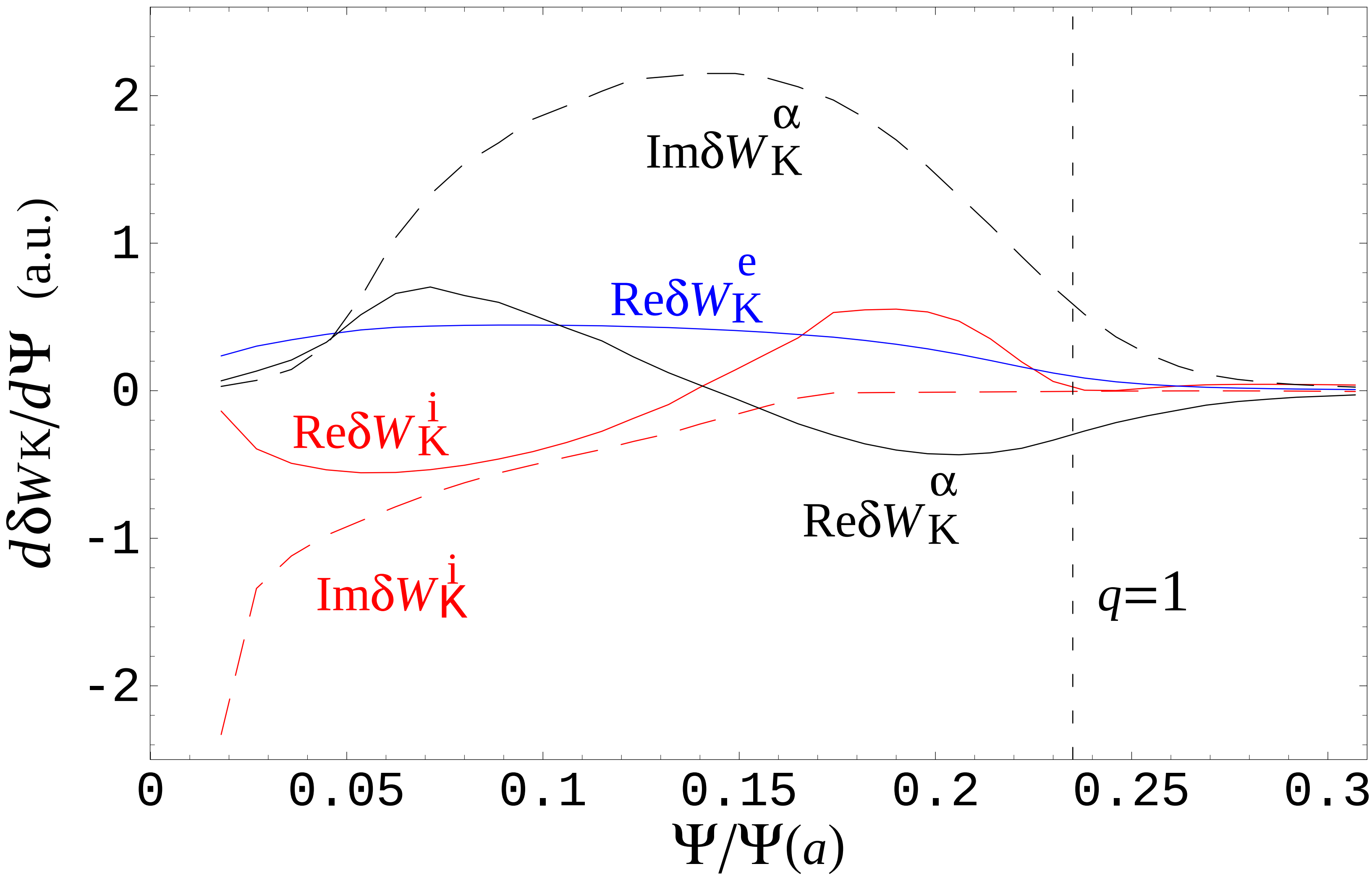
Fishbone at Marginal Stability





- Real frequency is a decreasing function of $\rho_{q=1}$.
- Bulk ion kinetic inertia enhancement is comparable to or larger than the adiabatic counterpart.

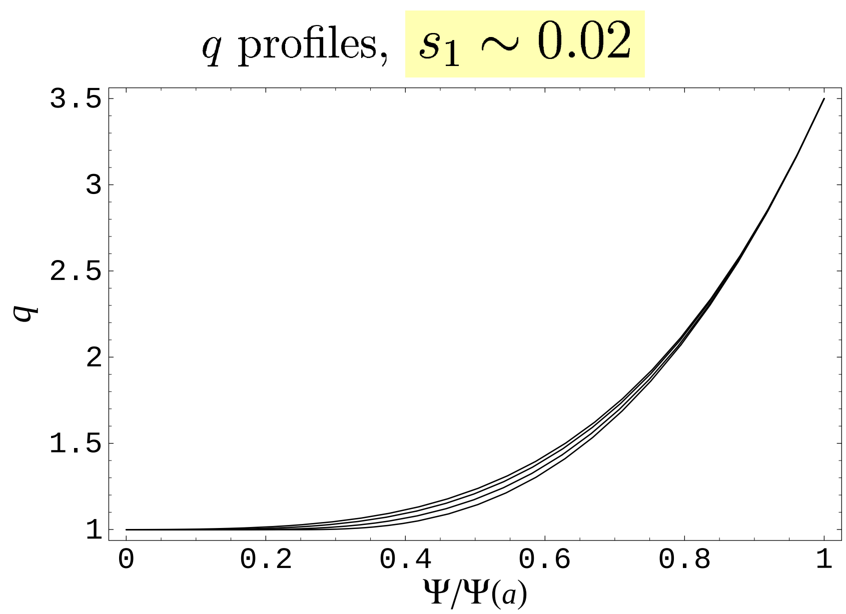
Kinetic contributions to δW for fishbone branch

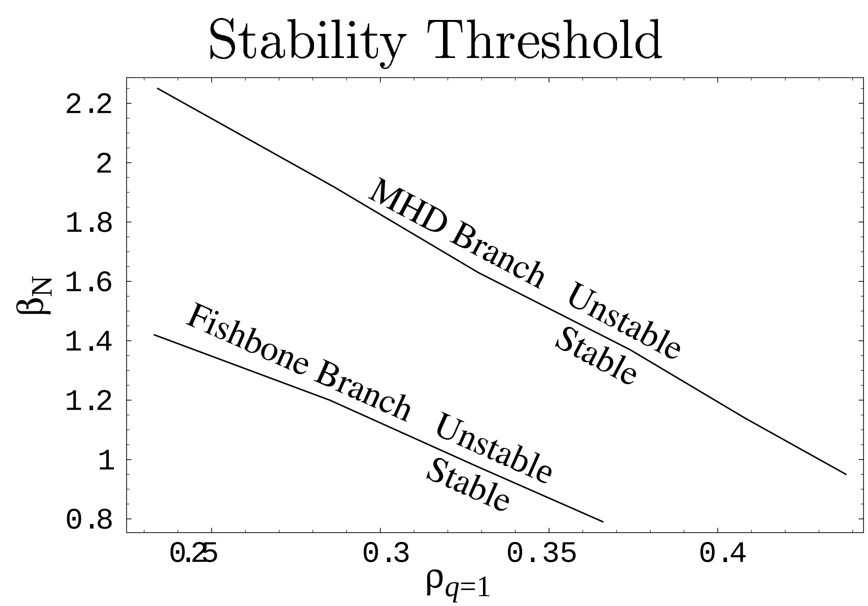


Kinetic contributions to δW

- Suprathermal electrons ($\hat{\varepsilon} \gtrsim 3, \hat{\varepsilon} = \varepsilon/T$) provide the largest kinetic electron contribution to δW_{K} because
  - integrand $\propto \hat{\varepsilon}^{5/2} \exp(-\hat{\varepsilon})$
  - collisionality $\nu_{\mathrm{eff}} \propto \hat{\varepsilon}^{-3/2}$
- Bulk ion kinetic compressibility is reduced by resonance of fishbone mode frequency with bounce/transit frequency and resonant contribution is increased. $\mathrm{Im}[\delta W_{\mathrm{K}}^{\mathrm{ion}}]$ is opposite to $\mathrm{Im}[\delta W_{\mathrm{K}}^{\alpha}]$, thus it is stabilizing.
- α particles can be stabilizing or destabilizing depending on $\rho_{q=1}$.

Quasi-interchange stability thresholds

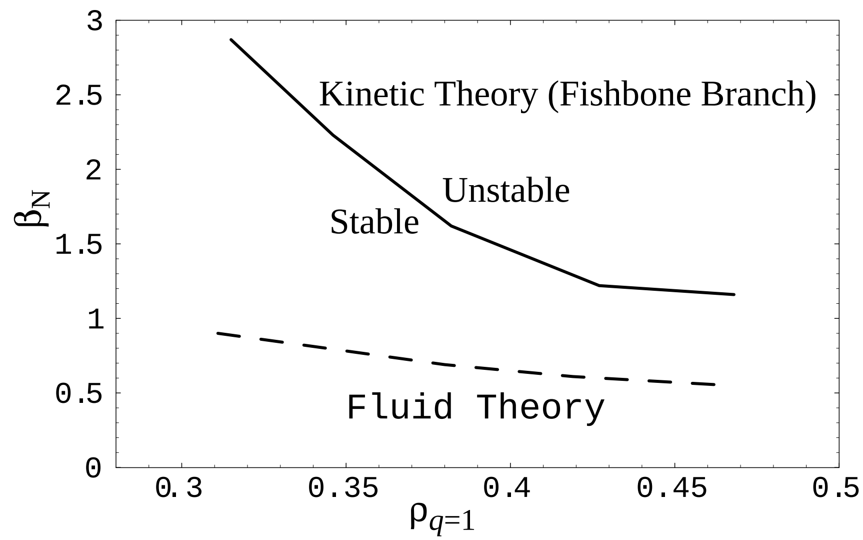




- Two branches can be simultaneously unstable.

Conclusions for internal kink in ITER

- For moderate shear ($s_1 \sim 0.3$), with kinetic effects, MHD branch is fully stabilized, while fishbone branch is destabilized at high β or large $\rho_{q=1}$.



- For low shear ($s_1 \lesssim 0.02$) quasi-interchange, with kinetic effects, fishbone branch threshold is low in β, and MHD branch threshold is high in β.