Active Feedback of Levitated Dipole Coil

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"Control of MHD Stability: Back to the Basics"

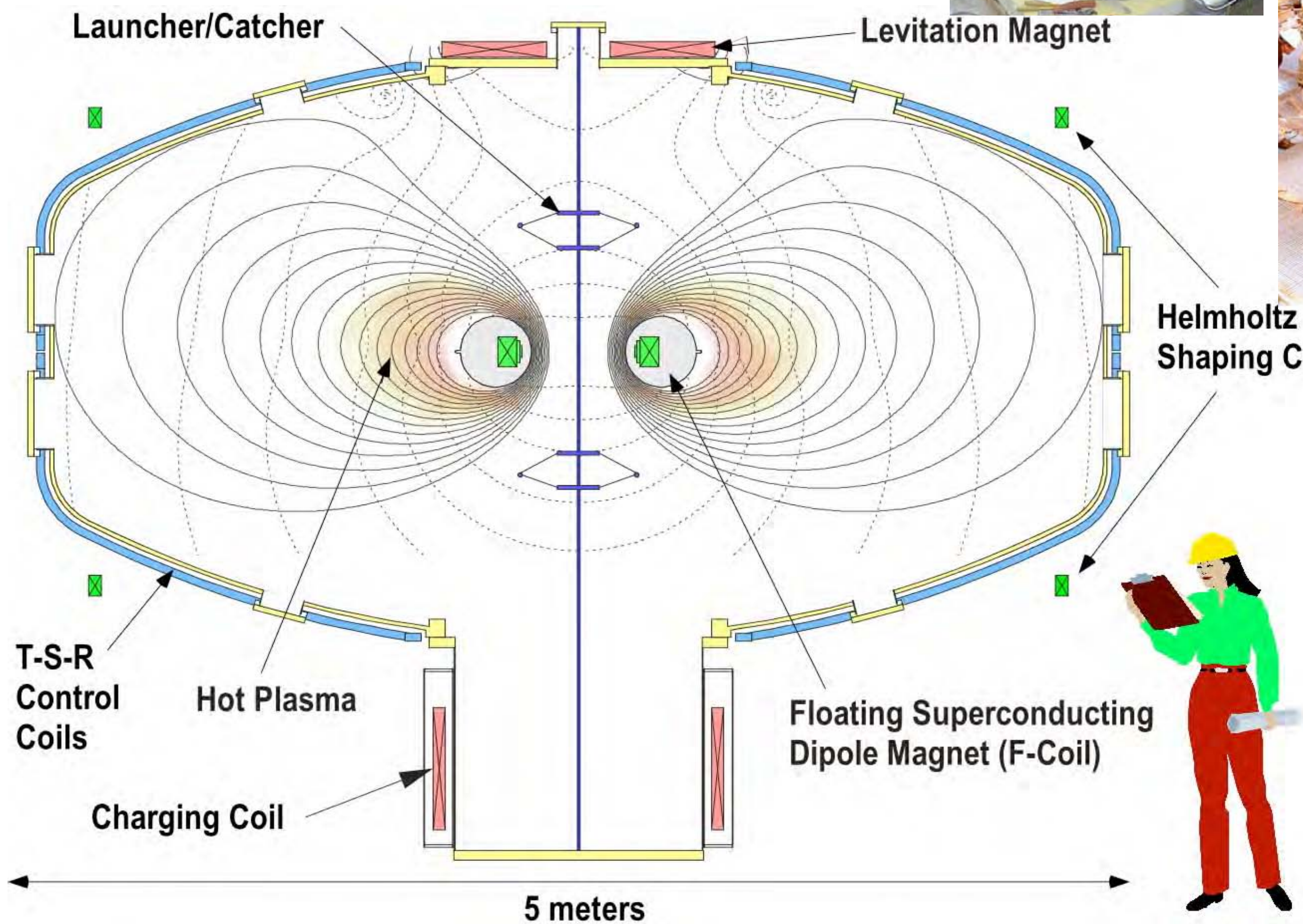
Outline

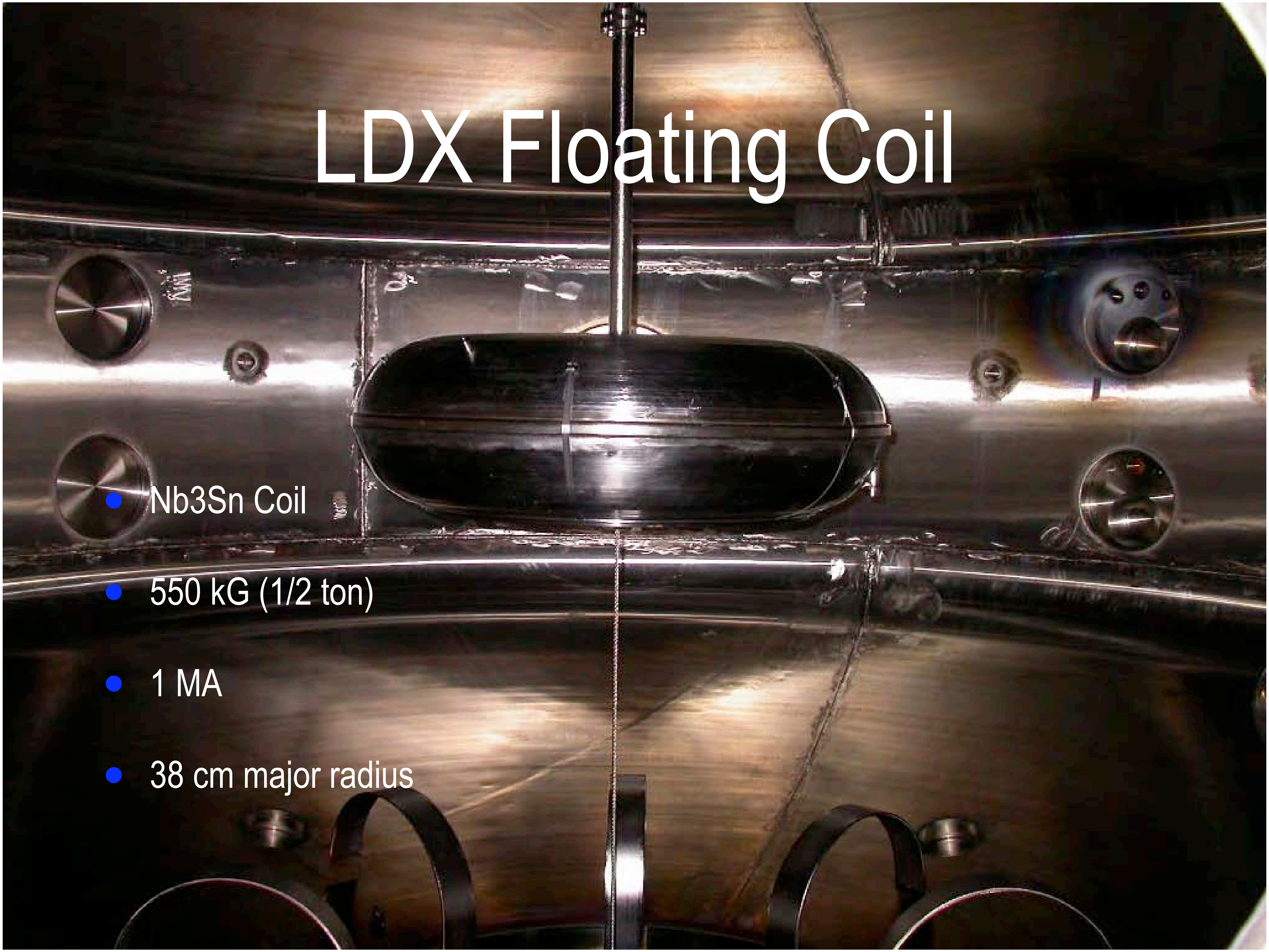
- Description of the problem: Can we use a High-T_c superconducting magnet to feedback control a $\frac{1}{2}$-ton, 1 MA, superconducting ring?

- The Kalman Filter (for "noise resistant" object tracking)
  - Introduction
  - Tracking a pendulum
  - Application to LDX

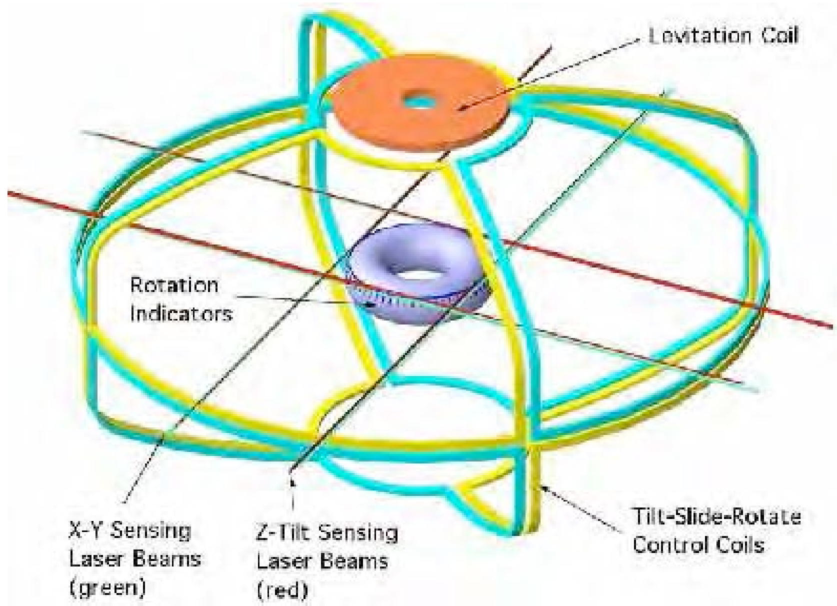
- Some initial thoughts on Kalman "tracking" of the external kink for "noise-resistant" mode control feedback.

LDX Experiment Cross-Section




LDX Floating Coil
Nb3Sn Coil
550 kG (1/2 ton)
1 MA
38 cm major radius

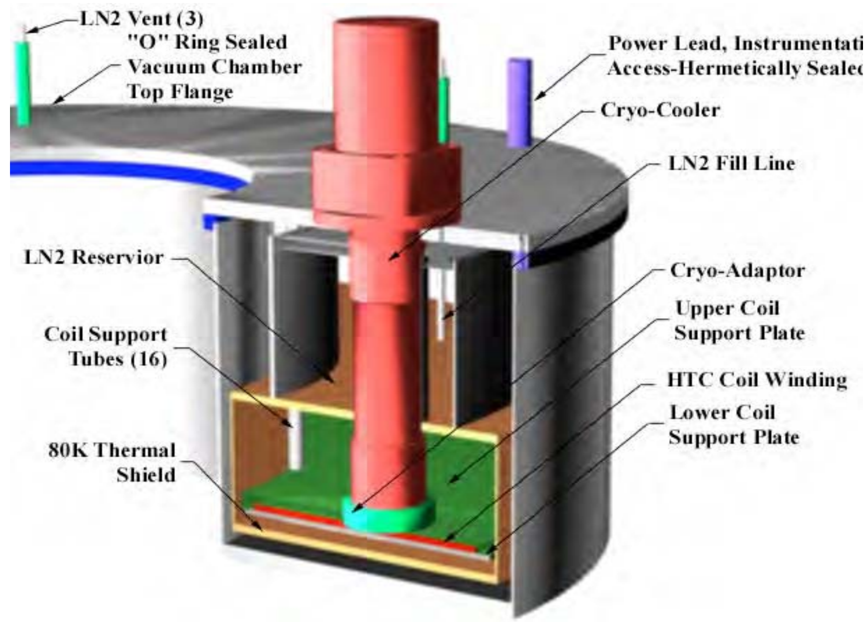
Laser Position System

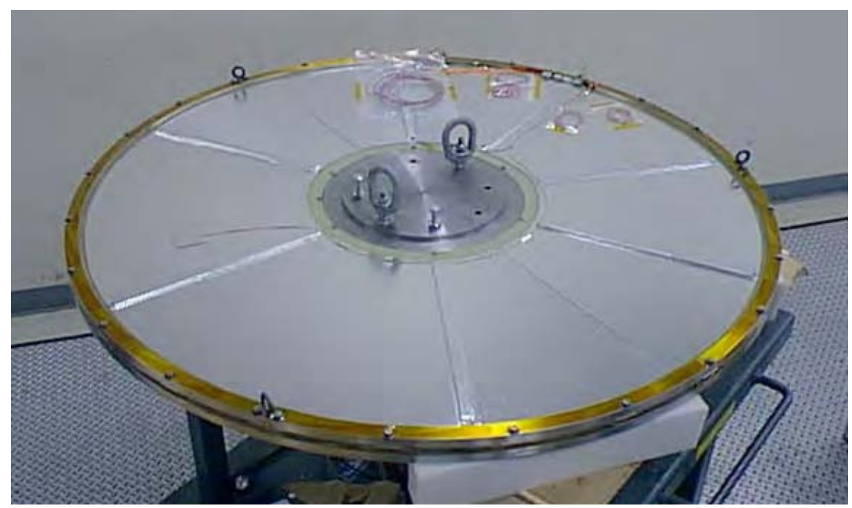


- The control system will use an optical position detection system designed specifically for LDX. Eight position detection channels will give information about the five degrees of freedom in the system.
- A digital feedback system will provide the control to the L-Coil current, stabilizing the vertical position of the F-Coil.
- Auxilliary coils will be used to damp oscillations in the other degrees of freedom.

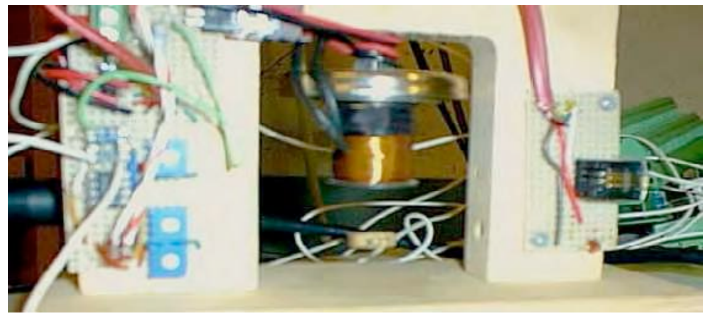
High T_c Superconducting Levitation Coil

- SBIR collaboration with American Superconductor
  - First HTS coil in the fusion community
  - Uses available BSSCO-2223 conductor
- Operational temp 20-25° K
- Feedback gain selected for 5 Hz mode frequency
  - < 20 W AC loss
- 20 kJ stored energy
  - Emergency dump in < 1 second.
- Coil Completed & Tested
  - 77° K superconducting tests successful
  - 20° K tests complete
  - Preliminary assessment: GOOD!





Levitated Cheerio Experiment



- The Levitated Cheerio Experiment (LCX) is a proof of concept experiment designed and built by Dr. Darren Garnier
- The physics for the Levitated Dipole Experiment are the same as those for the Levitated Cheerio Experiment, but on a much larger scale
- LCX proves the experimental feasability of a vertical-stabilizing feedback system

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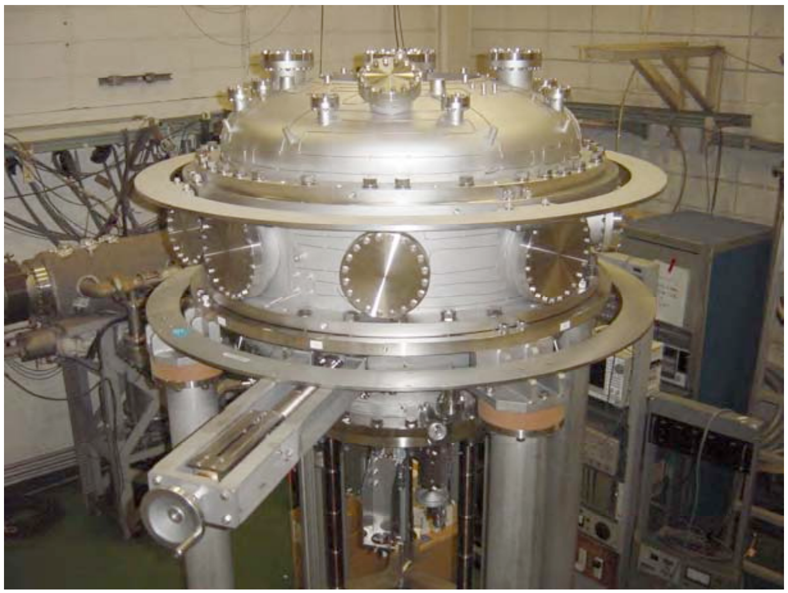
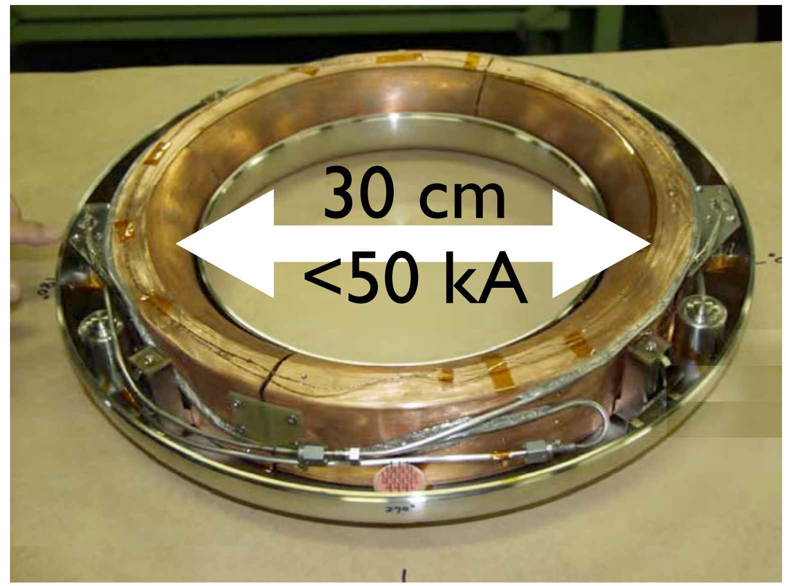
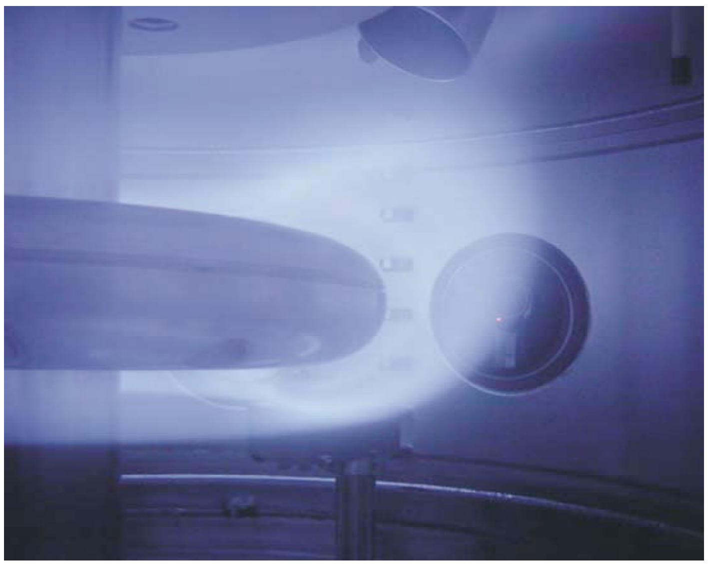


Vacuum Vessel



HTS Coil

Mini-Levitation!



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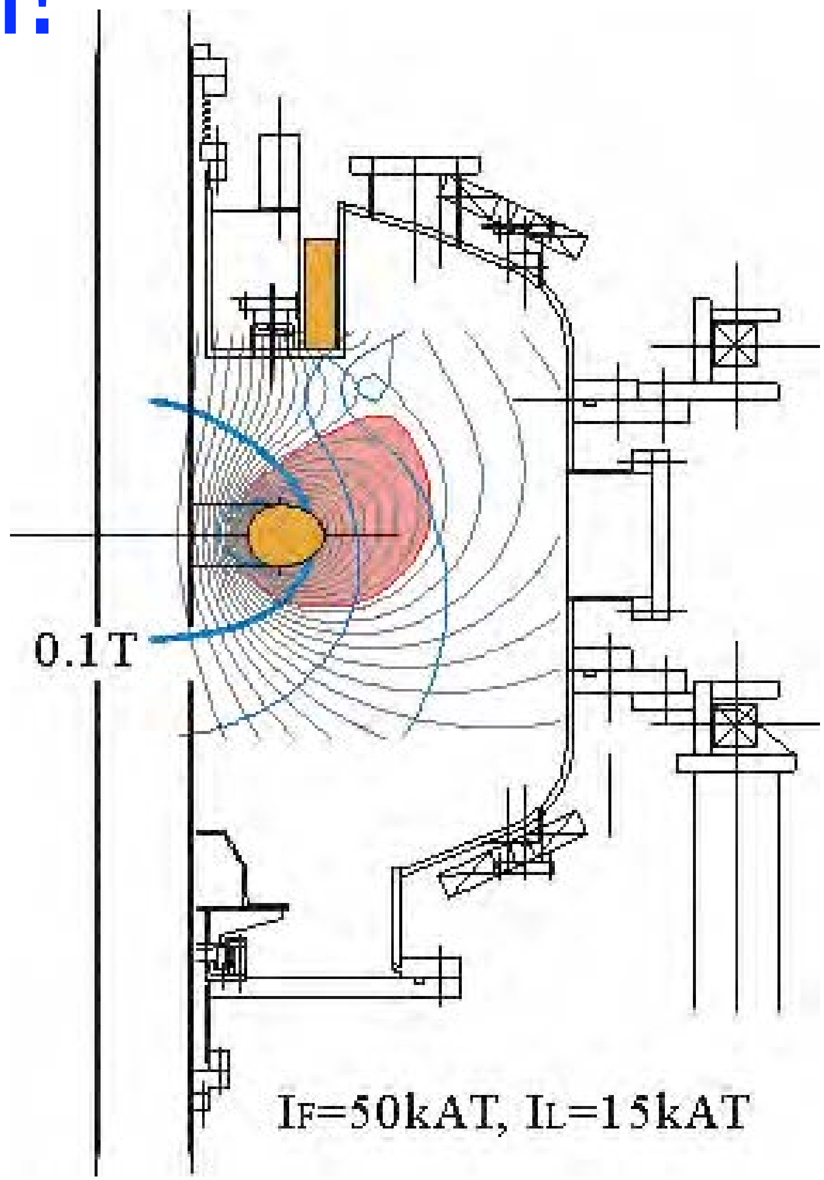


Dynamical Equations for Inductive Feedback Control for the Vertical Motion of F-Coil

$$\frac{dz}{dt} = v_z \tag{1}$$

$$\frac{dv_z}{dt} \approx g(c-1) + \gamma^2 z \left(1 + \frac{z}{d_{z1}} + \left(\frac{z}{d_{z2}}\right)^2 + \left(\frac{z}{d_{z3}}\right)^3 + \ldots\right) \tag{2}$$

$$\frac{dc}{dt} = \frac{V}{L_l I_0} \tag{3}$$

$$V = G_p z^m + G_d \frac{dz^m}{dt} + G_{d2} \frac{d^2 z^m}{dt^2} \tag{4}$$

The three gain parameters are used to (i) define the equilibrium location, G_p, (ii) stabilize vertical displacements, G_d, and (iii) damping vertical oscillations, G_{d2}. They translate measurements of the f-coil position, $z^m \approx z$, into the control voltage, V, applied to the l-coil. **With the f-coil charged to 1.18 MA·turns, 103.7 A in the l-coil, and 16 A in the c-coil, $\gamma = 3.8$ s^{-1} when $c \approx 1$.**

The Need for Noise Reduction

- Derivative gain is highly susceptible to noise.

- The measured f-coil position must be filtered to keep the control voltage on the l-coil power supply to be be well within limits, ± 150 V.

- Voltage fluctuations cause small current oscillations that heat the l-coil.

These can minimized by filtering the output voltage of the feedback controller or by filtering the measurement data. The previous approach taken for an “ideal” L-coil was to apply the filter to the measurements.

Example Digital Controller with Simple Averaging

For example, a digital controller with a sample period and latency both equal to $\delta t = 1$ ms. Single pole filters are used to compute the position, $\bar{z}_n$, velocity $\bar{v}_n$, and acceleration, $\bar{a}_n$, used to output a voltage, $V_{n+1} = G_p \bar{z}_n + G_d \bar{v}_n + G_{d2} \bar{a}_n$, applied at the *end* of the sample period. Digital filters with unity DC gain are

$$\bar{z}_n = \bar{z}_{n-1} + \frac{\delta t}{\tau_z}(z_n^m - \bar{z}_{n-1}) \tag{5}$$

$$\bar{v}_n = \bar{v}_{n-1} + \frac{\delta t}{\tau_v}\left(\frac{(\bar{z}_n - \bar{z}_{n-1})}{\delta t} - \bar{v}_{n-1}\right) \tag{6}$$

$$\bar{a}_n = \bar{a}_{n-1} + \frac{\delta t}{\tau_a}\left(\frac{(\bar{v}_n - \bar{v}_{n-1})}{\delta t} - \bar{a}_{n-1}\right) \tag{7}$$

Integration times were equal to $\tau_z/\delta t = \tau_v/\delta t = 20$ and $\tau_a/\delta t = 50$. With z_n^m equal to the actual f-coil position plus "white noise" with a magnitude of ± 20 μm, the noise caused the output voltage to fluctuate about ± 3 V, and this drove fluctuations of the I-coil current at ± 10 mA. (If the filters shown in Eqs. 8-10 are removed, the voltage fluctuations are are approximately ± 5 kV!)

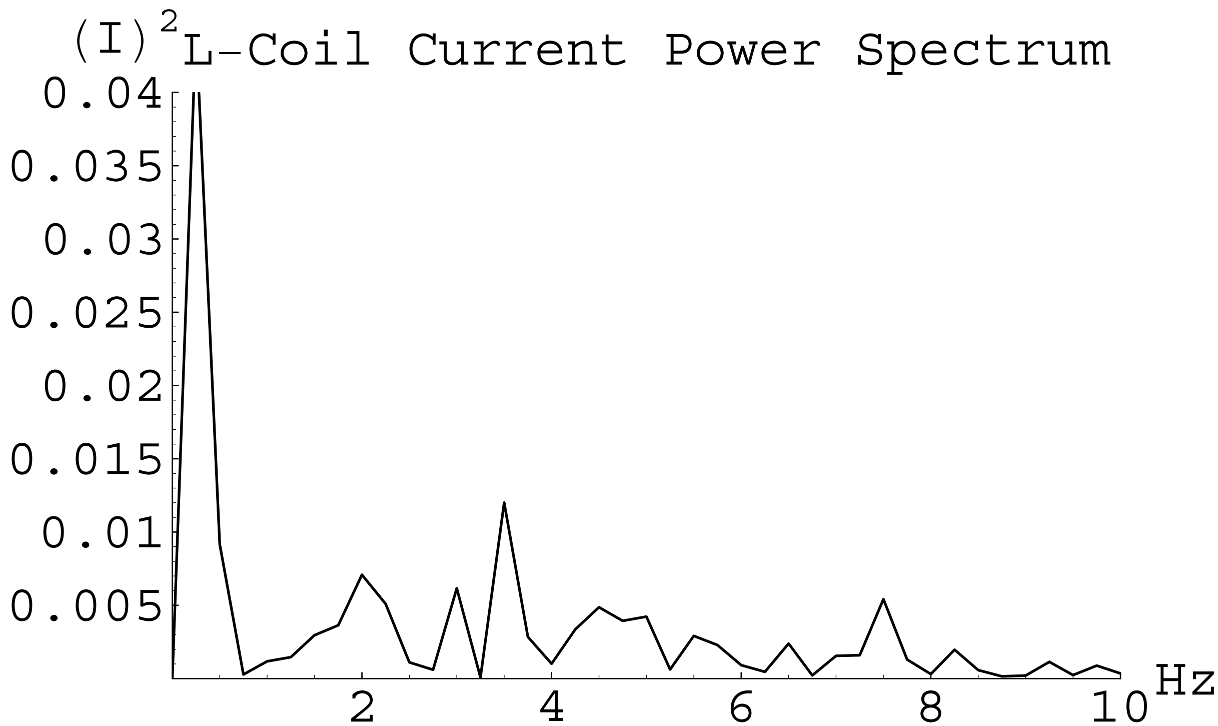


Figure 1

The DFT power spectrum from a simulation of the current fluctuations from an "**ideal**" l-coil due to $\pm 20\ \mu$m random measurement noise. The fluctuations of the l-coil current were acceptable: ± 10 mA.

Inside the L-Coil

The Empirical L-Coil Model

The measured response of the LDX l-coil to a voltage change couples the current through the superconductor to eddy currents in the support plate and currents passing through a short in the coil's insulation. The l-coil model is

$$\begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} L_l & M_{ls} & M_{lp} \\ M_{ls} & L_s & M_{sp} \\ M_{lp} & M_{sp} & L_p \end{pmatrix} \cdot \begin{pmatrix} \dot{I}_l \\ \dot{I}_l + \dot{I}_s \\ \dot{I}_p \end{pmatrix} + \begin{pmatrix} Q_l - R_s & R_s & 0 \\ 0 & Q_s + R_s & 0 \\ 0 & 0 & R_p \end{pmatrix} \cdot \begin{pmatrix} I_l \\ I_l + I_s \\ I_p \end{pmatrix}$$

where Q_l and Q_s represent frequency dependent loss factors empirically determined from the AC l-coil test.

The levitation field results from the l-coil current, I_l, shielded by eddy currents in the support plate, I_p, and by eddy currents in the LDX vacuum chamber. Eq. 3 must be modified to incorporate these effects. Defining c_{vac} to be the control field at the f-coil *in the absence* of the LDX vacuum vessel, and setting $\tau_{vac} \sim 15$ ms (*e.g.* the value from the SPARK code), the new dynamical control equation is

$$c = c_{vac} - \tau_{vac}\frac{dc}{dt} = \frac{I_l}{I_0} + \frac{2I_p}{2796 I_0} - \tau_{vac}\frac{dc}{dt} \qquad (8)$$

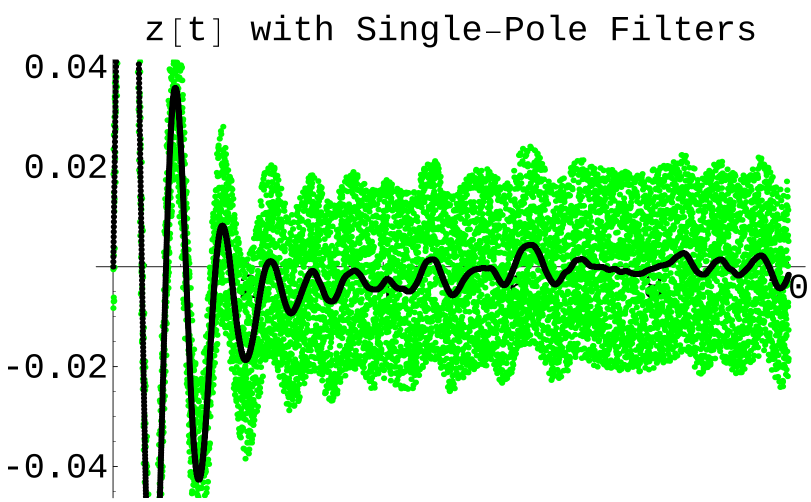
where we used the number of turns in the l-coil (2796) and the effective "turns" in the support plate (2, for both sides) is used to compute c_{vac}.

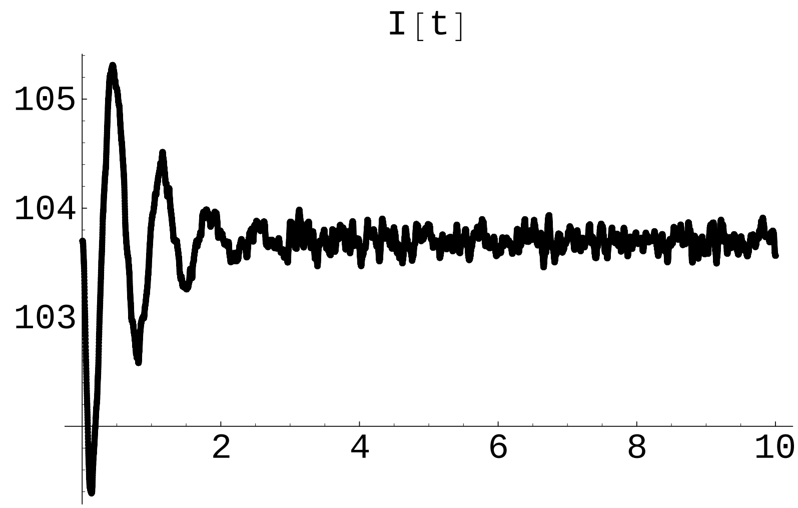
Simple Digital Controller with Time-Averaging Fails

The Laplace transform of Eqs. 1, 2, 4, 11, and 12 gives six poles (and one unstable mode, $\exp(\gamma t)$, when $G_p = G_d = G_{d2} = 0$.) The same gain vector discussed previously, $(G_p, G_d, G_{d2}) = (-1.0, -12, -1.7)$, stabilizes the f-coil using a "real" l-coil, but damps vertical displacements about 5 times more slowly, $\eta_{fb} > 0.11$ s^{-1}. Adjusting the gain vector with additional proportional gain, decreases the the settling time. When $(G_p, G_d, G_{d2}) = (-10, -17, -3.0)$, then $\eta_{fb} > 0.5$ s^{-1}.

The primary consequences of the support plate eddy currents are (1) to require an increase in the gain vector, and (2) the overall performance deteriorates.

- With noise of ± 20 μm, the l-coil current fluctuations increase nearly 10-fold to ± 91 mA.
- The higher gain vector and larger number of (stable) poles makes the digital response more problematic. For the example controller, the vertical displacements were underdamped–even though the analog response was nearly critical.





The DFT power spectrum from a simulation of the l-coil current fluctuations including the effects from plate eddy currents, the l-coil short, and vacuum vessel eddy currents. **The current fluctuations increased significantly (from Fig. 1) to ± 91 mA.**

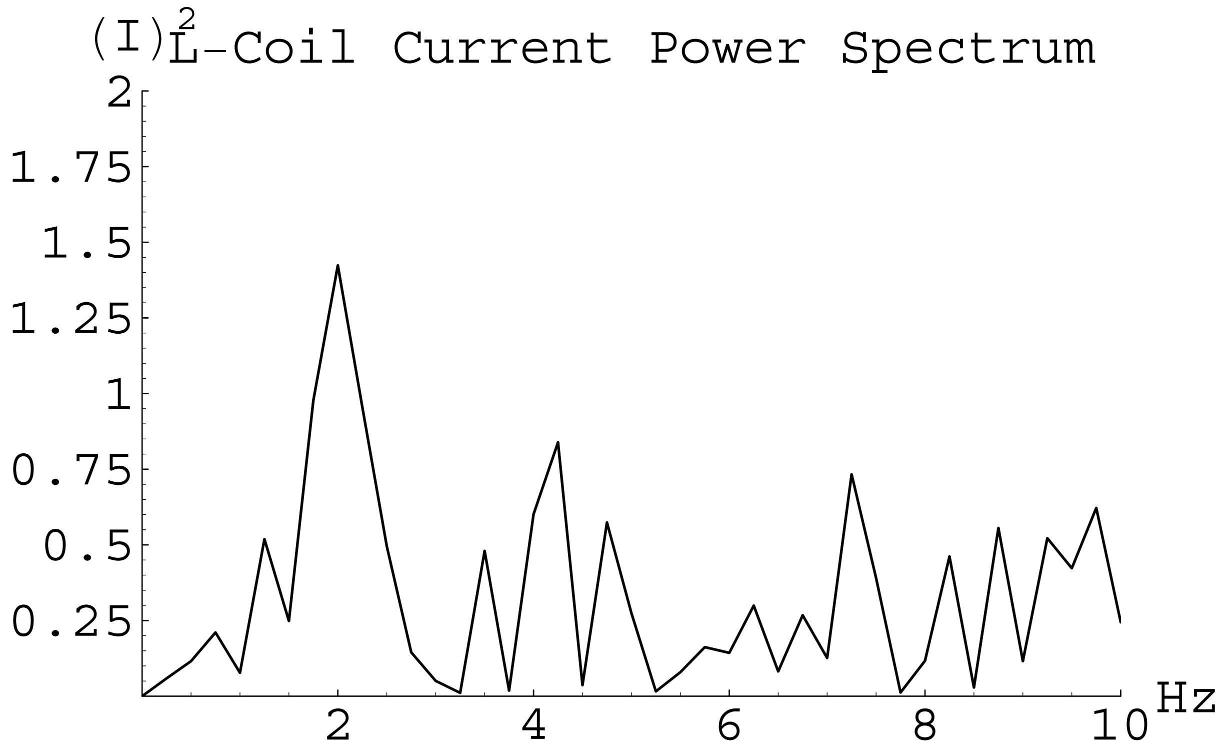


Figure 2.

The DFT power spectrum from a simulation of the l-coil current fluctuations including the effects from plate eddy currents, the l-coil short, and vacuum vessel eddy currents. **The current fluctuations increased significantly (from Fig. 1) to ± 91 mA.**

Rudolf Emil Kalman (May 19, 1930 -) is most famous for his invention of the Kalman filter, a mathematical digital signal processing technique widely used in control systems and avionics to extract meaning (a signal) from chaos (noise).

Kalman's ideas on filtering were initially met with scepticism. He had more success in presenting his ideas, however, while visiting Stanley Schmidt at the NASA Ames Research Center in 1967. This led to the use of Kalman filters during the Apollo program.

He was born in Budapest, Hungary. He obtained his bachelor's (1953) and master's (1954) degrees from MIT in electrical engineering. His doctorate (1957) was from **Columbia University**. His worked as Research Mathematician at the Research Institute for Advanced Study, in Baltimore, from 1958-1964, Professor at Stanford University from 1964-1971, and Graduate Research Professor, and Director, at the Center for Mathematical System Theory, University of Florida, Gainesville from 1971 to 1992. Starting in 1973, he simultaneously filled the chair for Mathematical System Theory at the Swiss Federal Institute of Technology, (ETH) Zurich.

He received the IEEE Medal of Honor (1974), the IEEE Centennial Medal (1984), the Inamori foundation's Kyoto Prize in High Technology (1985), the Steele Prize of the American Mathematical Society (1987), and the Bellman Prize (1997).

He is a member of the National Academy of Sciences (USA), the National Academy of Engineering (USA), and the American Academy of Arts and Sciences (USA). He is a foreign member of the Hungarian, French, and Russian Academies of Science. He has many honorary doctorates.

The Kalman Filter

A simple Kalman filter can remove measurement noise from a "tracking problem" (*i.e.* the f-coil position) and allow straight-forward application of the feedback gain vector during position control.

The Kalman filter is applied in two steps: the "prediction" or time update step and the "correction" or measurement update step. Design steps include:

- Definition of object "state" position, $\mathbf{x}_n$, and object "process model"
- Definition of "state" $\Leftrightarrow$ "measurement map, $\mathbf{H}$.
- "Standard" Kalman rules to advance $\mathbf{x}_n$ and "estimate error covariance", $\mathbf{P}_n$.

Tracking a Pendulum

$$\frac{d^2\theta}{dt^2} = \omega_0^2\theta(t)$$

State and process model is just a finite-difference approximation:

$$\mathbf{x}_n \equiv \{\theta_n,\ \theta_{n-1},\ \theta_{n-2}\}$$

$$\mathbf{x}_{n+1} = \mathbf{A} \cdot \mathbf{x}_n$$

$$\mathbf{A} \equiv \begin{pmatrix} 2 - \omega_0^2\delta t^2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

With the measurement matrix, $\mathbf{H} = \mathbf{I}$, and $\mathbf{x}_n = \mathbf{H} \cdot \mathbf{x}_n^m$.

Pendulum Tracking

The two-step Kalman filter can now be defined. The **"prediction"** step is

$$\begin{aligned}\mathbf{x}_n^* &= \mathbf{A} \cdot \mathbf{x}_{n-1} + \mathbf{u}_n \\ \mathbf{P}_n^* &= \mathbf{A} \cdot \mathbf{P}_{n-1} \cdot \mathbf{A}^T + \mathbf{Q}\end{aligned}$$

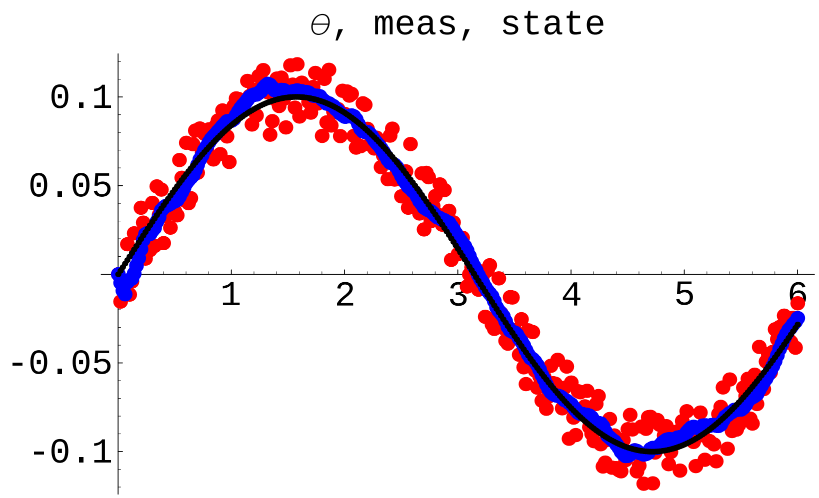
where $\mathbf{x}_n^*$ and $\mathbf{P}_n^*$ are predictions of the next step state vector and error covariance. (The matrix $\mathbf{Q}$ is user-defined parameter for the intrinsic noise. For the simulations described here, I took $\mathbf{Q}$ to be small: the identity matrix $\times 10^{-5}$.)

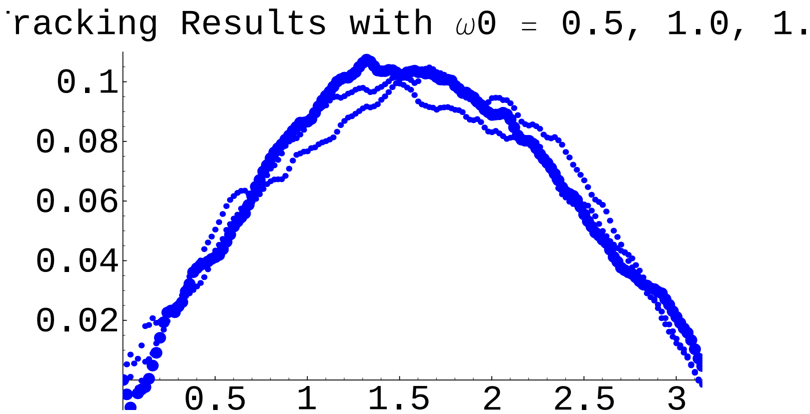
The **"correction"** step is

$$\begin{aligned}\mathbf{K}_n &= \mathbf{P}_n^* \cdot \mathbf{H}^T \cdot (\mathbf{H} \cdot \mathbf{P}_n^* \cdot \mathbf{H}^T + \mathbf{R})^{-1} \\ \mathbf{x}_n &= \mathbf{x}_n^* + \mathbf{K}_n \cdot (\mathbf{z}_n^m - \mathbf{H} \cdot \mathbf{x}_n^*) \\ \mathbf{P}_n &= (\mathbf{I} - \mathbf{K}_n \cdot \mathbf{H}) \cdot \mathbf{P}_n^*\end{aligned}$$

$\mathbf{K}_n$ is the "Kalman Gain". It **minimizes on average the corrected error covariance**. $\mathbf{R}$ is the measurement noise covariance. With $\mathbf{R}$ large, the tracking is less sensitive to noise.

With $\mathbf{H} = \mathbf{H}^T = \mathbf{I}$, these are especially simple. Even with a more complicated measurement matrix (*e.g.* when we use multiple laser detectors to simultaneously measure tilt, slide, and vertical position), these equations are less complicated than the sequential application of single-pole digital filters used in the example of the previous section.





Kalman filter results for pendulum tracking with ± 20 % measurement noise. $\mathbf{R} = 1.0$.

Tracking appears good *even when the model frequency varies,* $\omega_0 = 0.5$, 1.0, and 1.5.

The Kalman Filter for LDX

A simple Kalman filter can remove most of the measurement noise of the f-coil position and allow straight-forward application of the gain vector during position feedback control.

I choose to model the "state" of the f-coil's position with a vector $\mathbf{x}_n \equiv \{z_n, z_{n-1}, z_{n-2}\}$, where z_n is the vertical position during the nth time-sample of the digital controller. The "prediction" step is an internal model describing how the f-coil advances one time-step from $\mathbf{x}_n$ to $\mathbf{x}_{n+1}$. A second-order accurate state map is

$$
\begin{aligned}
\mathbf{x}_{n+1} &= \begin{pmatrix} z_{n+1} \\ z_n \\ z_{n-1} \end{pmatrix} = \begin{pmatrix} \gamma^2 \delta t^2 + 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} z_n \\ z_{n-1} \\ z_{n-2} \end{pmatrix} + \begin{pmatrix} g \delta t^2 (I_l(n)/I_0 - 1) \\ 0 \\ 0 \end{pmatrix} \\
&\equiv \mathbf{A} \cdot \mathbf{x}_n + \mathbf{u}_n
\end{aligned}
$$

where $I_l(n)/I_0$ is the measured normalized current in the l-coil, and $\mathbf{A}$ is called the "process matrix".

The "correction" step contains the important matrix, $\mathbf{R}$, called the "**measurement noise covariance**" matrix. This matrix is used by the filter designer to control the evolution of the state vector. When $\mathbf{R}$ is large, the process state is less sensitive to noise fluctuations. When $\mathbf{R}$ is small, the state vector's response is more sensitive to measurement noise. For the simulations I performed, I took $\mathbf{R}$ to be the identity matrix times a single parameter. This is appropriate if the noise for each measurement is independent or each other.

The Control Computation

The final step in the digital controller is to compute the control voltage to be applied to the I-coil power supply. A second order formula is

$$V_n = \begin{pmatrix} G_p & G_d & G_{d2} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1/\delta t & -1/\delta t & 0 \\ 1/\delta t^2 & -2/\delta t^2 & 1/\delta t^2 \end{pmatrix} \cdot \mathbf{x}_n$$

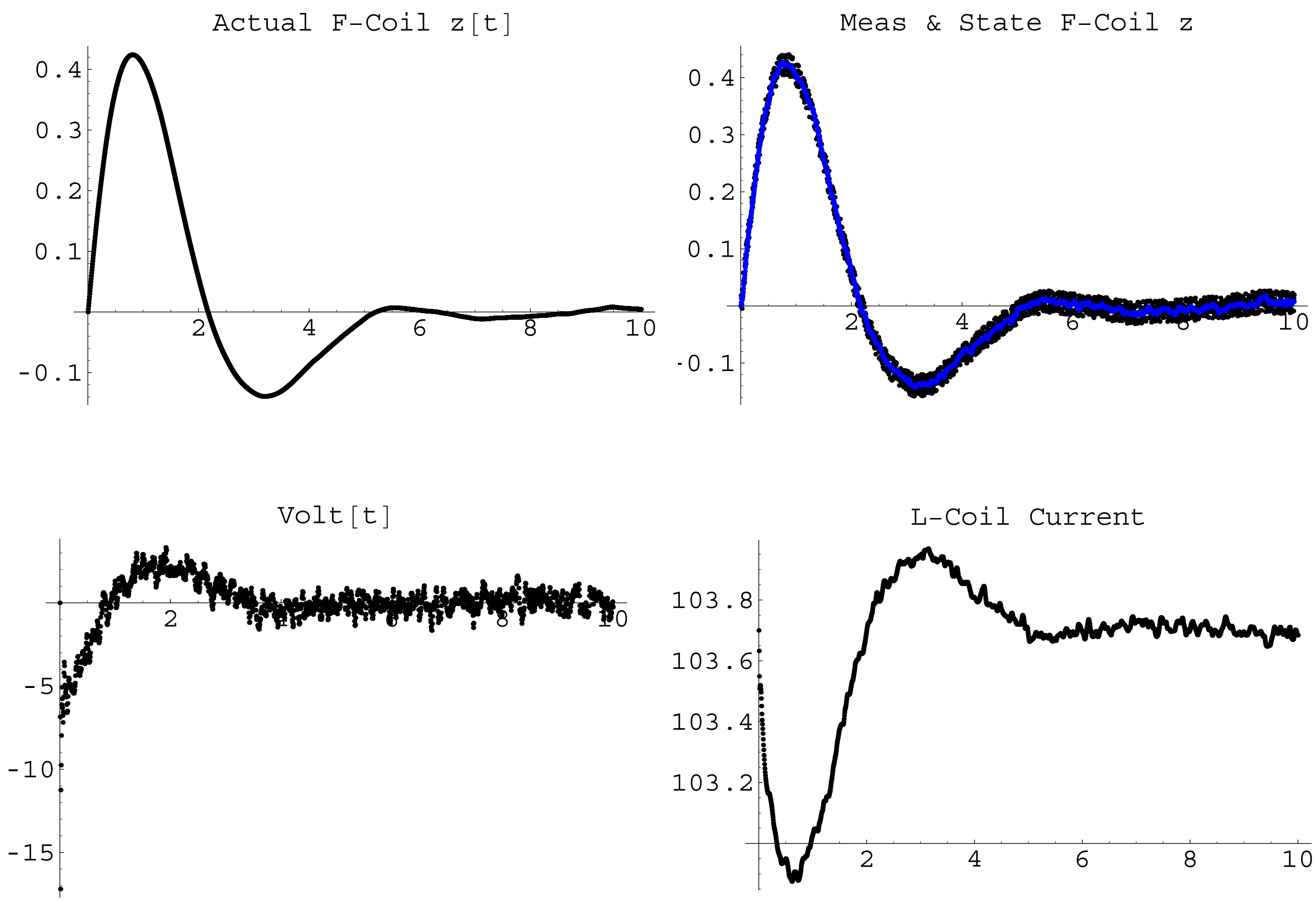


Figure 3. Simulation of F-coil position control using a digital controller and a Kalman filter. (“**Wow!**”)

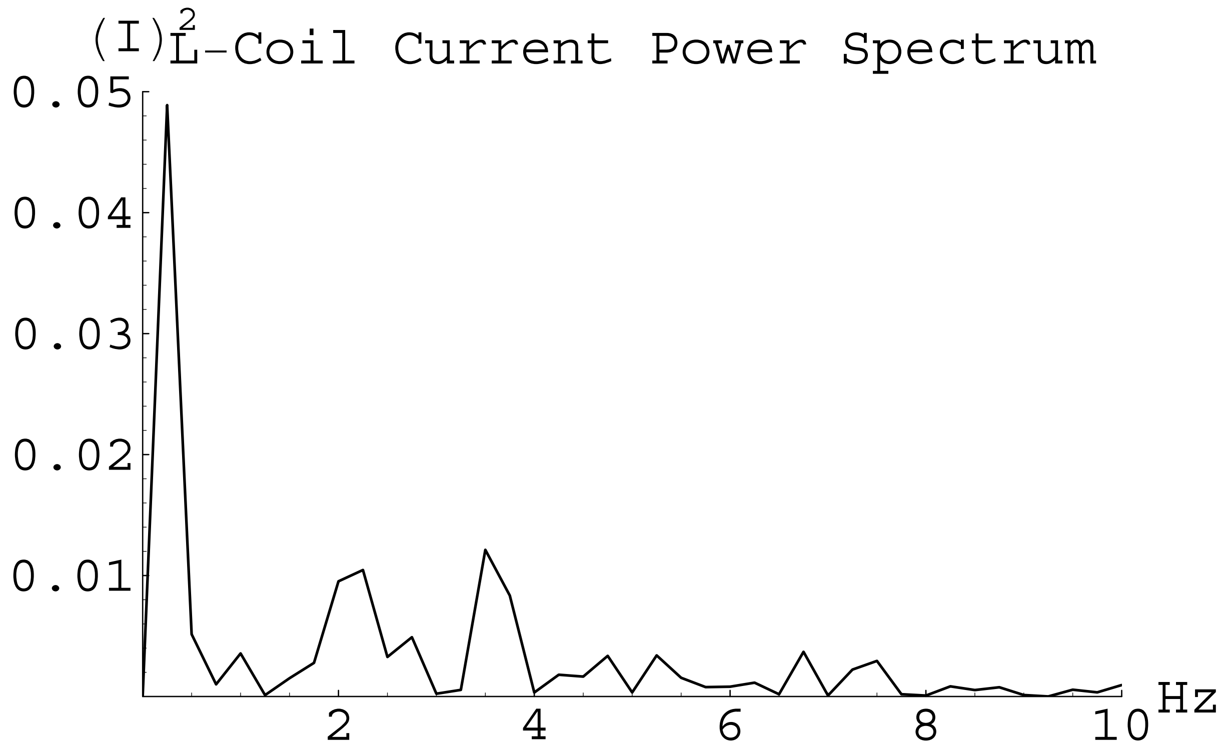


Figure 4.

The DFT of the l-coil current fluctuations from a simulation of the digital control of f-coil using a Kalman filter with $\mathbf{R} = \mathbf{I} \times 1$.

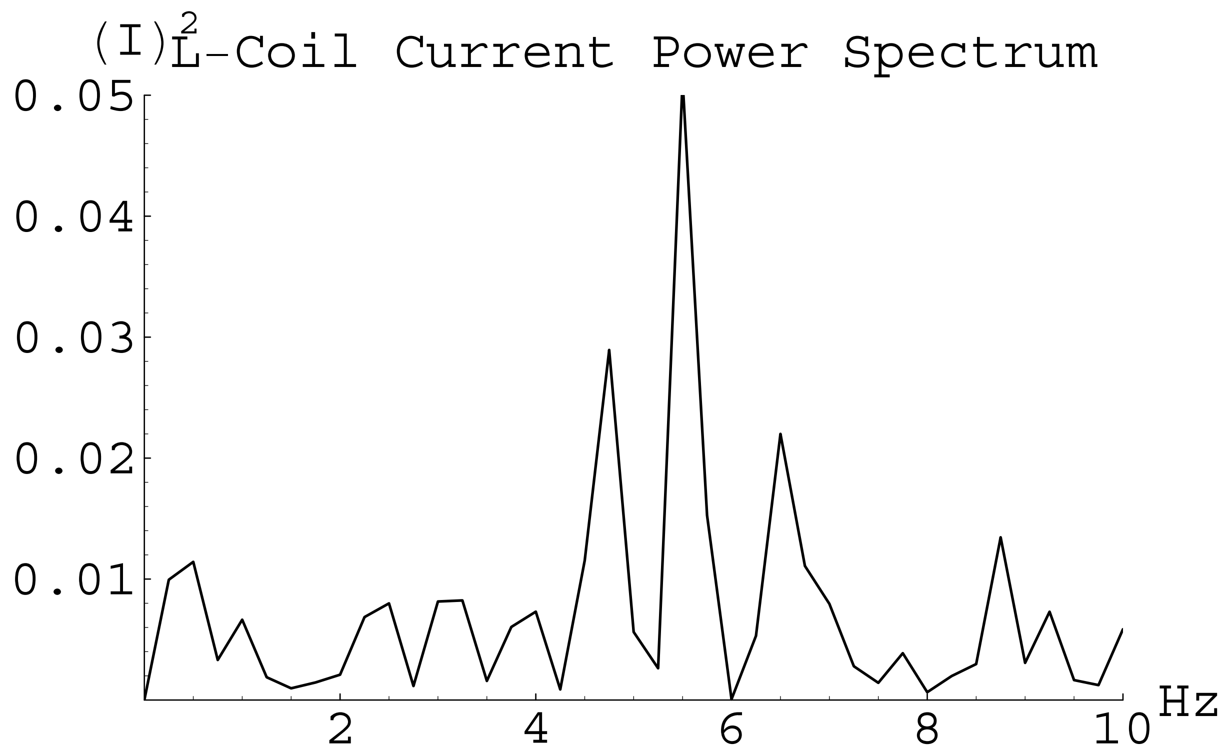


Figure 5.

The DFT of the l-coil current fluctuations from a simulation of the digital control of f-coil using a Kalman filter with $\mathbf{R} = \mathbf{I} \times 0.1$. The l-coil current fluctuations have increased to ± 29 mA.

Simulations show Kalman filter meets our requirements

- Following an "instantaneous" step in the f-coil velocity, at $t = 0$, the f-coil's upward velocity is set to be 1 cm/s. The controller operated with $\delta t = 5$ ms, and the tuning parameter was: $\mathbf{R} = \mathbf{I} \times 1$.
- The f-coil returned to equilibrium in about 5 s, comparable to the response obtained with an ideal analog controller.
- The steady fluctuations of the l-coil current had a standard deviation of ± 19 mA, but with a power spectrum dominated by low-frequencies, < 1 Hz, as shown in Fig. 4.
- With these filter/controller settings, the l-coil current fluctuations are nearly the same as obtained with a "bare" l-coil (shown in Fig. 1.)
- If the tuning parameter is made smaller, $\mathbf{R} = \mathbf{I} \times 0.1$, then the l-coil current fluctuations increase to ± 29 mA, and the power-spectrum has higher-frequency components.

“Mode Tracking” for External Kink Control

Let the kink state be $\mathbf{x}_n \equiv \{\psi_{a,n},\ \psi_{w,n},\ \psi_{a,n-1},\ \psi_{w,n-1}\}$. Measurements made by using δB_p, δB_r, or both. This representation allows proportional and derivative, flux-flux gain.

Process model:

$$\frac{d}{dt}\begin{pmatrix}\Psi_a\\ \Psi_w\end{pmatrix} = \begin{pmatrix}\frac{\Omega}{\bar{\bar{\alpha}}}(1-\bar{s}) - i\Omega & -\frac{\Omega}{\bar{\bar{\alpha}}}\sqrt{c}\\ \gamma_w\frac{\sqrt{c}}{1-c} & -\gamma_w\frac{1}{1-c}\end{pmatrix}\cdot\begin{pmatrix}\Psi_a\\ \Psi_w\end{pmatrix} + \begin{pmatrix}-\frac{\Omega}{\bar{\bar{\alpha}}}\frac{c_f}{c}\\ \gamma_w\left(1-\frac{c_f}{1-c}\right)\end{pmatrix}\Psi_c$$

where

$$\Psi_c = \text{Gain}\times\begin{cases}(0\ \ 1)\cdot\begin{pmatrix}\Psi_a\\ \Psi_w\end{pmatrix} & \text{RadialFieldSensors}\\[2ex] \left(\frac{6\sqrt{c}}{1-c}\quad -3\,\frac{1+c}{1-c}\right)\cdot\begin{pmatrix}\Psi_a\\ \Psi_w\end{pmatrix} & \text{PoloidalFieldSensors}\end{cases}$$

Plasma parameters, $\bar{s}$, Ω, (and possibly $\bar{\alpha}$) may slowly evolve with the discharge. (And this may not be too critical.)

Summary

- Vertical dynamics of the f-coil was re-examined using the empirical model of the actual l-coil.

- Using computer simulations, an active feedback controller using an adaptive Kalman filter was described that meets our requirements.

- The Kalman Filter is "magic", and it should be very useful for the tracking of external kink mode dynamics.