Advantages of Poloidal Field Sensors in RWM Feedback

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"CONTROL OF MHD STABILITY: BACK TO THE BASICS"

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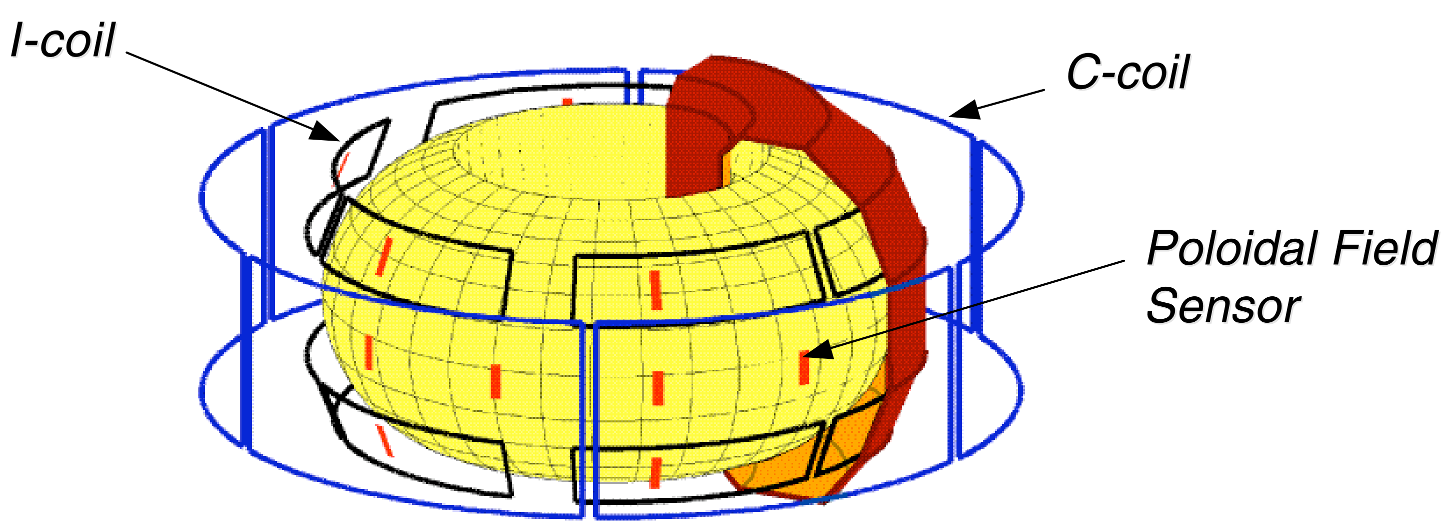
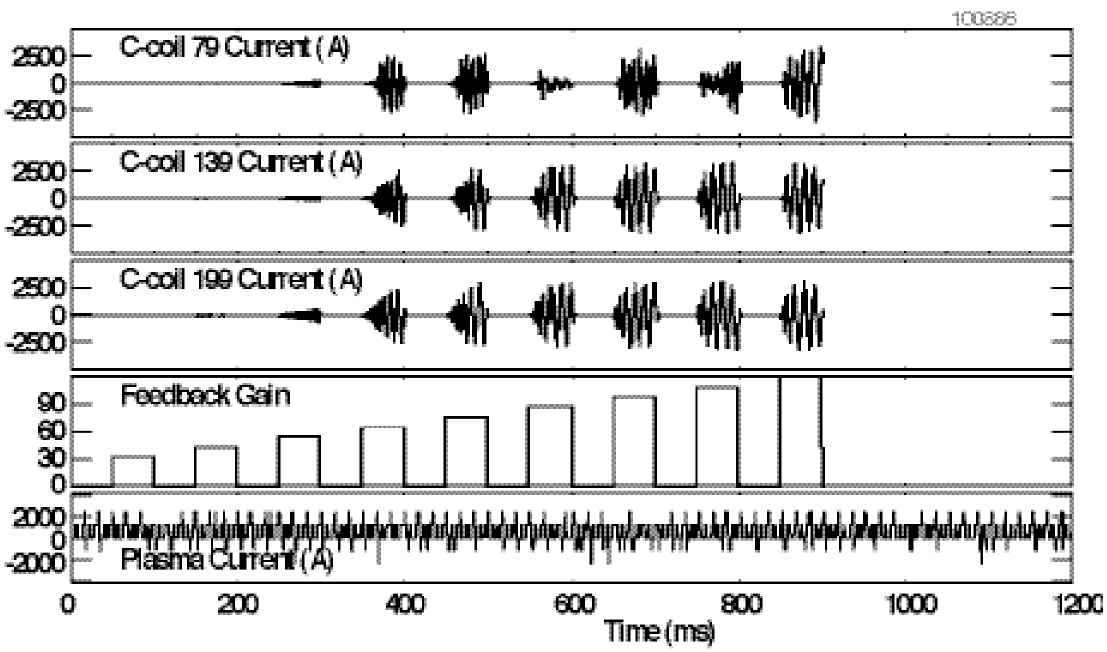
Introduction

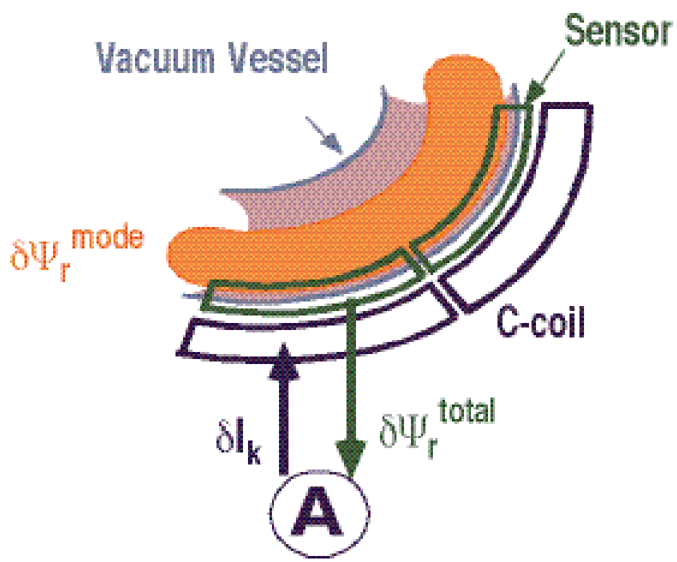
- A fundamental stability limit for Smart Shell feedback with external coils, radial field sensors and open-loop transfer function with upper cutoff Ω:
  - the upper cutoff frequency of the feedback system must be at least as large as the open-loop growth rate of the instability
- Modeling shows that a system using external feedback coils and poloidal field sensors can stabilize a mode with growth rate exceeding the “speed” of the system itself, i.e. Ω
  - Only with proper arrangements of the coupling between coils and sensors
- Using internal feedback coils, the stabilization “speed” of the system can be exceeded with less stringent requirements on the coil-sensor coupling



Maximum stable gain observed in DIII-D feedback experiments

- Maximum stable gain and frequency of the oscillation that occurs when that maximum is exceeded were measured for both vacuum and stable plasma cases
- Using only proportional gain and feedback algorithm varying from Smart Shell to Simple Mode Control





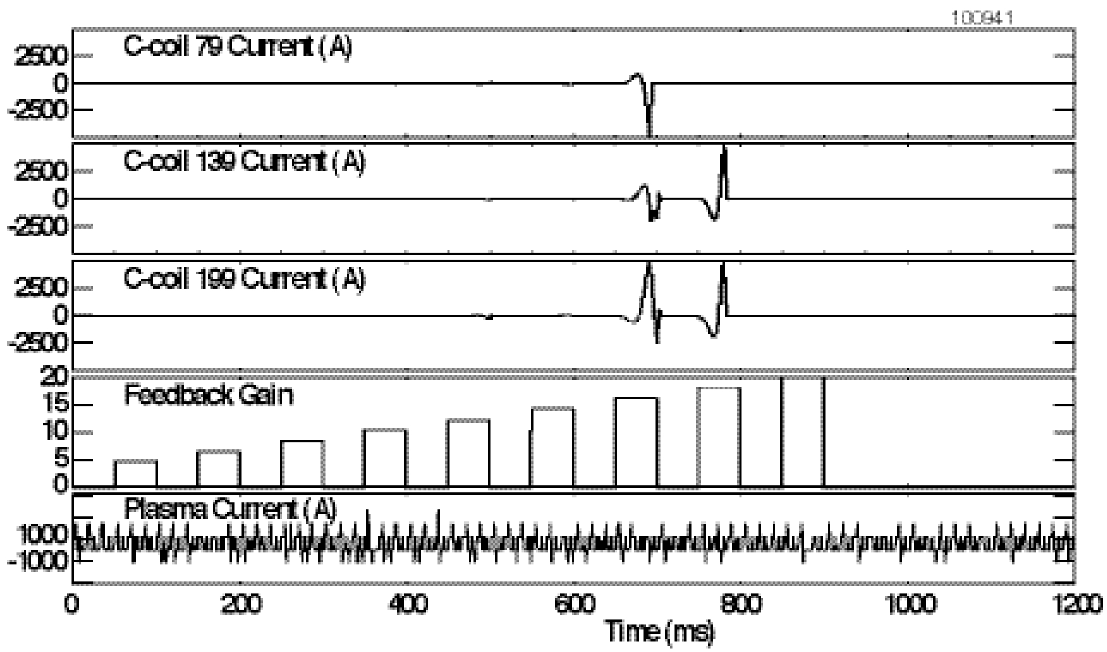
SMART SHELL

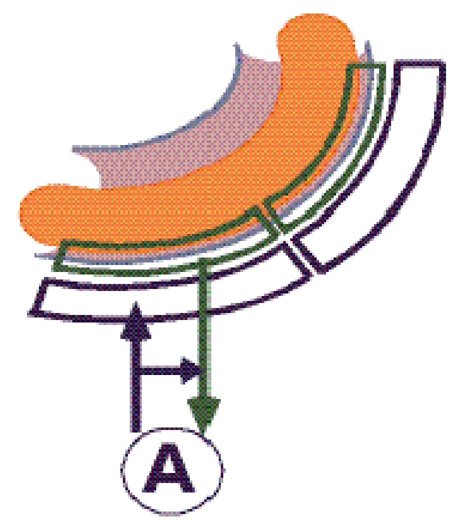
nulls out $\delta\Psi_r^{total}$ at wall

(Bishop, '89)

Maximum stable gain and frequency of oscillation vary widely with feedback algorithm

- Different types of feedback algorithm possible on DIII-D:
  - Smart Shell feedback = strongly coil-sensor coupling
  - “Full” Mode Control = no vacuum coil-sensor coupling
  - “Simple” Mode Control = no direct coil-sensor coupling, retains coupling through eddy currents





MODE CONTROL

nulls out $\delta\Psi_r^{mode}$ at wall

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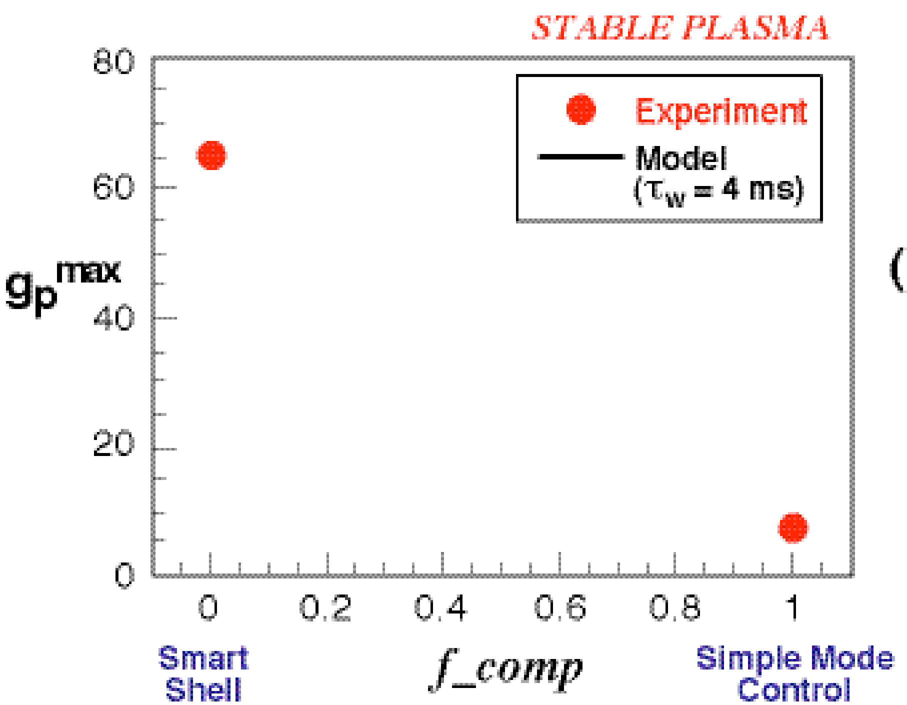
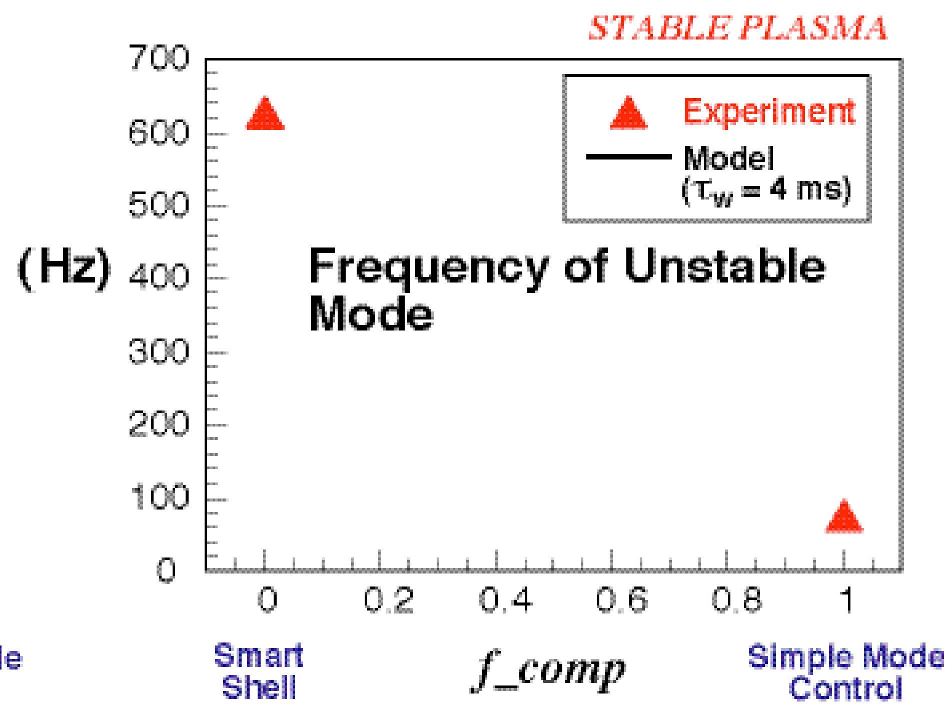
Maximum stable gain is smaller in presence of plasma

- Using only proportional gain and feedback algorithm varying from Smart Shell to Simple Mode Control





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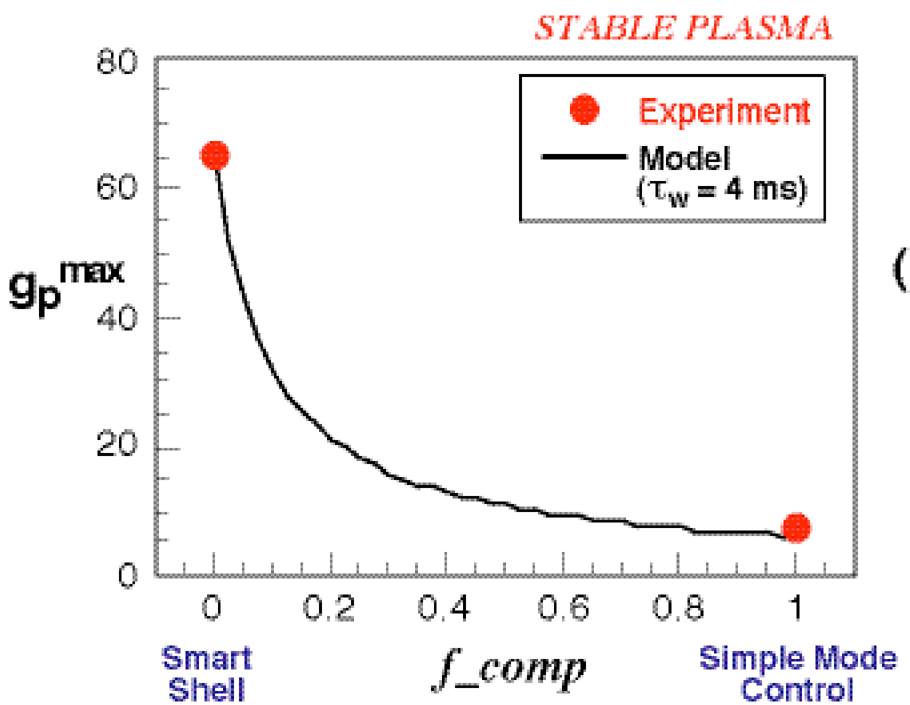
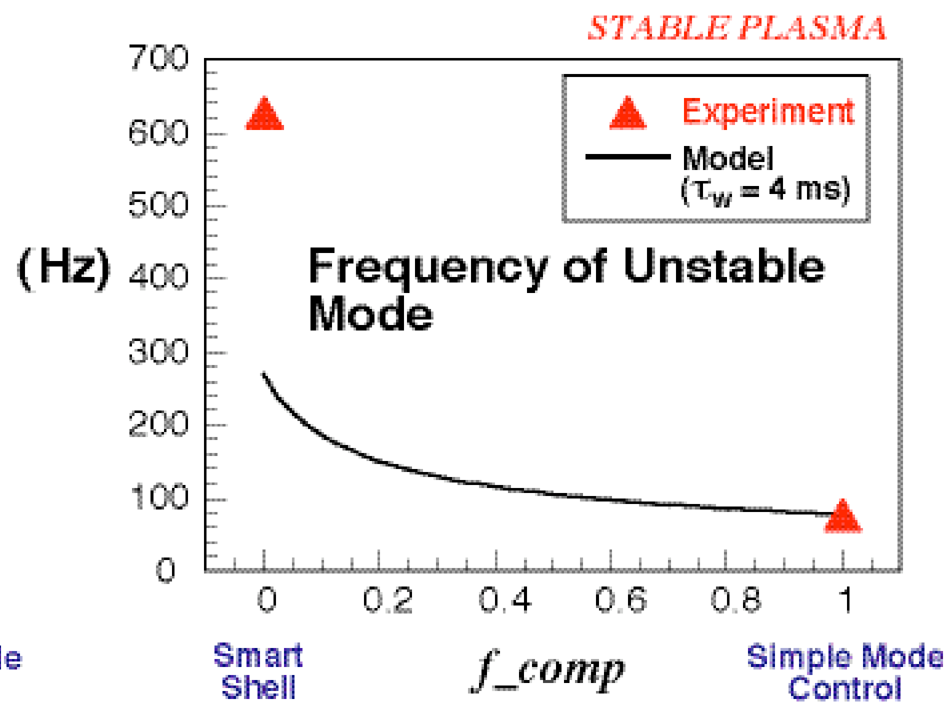
Semiquantitative model includes the effects of realistic electronics

- Model can be used for quantitative predictions through a conversion factor accounting for differences in mutual inductance values between model and experiment
  - A.M. Garofalo, T.H. Jensen, and E.J. Strait, *Phys. Plasmas* 9, 4573 (2002)





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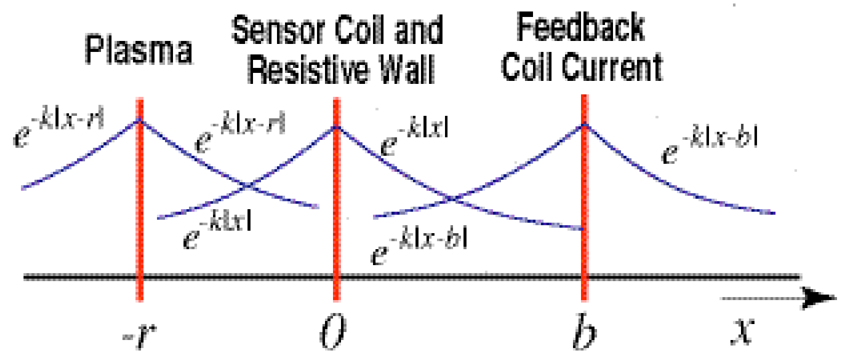
Slab model treats all currents as sheet current distributions

- **Assume:**

$$\partial/\partial t = i\omega t$$

$$\partial/\partial y = ik_t \ , \qquad k_t = n/R$$

$$\partial/\partial z = ik_p \ , \qquad k_p = m/r \ , \qquad k = \sqrt{k_t^2 + k_p^2}$$



- **Perturbed magnetic field:**

$$\bar{b} = \bar{\nabla} \times \bar{a} \ , \qquad \bar{a} = \left(\hat{z} - \frac{k_p}{k_t} \hat{y} \right) \varphi(x) e^{i(k_t y + k_p z)}$$

- **The value of $\bar{a}$ at x can be calculated using the Green's functions for a current sheet J_i at x_i:**

$$\bar{a}(x,y,z) = \sum_i \frac{\mu_0}{2\sqrt{k_t^2 + k_p^2}} J_i(x) e^{i(k_t y + k_p z)} \left(\hat{z} - \frac{k_p}{k_t} \hat{y} \right) e^{-\sqrt{k_t^2 + k_p^2}\,|x - x_i|}$$

Only one mode involved => plasma response given by only one parameter (e.g. instability strength)

- Boundary condition (external feedback coils): $$\left.\frac{1}{\varphi(x)}\frac{\partial\varphi(x)}{\partial x}\right|_{x=0^-} \equiv \Lambda$$
  - Equivalent to the assumption for the plasma current:

$$\bar{J}_P = -\frac{1}{\mu_0} A\bar{a}(0)\ , \quad A = (k-\Lambda)e^{ka}$$

- Wall currents: $$\bar{J}_W = -\frac{1}{\mu_0} i\omega\bar{a}(0)2k\tau_W\ , \quad \tau_W \equiv \frac{\delta\mu_0}{2k\eta}$$
- For Smart Shell feedback with sensors measuring the flux at the resistive wall the feedback current is: $$J_F = -G(i\omega)\varphi/M$$
- Dispersion relation: $$\alpha - i\omega\tau_W - G(i\omega) = 0\ , \quad \alpha = -\frac{1}{2}\left(\frac{\Lambda}{k}+1\right), \quad \alpha = \gamma_0\tau_W$$

Broader amplifier bandwidth improves feedback stabilization effectiveness

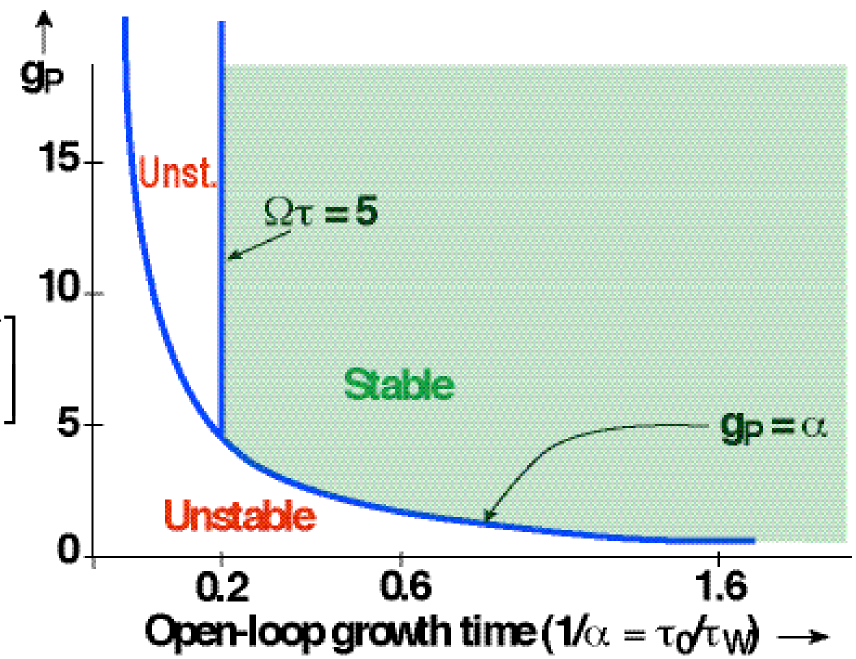
- **Simple example:** $G(i\omega) = g_P \dfrac{\Omega}{\Omega + i\omega}$, $(\Omega = \Omega^*)$

- **Dispersion relation:**

$$\alpha - i\omega\tau - g_P \frac{\Omega}{\Omega + i\omega} = 0$$

  – **With solutions:**

$$i\omega\tau = \tfrac{1}{2}\left[\alpha - \Omega\tau \pm \sqrt{(\alpha - \Omega\tau)^2 + 4\Omega\tau(\alpha - g_P)}\right]$$



- **Stability requires $\mathcal{R}e\{i\omega\} < 0$, therefore:**

$$\Omega\tau \geq \alpha = \gamma_O\tau$$

  – **i.e. bandwidth of the feedback system must be at least as large as the open-loop growth rate of the instability**

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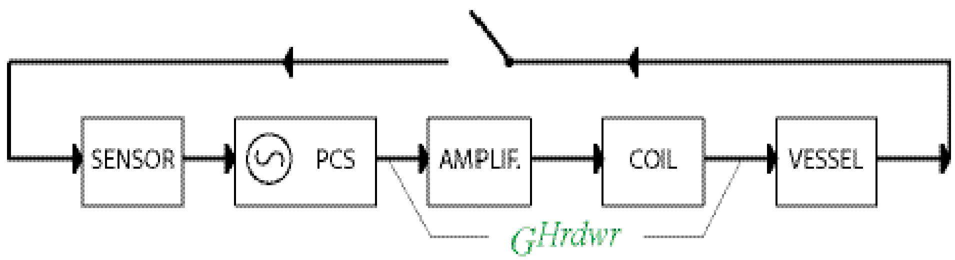
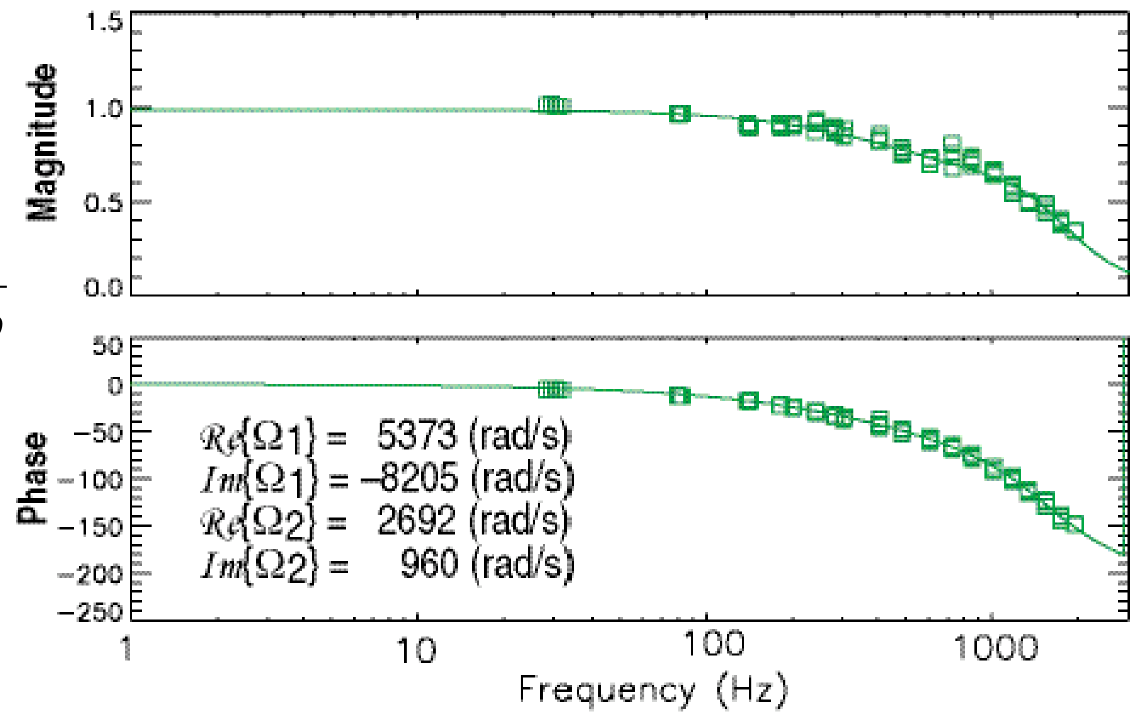
Measured DIII-D "hardware" gain is fitted to analytic function and inserted into dispersion relation

$$G(i\omega) = G^{PCS}(i\omega) \times G^{Hrdwr}(i\omega)$$

$$G^{PCS}(i\omega) = \frac{1}{1 + i\omega\tau_{\mathrm{P}}}\left(g_{\mathrm{P}} + \frac{g_{\mathrm{D}} i\omega\tau_{\mathrm{D}}}{1 + i\omega\tau_{\mathrm{D}}}\right) e^{i\omega\tau_{\mathrm{delay}}}$$



$$G^{Hrdwr}(i\omega) = \frac{\Omega_{U_1}}{\Omega_{U_1} + i\omega} \times \frac{\Omega_{U_2}}{\Omega_{U_2} + i\omega}$$



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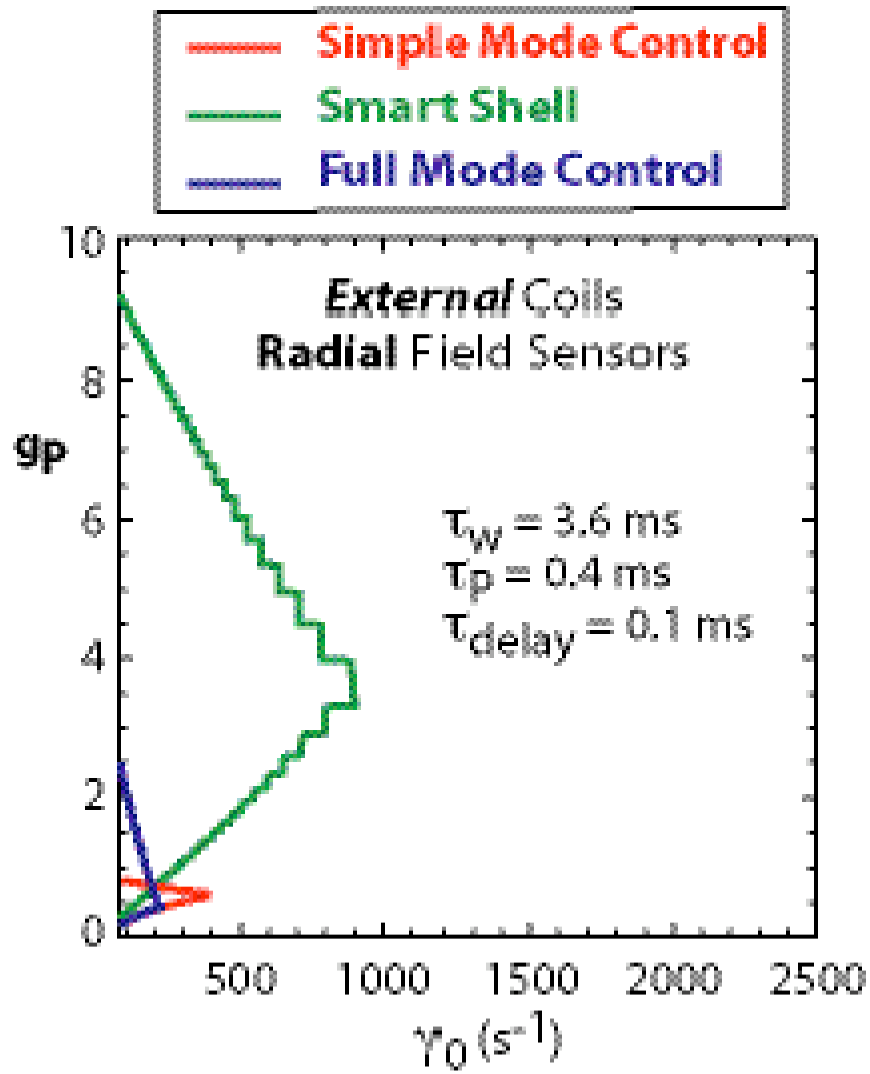
Smart Shell algorithm yields best performance with C-coil feedback using radial field sensors in DIII-D

- Simple Mode Control is obtained by removing from the sensor signal the direct coupling between Br sensors and feedback currents
- Dispersion relation:

$$\alpha - i\omega\tau_W - \frac{G(i\omega)}{1 - G(i\omega)} = 0\ , \qquad \alpha = \gamma_0\tau_W$$

- Full Mode Control is obtained by removing from the sensor signal the low-pass filtered (τ_W) contribution from the feedback currents
- Dispersion relation:

$$\alpha - i\omega\tau_W - \frac{G(i\omega)}{1 - \dfrac{G(i\omega)}{1 + i\omega\tau_W}} = 0$$



Slab model extended to simulate feedback using poloidal field sensors

- **Feedback current with poloidal field sensors inside the vessel:**

$$J_F(t) = -\frac{G(i\omega)}{M'}\left[-\frac{k_p}{k_t}\frac{\partial\varphi(x,t)}{\partial x}\bigg|_{x=0^-}\right] = -\frac{G(i\omega)}{kM}\lambda\varphi(0,t) = (1+2\alpha)\frac{G(i\omega)}{M}\varphi(0,t)$$

Dispersion relation: $\alpha - i\omega\tau_W + (1+2\alpha)G(i\omega) = 0$

- **Mode Control is achieved by removing from the sensor signal the direct coupling between Bp sensors and feedback coils:**

Simple Mode Control: $\alpha - i\omega\tau_W + \dfrac{(1+2\alpha)G(i\omega)}{1-G(i\omega)} = 0$

Full Mode Control: $\alpha - i\omega\tau_W + \dfrac{(1+2\alpha)G(i\omega)}{1-G(i\omega)/(1+i\omega\tau)} = 0$

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Feedback with Bp sensors can stabilize mode with growth rate exceeding "speed" of system itself

- Back to simple example for analytical demonstration:

$$G(i\omega) = g_P \frac{\Omega}{\Omega + i\omega} \;, \qquad (\Omega = \Omega^*)$$

- For Simple Mode Control feedback with either radial or poloidal field sensors, the condition for stability is:

$$\Omega \geq \frac{\gamma_O}{1 - g_P}$$

- Note stable values of g_P are:
  - Negative, for poloidal field sensors (stabilizing feedback tries to increase the perturbed poloidal field)
  - Positive, for radial field sensors (stabilizing feedback tries to reduce the perturbed radial field)
    - The requirements on the upper cutoff frequency of the system are lower with poloidal field sensors!!

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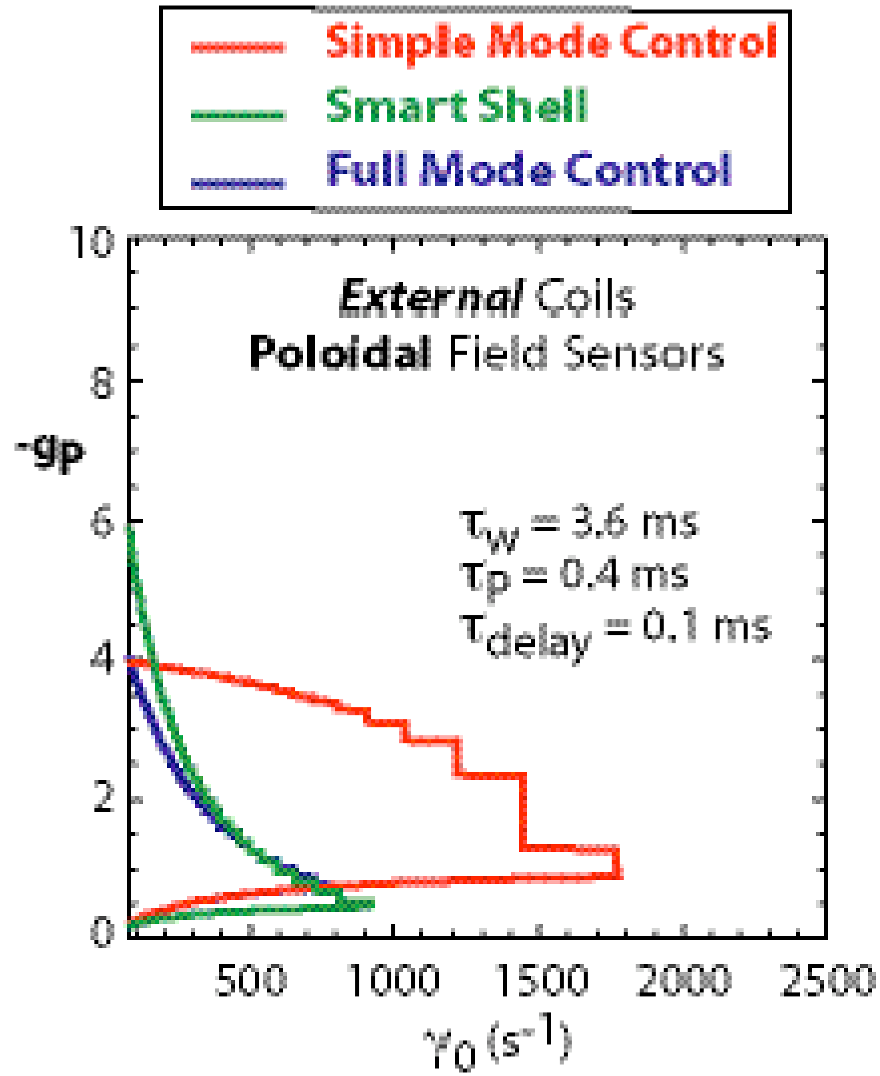
Advantage of poloidal field sensors (over radial) apparent also with realistic feedback transfer function

- Largest stabilizable growth rate using radial field sensors was < 1000 s^{-1}

- Simple Mode Control yields best performance with C-coil feedback using poloidal field sensors in DIII-D



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New boundary condition is necessary to simulate feedback using internal feedback coils

- Boundary condition (feedback coils at x = -b < 0): $\left.\frac{1}{\varphi(x)}\frac{\partial\varphi(x)}{\partial x}\right|_{x=-b^-} \equiv \Lambda$
  - Equivalent to the assumption for the plasma current:

$$\bar{J}_P = -\frac{1}{\mu_0} Q\bar{a}(-b) \ , \qquad Q = (k-\Lambda)e^{k(a-b)}$$

- The feedback current with sensors measuring the poloidal field between the feedback coils and the resistive wall is:

$$J_F(t) = -\frac{G(i\omega)}{M'}\left[-\frac{k_p}{k_t}\left.\frac{\partial\varphi(x,t)}{\partial x}\right|_{x=-b^+}\right] = -\frac{G(i\omega)}{kM}\lambda e^{-kb}\varphi(-b,t) = (1+2\alpha)e^{-kb}\frac{G(i\omega)}{M}\varphi(-b,t)$$

Dispersion relation: $\theta - i\omega\tau_W\left[(\theta+1)\Gamma - 1\right] + (1+2\theta)G(i\omega)\left[i\omega\tau_W\Gamma + 1\right] = 0$

$$\theta = \frac{\alpha}{(\alpha+1)e^{2kb} - \alpha}$$

$$\Gamma = 1 - e^{-2kb}$$

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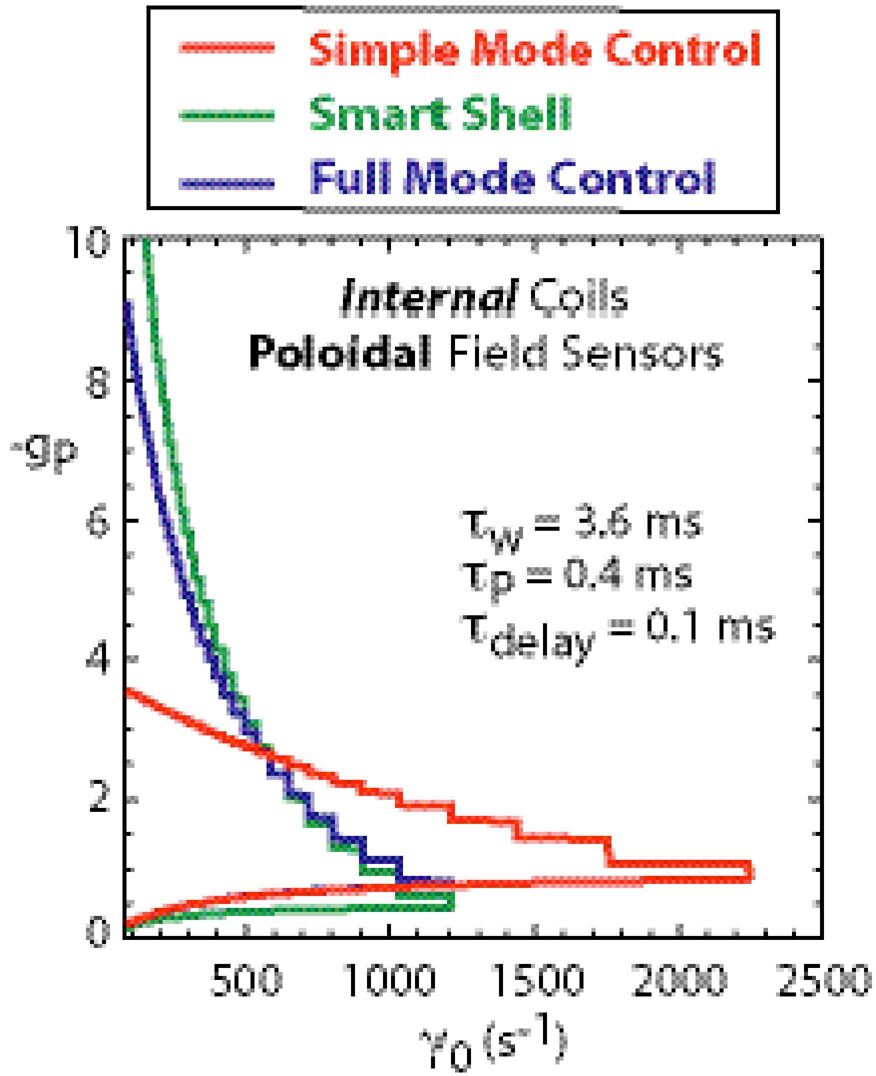
Easier to beat system "speed" using internal instead of external feedback-coils

- With external feedback coils, the advantage of poloidal field sensors applies only to Simple Mode Control feedback, i.e. sensors and feedback coils partailly decoupled
- With internal feedback coils, the advantage applies even to feedback with strongly coupled sensors and feedback coils:
  - Assume coils at x = -b < 0 and sensors measuring the poloidal field at x = -b$^+$
  - The condition for stability is: $$\Omega \geq \frac{\gamma_O}{1-\left(1-e^{-2kb}\right)\left(1+\alpha+g_P+2\alpha g_P\right)}$$
    - The requirement on the upper cutoff frequency of the system is eased if: $$g_P \leq -\frac{1+\alpha}{1+2\alpha}$$

Performance of all feedback algorithms is improved with I-coil and poloidal field sensors in DIII-D

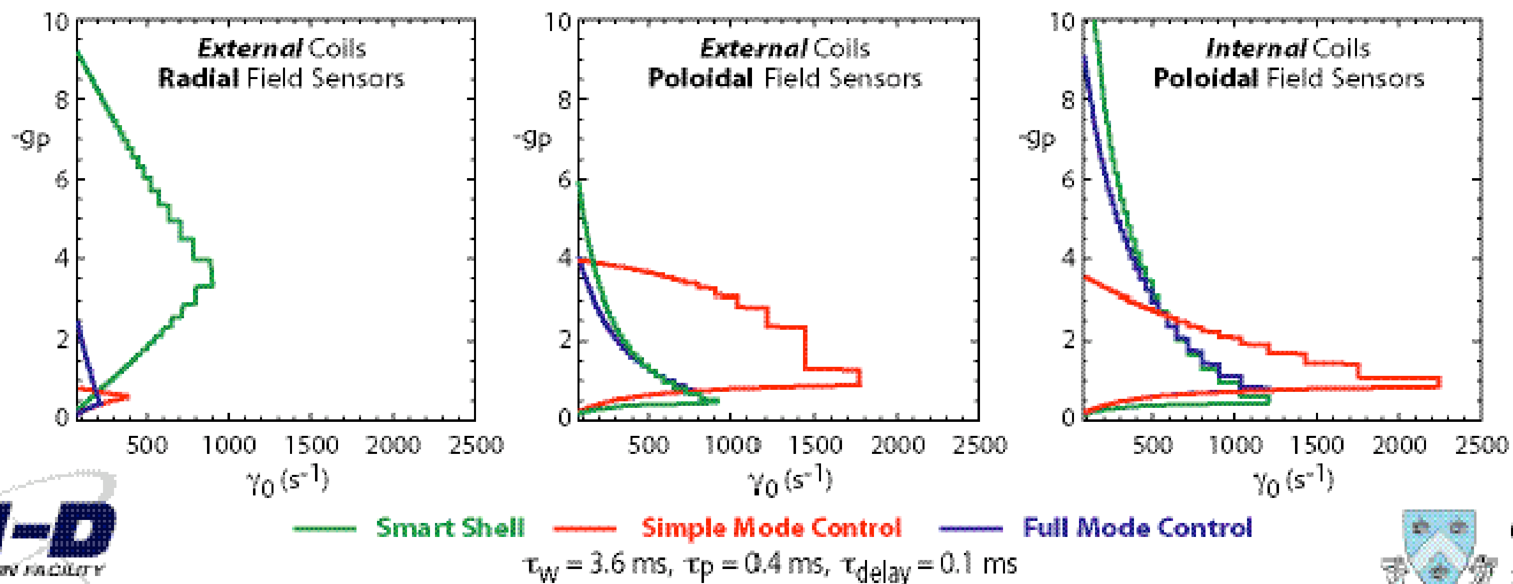
- Largest stabilizable growth rate using radial field sensors and external coils was < 1000 s^{-1}

- Simple Mode Control yields best performance with I-coil feedback using poloidal field sensors in DIII-D



Summary

- Modeling a current-controlled feedback system with realistic open-loop transfer function
  - Feedback with <u>external</u> coils and radial field sensors
    - ✓ Necessary condition for stability is that the “speed” of the system itself must be at least as large as the open-loop growth rate of the instability
  - Feedback with <u>external</u> coils and poloidal field sensors
    - ✓ Partially decoupled feedback can stabilize a mode with growth rate exceeding the “speed” of the system itself
  - Using <u>internal</u> feedback coils, the stabilization “speed” of the system can be exceeded with less stringent requirements on the coil-sensor coupling



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Some definitions

$k_t = n/R$ Wavenumber in toroidal direction

$k_p = m/r$ Wavenumber in poloidal direction

$$k = \sqrt{k_{\mathrm{t}}^2 + k_{\mathrm{p}}^2}$$

$M = \dfrac{\mu_0 e^{-kb}}{2k}$ Mutual inductance between control coils (at x=b 0) and radial field sensors (at x=0)

$b_z = -\dfrac{k_p}{k_t} \dfrac{\partial \varphi(x,t)}{\partial x}\Big|_{x=0^-}$ Poloidal field at the wall (x=0$^-$)

$M' = -kM \dfrac{k_p}{k_t}$ Mutual inductance between control coils (at x=b 0) and poloidal field sensors (at x=0$^-$)

τ Resistive wall time constant (~3.5 ms for RWM in DIII-D)

α Open-loop growth rate in units of τ^{-1} ( ) $\alpha = \gamma_O \tau$

g_{P} Proportional feedback gain

g_{D} Time-derivative feedback gain

τ_{P} Proportional-gain time constant (sets frequency cutoff for low-pass filter): noise reduction

τ_{D} Derivative-gain time constant (sets frequency cutoff for high-pass filter)