Neoclassical Tearing Mode Issues

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Outline

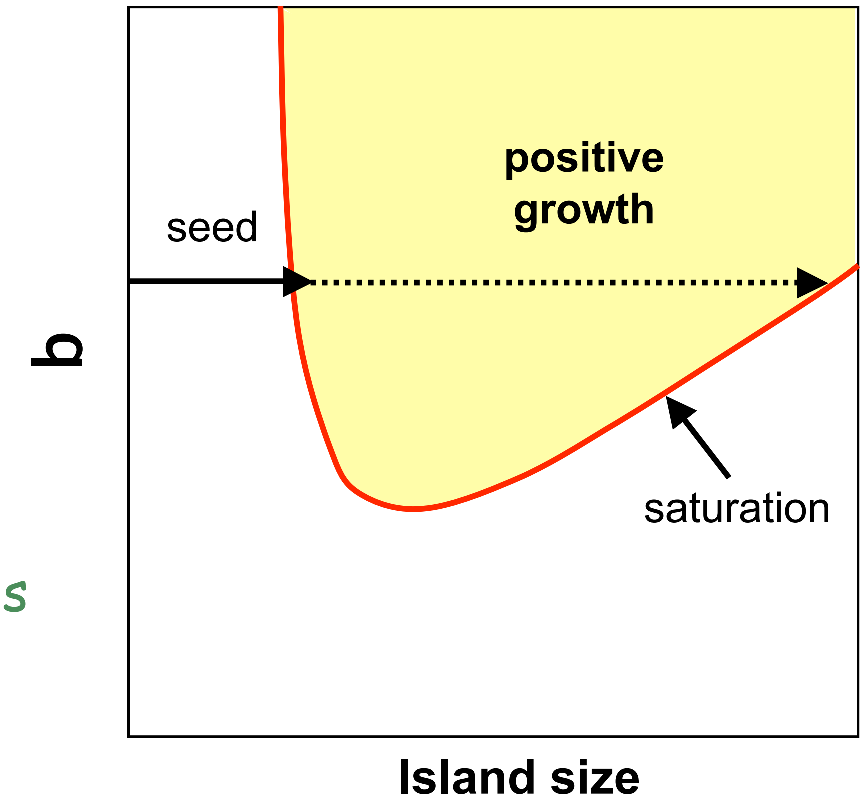
In this talk, progress on various topics in neoclassical tearing mode theory will be reviewed.

- Threshold physics
  - MHD Polarization currents
  - Neoclassical Polarization currents
  - Rotation velocity
- Seed Island physics
  - Equilibrium evolution - Role of Δ'
  - Electromagnetic coupling mechanisms
  - Nonelectromagnetic coupling

The modified Rutherford equation models the nonlinear evolution properties of magnetic islands in tokamaks

- The modified Rutherford equation

$$k_0 \frac{\mu_o}{\eta}\frac{dw}{dt} = \Delta' + k_1 \frac{D_{nc} w}{w^2 + w_d^2} + k_1 \frac{D_R^+}{\sqrt{w^2 + w_d^2}}$$
$$- \frac{D_{pol}}{w^3} - \frac{D_{ECCD}}{w}\eta(\frac{w}{\delta_D})$$



  - Saturated islands
  - Threshold physics
  - Seeding magnetic islands
  - ECCD Stabilization

Island evolution is derived from a helical "equilibrium" state

- Slowly evolving helical equilibrium - equations

$$\nabla \cdot \vec{J} = 0 \Leftrightarrow \vec{B} \cdot \nabla \frac{J_{\parallel}}{B} = -\nabla \cdot J_{\perp} \Rightarrow \frac{J_{\parallel}}{B} = \sigma(\Psi^*) - \int \frac{dl}{B} \nabla \cdot \vec{J}_{\perp}$$

$$\vec{J}_{\perp} = \frac{\vec{B}}{B^2} \times [\nabla p + \rho \frac{D\vec{v}}{Dt} + \nabla \cdot \vec{\Pi}]$$

Resistive Interchange Effects

MHD polarization

Neoclassical Polarization effects parallel and gyro-viscosity

Ohm's law determines $\sigma(\Psi*)$

$$< \vec{E} \cdot \vec{B} >_* = \eta < \vec{J} \cdot \vec{B} >_* + \frac{< \vec{B} \cdot \nabla \cdot \Pi_{\parallel e} >_*}{ne} - \eta < \vec{J} \cdot \vec{B}_{aux} >_*$$

Bootstrap current

ECCD current drive

Neoclassical ion viscosity "enhances" the polarization effect

- The conventional polarization current gives rise to terms of order ρ_s^2/w^3 in the island evolution equation

$$\nabla\cdot\vec{J}_\perp = \nabla\cdot\frac{\vec{B}}{B^2}\times[\rho\frac{D\vec{v}}{Dt}+\nabla\cdot\vec{\Pi}]+\ldots=\nabla\cdot\vec{J}_{pol}+\nabla\cdot\vec{J}_{neo}$$

  - In the collisional regime, the neoclassical enhancement can be seen when parallel momentum balance is used (Smolyakov et al)

$$\nabla\cdot\frac{\vec{B}\times\nabla\cdot\Pi}{neB^2}\cong-\frac{\partial}{\partial\psi}\frac{I}{neB^2}\vec{B}\cdot\nabla\cdot\Pi=\frac{\partial}{\partial\psi}\frac{I\rho}{B^2}B\cdot\frac{d}{dt}\vec{v}=-\frac{\partial}{\partial\psi}\frac{I^2\rho}{B^2}\frac{d}{dt}\frac{\partial\phi}{\partial\psi}\cong-\frac{B_\xi^2}{B_\theta^2}\nabla\cdot\vec{J}_{pol}$$

  - In the low collision limit $|\omega| > \nu/\varepsilon$, the neoclassical enhancement is reduced by $\varepsilon^{3/2}$ (Wilson, et al '97)

$$\frac{\partial f}{\partial t}+\ldots=C(f)\cong\nu\frac{\partial^2 f}{\partial\lambda^2}\sim\frac{\nu}{\varepsilon}f$$

A number of authors have addressed the effect of polarization currents in two-fluid models of islands

- Most authors consider the regime $w > \rho_s$

$$\frac{D_{pol}}{w^3} = g\frac{\omega(\omega-\omega_i^*)}{\omega_e^{*2}}\frac{\hat{\beta}}{w^3}$$

  - The sign of this term is controlled by a thin region near the island separatrix- Waelbroeck and Fitzpatrick '97, $g > 0$ corresponds to stabilizing polarization effects for $0 < \omega < \omega_i^*$
- Conventional model - sheared slab geometry, cold ions - Smolyakov '93; Connor et al '01, Mikhailovskii et al …

$$\frac{Dn}{Dt} = \frac{1}{e}\nabla_\parallel J_\parallel + D\nabla_\perp^2 n,$$

$$\frac{D\nabla^2\phi}{Dt} = \frac{4\pi v_a^2}{c^2}\nabla_\parallel J_\parallel + \mu\nabla_\perp^4\phi$$

$$E_\parallel + \frac{\nabla_\parallel p}{ne} + \alpha\frac{\nabla_\parallel T_e}{e} = \eta J_\parallel$$

$$\frac{3}{2}n\frac{DT_e}{Dt} = \nabla_\parallel(\kappa_\parallel\nabla_\parallel T_e) + \kappa_\perp\nabla_\perp^2 T_e + (1+\alpha)\nabla_\parallel\left(\frac{T_e J_\parallel}{e}\right)$$

The natural frequency of the island is determined by dissipative processes

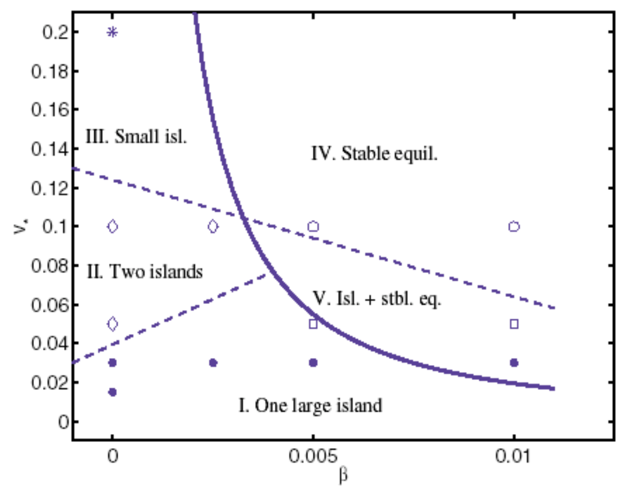
- The mode rotation frequency depends upon the profiles of flow, density and temperature in the island region .
  - These profiles are governed by transport equations - Connor et al '01
    - Solutions with $\omega \sim 0$? $\omega \sim \omega^*$?
    - Role of sound waves - Ottaviani et al '04 - multiple solutions

$$dU/dt = [J, \psi] + \mu \nabla^2 U,$$

$$d\psi/dt + v_* \partial\psi/\partial y = [n, \psi] + \eta \nabla^2 (\psi - \psi_{\mathrm{eq}}),$$

$$dn/dt + v_* \partial\varphi/\partial y = \rho_*^2 [J, \psi] + D \nabla^2 n - \beta [v, \psi],$$

$$dv/dt - v_* \partial\psi/\partial y = -[n, \psi] + \chi \nabla^2 v,$$



Analytic calculations of the rotation frequency are required

- The conventional technique for calculating the island frequency uses asymptotic matching

$$\Delta' = \int dx \oint d\alpha \frac{\cos(m\alpha)}{\pi} J_{\parallel}$$

Island growth - Rutherford equation

$$0 = \int dx \oint d\alpha \frac{\sin(m\alpha)}{\pi} J_{\parallel}$$

Island frequency

  - For localized velocity profiles, the asymptotic matching technique fails to find a unique frequency for Braginskii-like viscosities- Kuvshinov and Mikhailovskii '98

$$\frac{D\nabla^2\phi}{Dt} = \dots \mu \nabla_{\perp}^4 \phi$$

Island induced neoclassical toroidal viscosity produces an addition dissipative effect

- Islands in tokamaks produce 3-D distortions of |B| - Shaing '02

$$\frac{B}{B_o} = 1 + \frac{r}{R_o}\cos\theta \Rightarrow \frac{B}{B_o} = 1 + \frac{r_o + w\sqrt{\Omega + \cos(m\theta - n\zeta)}}{R_o}\cos\theta$$

  - Produces "stellarator-like" neoclassical transport depending on collisionality regime - Shaing '03
  - Also influences the determination of the island rotation frequency - not in the form of cross-field viscosity -

$$\nabla \cdot \Pi \sim \mu_{i\zeta} v^{\zeta} + \mu_{i\theta} v^{\theta},$$

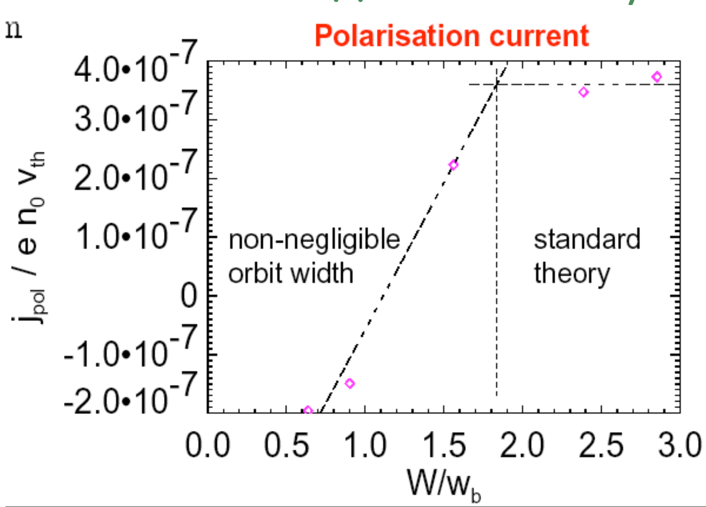
$$\nabla \cdot \Pi \neq \mu_{\perp} \frac{\partial v^{\zeta}}{\partial \psi}$$

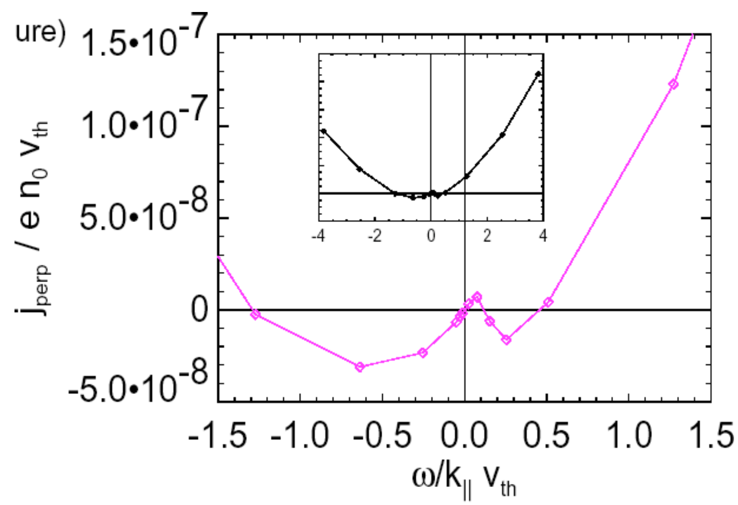
More general forms in kinetic treatment - CCH '03

Different treatments of fluid theory - Fitzpatrick and Waelbroeck '04

Efforts are underway to treat the case of more general magnetic island width

- For toroidal geometry, convential analytic treatments require w > $\rho_{I\theta}\varepsilon^{0.5}$ (banana width) - the regime of interest for threshold physics is w ~ $\rho_{I\theta}\varepsilon^{0.5}$
  - For small island width, the ion component of the bootstrap current plays no role, Poli '02
  - Role on polarization currents - size and frequency dependence are different - Poli, et al '04





The seeding process of magnetic islands is not well understood

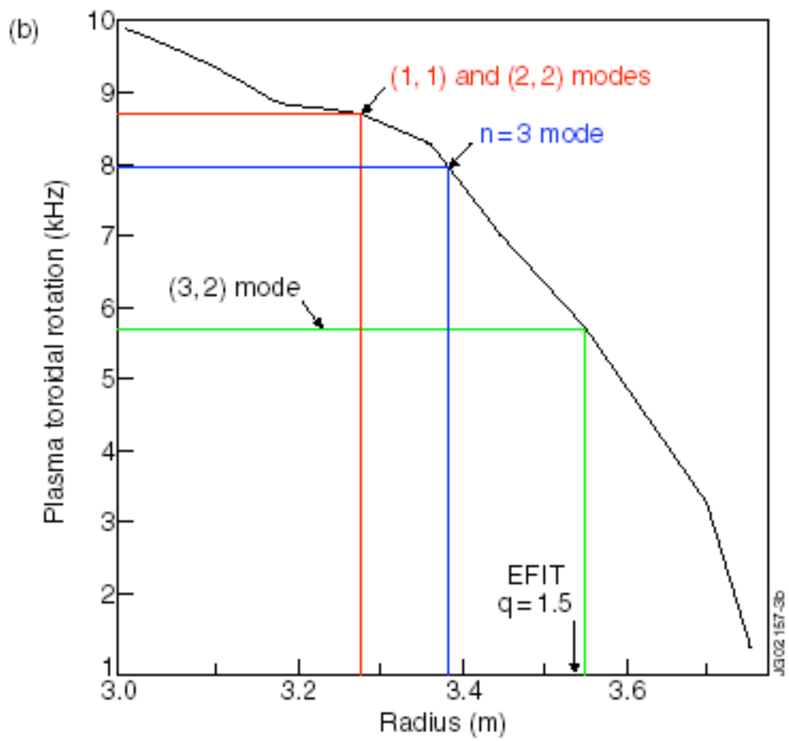
- Seeding process - if a nonlinear threshold needs to be exceeded, how does a nonlinearly island get established above the threshold?
  - The seeding mechanism has received much less theoretical attention.

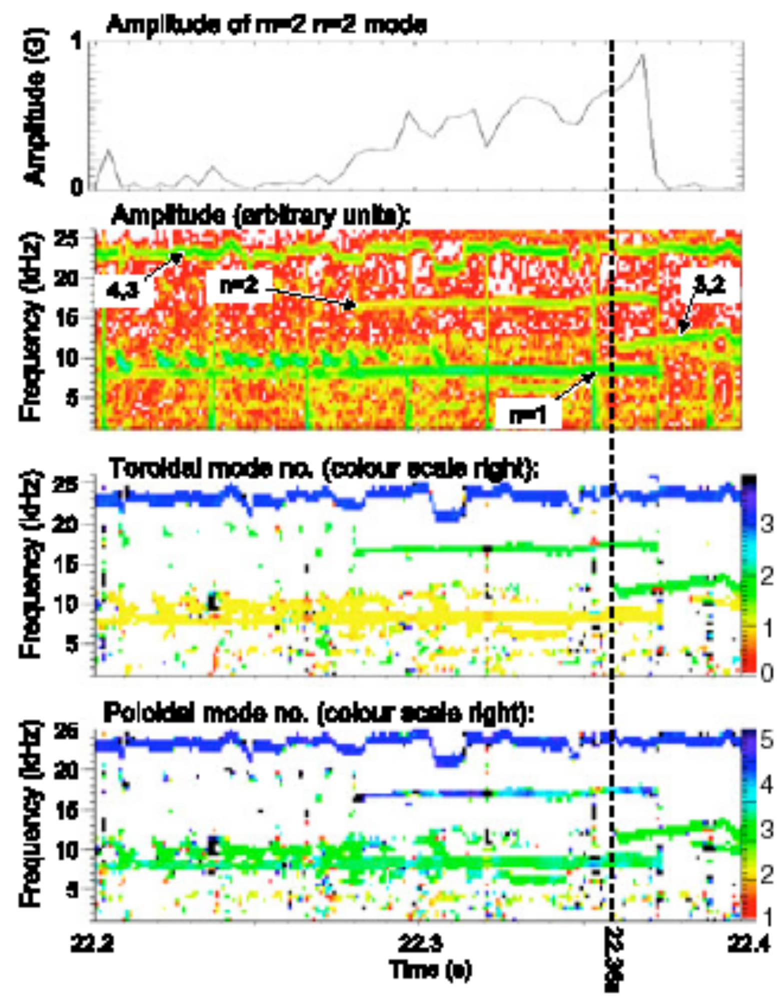
- Many times, but not always, the onset of an NTM corresponds to the occurrence of some other MHD activity
  - Sawtooth crash
  - ELM
  - Fishbones

The conventional picture for seeding has focused on magnetic coupling of MHD perturbations

- Previous theoretical work has focused on the electromagnetic coupling (CCH et al, '99).
  - Magnetic perturbation from some other MHD + geometric or nonlinear coupling leads to a magnetic signal that corresponds to a susceptible rational surface
  - Example: m/n = 2/2 harmonic associated with a sawtooth precursor + toroidal coupling = m/n = 3/2 'seed' magnetic perturbation ¿ 3/2 NTM - there are clear examples where this provides a reasonable explanation - nonlinear couplings 1/1,3/2,4/3 - Nave et al '02
- However, not all of the observed NTMs are seeded this way
  - Seedless NTMs on TFTR (Fredrickson, '01)
  - Non-magnetic coupling of NTMs with sawteeth precursors on JET (Buttery '03)

Data from JET does not support the electromagnetic coupling hypothesis for the seeding





Buttery et al, NF ‘03

Initiation of an island may correspond to the evolution of Δ'

- The matching data quantity Δ' is crucial in evaluating island evolution - $dw/dt = (\eta/\mu_o)\Delta' + \ldots$
- With increasing β, Δ' approaches a pole - Brennan et al '03

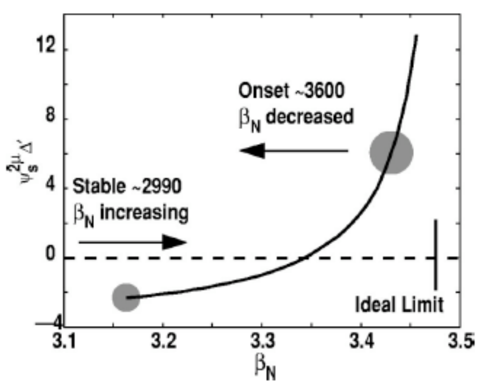
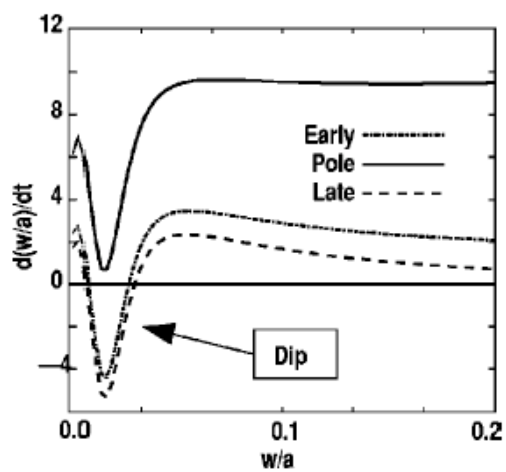


FIG. 6. The Δ' at the 3/2 surface as a function of β_N from equilibrium reconstruction at 2990 ms with a modification to the core pressure profile (which increases β_N) done so as to eventually closely match the profile reconstructed at 3600 ms. The structure of the pole in Δ' is evident.

Island evolution on DIII-D correctly modeled by modified Rutherford equation - Brennan '04



A different model for island seeding is based on transient transport events

- A model for seed island formation is presented based on the transient changes in the transport properties associated with other MHD events - Elements of the theory
  - Neoclassical polarization effect provides threshold physics
  - Evolution of island frequency as electron transport responds to outside MHD events
  - Ion viscosity is mostly unaffected by outside MHD events - as an Shaing '02-'03
  - Scenario
    - MHD enhanced D_e tends to move the island rotation frequency towards electron diamagnetic frequency
    - Polarization currents are destabilizing (or less stabilizing)
    - Bootstrap current destabilization occurs and island grows to large size

A kinetic theory is used to island currents

- Drift kinetic theory for both species

$$\frac{\partial f_s}{\partial t} + \vec{v}_{\parallel} \cdot \nabla f_s + \vec{v}_{\vec{E}\times\vec{B}} \cdot \nabla f_s + \vec{v}_{md} \cdot \nabla f_s$$

$$+ \frac{q_s}{m_s} \frac{\vec{v} \cdot \vec{E}}{v} \frac{\partial f_s}{\partial v} + \nabla \cdot (D_s \nabla f_s) = C(f_s)$$

$$\vec{E} \cdot \vec{B} = 0 \rightarrow \phi = -\omega[x + h(\Psi^*)]$$

$$\omega[h, f_s] + \frac{v_{\parallel}}{\sqrt{g}B}\left(\frac{\partial f_s}{\partial \theta} + [\Psi^*, f_s]\right) + \vec{v}_{md} \cdot \nabla\left(f_s - \frac{q_s \phi}{m_s v} \frac{\partial f_s}{\partial v}\right)$$

$$+\nabla \cdot (D_s \nabla f_s) = C(f_s)$$

D = phenomenological model for fluctuation induced transport

$D \sim v_s \delta B^2 L_c$

A kinetic theory for the island-ion interaction can be worked out

- Kinetic equation ordered using small gyroradius and island width
  - $f_s = \Sigma_{mn} f^{(m,n)} \delta^m \Delta^n$ $\delta = \varepsilon^{0.5}\rho_\theta/w$, $\Delta = w/a$
- To leading order, $f^{(0,0)} = f_M(\Psi^*)$ - equilibration on helical flux surfaces
- To next order in ρ_i, ions drift off magnetic surfaces

$$\frac{v_\parallel}{\sqrt{g}B}\frac{\partial f^{(1,0)}}{\partial \theta} + \vec{v}^{\,0}_{md} \cdot \nabla f^{(0,0)} = 0,$$

$$f^{(1,0)} = -\frac{I v_\parallel}{\Omega_s}\frac{\partial f^{(0,0)}}{\partial x}\frac{\omega^{*T} - \omega}{\omega^{*T}} + g_s$$

- g can be determined from bounce averaging next order equation

$$\omega < [h, f^{(1,0)}] > + < \frac{v_\parallel}{\sqrt{g}B}[\Psi^*, f^{(1,0)}] > - < C(f^{(1,0)}) > = - < \vec{v}_{md} \cdot \nabla \psi > \frac{\partial f^{(0,0)}}{\partial x}$$

First order solution has banana width and island induced radial drifts

- Quasineutrality calculated from kinetic equation

$$[\Psi^*, \frac{J_\parallel}{B}] + \sum_s q_s < \int d^3v \vec{v}_{md} \cdot \nabla f_s >$$
$$= - < \nabla \cdot \sum_s q_s \int d^3v D_s \nabla f_s >$$

- Current responses can be distinguished by the dissipative and non-dissipative contributions
  - Nondissipative - contributes to island evolution ~ cosα
  - Dissipative - contributes to island frequency ~ sinα

$$[\Psi^*, \frac{J_{\parallel c}}{B}] \cong -\sum_s q_s < \int d^3v \frac{Iv_\parallel}{\Omega_s} \frac{\partial}{\partial x} \frac{v_\parallel}{\sqrt{g}B} \frac{\partial f_{odd}^{(1,1)}}{\partial \theta} >$$

$$[\Psi^*, \frac{J_{\parallel s}}{B}] \cong -\sum_s q_s \int d^3v \frac{\partial}{\partial x} < \vec{v}_{md} \cdot \nabla \psi > f_{even}^{(1,1)}$$
$$- < \nabla \cdot e \int d^3v (D_i - D_e) \nabla f^{(0,0)} >$$

$$\frac{v_\parallel}{\sqrt{g}B} \frac{\partial f^{(1,1)}}{\partial \theta} \cong -\omega [h, f^{(1,0)}],$$

$$f^{(1,0)} \cong I(< \frac{v_\parallel}{\Omega_s} > - \frac{v_\parallel}{\Omega_s}) \frac{\partial f^{(0,0)}}{\partial x} \frac{\omega^{*T} - \omega}{\omega^{*T}}$$

$$f_{even}^{(1,1)} \cong -\frac{\nu}{\omega^2 \rho^2 / \varepsilon^2 + \nu^2} < \vec{v}_{md} \cdot \nabla \psi > \frac{\partial f^{(0,0)}}{\partial x}$$

Parallel currents are used in asymptotic matching

- cos α component contributes to island evolution

$$k_o \frac{\tau_r}{r_s^2}\frac{dw}{dt} = \Delta' + \frac{D_{nc}}{w} - \frac{D_{pol}}{w^3}$$

$$D_{pol} = \varepsilon^{3/2}\rho_\theta^2\beta_\theta \frac{L_q^2}{L_p^2}\frac{\omega(\omega_i^{*T}-\omega)}{\omega_e^{*2}}$$

- Sin α component controls island frequency

$$\int d\Psi^* \oint \frac{d\alpha}{2\pi}[\Psi^*, \frac{J_{\parallel s}}{B}] = \sum_s q_s \int d^3v \frac{\partial}{\partial x} < \vec{v}_{md} \cdot \nabla\psi >^2 \frac{\nu}{\omega^2\rho^2/\varepsilon^2 + \nu^2}(\omega - \omega_s^{*T})\frac{\partial h}{\partial x}$$

$$+\sum_s q_s \int d^3v \frac{\partial}{\partial x}(< D_s |\nabla\psi|^2 > \frac{\partial h}{\partial x})(\omega - \omega_s^{*T}) = 0$$

  - Island frequency equation takes the form

$$\sum_s \gamma_s w^2(\omega - \omega_s^{*T}) + \sum_s D_s(\omega - \omega_s^{*T}) = 0$$

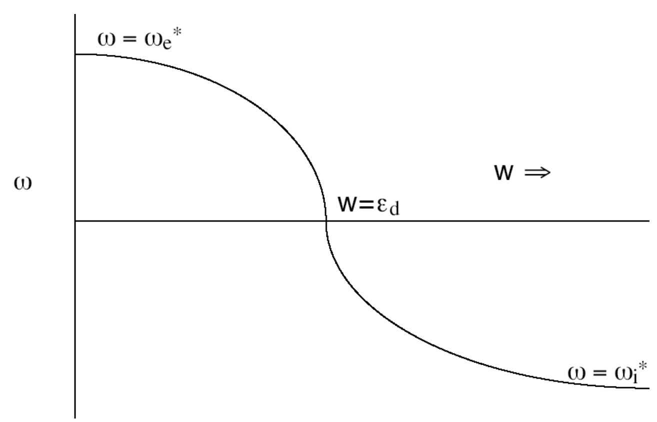
Different dissipation models introduce an additional length-scale in the problem

- In the limit the ions are dominated by neoclassical toroidal viscosity and electrons by anomalous viscosity

$$D_e(\omega - \omega_e^*) + \gamma_i w^2(\omega - \omega_i^*) = 0$$

  - In steady-state, the frequency is determined by island width

$$\omega = \frac{\frac{D_e}{\gamma_i}\omega_e^* + w^2\omega_i^*}{\frac{D_e}{\gamma_i} + w^2}$$



Stability properties of the polarization terms are determined by the transport properties

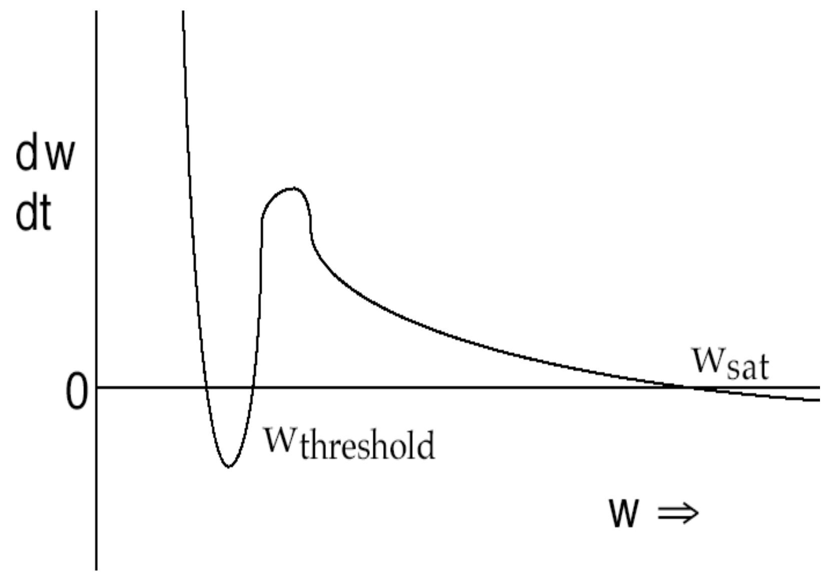
- Modified Rutherford equation

$$\frac{\tau_r}{r_s^2}\frac{dw}{dt} = \Delta' + k_o\sqrt{\varepsilon}\frac{L_q}{L_p}\frac{\beta_p}{w}[1+\frac{w_{pol}^2}{w^2}\frac{\varepsilon_d^2(\varepsilon_d^2-\tau w^2)}{(\varepsilon_d^2+w^2)^2}]$$

  - Stability determined by the dimensionless parameter

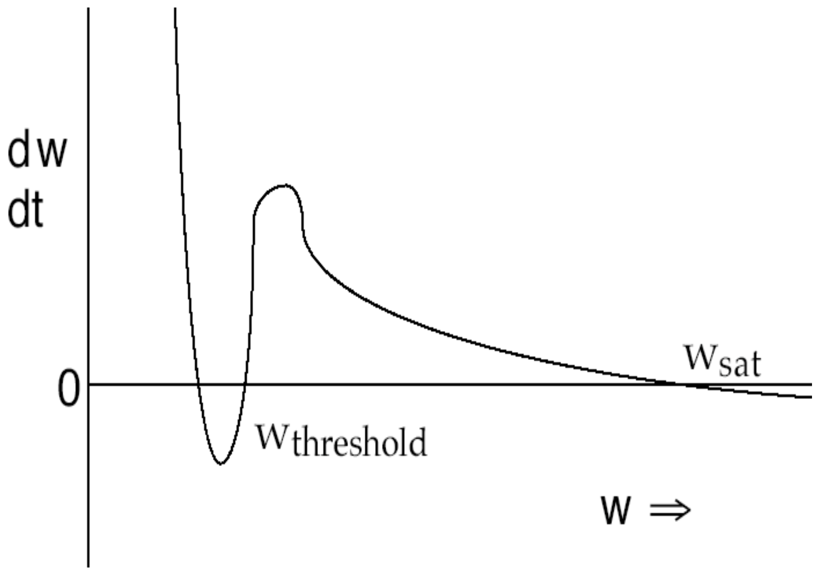
$$\delta \equiv \frac{\varepsilon_d}{w_{pol}} \sim \sqrt{\frac{D_e}{\gamma_i \varepsilon \rho_\theta^2}}$$

For $\delta < \delta_{crit} \sim 0.24$



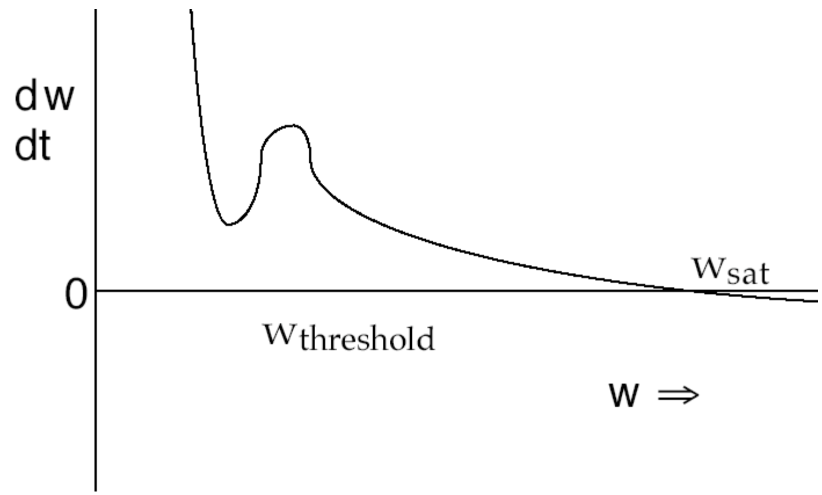
Larger δ eliminates the stabilizing effect of the neoclassical polarization currents

- For $\delta < \delta_{crit} \sim 0.24$



Nonlinear island threshold present

For $\delta > \delta_{crit} \sim 0.24$



No nonlinear island threshold polarization effect does not stabilize NTMs

Summary

- Several aspects of neoclassical tearing mode theory seem to be in good shape
  - Saturated island width - particularly when local parameters are used - Buttery '04
  - ECCD stabilization$_b$ routinely used - largely in agreement with expectations - ω_b , ω_t >> ω, ν_e -driven current conforms to flux surfaces - Sauter , '04
- Several issues still being investigated
  - Threshold physics
  - Seeding processes

Summary (continued)

- Neoclassical polarization threshold physics
  - Size and sign of polarization currents with different models
  - Response with island width comparable to banana width - nonlocal response - Poli et al '04, Liu et al, 04
  - Island frequency
    - Eigenvalue of the transport equations- Connor,et al '01
    - Minimum dissipation theorem? - Smolyakov and CCH
- Anisotropic heat conduction - $\chi_{perp}/\ \chi_{||}$ model
  - Parallel heat conduction is non-local at high temperature - Held '01
  - Perpendicular transport in slowly evolving island topology with stochasticity,etc.?

Summary (continued)

- Seeding mechanisms
  - Delta' evolution
  - Produced via outside MHD events
    - Electromagnetic coupling - proper modeling requires numerical treatment - NIMROD, etc.
    - Transient transport events?
- Other physics
  - Sawtooth control - JET
  - Resistive interchange effects - a threshold mechanism? Lutjens and Luciani '02
  - Neoclassical currents due to pressure variations within flux surfaces - Smoluakov '04
  - Flow and flow shear effects' - CCH '04