Magnetic Island Rotation

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Motivation

- Island rotation frequency □ is required to complete the description of a magnetic island.

- Island stability depends on the island rotation frequency through the polarization term.

- Plasma flow also depends on □. At island O-point, plasmas flow at the same rate as islands.

Island Rotation Frequency

- Island rotation frequency is determined from the *sine* component of Ampere's law:

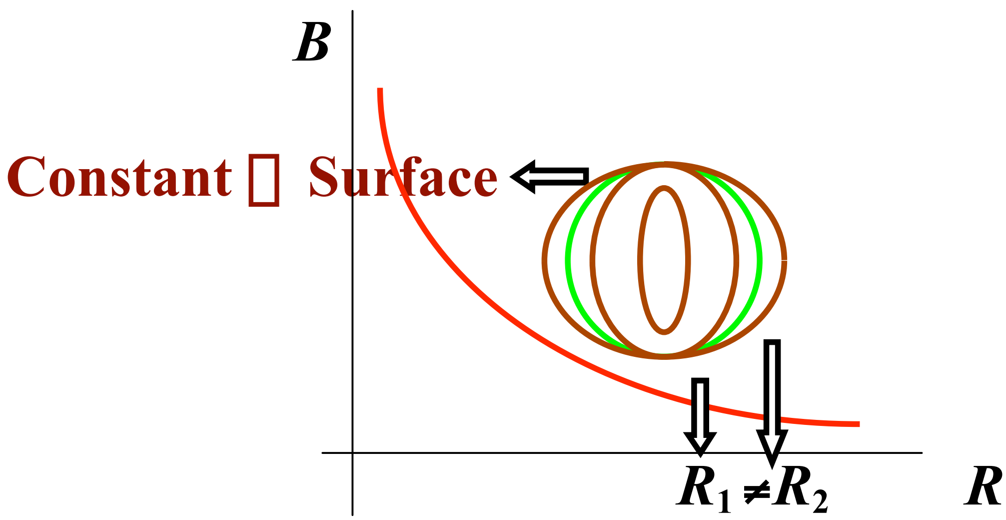
$$\oint d\xi \int_{-\infty}^{\infty} dx\, J_{\parallel}\, \sin\xi = c\,\tilde{\psi}\,\Delta_s'/(4\pi R).$$

- Parallel current density $J_{\parallel}$ can be determined from $\nabla \bullet \boldsymbol{J} = 0$.

- What is (are) the physical mechanism(s) that can drive a perpendicular current density $J_{\perp}$ which can give rise to a $J_{\parallel}$ with the right parity to contribute to the integral?

Broken Toroidal Symmetry in |*B*|

- In the vicinity of a magnetic island, the toroidal symmetry in |*B*| is broken:



- |*B*| on the island surface:

$$B/B_0 = 1 - \left[\frac{r_s}{R} \pm \frac{r_w}{R}\left(\overline{\Box} + \cos\Box\right)^{1/2}\right]\cos\Box$$

$\overline{\Box}$ **:** Normalized helical flux function,
□: m (□ - □/ q_S), helical angle,
***m* :** Poloidal mode number,
□: Toroidal angle,
r_w **:** A measure of the width of the island.

Symmetry Breaking Induced Viscosity

- Broken toroidal symmetry leads to an enhanced toroidal viscous force.

- This toroidal viscous force induces a radial current and thus a parallel current density that has the right parity to contribute to the integral.

- The flux surface averaged toroidal momentum balance equation, which includes the enhanced toroidal viscosity, determines a radial electric field.

- The conventional parallel momentum balance equation determines a poloidal flow, or a parallel flow.

- Thus, plasma flows are completely determined by introducing symmetry-breaking viscosity in the momentum equation.

Flux-Force Relation

- Radial current is

$$J_{\perp} \bullet \nabla\psi = \sum_{\mathrm{j}} e_{\mathrm{j}} \int d\mathrm{v}\, f_{\mathrm{j}}\, \mathrm{v}_{\mathrm{B}} \bullet \nabla\psi$$

$$= \sum_{\mathrm{j}} e_{\mathrm{j}}\, \Gamma_{\mathrm{j}} \,,$$

$\mathbf{v}_{\mathrm{B}}$: ∇B and curvature drifts.

- Decompose radial flux Γ_{j} into:

<u>Non-axisymmetric flux</u>:

$$(\Gamma \bullet \nabla\psi)_{\mathrm{na}} = - \int d\mathbf{v}\, f\; (\mathrm{v}_{\|}/B)\; \nabla \bullet [(\mathrm{v}_{\|} B^2 \boldsymbol{B}_{\mathrm{t}})/\Omega)],$$

<u>Banana-Plateau flux</u>:

$$(\Gamma \bullet \nabla\psi)_{\mathrm{bp}} = \int d\mathbf{v}\, f\; (\mathrm{v}_{\|}/B)\; \nabla \bullet [(\mathrm{v}_{\|} B^2/\Omega)\, \square\, (\square I_{\mathrm{c}}\, \square / \square B^2 \square) \boldsymbol{B}],$$

Pfirsch-Schluter flux:

$$(\square \bullet \square\square)_{ps} = \square d\mathbf{v}\, f\ (v_{\parallel}/B)\ \square \bullet [(v_{\parallel} B^2/\square)\square$$
$$(I_c /B^2 - \square I_c \square/\square B^2 \square)\boldsymbol{B}].$$

where $I_c = B_t \bullet B$,

$$\boldsymbol{B}_t = \square\square\ \square\square\square / (B \bullet \square\square).$$

- Banana-plateau flux is ambipolar after poloidal flow is relaxed. Pfirsch-Schluter flux is intrinsically ambipolar. Both of these fluxes do not contribute to the radial current.

- Only the non-axisymmetric flux

$$(\square \bullet \square\square)_{na} = - \square d\mathbf{v}\, f\ (v_{\parallel}/B)\ \square \bullet [(v_{\parallel} B^2 \boldsymbol{B}_t)/\square)],$$

contributes to the radial current. Note this flux vanishes if the system is toroidally symmetric.

Perturbed Particle Distribution Function

- To calculate the radial current, linear drift kinetic equation is solved for the perturbed particle distribution function.

- For example in the 1/□ regime, *i.e.*, banana particles are collisionless, the perturbed perpendicular pressure is

$$p_{\square 1/\square} = -(I/M\square)(n_0 \bullet \square\,\square)(q_S \square W_{\square}/q_S)(\square_w/2)\square$$

$$(m \sin\square)(4/M^{3/2})\square^{3/2} I_{1/\square} \square_{0} dE\ (E^{5/2}/\square)\, f_M\ \square$$

$$[p\square/p + e\mathcal{F}\square/T\ + (x^2 - 5/2)\ T\ \square/T],$$

where □ □ □, $\square_w = r_W/R$, and $E = M\mathrm{v}^2/2$.

Toroidal Viscosity

- Using the perturbed pressure, the flux surface averaged toroidal viscosity is

$$\Box_j = C_1[N_j v_{tj}^4/(2^{7/2}\Box^{3/2}\Box_j)](\boldsymbol{IB}\bullet\Box\Box/\Box_j B)^2 m^2$$

$$\Box_w{}^2\Box^{3/2}[(dq/dr)r_W/q_S]^2[F(\Box)(1+\Box)^{1/2}/K(\Box_f)]$$

$$[\ \Box_{1j}(\ p_j\Box/p_j + e_j\mathcal{F}\Box/T_j) + \Box_{2j}\ T_j\Box/T_j\ ].$$

- Because ion viscosity is larger than electron viscosity in this regime, $\Box_i \approx 0$, to maintain ambipolarity. This equation determines a radial electric field $e_i\mathcal{F}\Box/T_i$:

$$e_i\mathcal{F}\Box/T_i = -\ (\ p_i\Box/p_i) - 2.37\ T_i\Box/T_i\ .$$

Radial Electric Field

- Because parallel electric field $E_{□}$ □ 0, the electrostatic potential □ has the form

$$□ = -(□ q/mc)\,(□ - □_S) + \mathcal{F}(□).$$

- Assuming the electric field away from the island is not perturbed, $\mathcal{F}(□)$ must have the form

$$\mathcal{F}(□) = (q/mc)(□ - □_{E0})\,\mathcal{H}(□),$$

where $□_{E0} = - mc□_0 □/q$, and

$\mathcal{H}(□)$ □ $(□ - □_S)$ far away from the island.

Profile Functions

- Both the profile function $\mathcal{H}(\square)$ and the island rotation frequency $\square$ need to be determined.

- For simplicity, assume

$$p\square = p_0\square\partial\mathcal{H}/\partial\square\,,\; T\square = T_0\square\partial\mathcal{H}/\partial\square\,,$$

and

$$\mathcal{H}(\square) = \pm\, W_{\square}\,[(\overline{\square})^{1/2} - 1] \text{ for } \overline{\square} > 1$$

$$= 0, \text{ inside the separatrix.}$$

- Because $\square$ is not yet determined, ambipolarity condition is not yet satisfied for a given set of profile functions.

Parallel Current Density

- Using $\nabla \bullet \boldsymbol{J} = 0$, parallel current density is

$$B\bullet\nabla(J_{\|}/B) \; \square$$

$$-c\,g^{-1/2}\,\partial[g^{1/2} p_{\square t\square}(B\square\square\square\bullet\square\square)(\partial B/\partial\square)/B^3]/\partial\square +$$

$$\square c\; g^{-1/2}\partial[g^{1/2} p_{\square t\square}(B\square\square\square\bullet\square\square)(\partial B/\partial\square)/B^3]/\partial\square\;\square.$$

- This parallel current density has the right parity to contribute to the integral.

- Using this $J_{\|}$, an equation for the island rotation frequency can be obtained.

Island Rotation Frequency

- Island rotation can be solved from

$$-(2\square)^{-5/2}\,(v_{ti}/Rq)^2\,(r_W/\square)\square^{3/2} m^2\,(\square_w{}^2/4)$$
$$I_{1/\square} C_{1/\square}\{\square_{T1i}\,[\square_{\square pi} + (\square - \square_{E0})] + \square_{T2i}\,\square_{\square Ti}\,\}$$
$$= n\square^2 (V_{AP}{}^2/4\square)(S^2/4)(\,r_W\,/\,r_S)^4\,\square_s\square$$

in the $1/\square$ regime.

$C_{1/\square}\,\square\,\square$, $\square_{\square pi} = (mcT/eq)(p_{0i}\square/p_i)$,
$\square_{\square Ti} = (mcT/e_j q)(T_{0i}\square/T_i)$,
$V_{AP} = B_p\,/(4\square NM)^{1/2}$,
n : toroidal mode number, and
$S = r_S\,(dq/dr)/q$.

- $\square$ in other regimes, such as the $\square$ regime, and the plateau-Pfirsch-schluter regime is also calculated. The main difference is in the coefficients $\square$s, and the collision frequency dependence on the left side of the equation.

- If island is not interacting with a conducting wall or helical coils, island rotation frequency is determined from

$$-(2\square)^{-5/2}(v_{ti}/Rq)^2 (r_W/\square)\square^{3/2} m^2 (\square_w{}^2/4)$$
$$I_{1/\square} C_{1/\square}\{\square_{T1i}[\square_{\square pi} + (\square - \square_{E0})] + \square_{T2i}\square_{\square Ti}\} = 0,$$

in the 1/□ regime.

- And

$$(\square - \square_{E0}) = -\square_{\square pi} - (\square_{T2i}/\square_{T1i})\square_{\square Ti},$$

in the 1//□ regime.

- In the plateau-Pfirsch-Schluter regime, $(\square - \square_{E0})$ has the same form as that in the 1/□ regime except that it has a different ratio of $(\square_{T2i}/\square_{T1i})$.

- In the collisionless □ regime:

$$(\square - \square_{E0}) = -\square_{\square pe} - (\square_{T2e}/\square_{T1e})\square_{\square Te}.$$

Schematic Collision Frequency Dependence for the Toroidal Viscous Force

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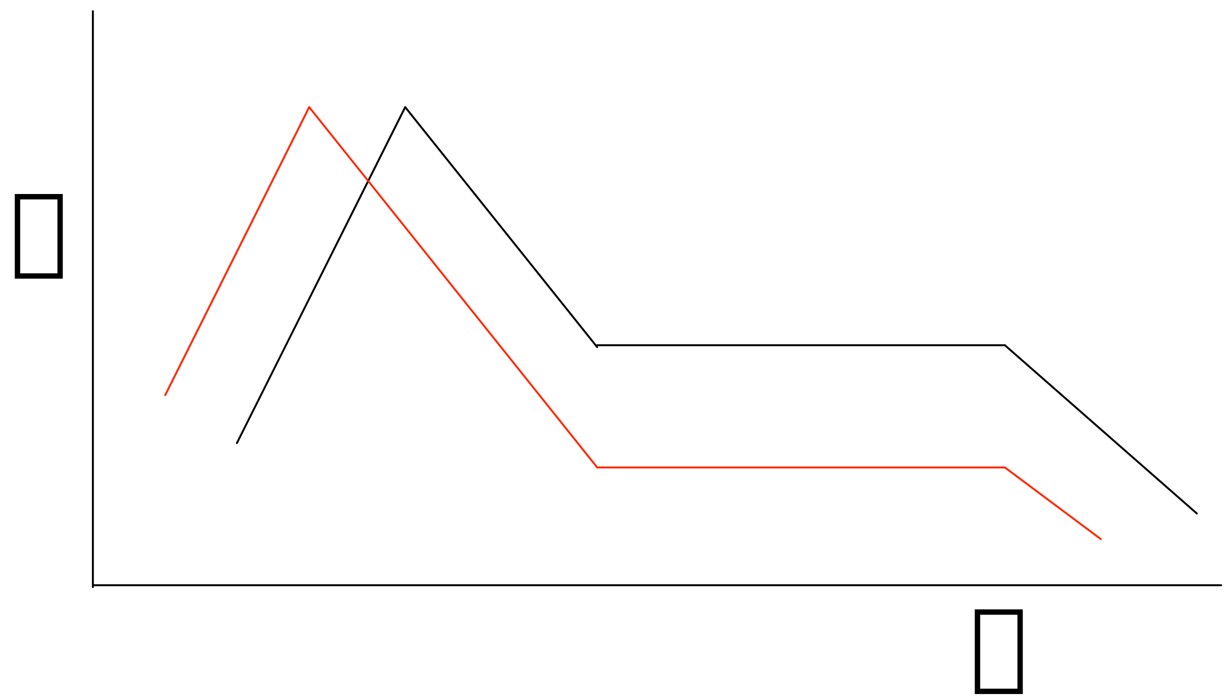

Electrons: ———

Ions: ———

- When the electron viscosity dominates in the low collisionality regime, island rotation frequency reverses the direction.

Parallel Flow

- From parallel momentum balance equation:

$$V_{\|}/B = -\,[IcT_i/(e_i \square B^2 \square)]\{-1.17(dT_i/d\square)/T_i$$

$$+\,[(dp_i/d\square)/p_i + e_i \mathcal{F}\square/T_i]\}d\square/d\square\ +$$

$$(Ic/\square B^2 \square)(\square q/mc).$$

  in the banana regime.

- In the plateau regime, and the Pfirsch-Schluter regime, the number '-1.17' is replaced by '1/2' and '1.69' respectively.

- At the island O-point, plasma gradients vanish, thus, plasmas rotate at the same rate as the island. One can use this property to measure the island rotation frequency.

Conclusions

- At the vicinity of a magnetic island, the toroidal symmetry of $|\boldsymbol{B}|$ is broken.

- This broken symmetry leads to an enhanced toroidal viscous force.

- Flux surface averaged toroidal momentum balance equation determines a radial electric field from the toroidal viscosity.

- A parallel current density induced by a toroidal viscosity driven radial current has the right parity to contribute to the integral in the *sine* component of the Ampere's law.

- Island rotation frequency in the collisional and collisionless regimes is calculated. In the ion viscosity dominant regimes, island rotates at a rate:

$$(\square - \square_{E0}) = - \square_{\square pi} - (\square_{T2i} / \square_{T1i}) \square_{\square Ti}.$$

- In the electron viscosity dominant regime, island rotation frequency reverses direction.

- At the island O-point, plasmas rotate at the same rate as the island.