RWM DAMPING DUE TO TOROIDAL VISCOSITY

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Integrated Modeling Meeting, General Atomics, 19 March 2003

Theses:

1) The wall-locked kink-like displacement $\tilde{\xi}$ induced in a tokamak plasma by a Resistive Wall Mode (RWM) causes a non-axisymmetric magnetic perturbation -- $\tilde{B}/B_0 \sim \tilde{\xi}/R_0 \sim 10^{-2}$

2) These non-axisymmetric magnetic field distortions damp the toroidal flow via two neoclassical processes --
ripple viscosity (TTMP), ★ banana drift ★

3) For low collisionality (banana regime) ions, the toroidal flow viscous damping rate is approximately (for $\nu_i/\epsilon > \omega_E$)

$$\mu_t \equiv \gamma_{damp} \sim \underbrace{\omega_{ti}}_{\sim 10^5} \; \frac{\# \, \delta^2}{\underbrace{\nu_{*i}}_{\gtrsim 10}} \; \underbrace{\left(\frac{\tilde{\xi}_{r\,n=1}}{R_0}\right)^2}_{\sim (10^{-2})^2} \gtrsim 10^2/s \sim 1/10\,ms$$

Outline:

- Non-axisymmetric magnetic perturbations induced by a RWM
- Neoclassical toroidal flow damping due to ripple viscosity, banana drift
- Toroidal flow damping induced by a RWM
- Possible tests to determine relevance, importance
- Summary

NON-AXISYMMETRIC $\tilde{\underline{B}}$ INDUCED BY A RWM

- A RWM Is An Ideal MHD Instability That Causes A Wall-Locked Kink-Like Perturbation In A Toroidal Plasma

$$\tilde{T}_e \sim \tilde{\underline{\xi}} \cdot \underline{\nabla} T_{e0}$$ -- see DIII-D data on next viewgraph

$$\Rightarrow \tilde{\xi}_r \sim \frac{\tilde{T}_e}{dT_{e0}/dr} \sim \frac{100\,\mathrm{eV}}{2000\,\mathrm{eV}/30\,\mathrm{cm}} \sim 1.5\,\mathrm{cm} \quad (@\,1332\,\mathrm{ms})$$

$$\sim 6\,\mathrm{cm} \quad (@\,1336.5\,\mathrm{ms})$$

- Since The Magnetic Field Moves With The Plasma In Ideal MHD, These Kink-Like Perturbations Also Cause Significant Wall-Locked Magnetic Perturbations In The Plasma:

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$$|\underline{B}(\underline{x})| \Rightarrow |\underline{B}(\underline{x}+\tilde{\underline{\xi}})| \simeq B_0(\underline{x}) + \tilde{\underline{\xi}} \cdot \underline{\nabla}|B_0| + \cdots$$

$$\simeq B_0\left[1 - \frac{r + \sum_{mn} \tilde{\xi}_{mn} \cos(m\theta - n\zeta)}{R_0} \cos\theta\right].$$

$$\Rightarrow \frac{\tilde{B}_{m\pm1/n}}{B_0} \sim \frac{\tilde{\xi}_{m/n}}{R_0} \sim \frac{1.5\,\mathrm{cm}}{170\,\mathrm{cm}} \sim 10^{-2}$$ -- large "field error"

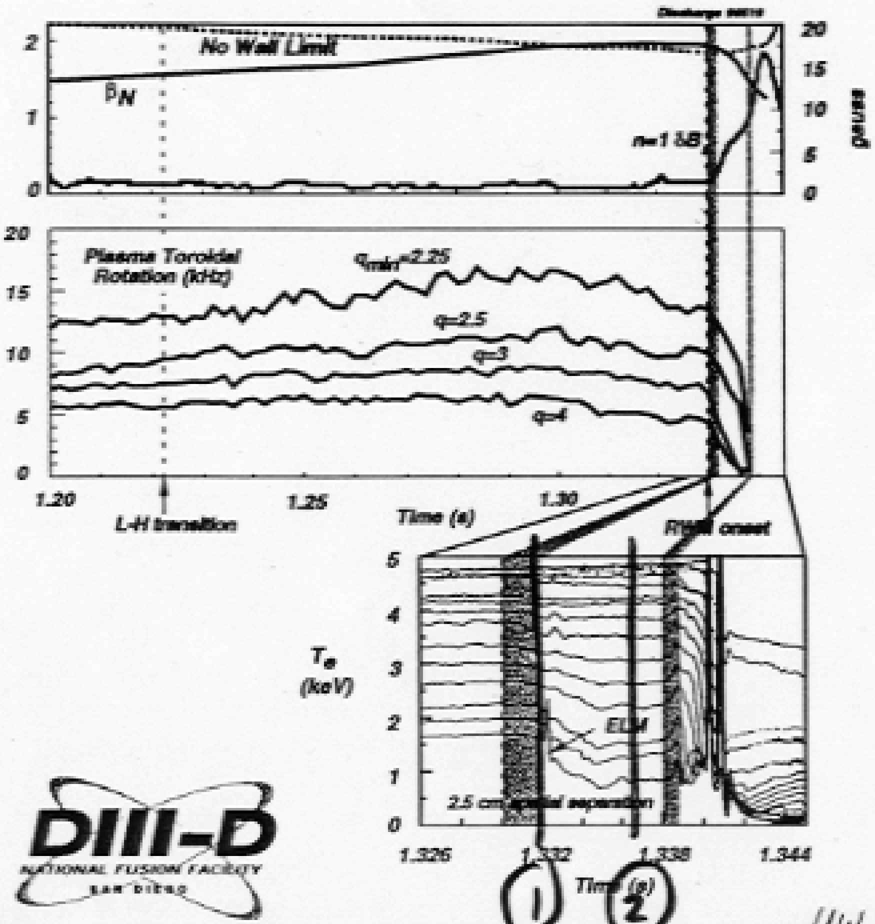
NOTE: For ideal MHD $\tilde{\xi}_\perp \sim (\underline{B}_0 \cdot \underline{\nabla}) \underline{\chi}$ which vanishes at rational surfaces where $\underline{B}_0 \cdot \underline{\nabla} \to 0$.

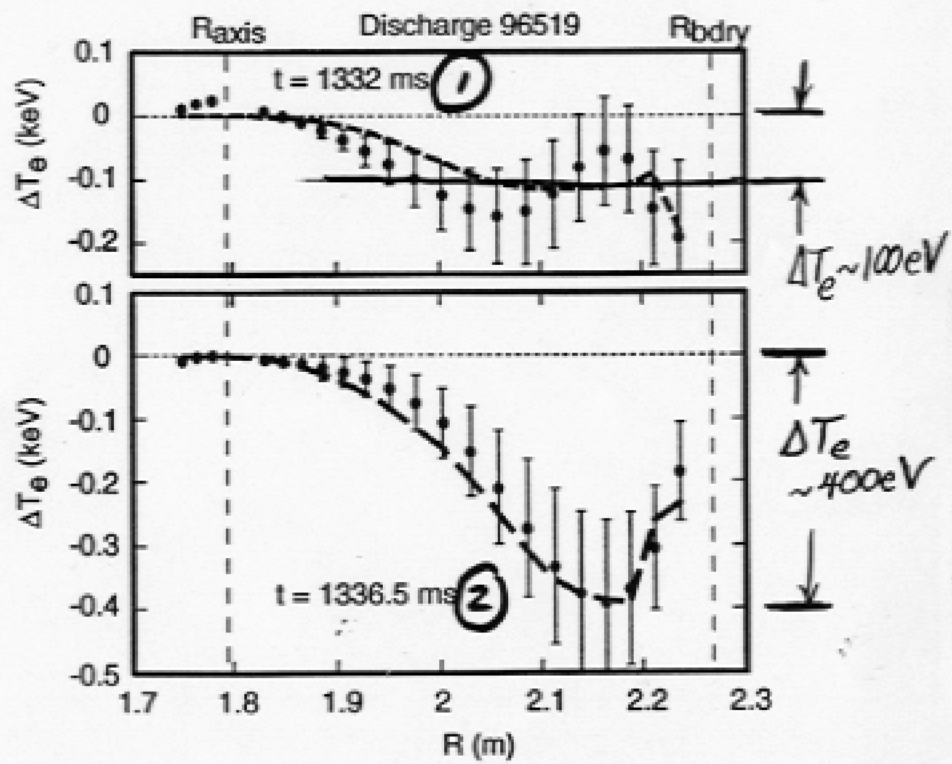
- These Non-Axisymmetric Magnetic Perturbations Produce Neoclassical Toroidal Flow Damping That Increases As RWM Grows -- see DIII-D data on 2nd viewgraph after this one

Toroidal rotation frequency: $\Omega_\zeta \equiv \Omega_\zeta(\psi) \equiv \underline{V} \cdot \underline{\nabla}\zeta \simeq V_\zeta / R_0$

Te PROFILE EVOUTION SHOWS IDEAL KINK-LIKE MODE STRUCTURE IN AGREEMENT WITH GATO PREDICTION

- Measured change in radial T_e profile from ECE analyzed at times where the n=1 displacement can be separated by the axisymmetric collapse
- T_e profile perturbation agrees with calculated perturbation from predicted ideal kink
  $\Delta T_e \propto \xi^{GATO} \times dT_e/dR$ (A. Garofalo, et al., Phys. Rev. Lett., 1999; E. Strait, et al., Nucl. Fusion, 1999)
  - ➢ Amplitude and toroidal phase of calculated eigenfuction scaled according to magnetic data



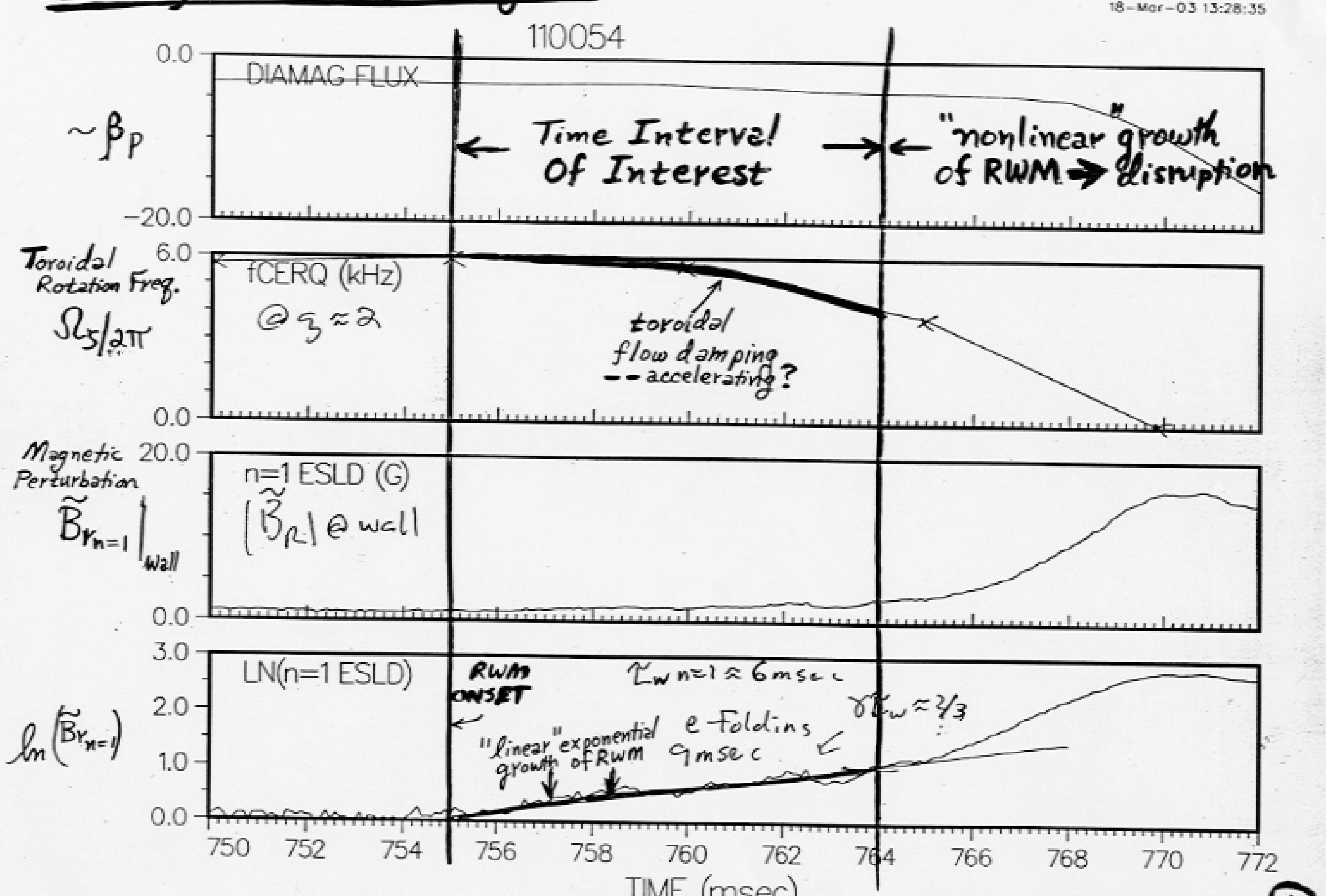


DIII-D NATIONAL FUSION FACILITY SAN DIEGO

Courtesy of A. Garofalo
UW-Madison Seminar, Sept. 9, 2002

Columbia University

DIII-D Dynamics With Growing RWM $B_{tor} = -1.05T$ RWM



Courtesy of R.J. La Haye

TOROIDAL ANGULAR ROTATION EQUATION IN A TOKAMAK

-- a number of effects can be important

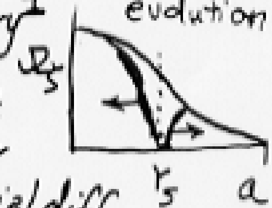
Inertia $\frac{\partial \Omega_\zeta}{\partial t} = \frac{\dot{n}_f}{n}\frac{V_f}{R}$ -- NBI fast ion momentum input

cylindrical

⊥ classical/neoclassical axisymmetric $+ \frac{1}{r}\frac{d}{dr} r (0.1 \nu_i \rho_i^2) \frac{d}{dr}(\Omega_\zeta - \cdots)$ negligible

⊥ anomalous $+ \frac{1}{r}\frac{d}{dr} r \chi_\zeta^{anom} \frac{d\Omega_\zeta}{dr}$ empirical, balances NBI torque

em torque on resistive layer $- \text{coeff.} \left(\frac{\tilde{B}_r}{B}\right)^2 \frac{1}{\tau_{A\theta}} \frac{\Omega_\zeta \tau_R}{\Delta'^2 + (\Omega_\zeta \tau_R)^2}$ Fitzpatrick theory[1] of error field locking -- at resistive layer plus $\chi_\perp^{anom}$ radial diffusion

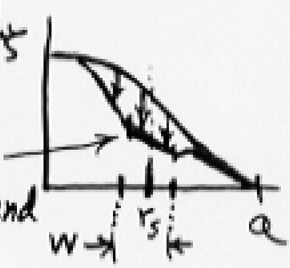


Nonaxisymmetric neoclassical processes:

toroidal nonaxisymmetric

Nonresonant[3,4] (no magnetic island) $- (\dots) \frac{v_{Ti}}{R_0 q} \left(\frac{\tilde{B}_r}{B}\right)^2 (\Omega_\zeta - \Omega_0^{NR})$ bulk flow damping

Resonant[5] (with magnetic island) $- (\dots) \frac{\omega_{Dr}^2}{\nu_i} \underbrace{\left(\frac{\tilde{B}_r}{B}\right)}_{\sim \left(\frac{W}{r_s}\right)^2} (\Omega_\zeta - \Omega_0^R)$ flow damping in vicinity of magnetic island



[1] R. Fitzpatrick, NF 33, 1049 (1993); Phys. Pl. 2002.

[2] M. Yokoyama, J.D. Callen, C.C. Hegna, NF 36, 1307 (1996).

[3] K.C. Shaing, S.P. Hirshman, J.D. Callen, PF 29, 521 (1986); K.C. Shaing et al. PRL 80, 5353 (1998)

[4] K.C. Shaing, Phys. Plasmas 10, 1443 (2003).

[5] K.C. Shaing, PRL 87, 245003 (2001); Phys. Plasmas 9, 3470 (2002).

NEOCLASSICAL TOROIDAL FLOW DAMPING -- physical origin

Fluid Picture

Magnetic bumps impede flow

$$\underline{B}_t \cdot \underline{\nabla} \cdot \underline{\underline{\pi}} \sim -mn\mu_t V_\zeta \frac{\partial B}{\partial \zeta}$$

$\Rightarrow$ viscous damping of V_ζ due to $\frac{\partial B}{\partial \zeta} \neq 0$

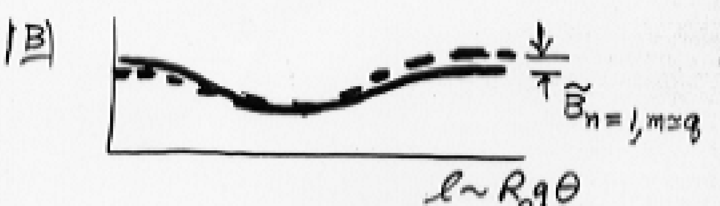
Particle Picture

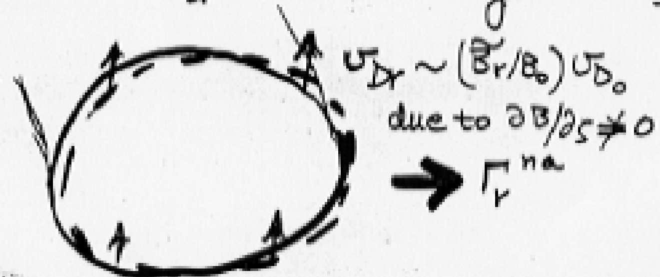
Symmetry breaking by $\frac{\partial B}{\partial \zeta} \neq 0$

$\Rightarrow$ radial drift $\Rightarrow$ nonambipolar radial particle flux Γ_r^{na}

$\Rightarrow J_r^{na} \times \underline{B}_\theta \Rightarrow$ toroidal viscous force

- Non-Resonant "Ripple" (TTMP) Viscosity[1] $\left[\left|n - \frac{m}{q}\right| \left(\frac{\tilde{B}_{mn}}{B_0}\right)^{3/2} < \frac{\nu_i}{\omega_{ti}} < 1 \Rightarrow \text{plateau collisionality regime} \right]$:
(e.g., $n/m = 1/3$ magnetic braking exp.)



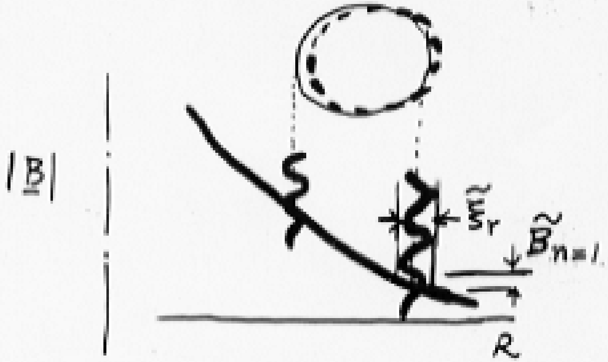


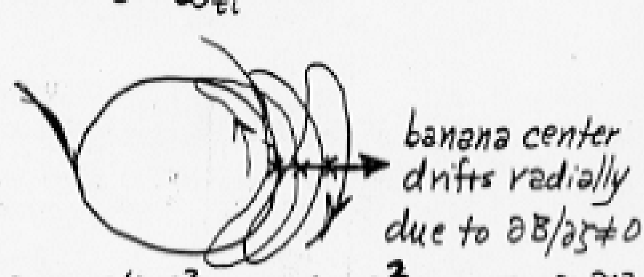
$$\frac{\partial \Omega_\zeta}{\partial t} \simeq -\frac{\sqrt{\pi}}{2} \sum_{m,n} \frac{\omega_{ti}\, q}{|n - m/q|} \left(\frac{n \tilde{B}_{mn}}{B_0}\right)^2 \left(\Omega_\zeta - \Omega_0^{\text{plateau}}\right), \quad \omega_{ti} \equiv \frac{v_{Ti}}{R_0 q} \text{ -- ion transit frequency}$$

$$\Omega_0^{\text{plateau}} = \frac{1.17 - 0.5}{q_i} \frac{dT_i}{d\psi} \simeq \frac{0.67}{z_i e B_\theta R} \frac{dT_i}{dr}$$

- Banana-Drift Transport Viscosity[2] $\left[\omega_E < \frac{\nu_i}{\epsilon} < \sqrt{\epsilon}\, \omega_{ti} \Rightarrow \text{banana collisionality regime} \right]$:

$$\nu_{*i} \equiv \frac{\nu_i}{\epsilon^{3/2} \omega_{ti}} < 1$$





$$D_r \sim f \frac{(\Delta x)^2}{\Delta t} \sim f_t \langle v_{Dr} \rangle^2 \Delta t \sim \sqrt{\epsilon}\, v_{D_0}^2 \left(\frac{n\tilde{B}}{B}\right)^2 \frac{\epsilon}{\nu}$$

$$\frac{\partial \Omega_\zeta}{\partial t} \sim -\omega_{ti} \frac{\# q^2}{\nu_{*i}} \left(\frac{n \tilde{B}_{mn}}{B_0}\right)^2 \left(\Omega_\zeta - \Omega_0^{1/\nu}\right) \quad \text{for } \nu_i/\epsilon > \omega_E$$

1 K.C. Shaing, S.P. Hirshman, J.D. Callen, Phys.Fl. 29, 521 (1986); K.C. Shaing et al, Phys.Rev.Lett. 80, 5353 (1998)

2 K.C. Shaing, "MHD-Activity-Induced Momentum Dissipation In Collisionless Regimes in Tokamaks," January 2003, accepted for publication in Phys. Plasmas 10, 1443 (2003)

Toroidal Flow Damping Induced By A RWM

-- summary, scaling of Shaing banana-drift formulas

- Magnetic Perturbation Induced By RWM Kink-like Distortion:

$$\frac{\tilde{B}_{m\pm 1/n}}{B_0} \sim \tilde{\underline{\xi}}_{m/n} \cdot \underline{\nabla} \ln B_0 \sim \frac{\tilde{\xi}_{rm/n}}{R_0}$$

- Toroidal Flow Damping Due To $\tilde{B}_{m\pm 1/n}$ Banana-Drift Effect:

$$\nabla\zeta \cdot \left(mn\frac{d\underline{V}}{dt} = + \cdots - \underline{\nabla}\cdot\underline{\underline{\pi}} \right) \Longrightarrow mn\frac{d\Omega_\zeta}{dt} = \cdots - \langle \nabla\zeta \cdot \underline{\nabla}\cdot\underline{\underline{\pi}} \rangle \simeq \cdots - \frac{e_i}{R^2}\chi' \langle \Gamma \cdot \nabla V \rangle$$

$$\Longrightarrow \frac{\partial \Omega_\zeta}{\partial t} \simeq -\mu_t \left(\Omega_\zeta - \Omega_0 \right)$$

$1/\nu$ regime

$\omega_E < \frac{\nu_i}{\epsilon} < \epsilon^{1/2}\omega_{ti}$

$$\mu_t^{1/\nu} \simeq \omega_{ti} \frac{\# q^2}{\nu_{*i}} \underbrace{\left[\frac{I_\lambda}{\epsilon^{3/2}} \right]}, \quad \Omega_0^{1/\nu} \simeq \frac{1.17 + \frac{32.4}{13.7}}{e_i} \frac{dT_i}{d\chi} \simeq \frac{3.5}{Z_i e B_\theta R_0} \frac{dT_i}{dr}$$

$\sim$ averages over $(n\tilde{B}_{mn}/B_0)^2$

ν regime

$\frac{\nu_i}{\epsilon} < \omega_E$

$$\mu_t^{\nu} \simeq \omega_{ti} \, \# q^2 \frac{\nu_i \omega_{ti}}{\omega_E^2} \left[\frac{G_\lambda}{\epsilon^{1/2}} \right], \quad \Omega_0^{\nu} \simeq \frac{1.17 - \frac{0.088}{0.354}}{e_i} \frac{dT_i}{d\chi} \simeq \frac{0.9}{Z_i e B_\theta R_0} \frac{dT_i}{dr}$$

Combination

$$\boxed{\mu_t \sim \omega_{ti} \frac{\# q^2}{\nu_{*i}} \frac{1}{1 + \left(\frac{\omega_E}{\nu_i/\epsilon} \right)^2} \left(n \frac{\tilde{\xi}_{rm/n}}{R_0} \right)^2}$$

- Some Definitions:

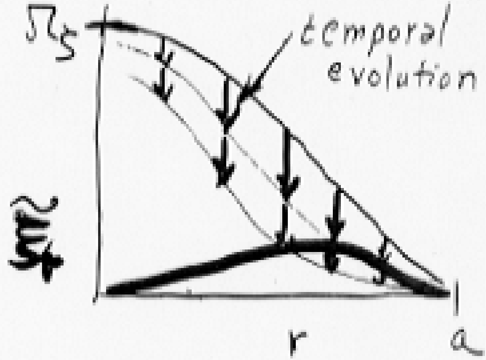
$$\omega_{ti} \equiv \frac{v_{Ti}}{R_0 q}, \quad \omega_E \equiv \frac{\Phi'}{\chi'} = \frac{\partial \Phi}{\partial \chi} \simeq \frac{1}{B_\theta R} \frac{d\Phi}{dr}, \quad d\chi \simeq B_\theta R\, dr, \quad \chi \equiv \text{poloidal flux function}$$

ω_{ti}: ion transit frequency; ω_E: $\underline{E}\times\underline{B}$ drift frequency

POSSIBLE TESTS TO DETERMINE RELEVANCE, IMPORTANCE

-- of neoclassical toroidal flow damping by a RWM

- Toroidal Flow Damping Properties:



Spatial -- across entire plasma, $\propto \tilde{\xi}_r^2(r)$

Temporal -- damping rate μ_t

$$\propto \tilde{\xi}_r^2 \sim \left[\tilde{B}_r(n=1)\big|_{wall}\right]^2 \sim e^{2\gamma_{RWM} t}$$

$$\propto \omega_{ti} \frac{q^2}{\nu_{*i}} \frac{1}{1+\left(\frac{\omega_E}{\nu_i/\epsilon}\right)^2} \sim \begin{cases} \frac{T_i^{5/2}}{n_i}, & \nu_i/\epsilon > \omega_E \\ \frac{n_i}{\sqrt{T_i}\,\omega_E^2}, & \nu_i/\epsilon < \omega_E \end{cases}$$

- Flow Evolution If Neoclassical Toroidal Flow Damping Is Dominant And $\Omega_\zeta \gg \Omega_0$ (result stimulated by R. J. La Haye):

$$\frac{1}{\Omega_\zeta}\frac{d\Omega_\zeta}{dt} \simeq -\mu_0 \left(\frac{\tilde{\xi}}{R_0}\right)^2 e^{2\gamma_{RWM} t}$$ when RWM is growing linearly

$$\Rightarrow \Omega_\zeta(t) \simeq \frac{\Omega_\zeta(t=0)}{\exp\left\{\frac{\mu_0}{2\gamma_{RWM}}\left[\left(\frac{\tilde{\xi}_r}{R_0}\right)_t^2 - \left(\frac{\tilde{\xi}_r}{R_0}\right)_{t=0}^2\right]\right\}}$$

in which $\mu_0 \sim \underbrace{\omega_{ti}}_{\sim 10^5/s} \underbrace{\frac{\# q^2}{\nu_{*i}} \frac{1}{1+\left(\frac{\omega_E}{\nu_i/\epsilon}\right)^2}}_{\sim 3?,\ \text{in } \nu_i/\epsilon < \omega_E \text{ regime?}} \sim 3\times 10^5/s$

SUMMARY

- A RWM instability induces a kink-like distortion $\tilde{\xi}_{r\,m/n}$ in a tokamak plasma

- This $\tilde{\xi}_r$ induces non-symmetric $|\mathbf{B}|$ magnetic field distortions in the plasma region:

$$\frac{\tilde{B}_{m\pm 1/n}}{B_0} \sim \frac{\tilde{\xi}_{r\,m/n}}{R_0} \gtrsim 10^{-2}$$

- These magnetic field distortions cause "banana-drift" neoclassical toroidal flow damping at rate

$$\mu_t \sim \omega_{ti} \left[\frac{\# q^2}{\nu_{*i}} \, \frac{1}{1+\left(\frac{\omega_E}{\epsilon \nu_i}\right)^2} \right] \left(\frac{n \tilde{\xi}_r}{R_0} \right)^2 \gtrsim 10^2/s \sim 1/10\,ms$$

(nonresonant "ripple" viscosity is comparable and should be added to obtain total μ_t)