MHD Stability Based on Transport Analysis

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Thanks to N.Hayashi, S.Tokuda, N.Aiba
JAERI

Workshop on
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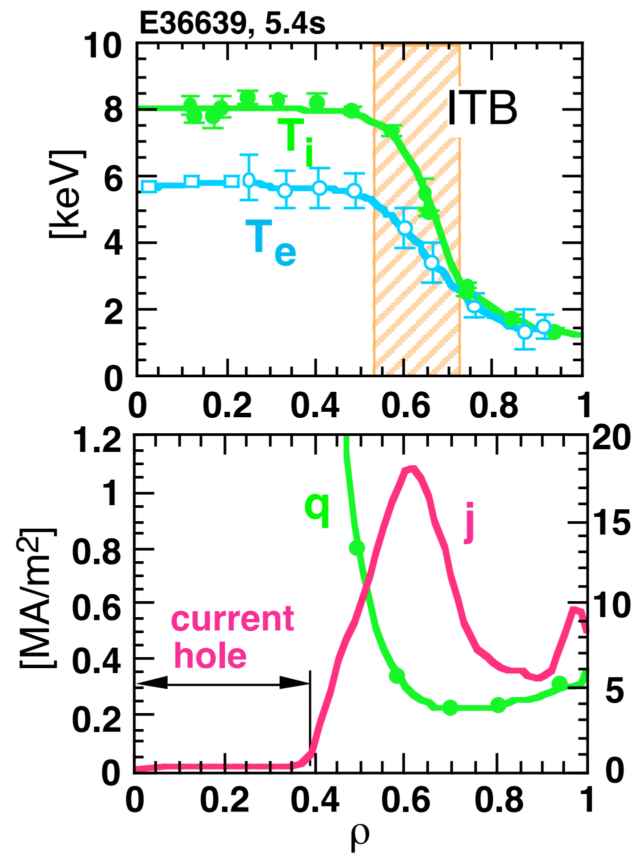
1. Introduction

The current hole with nearly zero toroidal current in the central region has been observed by the MSE measurement.

The current hole plasma was produced by the local large bootstrap current and the strong internal transport barrier (ITB) just inside the q_{min} region.



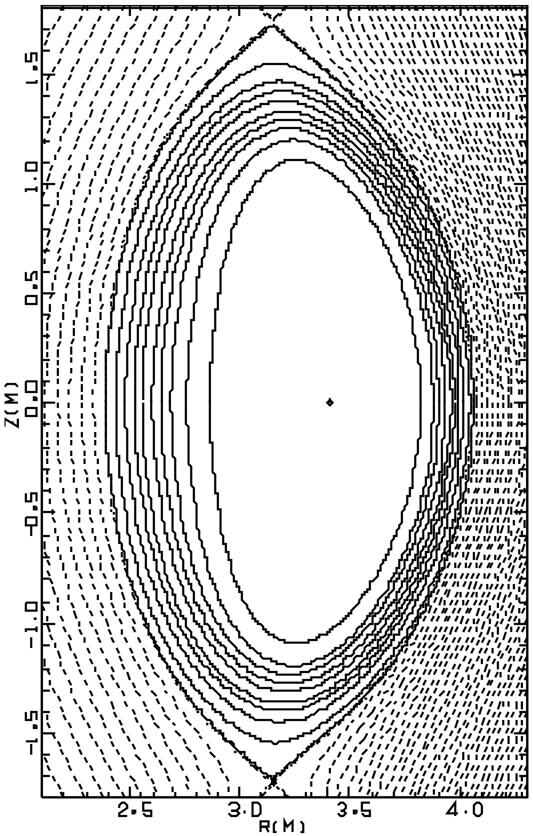
In the CH plasma, the current profile and the pressure profile are strongly coupled each other, and finally the discharge is terminated by the disruption.
Then, this is a good example to explore the stability of the autonomous property and burning plasmas.

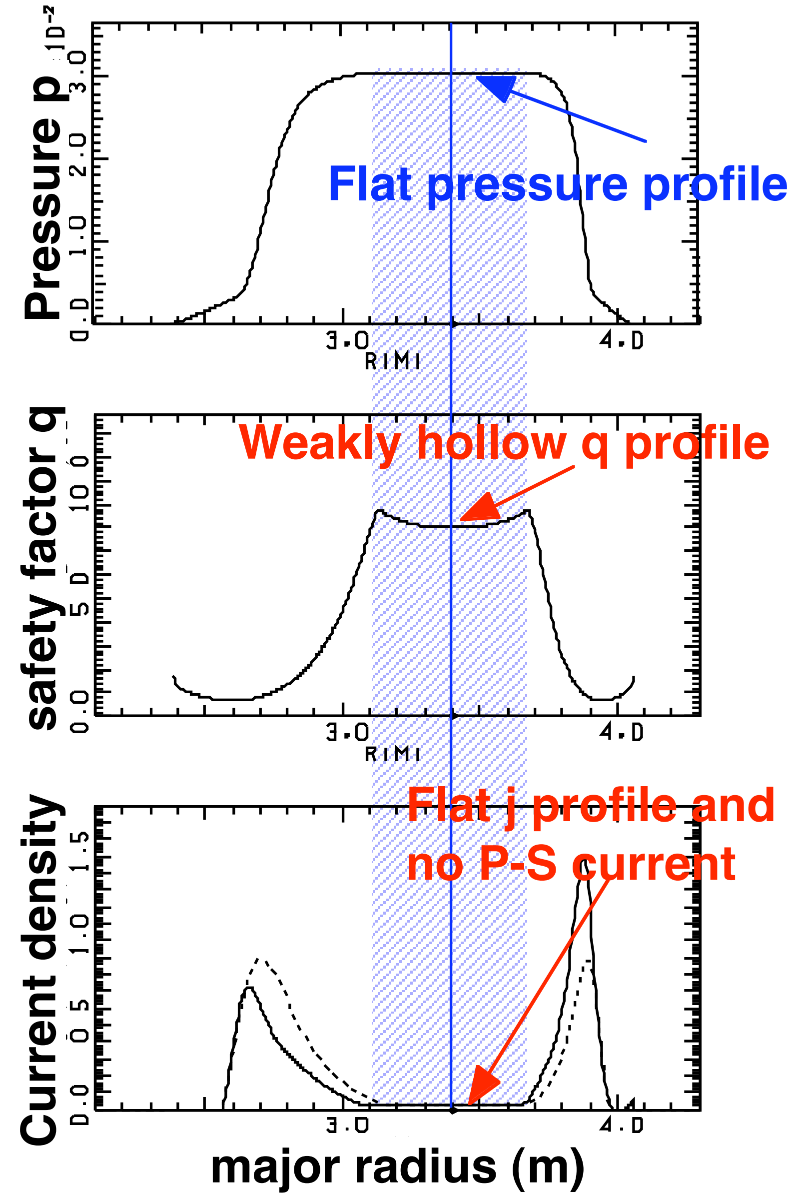
T.Fujita, et al., Phys. Rev. Lett. **87**(2001)245001.

Equilibrium of strongly hollow current plasmas

Grad-Shafranov equation was solved assuming
dp/dp=0, in the hole (r<0.4) and
Extremely small but positive $j_{//}$

q0 of ~80, qmin~6 and βp~1.5
Small Shafranov shift (εβp(core)=0.0).
No Pfirsch-Schluter current in the central region.





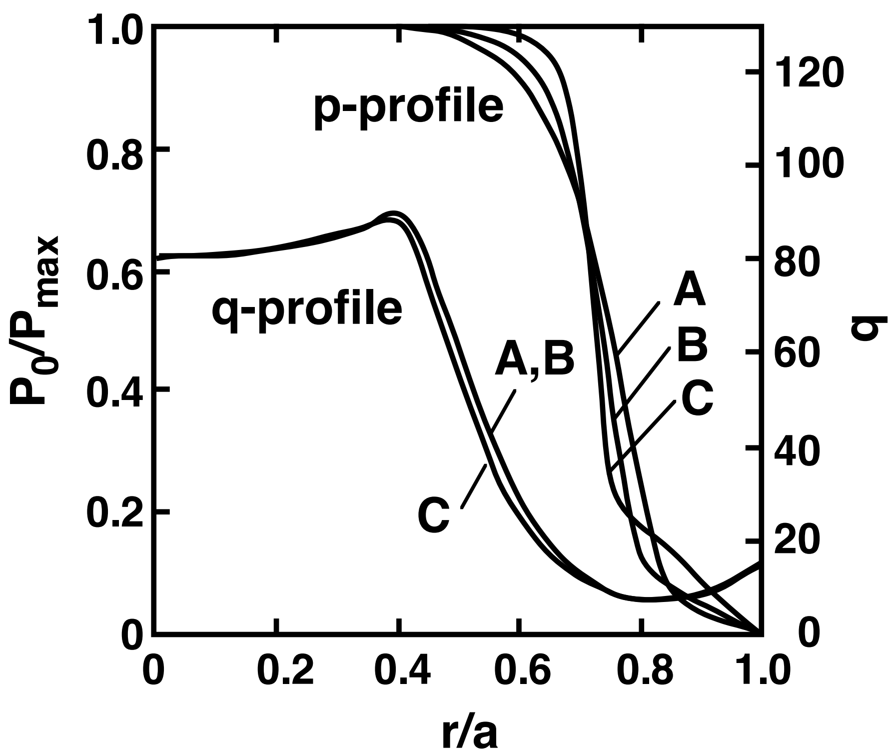
Beta limit of high q0/qmin plasmas

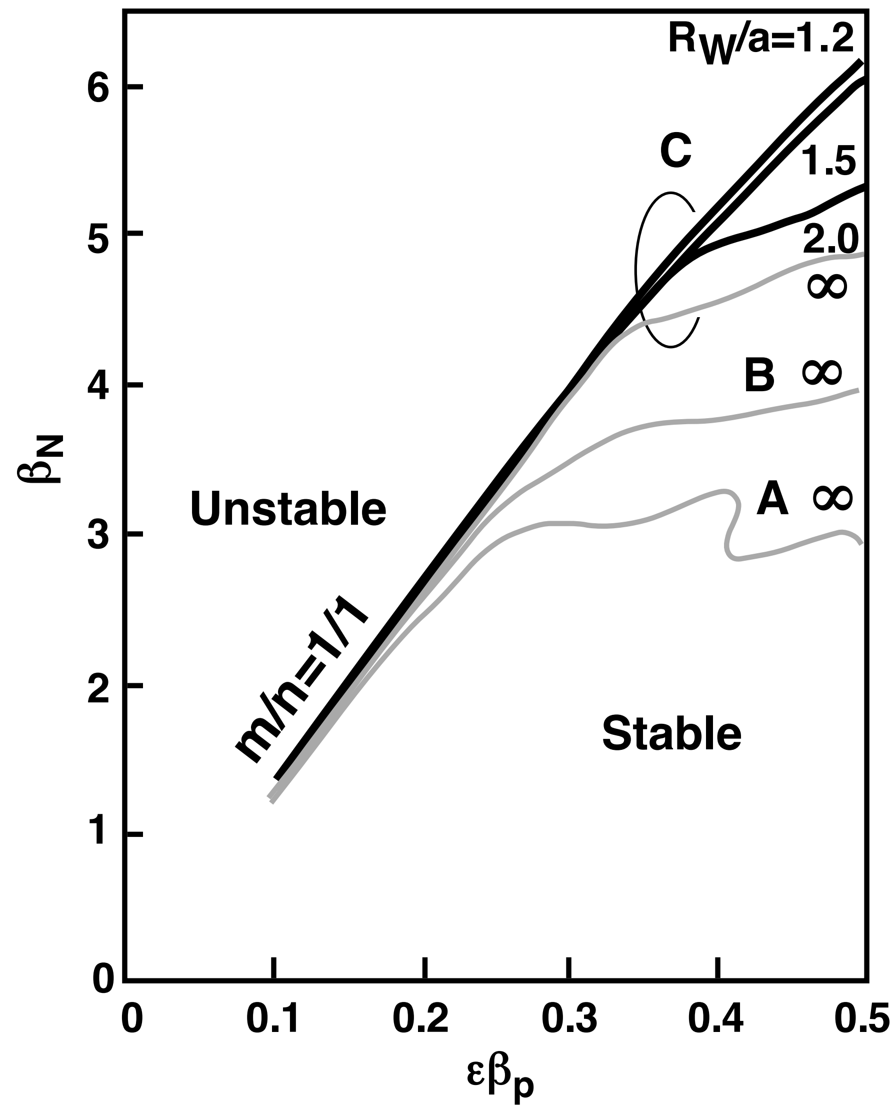
Low n ideal MHD stability : ERATO-J

Stability boundaries are improved by tailoring the pressure profile and improved more by the wall.

Analyses were done for the prescribed profiles.

q_0/q_{min}~12





Self-consistent Analysis

- The current hole plasma has the autonomous property, then it is impossible to tailor the pressure profile and the current profile separately.
- Then, here, MHD stability analysis was done based on the transport simulation, where the pressure and current profiles are consistent.
- The method obtained here is important for estimations and analyses of burning plasmas, such as ITER.
  - High beta steady state, autonomous property, ..

Burning Plasma Simulation Code Cluster in JAERI

Transport code TOPICS

Tokamak Prediction and Interpretation Code
Time dependent/Steary state analyses
1D transport and 2D equilibrium
Matrix Inversion Method for NeoClassical Trans.

- **Current Drive** — ECCD/ECH (Ray tracing, Relativistic F-P), NBCD(1 or 2D F-P)
- **Impurity Transport** — 1D transport for each impurities, Radiation: IMPACT
- **Edge Pedestal**
- **Divertor** — Perp. and para. transport in SOL and Divertor, Neutral particles, Impurity transport on SOL/Div. : SOLDOR, NEUT2D, IMPMC
- **MHD** — Tearing/NTM, High-n ballooning, Low-n: ERATO-J, Low and Mid.-n MARG2D
- **High Energy Behaviour** — OFMC

MHD Stability and Modeling

MHD Behavior	Stability	Modeling
Sawtooth	Ideal/Resistive m/n=1/1 mode	Kadomtsev Model
Island Evolution	Tearing/NTM	Modified Rutherford Eq.
Beta Limits/ Disruption	low n kink high n ballooning	ERATO-J Ballooing Eq.
ELM	Medium n modes high n ballooning	MARG2D Ballooning Eq.
High energy induced instability & particles	TAE/EAE/EPM... Particles loss	Under consideration

2. Simulation model

1.5D transport code

1D transport equations :

Normalized minor radius defined by toroidal flux :

$$\rho = (\Phi(\rho)/\Phi(1))^{0.5} \ (0 < \rho < 1)$$

$$\frac{\partial n_i}{\partial t} = \frac{\partial}{V'\partial\rho}\left(V'\left\langle|\nabla\rho|^2\right\rangle D_i \frac{\partial n_i}{\partial\rho}\right) + S$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_j T_j\right) = \frac{\partial}{V'\partial\rho}\left(V'\left\langle|\nabla\rho|^2\right\rangle n_j \chi_j \frac{\partial T_j}{\partial\rho}\right) + P_j \qquad (j = e,i) \qquad V' = \frac{dV}{d\rho}$$

1D current diffusion equation :

$$\frac{\partial}{\partial t}\left(\rho\frac{\partial\Psi}{\partial\Phi}\right) = \frac{\partial}{\partial\rho}\left\{D\frac{\partial}{\partial\rho}\left(E\frac{\partial\Psi}{\partial\Phi}\right) - S(j_{BS})\right\}$$

Parallel current density

$$j = \frac{2\Phi_1}{\mu_0} D\left\langle R^{-2}\right\rangle^{1/2} \rho\frac{\partial}{\partial\rho}\left(E\frac{\partial\Psi}{\partial\Phi}\right)$$

Impurity : C^{6+}, $T_{imp}=T_i$, assumed profile Z_{eff}

NBI source : Fixed profile, Given deposition ratio of ion to electron, R_{NB}

Neutral : Monte-Carlo method, Given recycling coefficient, R

2D MHD equilibrium : Grad-Shafranov equation (Fixed boundary)
Model of current limit inside CH is applied. (q_{limit}= constant)

2D Equilibrium data

low n/High n MHD stability: ERATO/MARG2D, Ballooning and Interchange

Model of CH was proposed: Axisymmetric Tri-Magnetic-Islands (ATMI) equilibrium

T.Takizuka, et al., J. Plasma Fusion Res. **78**(2002)1282.

ATMI has three islands along the R direction (a central-negative-current island and two side-positive-current islands) and two x-points along the Z direction.

ATMI equilibrium is stable with the elongation coils when the current in the ATMI region is limited to be small.

Stable condition :

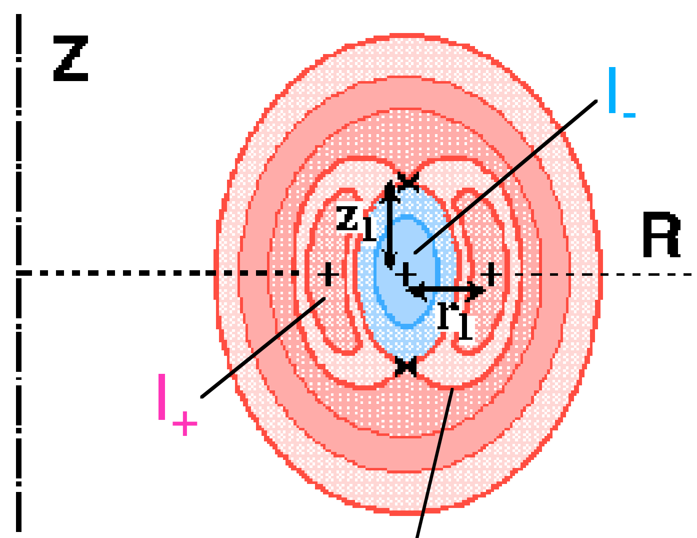
$$\frac{q_{ATMI}}{q_a} > \kappa_1^{\,2} \frac{Z_c^{\,2}}{a(Z_c - Z_x)} \qquad \kappa_1 = \frac{z_1}{r_1}$$

q_{ATMI} : Effective safety factor at the surface of ATMI

q_a : Engineering safety factor at the surface of a whole plasma

Z_x, Z_c : Position of x-points and elongation coils

$$\frac{2I_+}{I_-} \approx \frac{1+\kappa_1^{\,2}}{\kappa_1^{\,2}}$$



$I_{ATMI} = 2I_+ - I_- \sim I_- > 0$

$(\kappa_1 \sim 1)$

JT-60U parameters : $q_{ATMI} > 30$ for $Z_c = 2$, $Z_c - Z_x = 0.6$, $a = 0.8$, $q_a = 4$, $\kappa_1 = 1$

Upper limit of q inside the current hole is modeled based on the ATMI model

N. Hayashi et al. IAEA 2004 TH/1-6

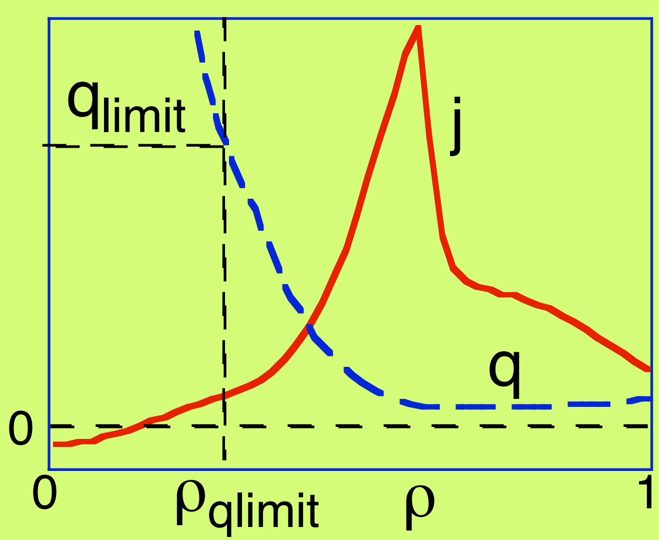
Safety factor at the surface of stable ATMI equilibrium : q_{ATMI}

Safety factor inside CH is limited by $q_{limit} = q_{ATMI}$ in the calculation of the MHD equilibrium and the neoclassical transport.

CH radius is at $q=q_{limit}$ and moves in the simulation.

1D Transport

Negative current inside CH



$q_{eq}(\rho), n(\rho), T(\rho)$ →

←

Geometry

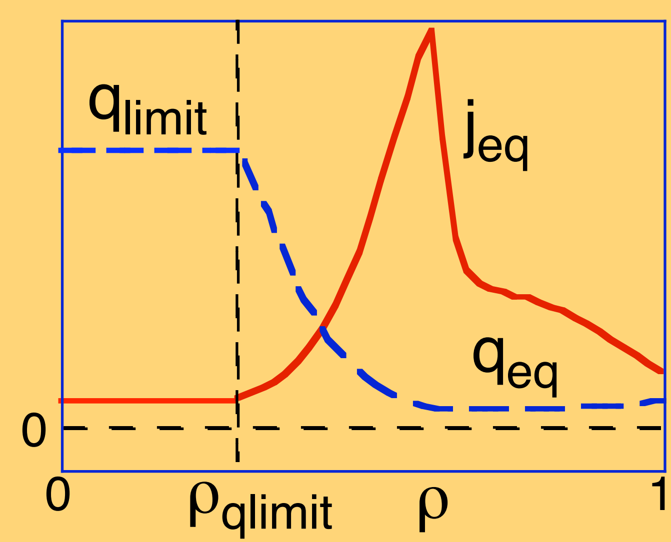
Bootstrap current

Neoclassical diffusivity

Neoclassical resistivity

2D MHD equilibrium

Neoclassical transport (Bootstrap current, diffusivity, resistivity)



Model of transport: Neoclassical inside of s~0 and anomalous outside of s~0

Diffusivities in the transport eqs. :

$$\chi_i = \chi_e = \chi_{neo,i} + \chi_{ano}$$
$$D_i = C_D D_{neo,i} + D_{ano}$$

Neoclassical transport :

Diffusivity and bootstrap current : Matrix inversion method for Hirshman & Sigmar formula (M.Kikuchi, et al., Nucl. Fusion **30**(1990)343.)

Neoclassical resistivity : Hirshman & Hawryluk model (Nucl. Fusion **17**(1977)611.)

Inside the CH region, model of current limit is applied. (q_{limit}= constant)

Anomalous transport : Negative magnetic shear is effective to stabilize the ballooning mode and micro-instabilities. **CDBM-type model**

$$\chi_{ano} = D_{ano} = \chi_0 F(s - k\alpha)$$

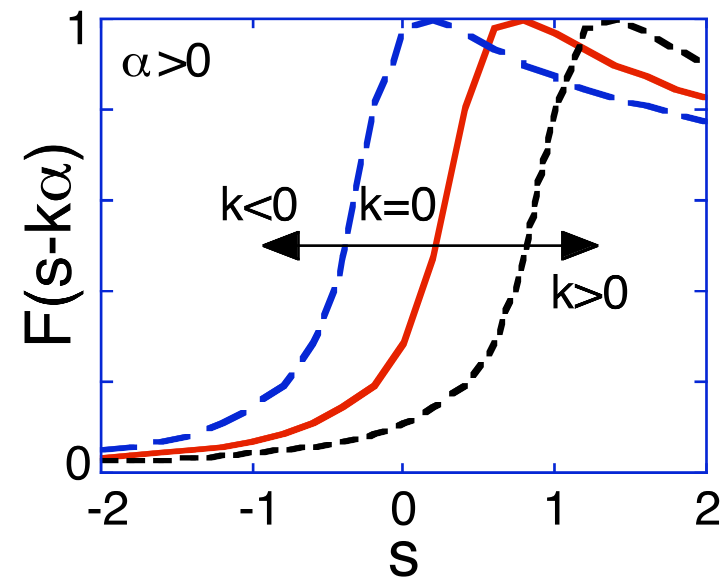
χ_0 : constant anomalous diffusivity

k : arbitrary constant

Function F depends on

s : magnetic shear

α : normalized pressure gradient



CDBM model : F with k=1 was originally developed for the ballooning mode turbulence. (A.Fukuyama, et al., Plasma Phys. Control. Fusion **37**(1995)611.)

3. Results: Comparison with experiments

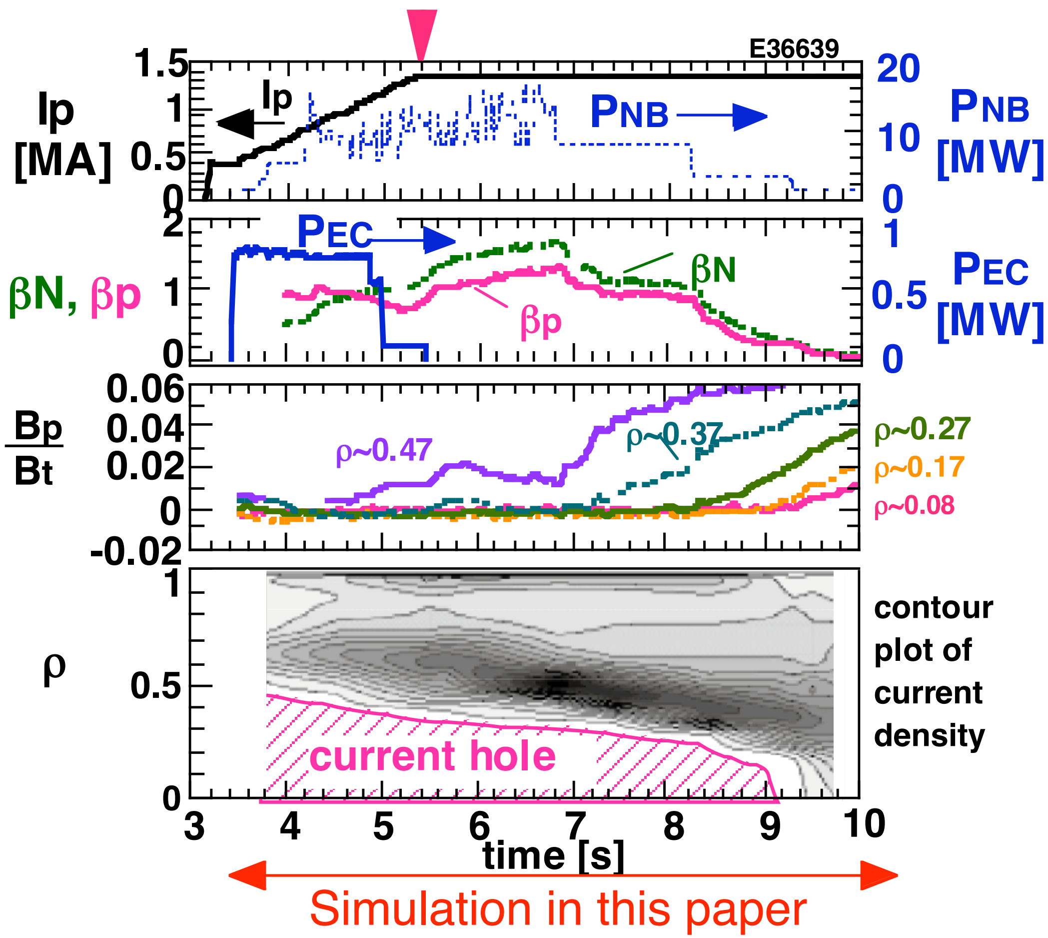
N. Hayashi et al.
IAEA 2004 TH/1-6

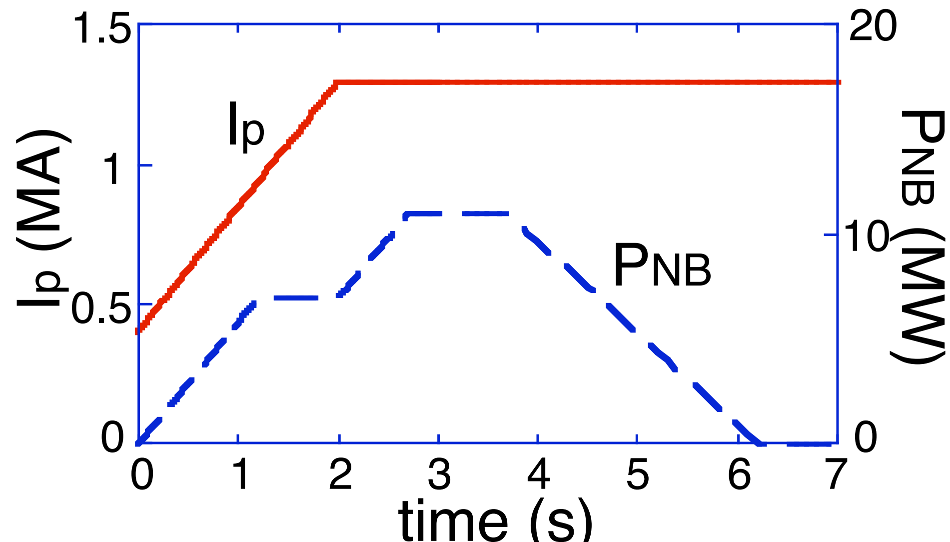
Simulation was done for the shot of E36639.

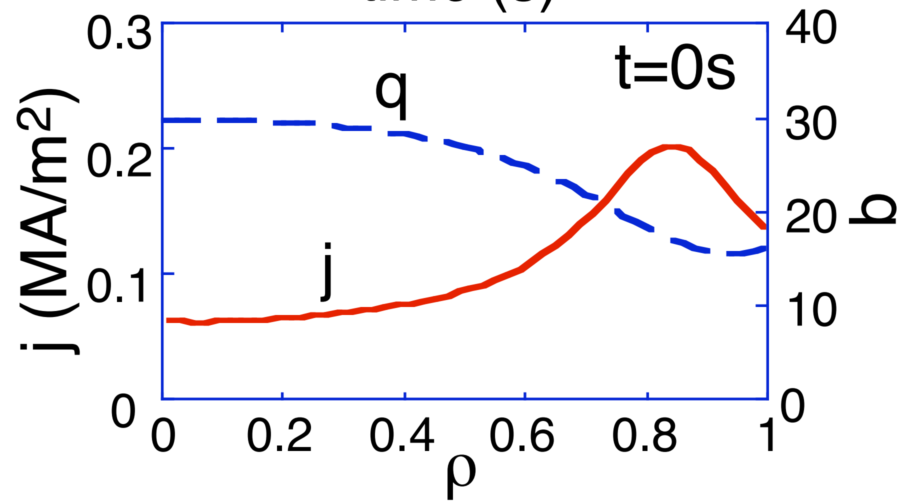
Neutral-beam (NB) is injected during the current ramp-up.

$B_{t0} = 3.7\ \mathrm{T},\ R_0 = 3.3\ \mathrm{m},\ a = 0.8\ \mathrm{m}$

Initial plasma profiles are assumed to be parabolic. ($n_{e0}=10^{19}\mathrm{m}^{-3}$, $T_{i0}=T_{e0}=1\mathrm{keV}$)





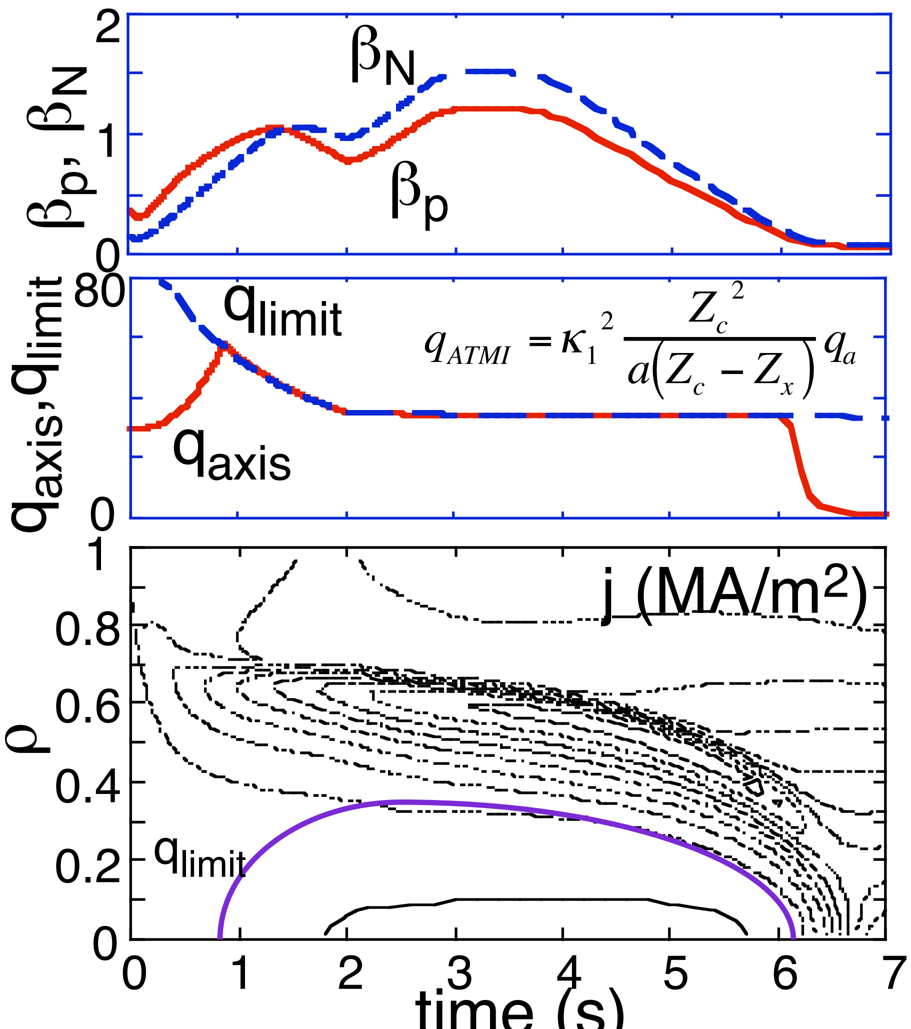


Validation of Transport Model

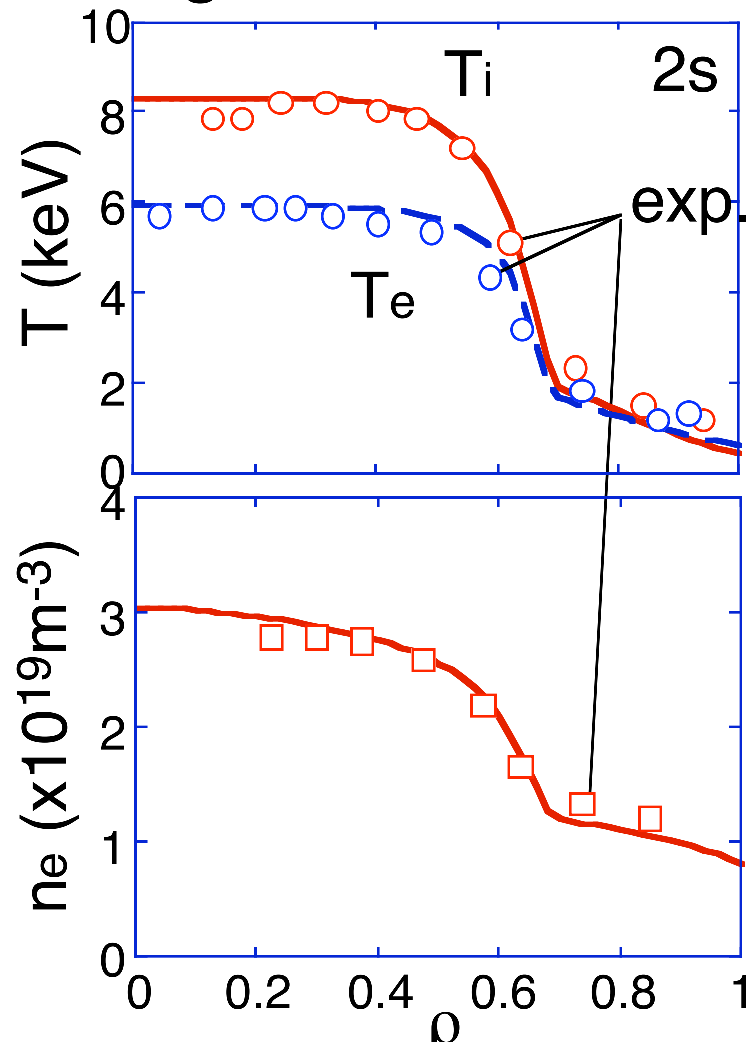
Parameters set to simulate the experiment : $k = 0,\ \chi_0 = 2.6\ \mathrm{m^2/s},\ C_D = 1,\ R_{NB} = 2,\ R = 0.978$

for ITB — for T_i / T_e — for n_e

Normalized beta, poloidal beta, contour plot of current density and profiles almost agree with those in E36639.
Transport is neo-calssical level in the RS region.



$$q_{ATMI} = \kappa_1^{\ 2} \frac{Z_c^{\ 2}}{a(Z_c - Z_x)} q_a$$

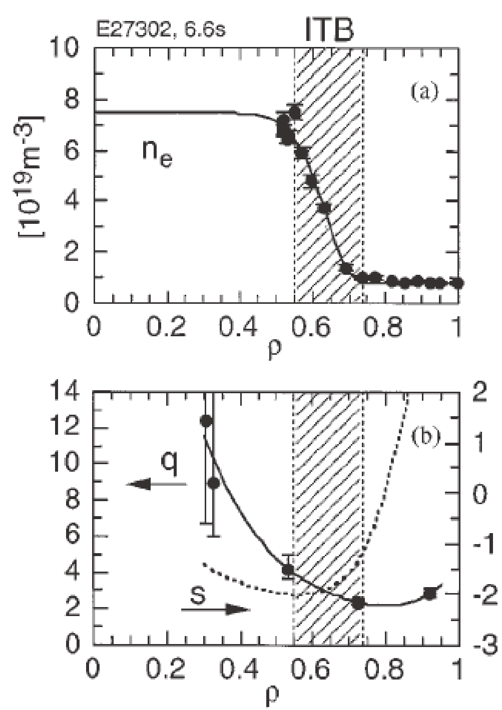


Stability Analysis for results of transport simulation

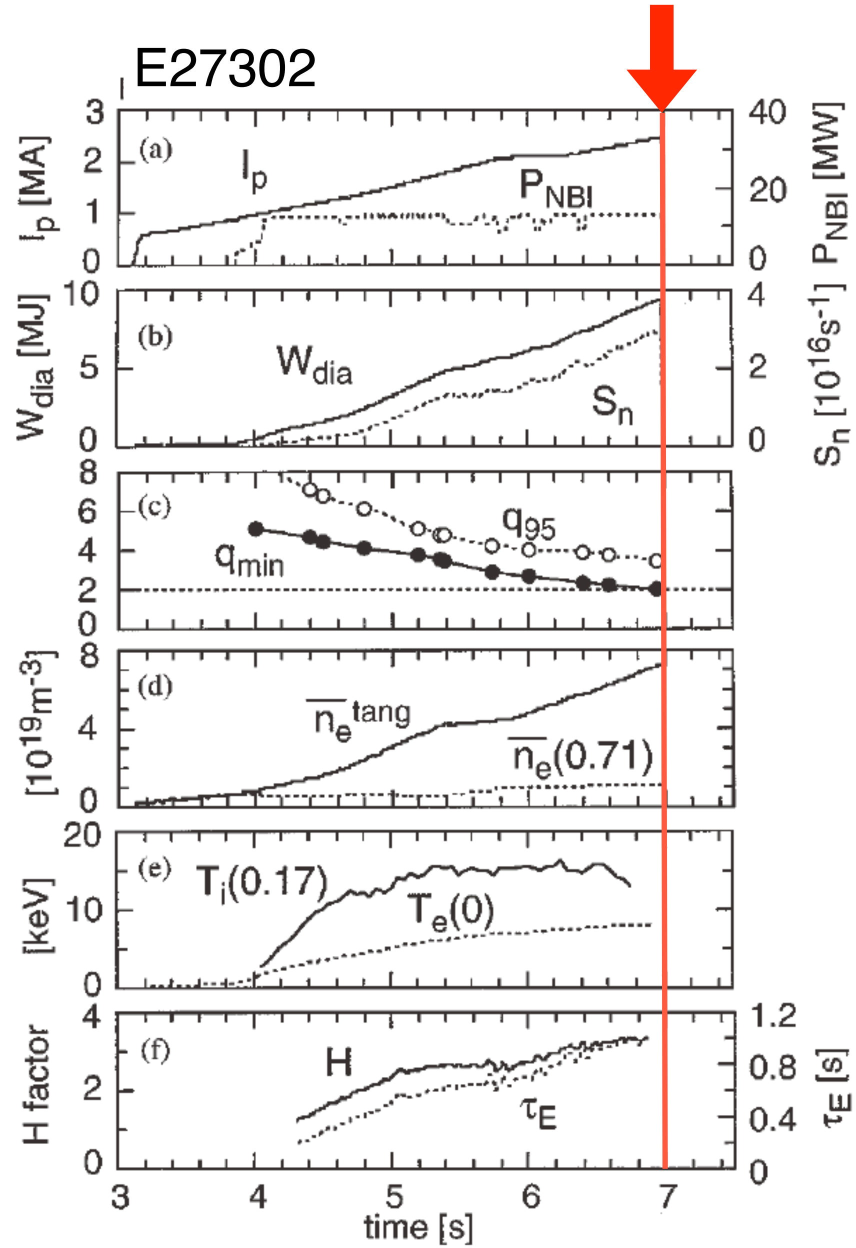
Simulation was done for the high accuracy which can be used for the stability calculation.
Low n modes stability was checked by ERATO-J:
Equilibrium of each time steps in E36639 (previous discharge) are stable for low n ideal modes.
Results are consistent to the observation.

The discharge of E27302 was terminated by the disruption.

Disruption:
βN~1.6
Steep pressure grad.
t~4sec

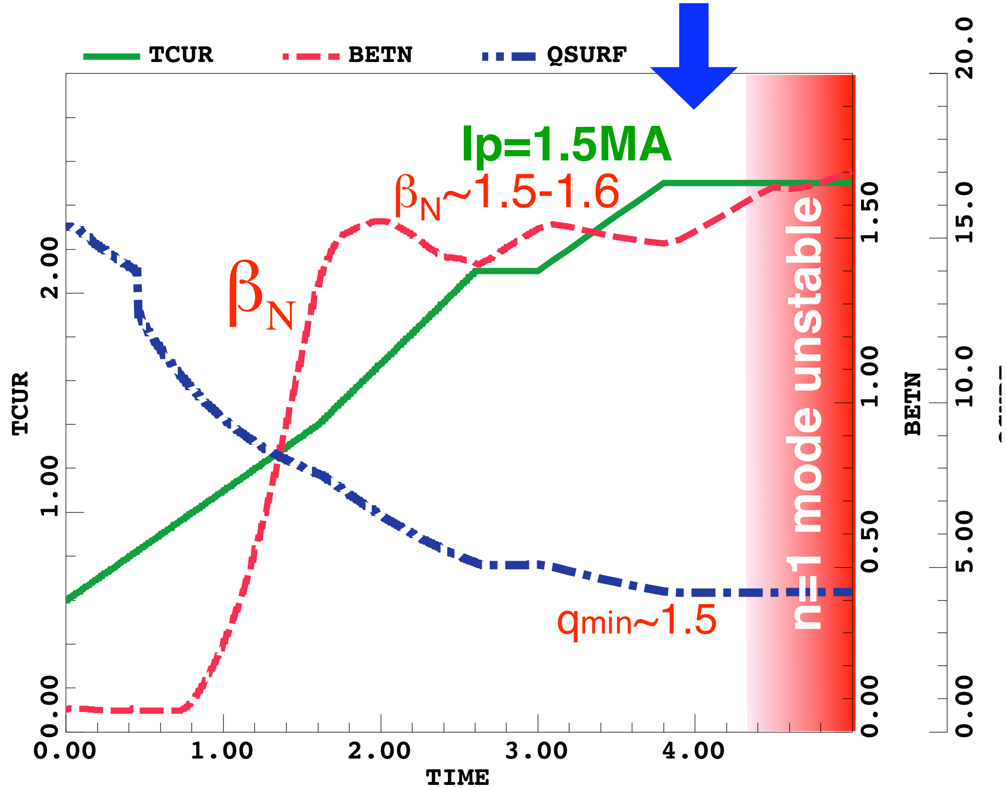


Disruption

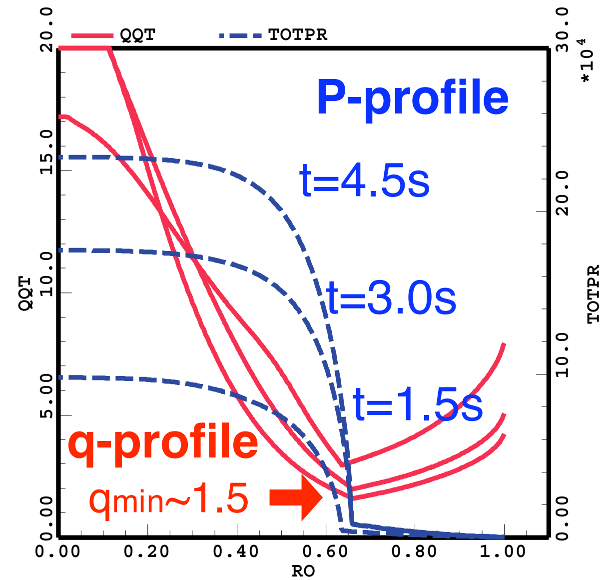


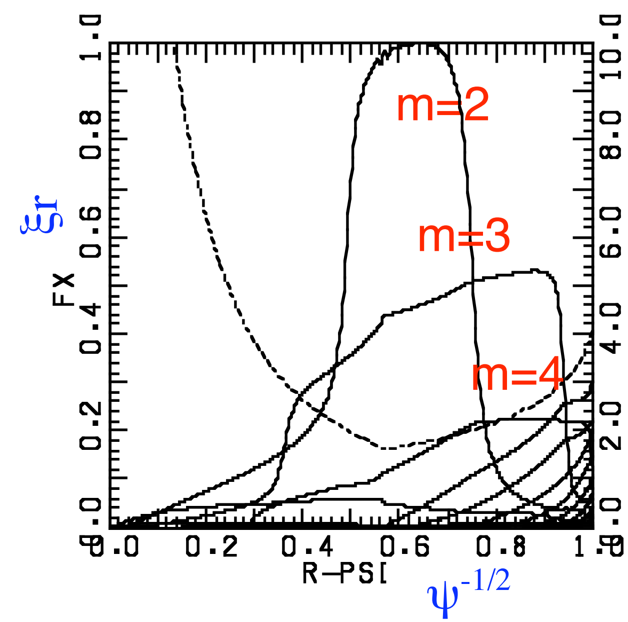
Stability analysis of low n modes

Result of the transport simulation of the case of disruption E27302



Disruption was observed at 4s and the unstable n=1 mode was obtained at 4.4s. Results are roughly consistent to the observation but depend on the profiles.



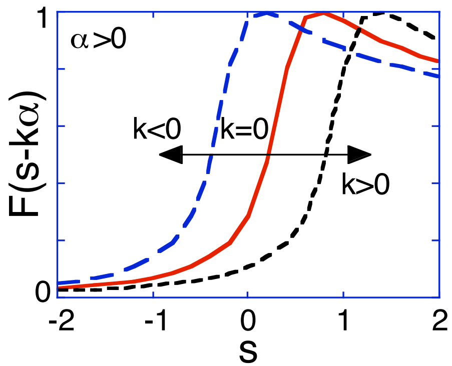


Eigen-function of unstable modes

Influence of transport property on the MHD Stability:

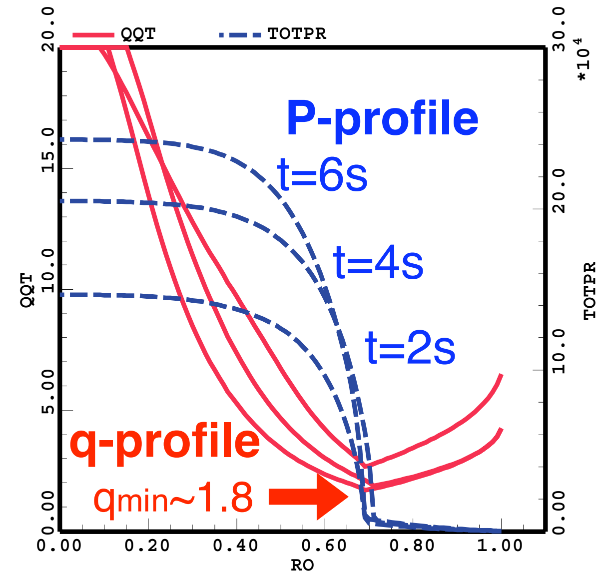
Case of $k\alpha$=0.3

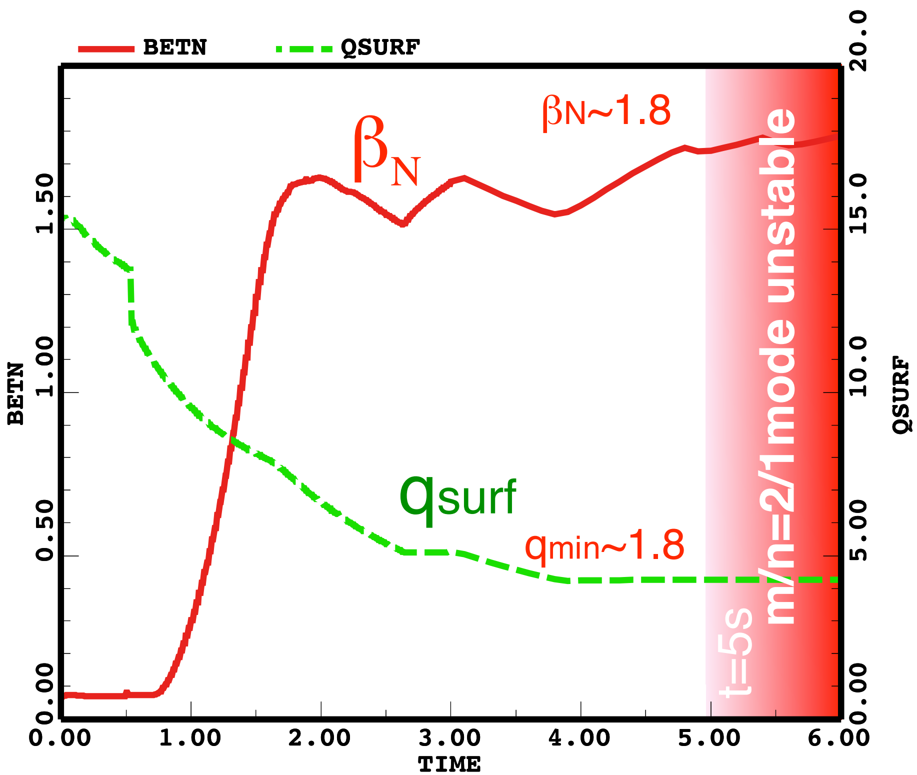
$$\chi_{ano} = D_{ano} = \chi_0 F(s - k\alpha)$$



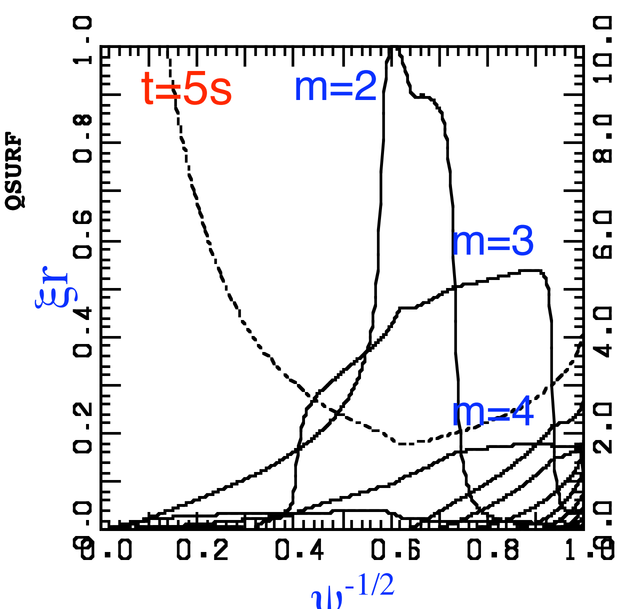
Position of q_{min} moves outward.

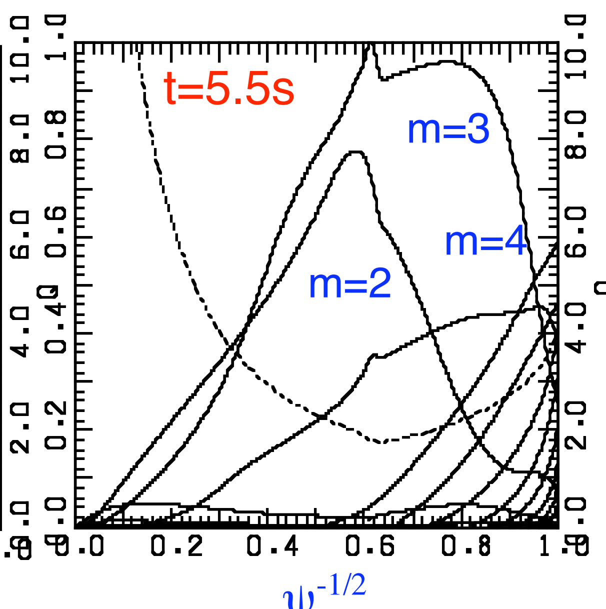
Pressure profile becomes broader, β_N increases up ~1.8 and q_{min} keeps ~1.8.





Global (Internal and external) n=1 mode becomes unstable ~5s.



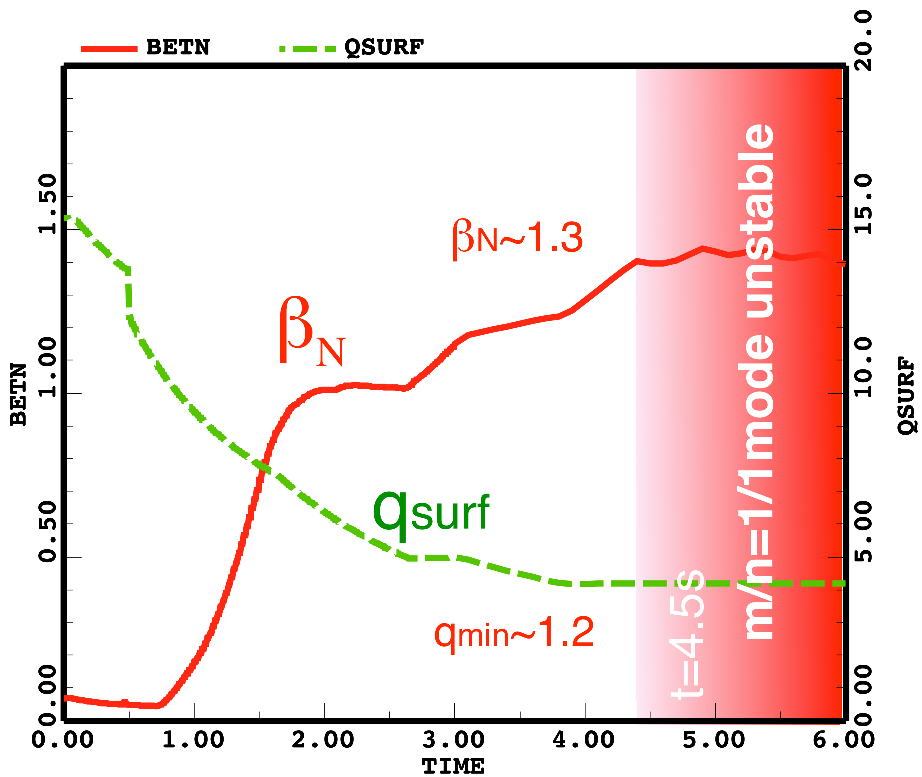


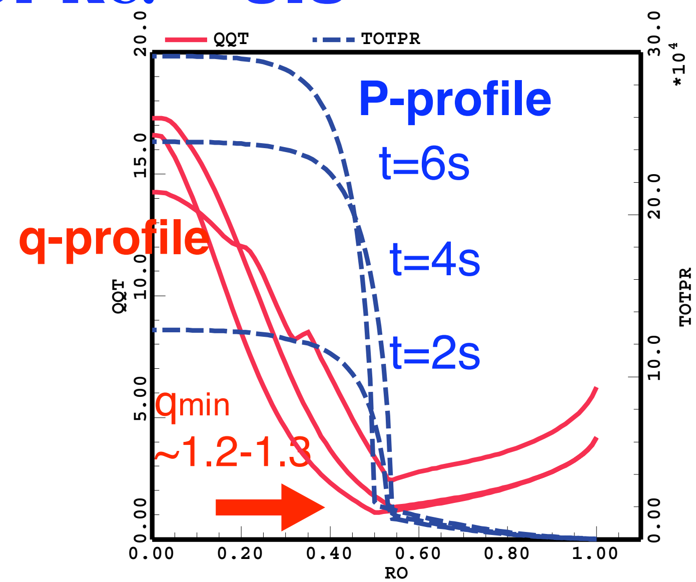
Influence of transport property on the MHD Stability: Case of $k\alpha$=-0.5

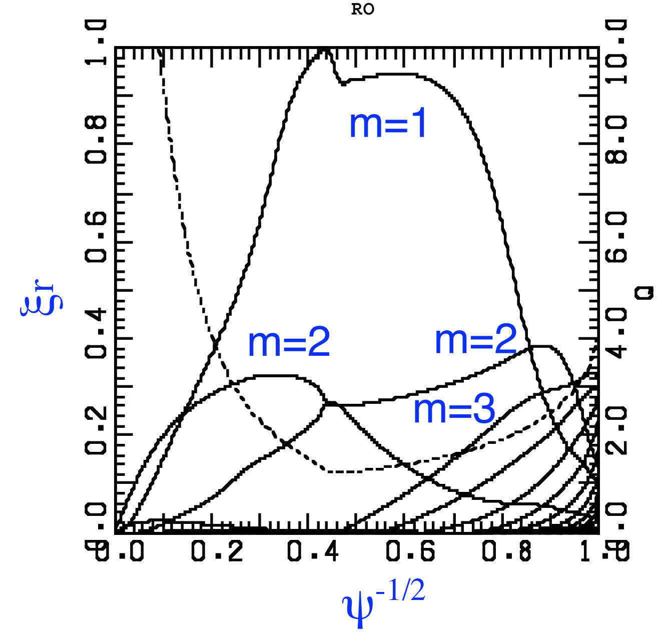
Position of qmin moves inward.

Pressure profiles become peaked and the qmin decreased down to ~1.3

Internal modes of m/n=1/1, etc. becomes unstable due to the low qmin (~1.2).





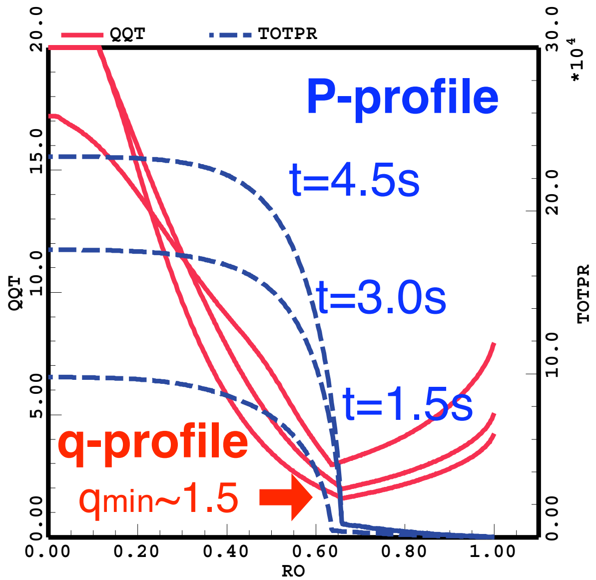


- Broad pressure profile is preferable for high β_N.

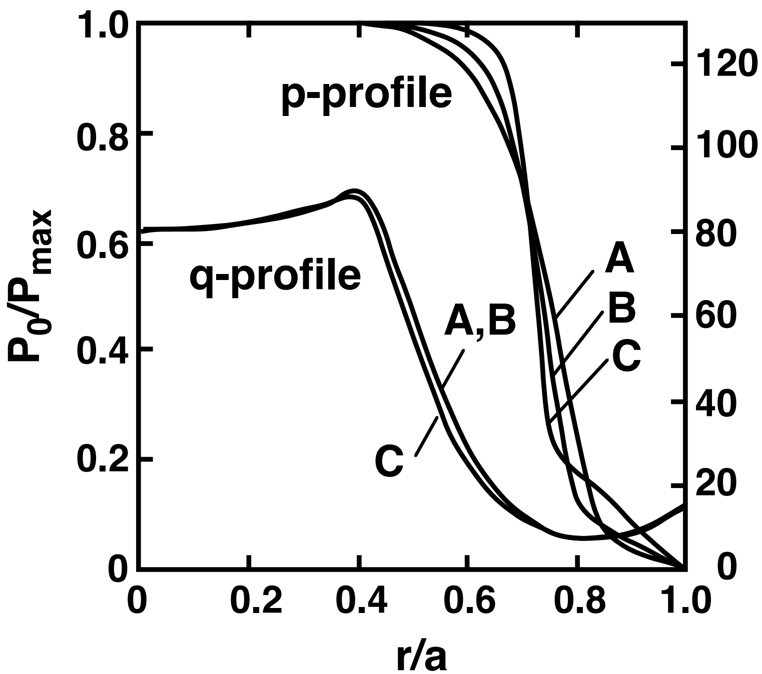
- Transport property is sensitive to making the profiles.

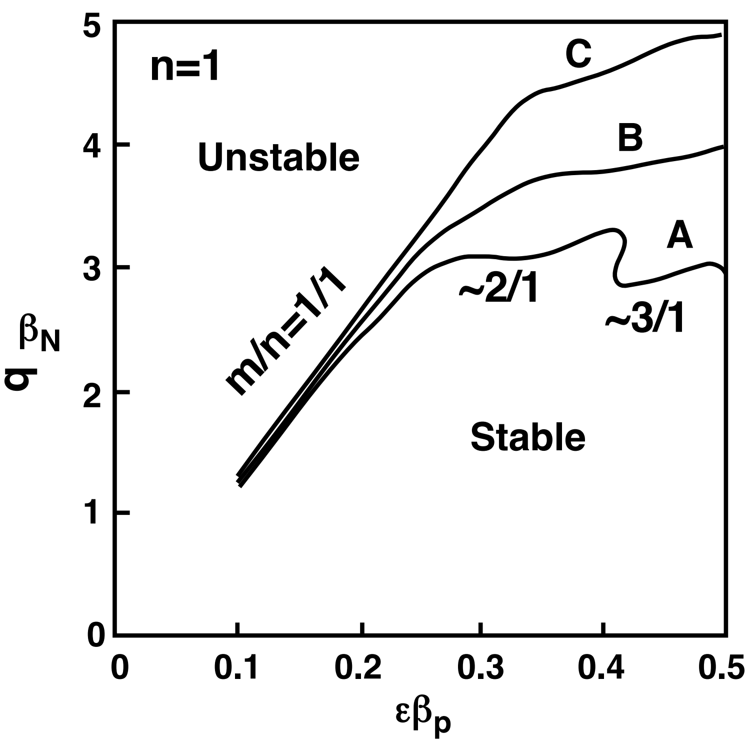
Profile control is important to sustain the plasma.

Discussion: Importance of detailed profiles on the MHD Stability



The q-profile has a sharp structure near q_{min}, which may be sensitive to the MHD stability. This is due to the sharp current density profile of the bootstrap current.
The current profile may be more relaxed, and it becomes the beta limit higher.
For k<0, the profile of case B or C can be expected, but not obtained yet.

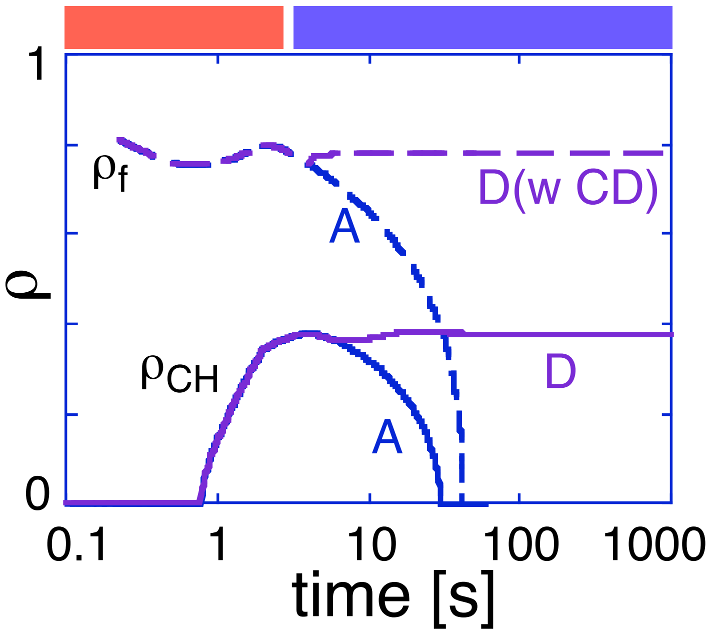




The relaxation of the current profile becomes the q-profile near q_{min} smooth. The location of ITB will moves inward due to the weak negative shear, and it improves the beta limits as shown in figures

4. Results : Simulation of Sustainment and Control of Current-Hole Plasma by Current Drive

[N. Hayashi et al. IAEA 2004 TH/1-6]

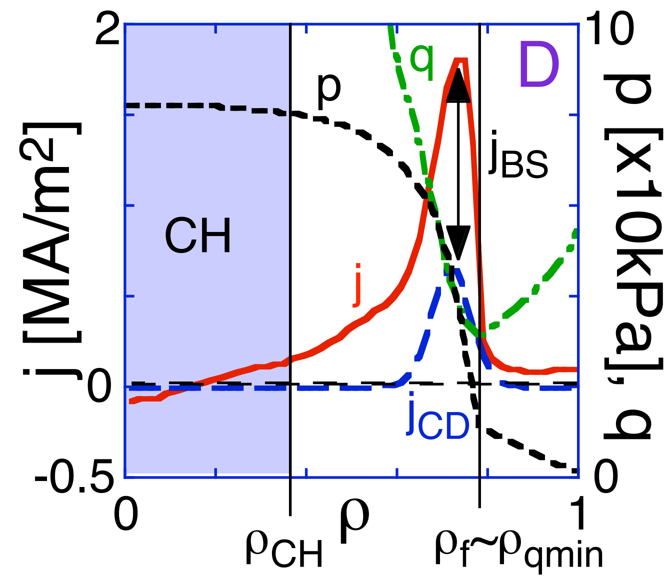


During the CH formation phase (t < 2 s), CH radius (ρ_{CH}) and ITB foot radius (ρ_f) can be expanded by increasing the heating power.

After the formation phase,

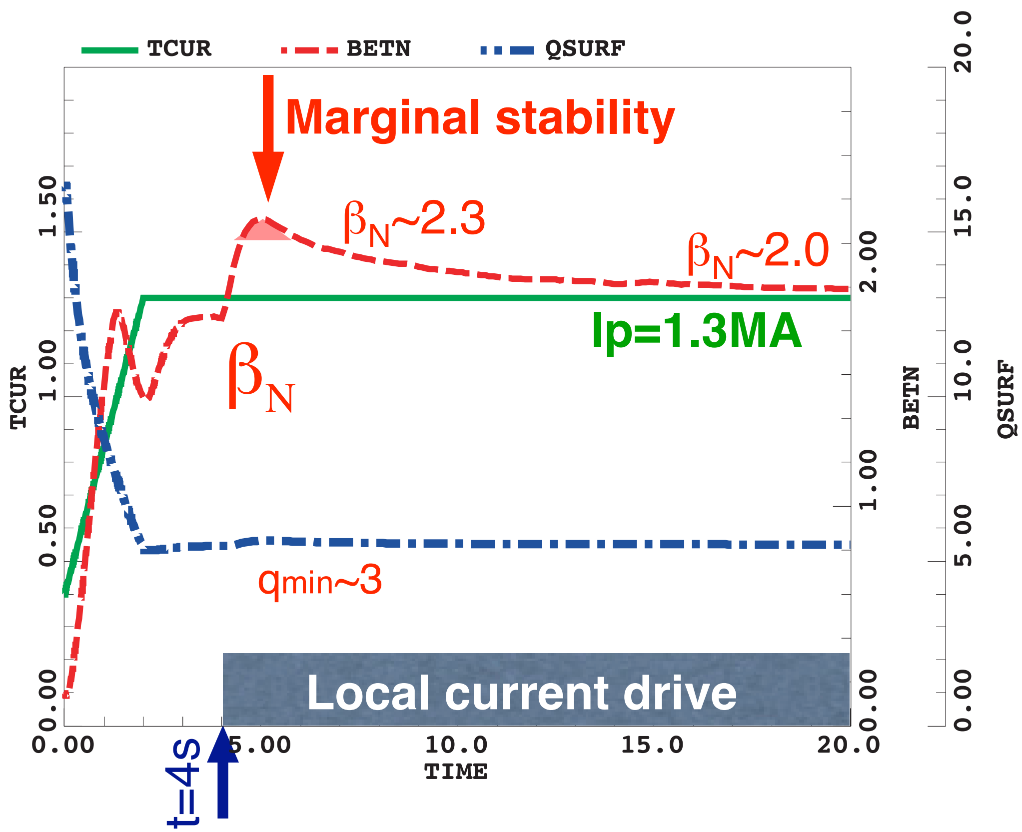
Plasmas with small ρ_{CH} and $f_{BS}<1$ shrinks due to the penetration of inductive current (case A, P_{NB}=12 MW).

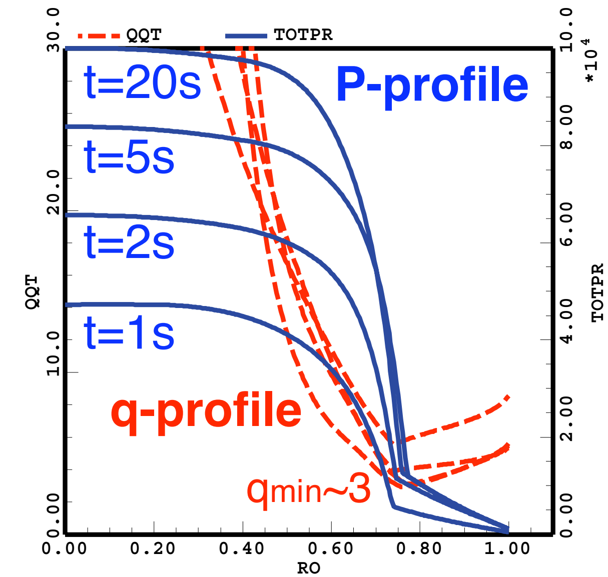
External CD (ex. EC) can prevent the shrinkage of case A by adding at the local current with $f_{BS}+f_{CD}$~1 (case D).

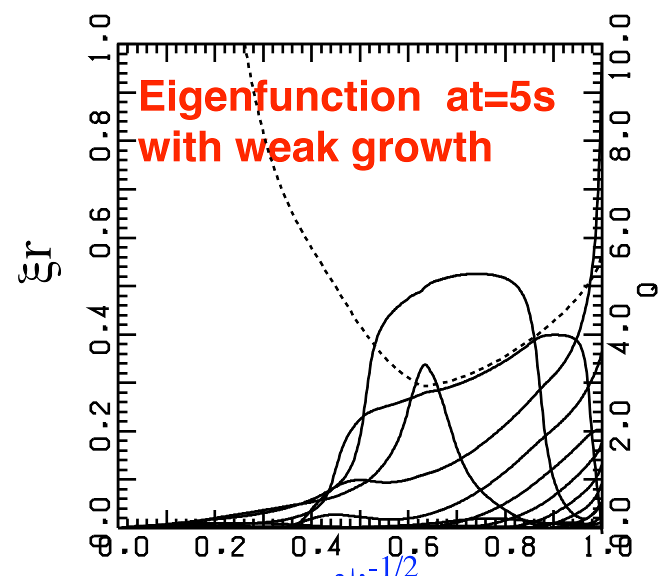


Stability analysis of the simulation

Result of low-n ideal MHD stability of the transport simulation of caseD.





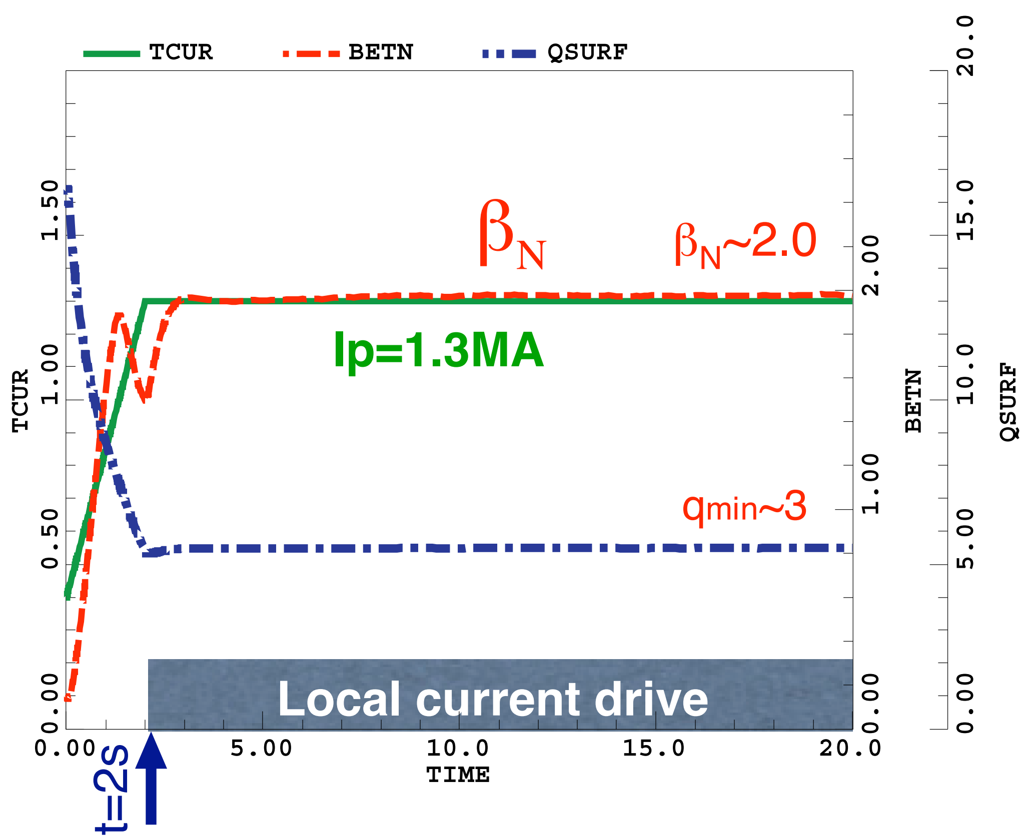


Local current is injected from 4s. The negative shear increases inside qmin, the χi decreases and the beta value increased.

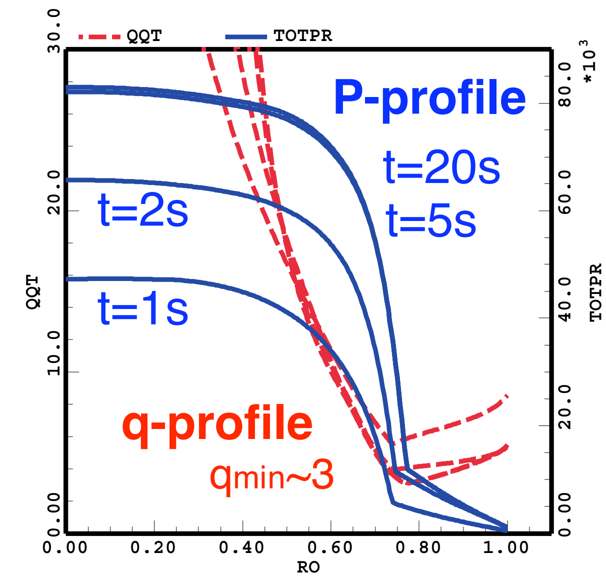
When the beta value overshoots, the plasma has the marginal stability.

Stable route by current drive and autonomous property

Early injection of the local current suppress the overshoot of beta_n.



By the early injection of the local current, the beta increases by the improvement of the transport, but the beta is saturated autonomously.



Autonomously property softly limits the beta near the plasma center. [N. Hayashi et al. IAEA 2004 TH/1-6]

$$\varepsilon_f \beta_{p,core} = C_1(\approx 0.25)$$

$\beta_{p,core}$: core poloidal beta inside ITB

The ideal low n modes are stable for these plasmas.

5. Summary

- Stability analysis consistent to the transport property was done for strongly reversed shear plasmas.
- The profiles for high beta plasmas were searched, using the transport results, but not obtained yet.
  - The detailed q-profile is slightly different to the experiment.
  - The transport model should be improved.
- This is the effective method to explore the scenario in burning plasmas, such as ITER
  - Key point is the use of soft limitation of the autonomous property and current profile control to avoid the hard MHD limit.
- Further modification of the modeling and the optimization of the profiles are need.
  - More integration of modeling are also need: Effect of MHD instability on the transport and effect of burning such as alpha heating, high energy particles, ...

Transport Model for Current Diffusive Ballooning Mode (CDBM)

A.Fukuyama, et al., Plasma Phys. Control. Fusion **37**(1995)611.

Turbulent thermal diffusivity

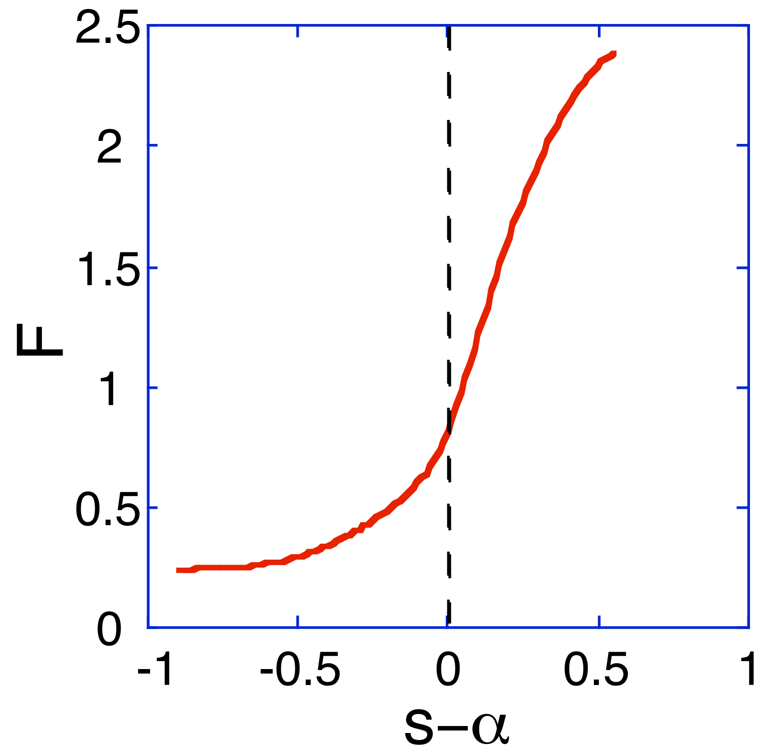
$$\chi_{CDBM} = F(s,\alpha)\alpha^{3/2}\frac{c^2}{\omega_{pe}^{\;2}}\frac{v_A}{qR}$$

Magnetic shear : $s = \dfrac{r}{q}\dfrac{dq}{dr}$

Pressure gradient : $\alpha = -q^2 R\dfrac{d\beta}{dr}$

Fitting formula :

$$F(s,\alpha) = \begin{cases} \dfrac{1}{\sqrt{2(1-2s')(1-2s'+3s'^2)}} & (s' = s-\alpha < 0) \\ \dfrac{1+9\sqrt{2}s'^{5/2}}{\sqrt{2}(1-2s'+3s'^2+2s'^3)} & (s' = s-\alpha > 0) \end{cases}$$



Weak / negative magnetic shear and Shafranov shift reduce thermal diffusivity.