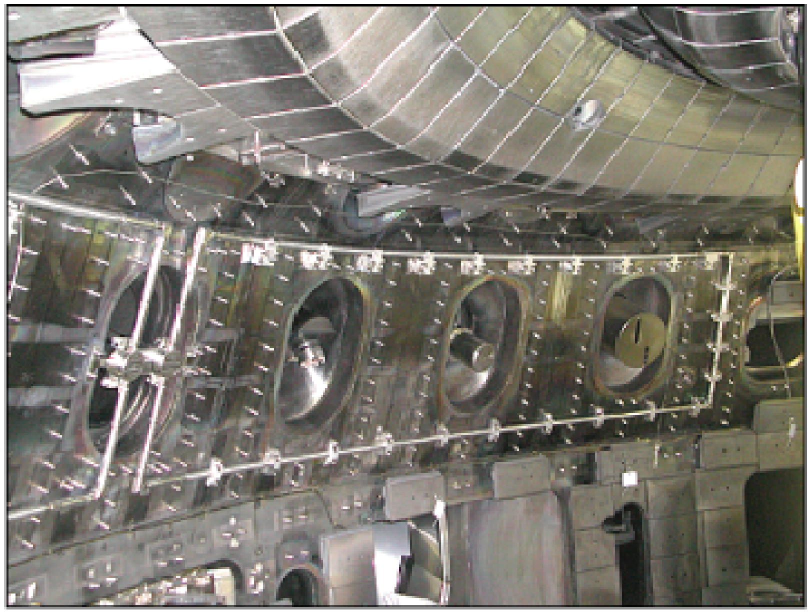
Time Domain VALEN Calculations



Jim Bialek
with
Allen Boozer, Oksana Katsuro-Hopkins & Gerald A. Navratil

Columbia University

MHD Mode Control Workshop
University of Texas-Austin
3-5 November, 2003

•Outline

•Calculations performed with **VALEN** computer code **in the time domain**.

•Resonant Field Amplification

•Time evolved calculations, basic passive performance and results with idealized feedback

•Time evolved calculation, adding realistic effects to feedback circuits: **noise and time delay**.

VALEN Formulation

VALEN uses a thin shell lumped circuit finite element approximation to model induced currents in passive structures. The resulting equations are matrix L & R 'mesh' **circuit equations**, conducting thin shells, arbitrary thin coils, and magnetic sensors may be included in this formulation. Sensors are modeled via a mutual inductance to the problem variables, i.e.,

$$\{\Phi_{sensor}\} = [M]\{I\}$$

feedback rules may be defined among the sensors and coils.

$$G_p \Phi_s(t - \tau_p) + G_d(\dot{\Phi}_s)(t - \tau_d) = V_{coil}(t)$$

A plasma mode can be included,

see Boozer, Physics of Plasmas, 5, page 3350, (1998)

$$\left[L_{ww}\right]\left\{I^{w}\right\}+\left[M_{wp}\right]I^{d}+\left[M_{wp}\right]I^{p}=\left\{\square^{w}\right\}$$

$$\left[M_{pw}\right]\left\{I^{w}\right\}+LI^{d}+LI^{p}=\square$$

$$LI^{p}=(1+s)\square$$

The plasma mode is represented by a current distribution in a control surface surrounding the unperturbed plasma, the strength of this mode is represented by the parameter s is the normalized mode energy, when s<0 stable, when s>0 unstable, and   marginal stability

$$s=\frac{\square\square W}{L_{s}I_{s}^{2}/2}$$

From a plasma equilibrium, we obtain $\square W$ and the mode's B-normal on the plasma surface from **DCON** (A. Glasser's code).

Resonant Field Amplification

A. Boozer predicted Resistive Wall Modes would amplify magnetic field (e.g. errors see Physics of Plasmas 10, pg 1458 (2003))

The VALEN formulation handles this via an extra field source

$$\left[L_{ww}\right]\left\{I^{w}\right\}+\left[M_{wp}\right]I^{d}+\left[M_{wp}\right]I^{p}+\left[M_{wD}\right]\left\{I^{D}\right\}=\left\{\Box^{w}\right\}$$

$$\left[M_{pw}\right]\left\{I^{w}\right\}+LI^{d}+LI^{p}+\left[M_{pD}\right]\left\{I^{D}\right\}=\Box$$

$$LI^{p}=(1+s)\Box$$

$$\left\{\dot{\Box}_{w}\right\}+\left[R_{ww}\right]\left\{I^{w}\right\}=\left\{V\right\}$$

$$\left\{\dot{\Box}\right\}+\left[R_{d}\right]\left\{I^{d}\right\}=\left\{0\right\}$$

(resonant field amplification) in the sensors is given by

$$\left[\left[M_{sw}\right]\Box\left[M_{sp}\right]\frac{(1+s)}{sL}\left[M_{pw}\right]\right]\left\{I^{w}\right\}+\left[\left[M_{sp}\right]\Box\left[M_{sp}\right]\frac{(1+s)}{sL}L\right]I^{d}+$$

$$\left[\left[M_{sD}\right]\Box\left[M_{sp}\right]\frac{(1+s)}{sL}\left[M_{pD}\right]\right]\left\{I^{D}\right\}=\left\{\Box^{s}\right\}$$

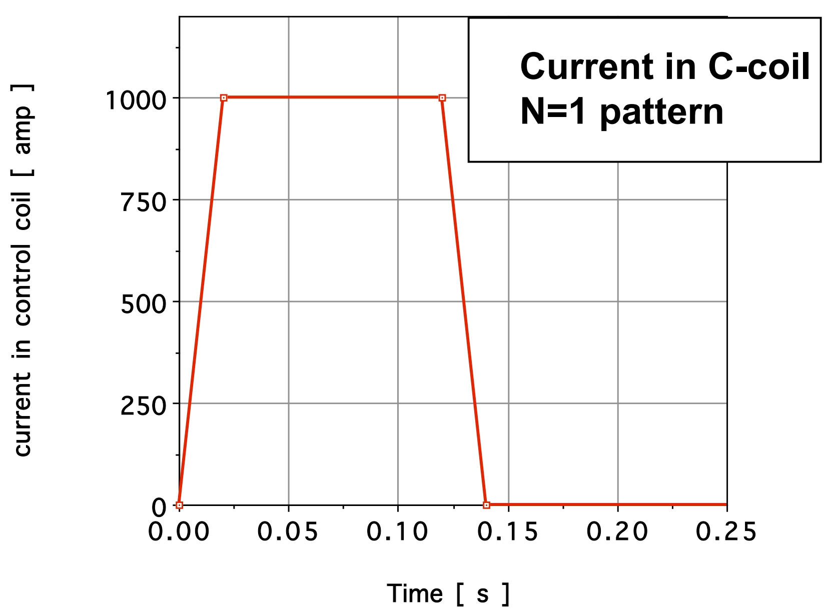
As ‘s’ approaches 0^- we have two effects:

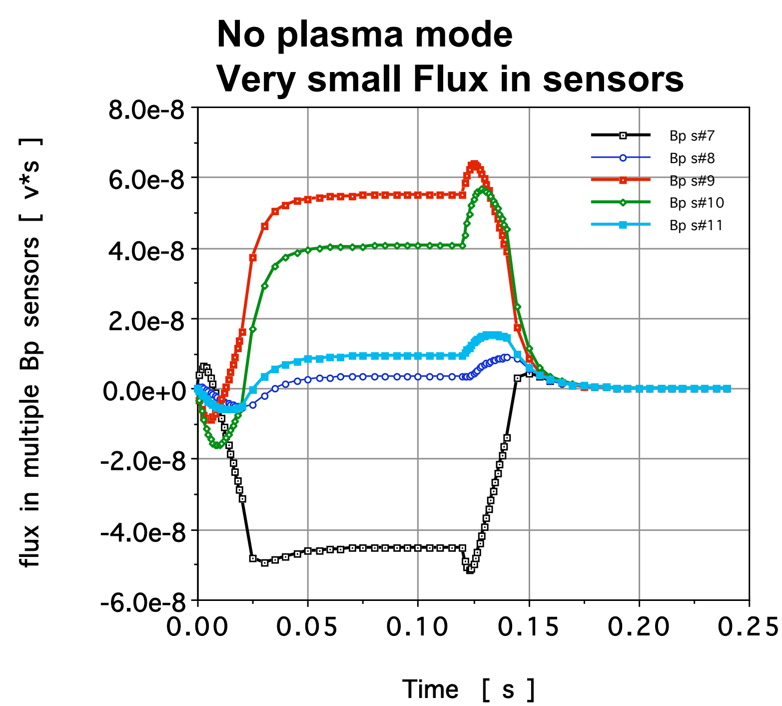
1) near singular behavior (amplification)
2) RWM mode response gets slower!

We expect similar behavior approaching marginal stability in a rotationally stabilized RWM above the no-wall limit.

We demonstrate RFA in a VALEN model of DIII-D

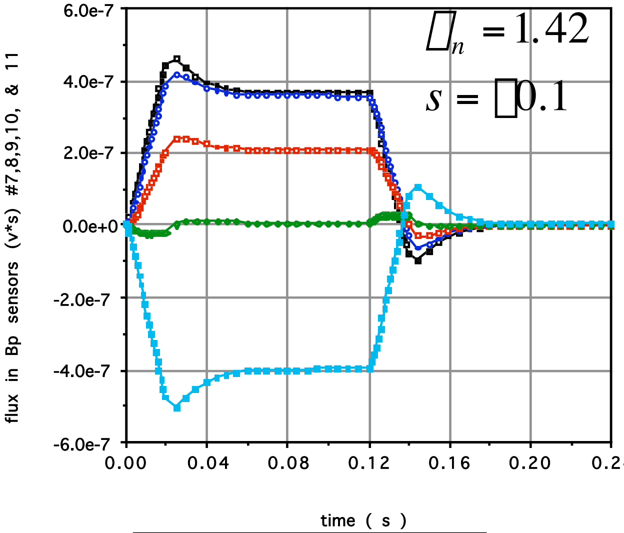
1) use ‘C-coil’ to generate a resonant field
2) use poloidal sensors with small direct coupling to the C-coil
3) examine sensor signal as ‘s’ varies



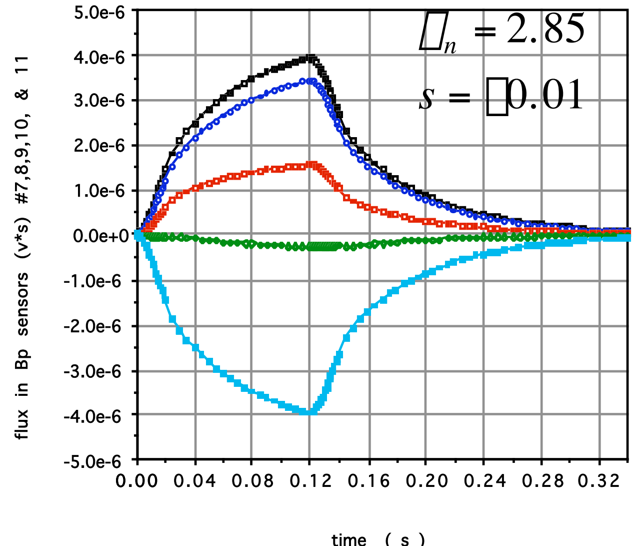


Weakly Damped Modes Slow to Approach Steady State Values

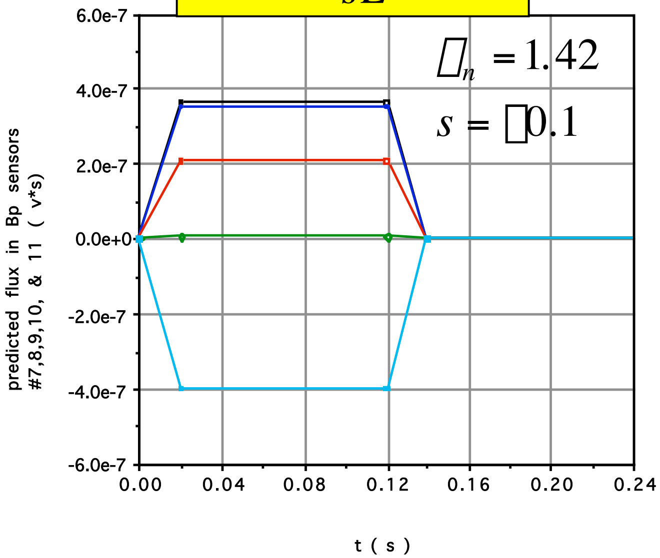
Sensor Signals



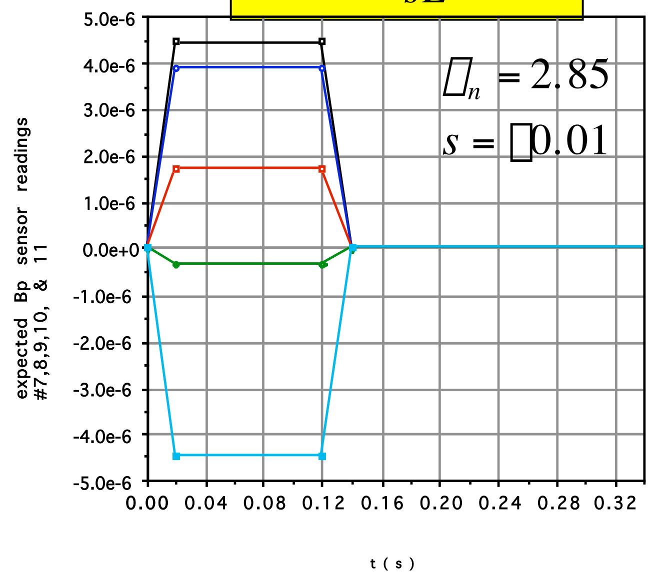
Sensor signals



$$\square\left[M_{sp}\right]\frac{(1+s)}{sL}\left[M_{pD}\right]$$

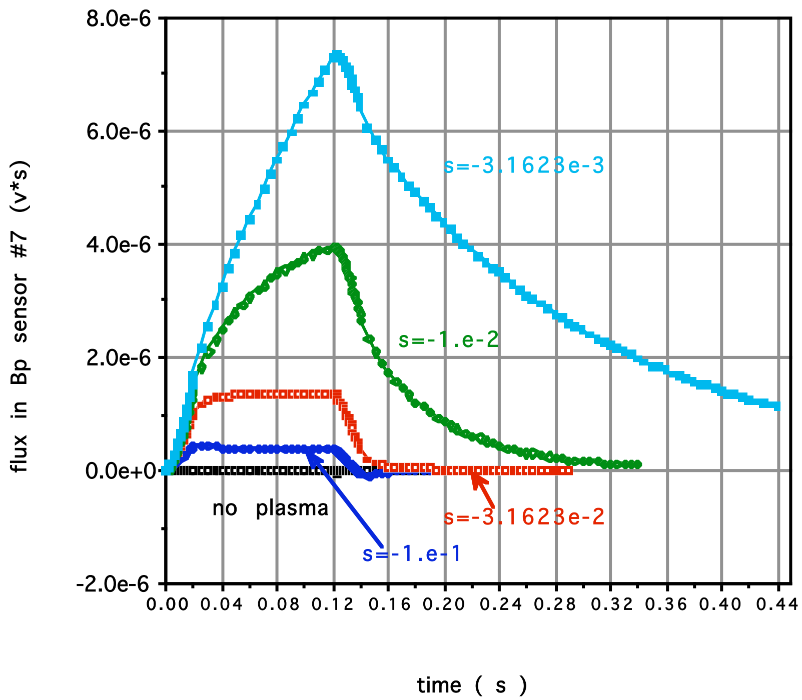


$$\square\left[M_{sp}\right]\frac{(1+s)}{sL}\left[M_{pD}\right]$$

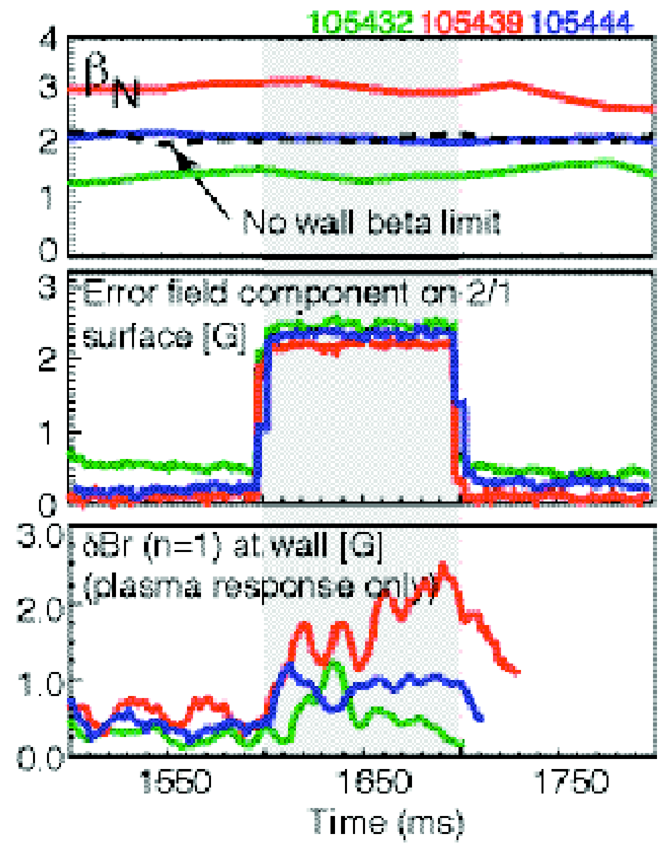


Sensor Flux for Damping Rates Showing Resonant Field Amplification

S	β_N	Damping Time
-0.1	1.42	-0.012 sec
-0.01	2.85	-0.058 sec
-0.001	2.98	-0.546 sec

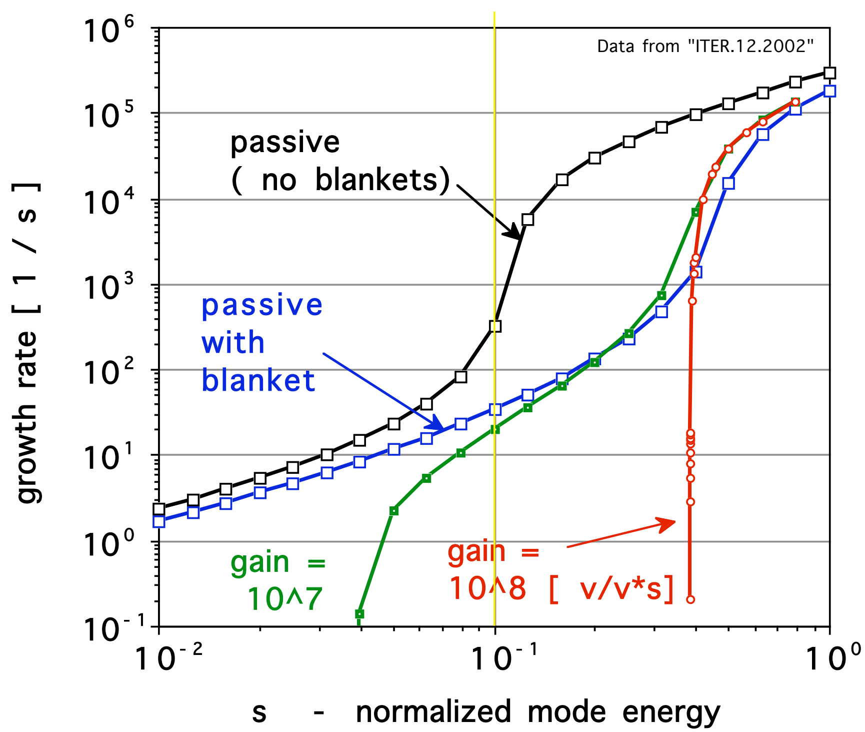


DIII-D Results

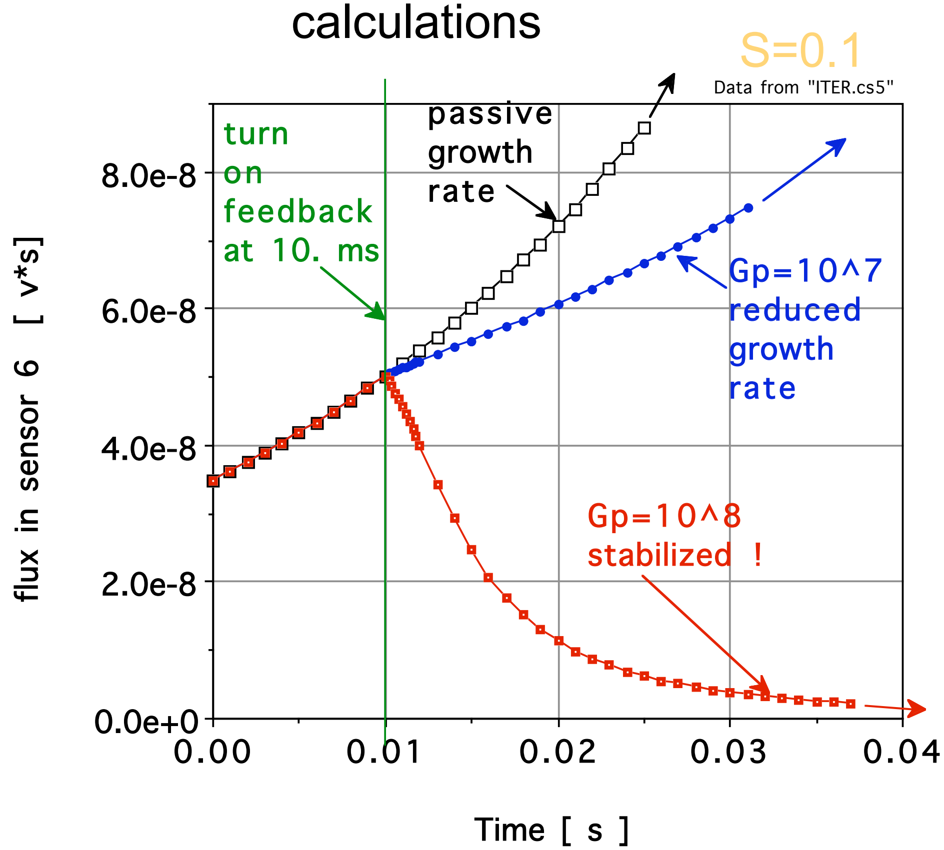


VALEN Eigenvalue (growth rate) and transient calculations are consistent for zero time delay in the feedback logic Example from VALEN ITER analysis

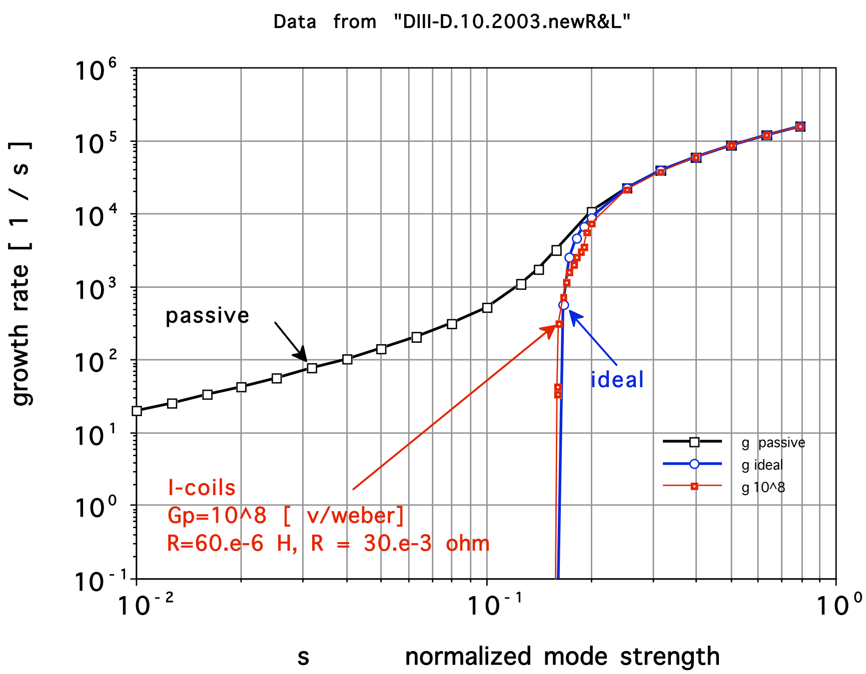
Growth rate predicted
By eigenvalue analysis
Do transients at s=0.1

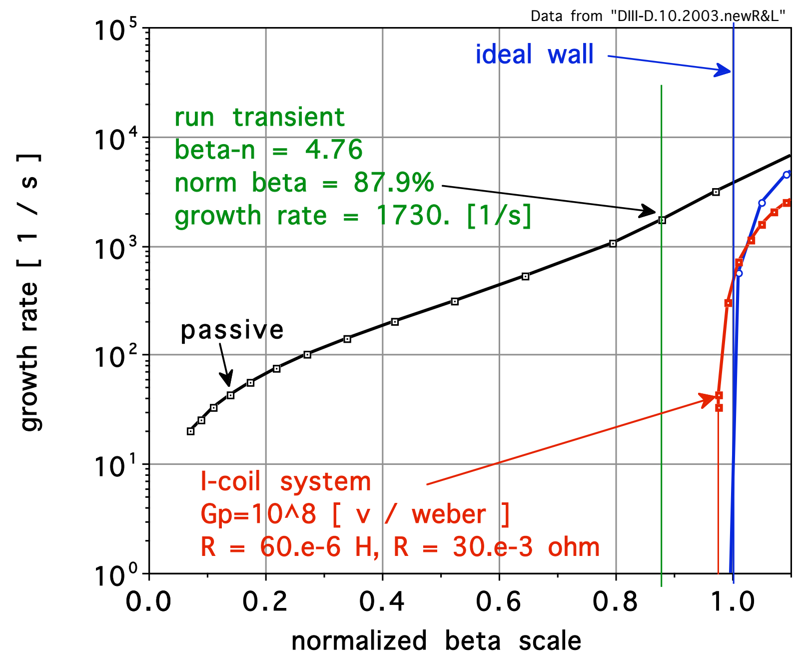


Expected growth rates
Observed in transient
calculations



DIII-D I-Coil Feedback Performance



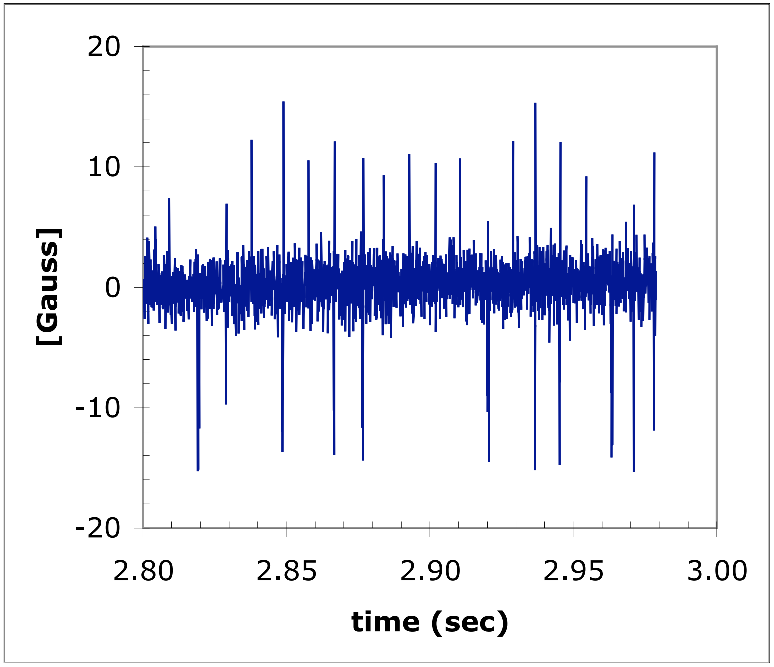


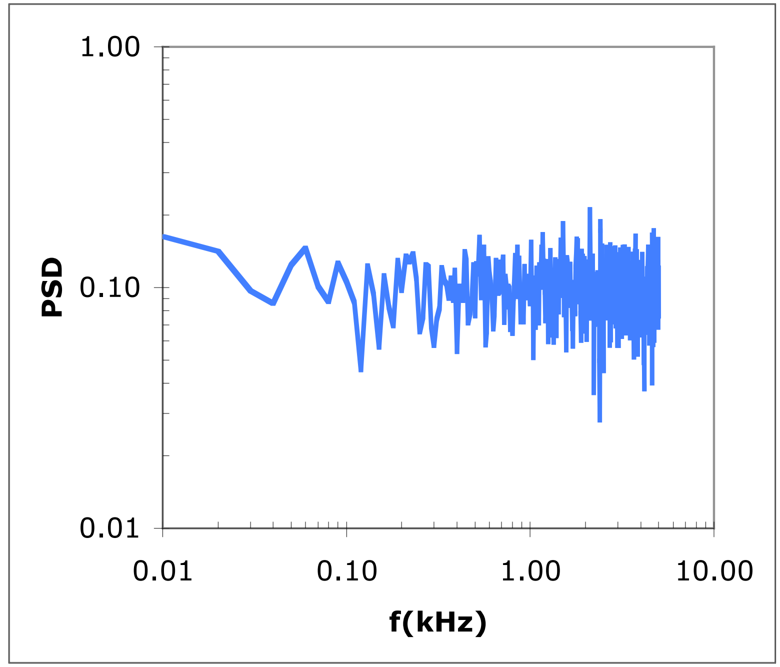
Conversion from s to □

$$\beta_{normal} = \frac{\beta - \beta_{no\_wall}}{\beta_{ideal\_wall} - \beta_{no\_wall}}$$

s	β	%
0.15849	4.49	97.0
0.15634	4.48	95.8
0.15418	4.48	94.6
0.14987	4.46	92.2
0.14125	4.43	87.4
0.13357	4.40	83.2
0.12589	4.38	78.9
0.11295	4.33	71.7
0.10000	4.29	64.5

Simulated Noise on DIII-D RWM Poloidal Sensors

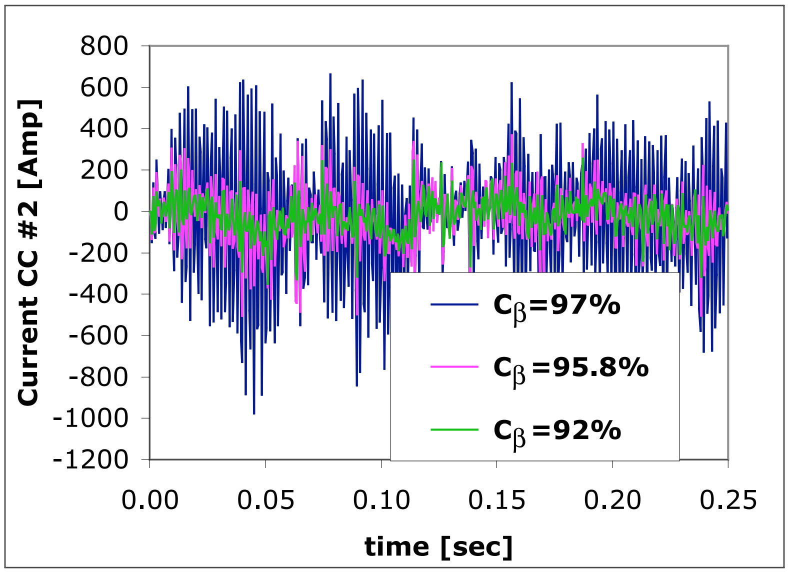


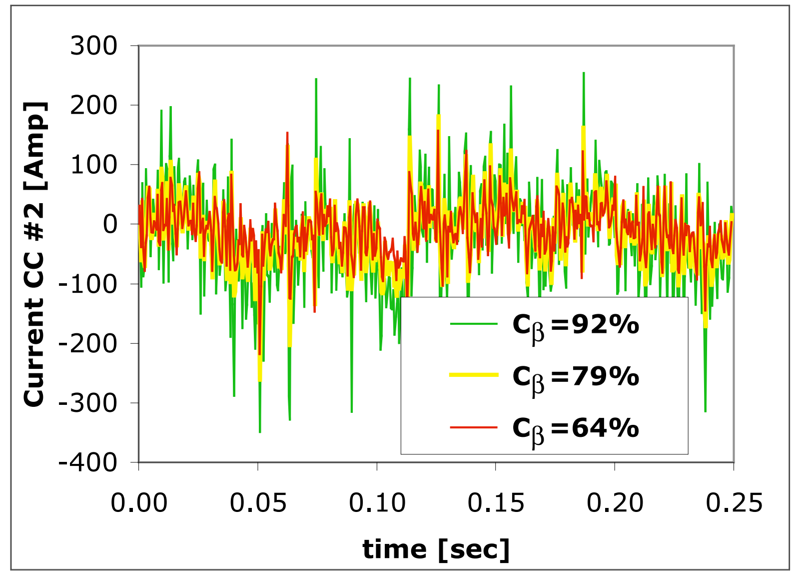


Broadband noise *was modeled as Gaussian random number with standard deviation* ***1.5 G*** *about 0 mean and frequency 10kHz.*

To the broadband noise ***ELMs*** *were added as additional Gaussian random numbers from* ***6 to 16 Gauss*** *approximately every 10 msec with +/- chosen with 50% probability.*

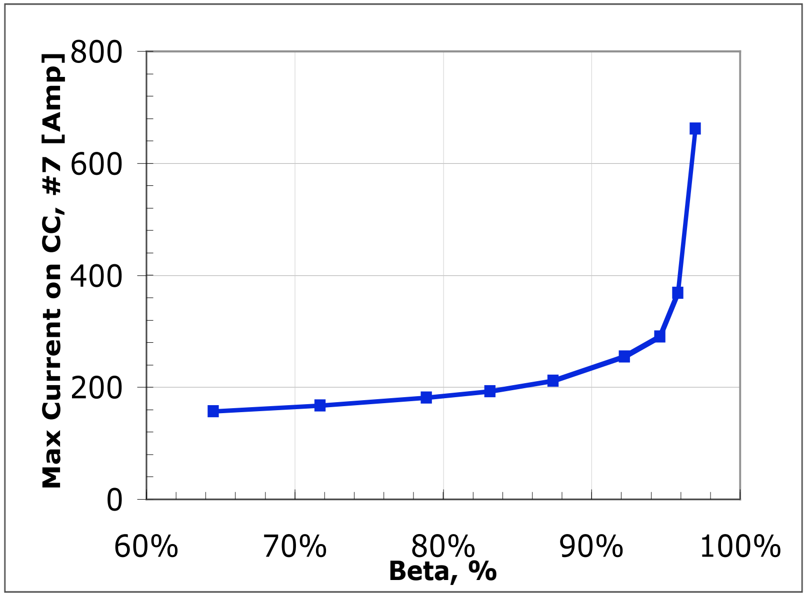
DIII-D I-Coil Feedback Current Simulation with Sensor Noise for Range of β_N

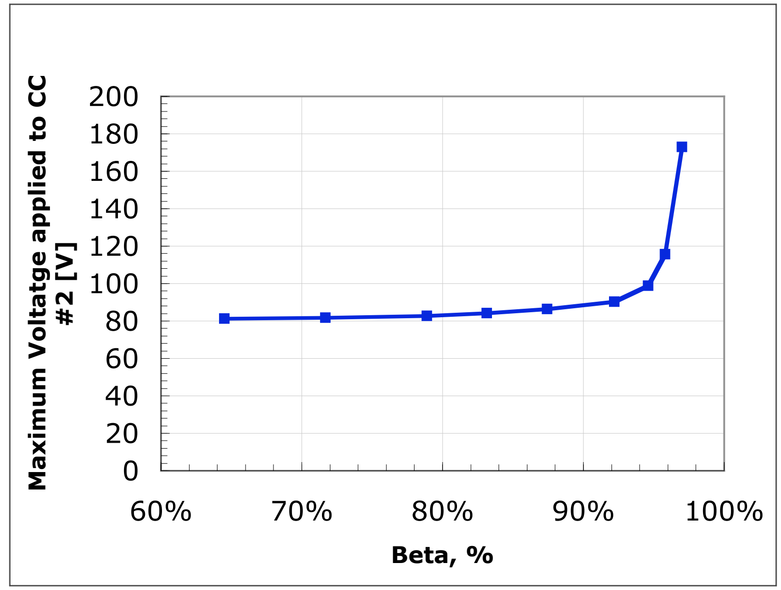




L=60 mH and R=30 mOhm
with Proportional Gain G_p=7.2Volts/Gauss

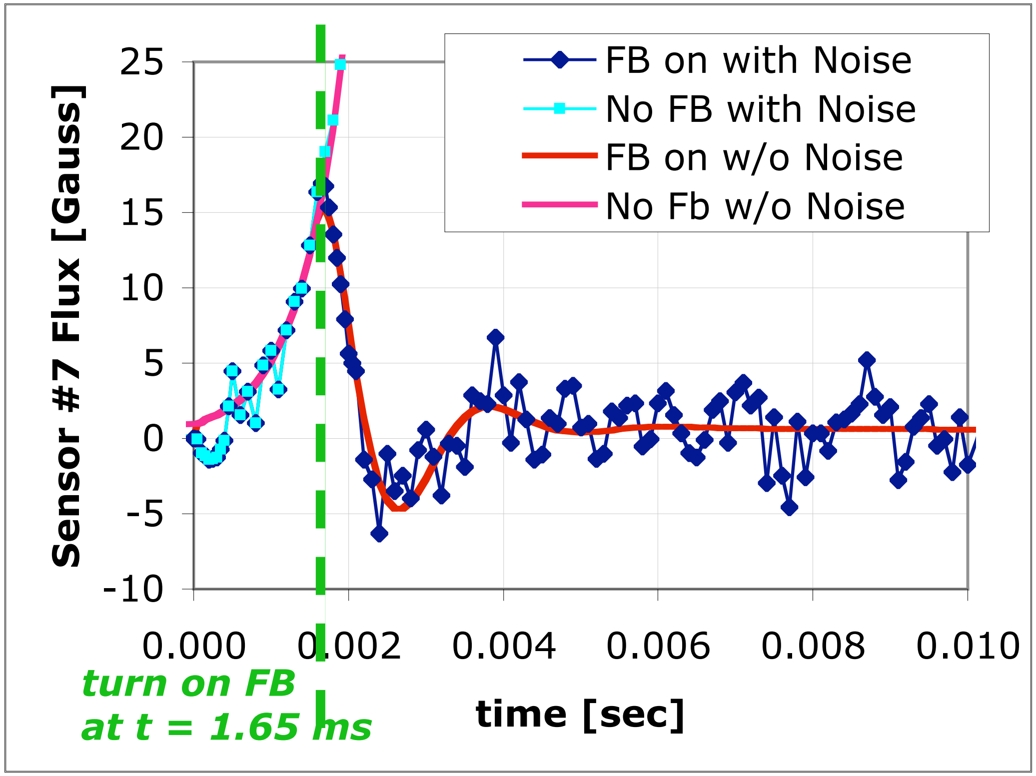
Resonant Amplification of Noise Limits Feedback when Approaching Ideal Limit





Maximum control coil current and voltage as function of β_{normal}

Effects of Noise on Feedback Dynamics

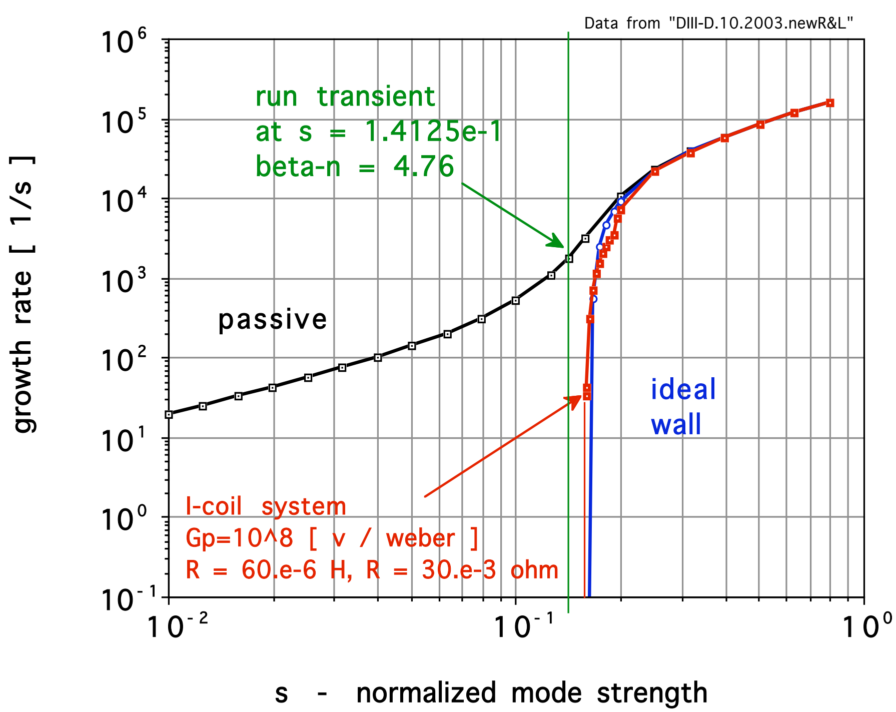


Sensor Flux

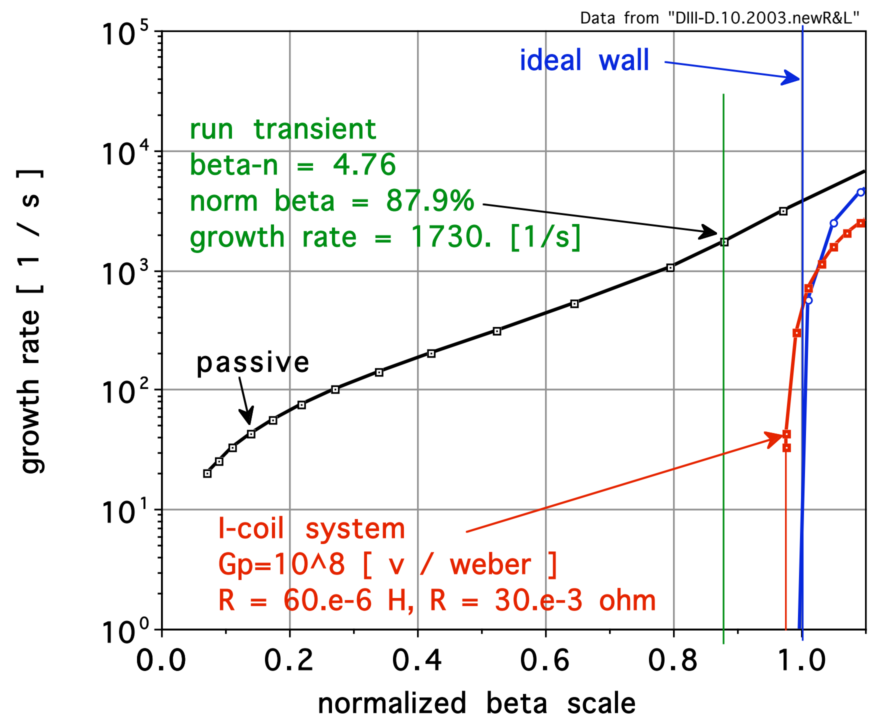
L=60 mH and R=30 mOhm DIII-D I-Coil Feedback model with Proportional Gain G_p=7.2Volts/Gauss

VALEN may model time delays in Feedback Performance
Examine performance of DIII-D I-coil system

VALEN predictions
Eigenvalue analysis
Zero delay time
Results vs. 's'



VALEN predictions
Eigenvalue analysis
Zero delay time
Results vs. normalized betan

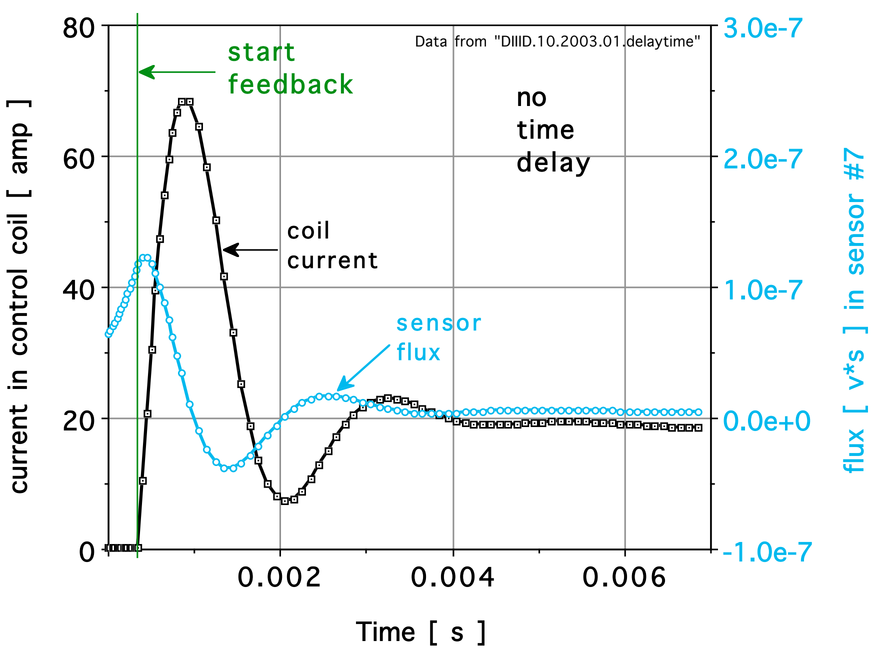


BAISC problem, start feedback at t = 0.35 ms, beta-n = 4.76
Feedback defined by:
Poloidal sensor flux * gain = control coil voltage
Stabilized behavior !

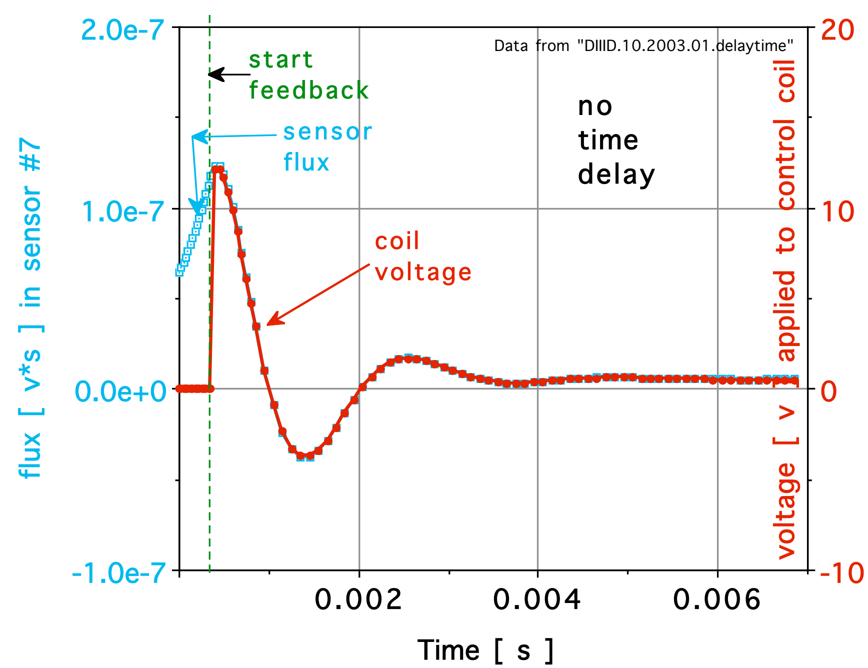
No time delay ! I.e.,

$$G_p \Phi_p(t) = V_{cc}(t)$$

Coil current & sensor flux



Sensor flux & coil voltage

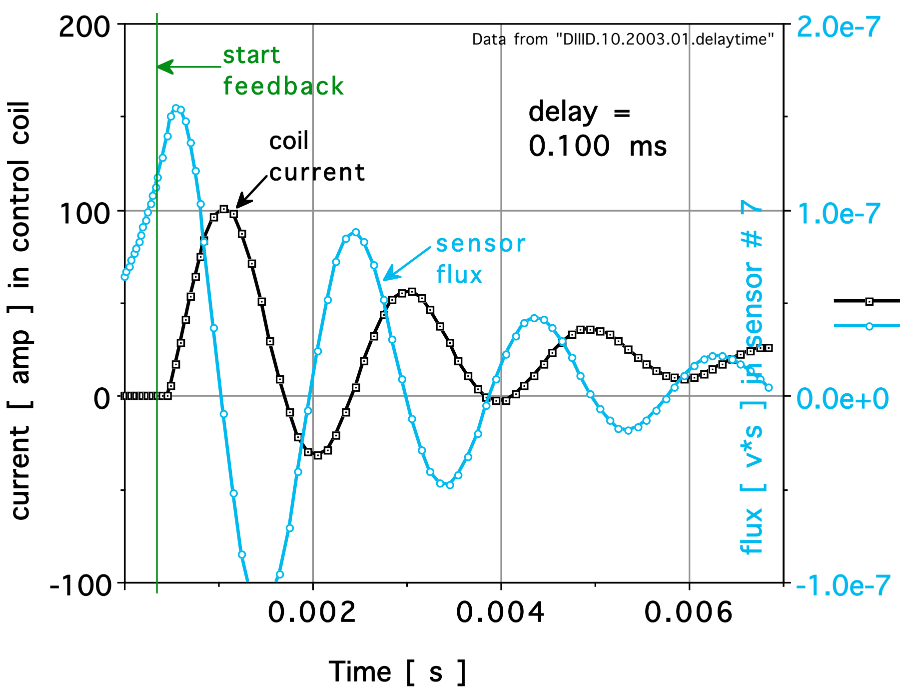


Small time delay, start feedback at t = 0.35 ms, beta-n = 4.76
Feedback defined by:
Poloidal sensor flux(delayed) * gain = control coil voltage

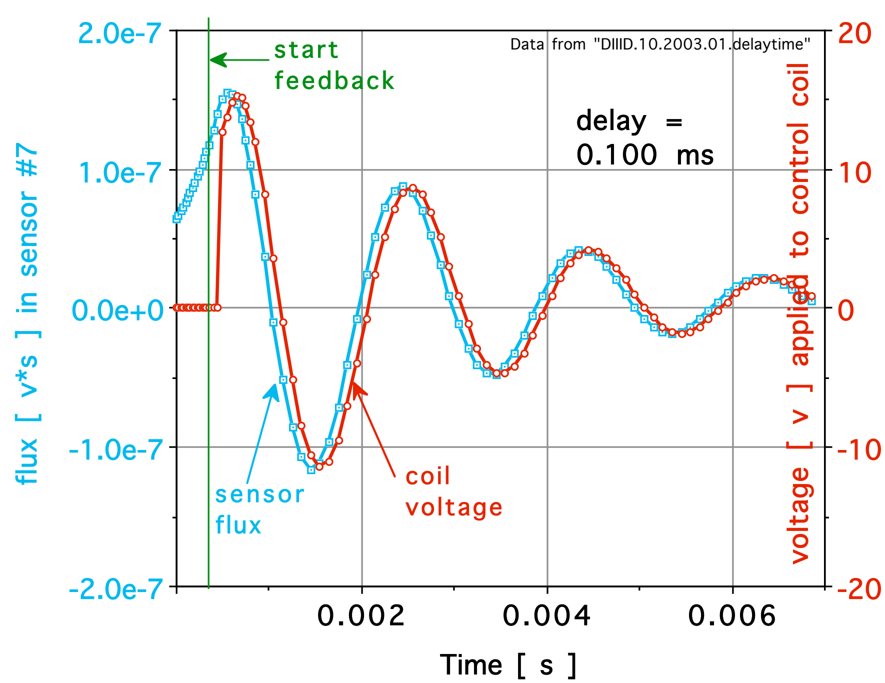
Delay = 0.10 ms

$$G_p \square_p (t \square .0001) = V_{cc}(t)$$

Coil current & sensor flux



Sensor flux & coil voltage



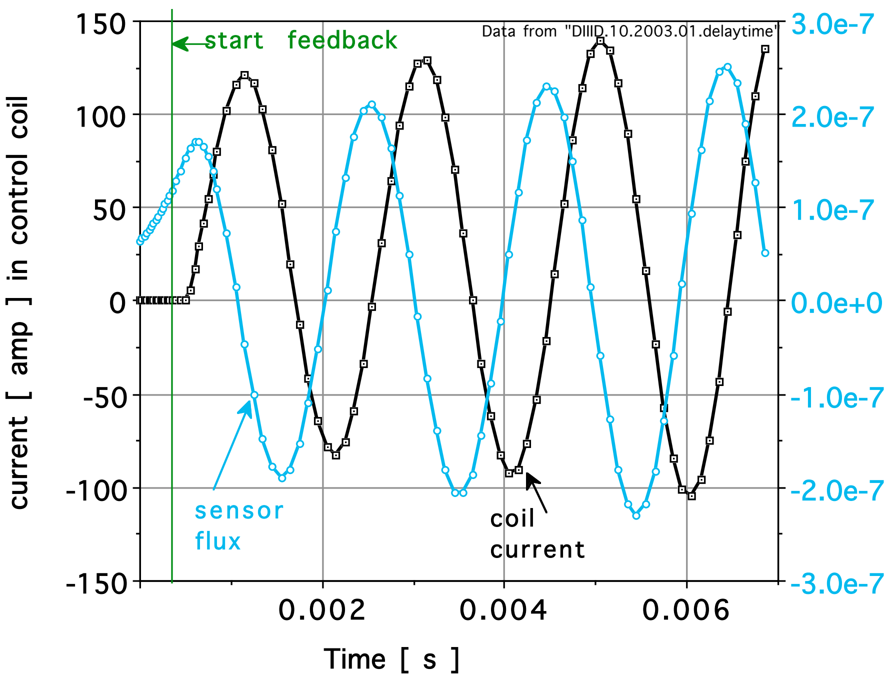
increase time delay, start feedback at t = 0.35 ms, beta-n = 4.76

Feedback defined by:
Poloidal sensor flux(delayed) * gain = control coil voltage

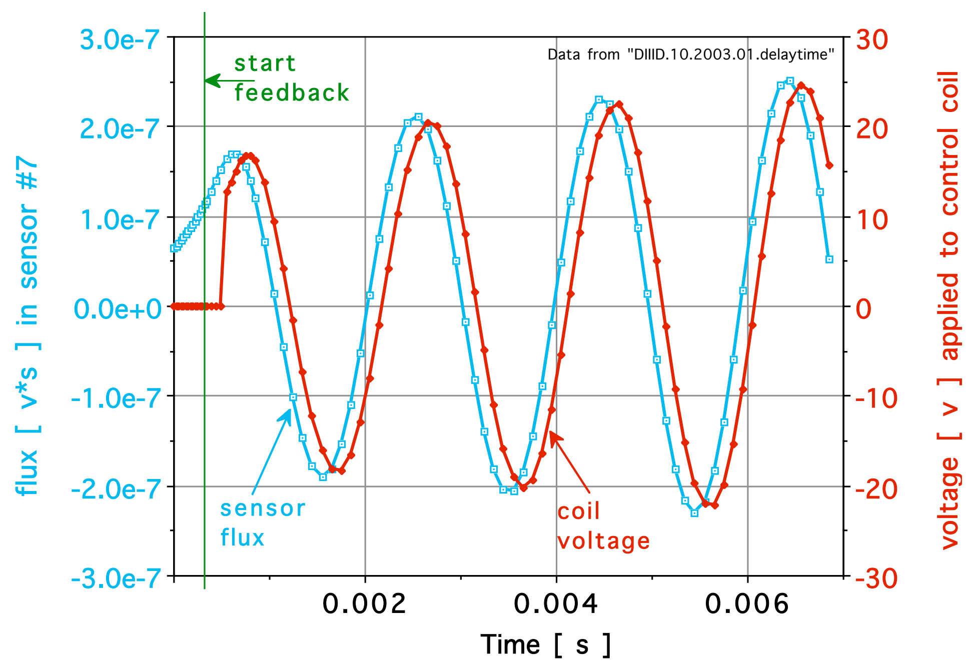
Delay = 0.15 ms

$$G_p \psi_p(t - .00015) = V_{cc}(t)$$

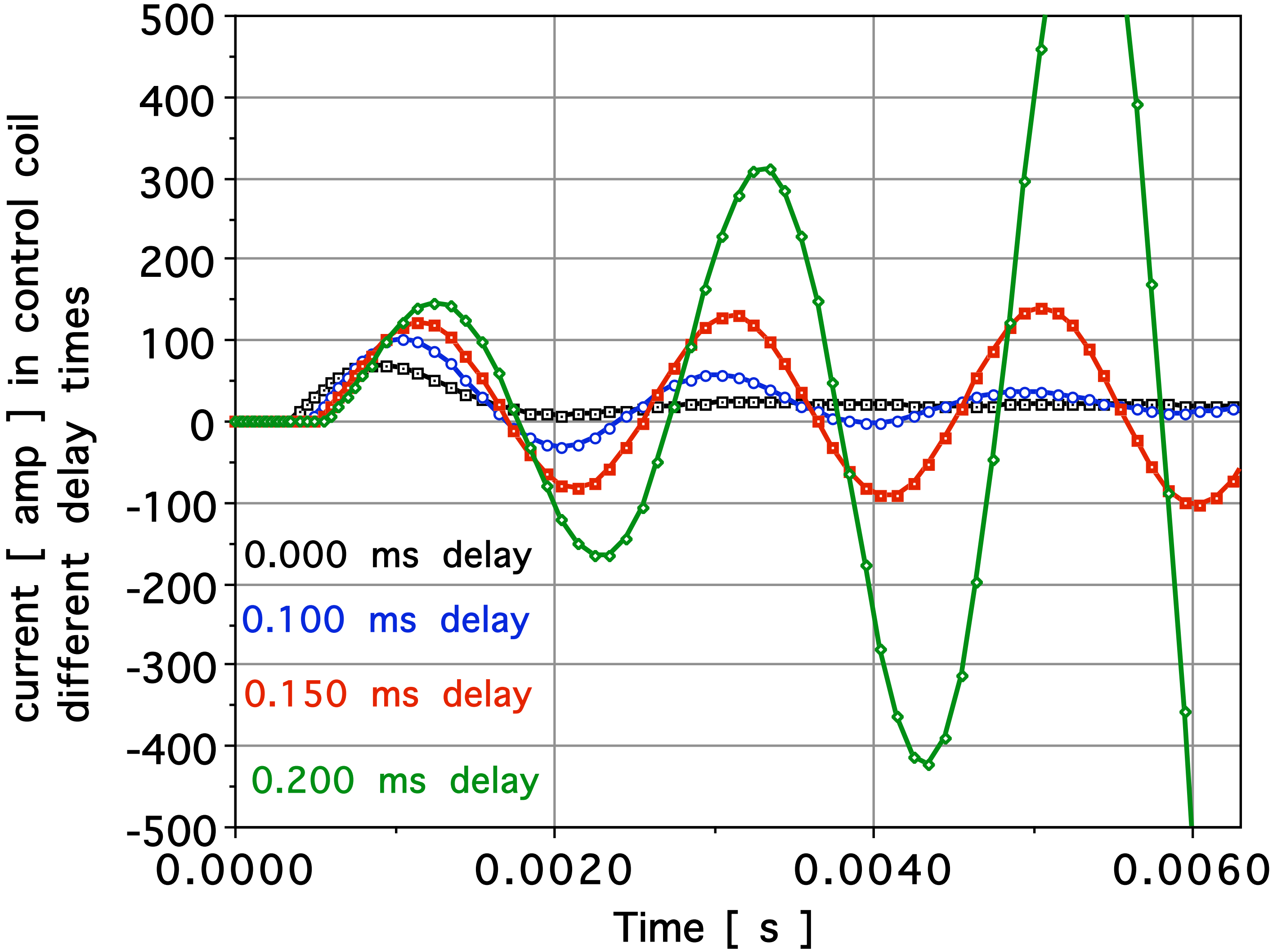
Coil current & sensor flux



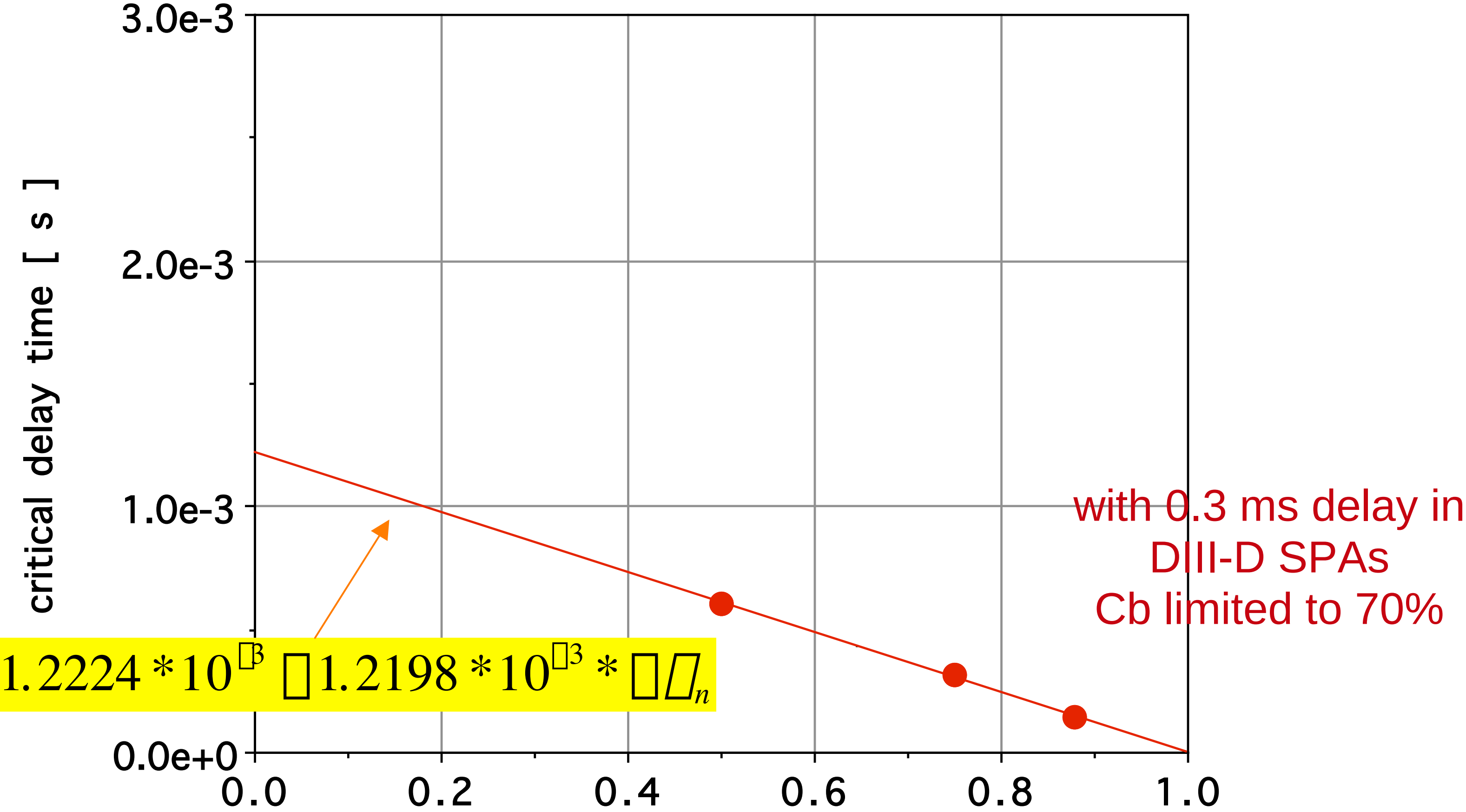
Sensor flux & coil voltage



Stability depends on plasma growth rate and time delay in feedback !



critical delay time as a function of plasma beta



$$\tau_{critical} = 1.2224 * 10^{-3} - 1.2198 * 10^{-3} * \Delta\beta_n$$

$$C_{\beta} = \frac{\beta_n - \beta_n|_{no-wall}}{\beta_n|_{ideal.wall} - \beta_n|_{no-wall}}$$

Summary

- RFA time dependent modeling in qualitative with DIII-D results.
- Time dependent feedback results consistent with eigenvalue analysis.
- Noise simulation show performance limitation when approaching feedback stabilized marginal stability limit.
- Time dependent feedback simulation predict critical time delay for RWM stabilization.